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Inspiring Minds

According to Current Test Pattern
at the Level of Class XI-XII

Leading Edge Texts

Understanding Physics

Mechanics

Part 2

DC Pandey

Contains All Types of
Questions Including Reasoning, Aptitude & Comprehension

According to Current Test Pattern
at the Level of Class XI-XII

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Understanding Physics

Mechanics

Part 2

DC Pandey

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Inspiring Minds.Inspiring Lives
Arihant Prakashan, Meerut

Contents

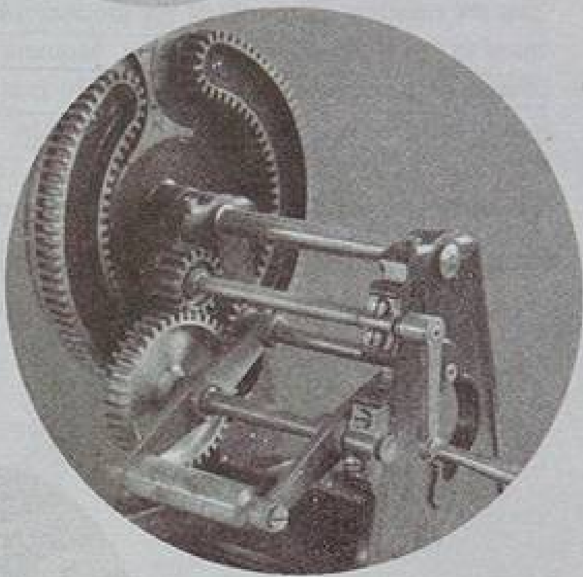
$I = MK^2.$

$\tau = F \times r$

$\tau = I\alpha$

$L = I\omega$

$K.E = \frac{1}{2} I\omega^2$



9

Mechanics of Rotational Motion

Chapter Contents

Solved Examples

Level 1

Example 1 *If the radius of the earth contracts to half of its present value without change in its mass, what will be the new duration of the day?*

Solution Present angular momentum of earth

$$L_1 = I\omega = \frac{2}{5}MR^2\omega$$

New angular momentum because of change in radius

$$L_2 = \frac{2}{5}M\left(\frac{R}{2}\right)^2 \omega'$$

If external torque is zero then angular momentum must be conserved

$$L_1 = L_2$$

$$\frac{2}{5}MR^2\omega = \frac{1}{4}\times\frac{2}{5}MR^2\omega'$$

i.e.,

$$\omega' = 4\omega$$

$$T' = \frac{1}{4}T = \frac{1}{4}\times 24 = 6\,\text{h}$$

Example 2 *A particle of mass m is projected with velocity v at an angle θ with the horizontal. Find its angular momentum about the point of projection when it is at the highest point of its trajectory.*

Solution At the highest point it has only horizontal velocity $v_x = v\cos\theta$

Length of the perpendicular to the horizontal velocity from 'O' is the maximum height, where

$$H_{\max} = \frac{v^2\sin^2\theta}{2g}$$

$\Rightarrow$ Angular momentum $L = \dfrac{mv^3\sin^2\theta\cos\theta}{2g}$

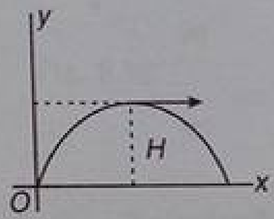


Fig. 9.86

Example 3 *A horizontal force F acts on the sphere at its centre as shown. Coefficient of friction between ground and sphere is μ. What is maximum value of F, for which there is no slipping ?*

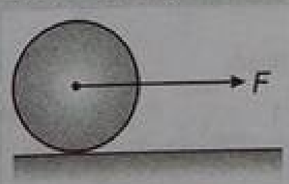


Fig. 9.87

Solution

$$F - f = Ma$$

$$f \cdot R = \frac{2}{5} MR^2 \frac{a}{R} \Rightarrow f = \frac{2}{5} Ma$$

$$\Rightarrow \quad f = \frac{2}{7} F$$

$$\frac{2}{7} F \le \mu mg \Rightarrow F \le \frac{7}{2} \mu mg$$

Example 4 *A tangential force F acts at the top of a thin spherical shell of mass m and radius R. Find the acceleration of the shell if it rolls without slipping.*

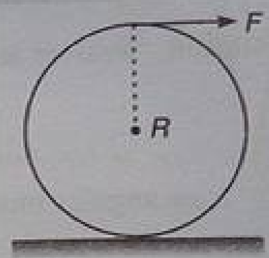


Fig. 9.88

Solution Let f be the force of friction between the shell and the horizontal surface.

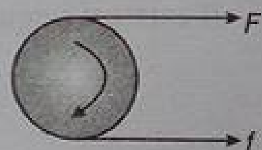


Fig. 9.89

For translational motion,

$$F + f = ma \qquad \ldots(i)$$

For rotational motion,

$$FR - fR = I\alpha = I\frac{a}{R} \qquad [\because a = R\alpha \text{ for pure rolling}]$$

$$\Rightarrow \quad F - f = I\frac{a}{R^2} \qquad \ldots(ii)$$

Adding Eqs. (i) and (ii), we get

$$2F = \left(m + \frac{I}{R^2}\right)a = \left(m + \frac{2}{3}m\right)a = \frac{5}{3}ma$$

$$\text{or} \quad F = \frac{5}{6}ma \qquad \left[\because I_{\text{shell}} = \frac{2}{3}mR^2\right]$$

$$\Rightarrow \quad a = \frac{6F}{5m}$$

Example 5 *A solid cylinder of mass m and radius r starts rolling down an inclined plane of inclination θ. Friction is enough to prevent slipping. Find the speed of its centre of mass when its centre of mass has fallen a height h.*

Solution Considering the two shown positions of the cylinder. As it does not slip hence total mechanical energy will be conserved.

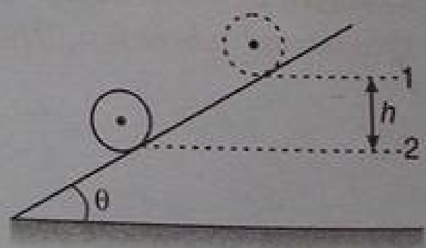


Fig. 9.90

Energy at postion 1 is $\quad E_1 = mgh$

Energy at position 2 is $\quad E_2 = \frac{1}{2}mv_{\text{COM}}^2 + \frac{1}{2}I_{\text{COM}}\,\omega^2$

$$\because \quad \frac{v_{\text{COM}}}{r} = \omega \quad \text{and} \quad I_{\text{COM}} = \frac{mr^2}{2}$$

$$\Rightarrow \qquad E_2 = \frac{3}{4} m v_{COM}^2$$

From COE, $E_1 = E_2$

$$\Rightarrow \qquad v_{COM} = \sqrt{\frac{4}{3} gh}$$

Example 6 *A disc starts rotating with constant angular acceleration of π rad/s^2 about a fixed axis perpendicular to its plane and through its centre. Find :*

(a) the angular velocity of the disc after 4 s.

(b) the angular displacement of the disc after 4 s and

Solution Here $\alpha = \pi \text{ rad/s}^2$, $\omega_0 = 0$, $t = 4\text{ s}$

(a) $\omega_{(4\text{s})} = 0 + (\pi \text{ rad/s}^2) \times 4\text{ s} = 4\pi \text{ rad/s}.$

(b) $\theta_{(4\text{s})} = 0 + \frac{1}{2}(\pi \text{ rad/s}^2) \times (16\text{s}^2) = 8\pi \text{ rad}$

Example 7 *A small solid cylinder of radius r is released coaxially from point A inside the fixed large cylindrical bowl of radius R as shown in figure. If the friction between the small and the large cylinder is sufficient enough to prevent any slipping, then find :*

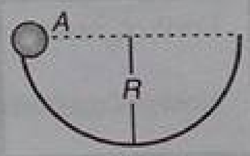


Fig. 9.91

(a) What fractions of the total energy are translational and rotational, when the small cylinder reaches the bottom of the larger one?

(b) The normal force exerted by the small cylinder on the larger one when it is at the bottom.

Solution (a)

$$K_{trans} = \frac{1}{2} m v^2$$

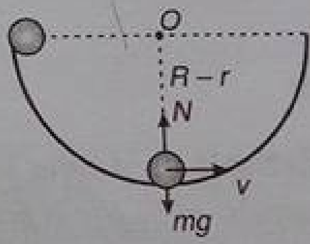


Fig. 9.92

$$K_{rot} = \frac{1}{2} I\omega^2 = \frac{1}{4} m v^2$$

$$K = K_{trans} + K_{rot} = \frac{3}{4} m v^2$$

$$\therefore \qquad \frac{K_{trans}}{K} = \frac{2}{3}$$

$$\frac{K_{rot}}{K} = \frac{1}{3}$$

(b) From conservation of energy,

$$mg(R - r) = \frac{3}{4} m v^2$$

$$\therefore \qquad \frac{mv^2}{R - r} = \frac{4}{3} mg$$

Now,

$$N - mg = \frac{mv^2}{R - r}$$

$$\therefore \qquad N = \frac{7}{3} mg$$

Example 8 *A wheel rotates around a stationary axis so that the rotation angle θ varies with time as $\theta = at^2$, where $a = 0.2 rad/s^2$. Find the magnitude of net acceleration of the point A at the rim at the moment $t = 2.5\,s$ if the linear velocity of the point A at this moment is $v = 0.65\,m/s$.*

Solution Instantaneous angular velocity at time t is

$$\omega = \frac{d\theta}{dt} = \frac{d}{dt}(at^2)$$

or $$\omega = 2at = 0.4t \qquad (\text{as } a = 0.2 \text{ rad/s}^2)$$

Further, instantaneous angular acceleration is,

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}(0.4t)$$

or $$\alpha = 0.4 \text{ rad/s}^2$$

Angular velocity at $t = 2.5\text{s}$ is

$$\omega = 0.4 \times 2.5 = 1.0 \text{ rad/s}$$

Further, radius of the wheel $R = \frac{v}{\omega}$ or $R = \frac{0.65}{1.0} = 0.65\text{ m}$

Now, magnitude of total acceleration is,

$$a = \sqrt{a_n^2 + a_t^2}$$

Here, $$a_n = R\omega^2 = (0.65)(1.0)^2 = 0.65 \text{ m/s}^2$$

and $$a_t = R\alpha = (0.65)(0.4) = 0.26 \text{ m/s}^2$$

$\therefore$ $$a = \sqrt{(0.65)^2 + (0.26)^2}$$

or $$a = 0.7 \text{ m/s}^2$$

Example 9 *A solid ball of radius 0.2 m and mass 1 kg is given an instantaneous impulse of 50 N-s at point P as shown. Find the number of rotations made by the ball about its diameter before hitting the ground. The ball is kept on smooth surface initially.*

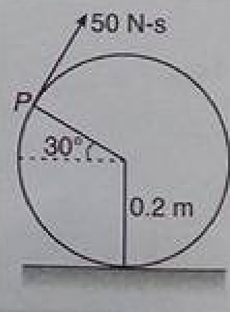


Fig. 9.93

Solution Impulse gives translational velocity

$$u = \frac{\text{Impulse}}{\text{Mass}} \text{ along impulse} = 50 \text{ m/s}$$

$$T = \text{time of flight of projectile}$$

$$= \frac{2u\sin\theta}{g} = \frac{2 \times 50 \times \sin 60^\circ}{10} = 5\sqrt{3}\text{ sec}$$

Impulse give angular impulse also

$$\omega = \frac{\text{Impulse} \times R}{I}$$

or

$$\omega = \frac{\text{Impulse} \times R}{\frac{2}{5} mR^2}$$

Number of rotations,

$$n = \frac{\omega T}{2\pi} = \frac{3125\sqrt{3}}{2\pi}$$

Level 2

Example 1 *A solid ball rolls down a parabolic path ABC from a height h as shown in figure. Portion AB of the path is rough while BC is smooth. How high will the ball climb in BC?*

Hint *In case of pure rolling mechanical energy is conserved.*

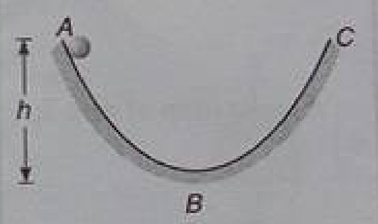


Fig. 9.94

Solution At B, total kinetic energy $= mgh$

Here, $m =$ mass of ball

The ratio of rotational to translational kinetic energy would be,

$$\frac{K_R}{K_T} = \frac{2}{5}$$

$$\therefore \qquad K_R = \frac{2}{7} mgh \quad \text{and} \quad K_T = \frac{5}{7} mgh$$

In portion BC, friction is absent. Therefore, rotational kinetic energy will remain constant and translational kinetic energy will convert into potential energy. Hence, if H be the height to which ball climbs in BC, then

$$mgH = K_T$$

or

$$mgH = \frac{5}{7} mgh \quad \text{or} \quad H = \frac{5}{7} h$$

Example 2 *A thread is wound around two discs on either sides. The pulley and the two discs have the same mass and radius. There is no slipping at the pulley and no friction at the hinge. Find out the accelerations of the two discs and the angular acceleration of the pulley.*

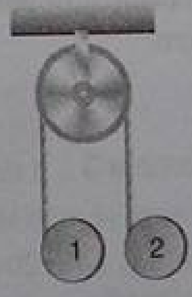


Fig. 9.95

Solution Let R be the radius of the discs and T_1 and T_2 be the tensions in the left and right segments of the rope.

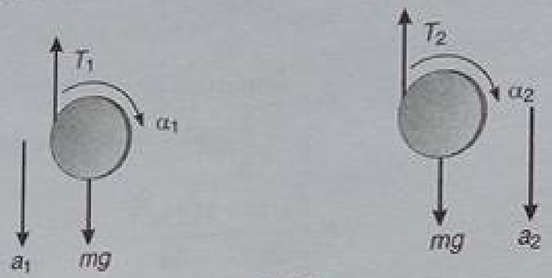


Fig. 9.96

Acceleration of disc 1,

$$a_1 = \frac{mg - T_1}{m} \quad \text{...(i)}$$

Acceleration of disc 2,

$$a_2 = \frac{mg - T_2}{m} \quad \text{...(ii)}$$

Angular acceleration of disc 1, $$\alpha_1 = \frac{\tau}{I} = \frac{T_1 R}{\frac{1}{2} mR^2} = \frac{2T_1}{mR} \quad \text{...(iii)}$$

Similarly, angular acceleration of disc 2, $$\alpha_2 = \frac{2T_2}{mR} \quad \text{...(iv)}$$

Both α_1 and α_2 are clockwise.

Angular acceleration of pulley,

$$\alpha = \frac{(T_2 - T_1)R}{\frac{1}{2} mR^2} = \frac{2(T_2 - T_1)}{mR} \quad \text{...(v)}$$

Fig. 9.97

For no slipping, $$R\alpha_1 - a_1 = a_2 - R\alpha_2 = R\alpha \quad \text{...(vi)}$$

Solving these equations, we get

$$\alpha = 0 \quad \text{and} \quad a_1 = a_2 = \frac{2g}{3}$$

Alternate Solution

As both the discs are in identical situation, $T_1 = T_2$ and $\alpha = 0$ *i.e.*, each of the discs falls independently and identically. Therefore, this is exactly similar to the problem shown in figure.

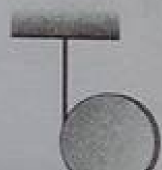


Fig. 9.98

Example 3 *A thin massless thread is wound on a reel of mass 3 kg and moment of inertia 0.6 kg-m^2. The hub radius is $R = 10$ cm and peripheral radius is $2R = 20$ cm. The reel is placed on a rough table and the friction is enough to prevent slipping. Find the acceleration of the centre of reel and of hanging mass of 1 kg.*

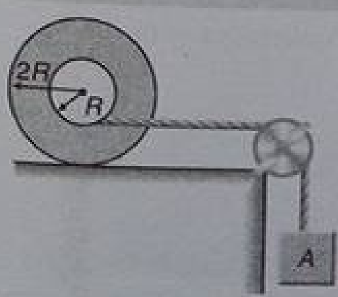


Fig. 9.99

Solution Let, a_1 = acceleration of centre of mass of reel

a_2 = acceleration of 1 kg block

α = angular acceleration of reel (clockwise)

T = tension in the string

and f = force of friction

Free body diagram of reel is as shown below: (only horizontal forces are shown).

Equations of motion are :

$$T - f = 3a_1 \qquad \text{...(i)}$$

$$\alpha = \frac{\tau}{I} = \frac{f(2R) - T.R}{I} = \frac{0.2f - 0.1T}{0.6} = \frac{f}{3} - \frac{T}{6} \qquad \text{...(ii)}$$

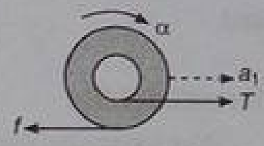


Fig. 9.100

Free body diagram of mass is,

Equation of motion is,

$$10 - T = a_2 \qquad \text{...(iii)}$$

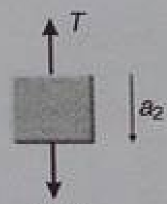


Fig. 9.101

For no slipping condition,

$$a_1 = 2R\alpha \quad \text{or} \quad a_1 = 0.2\alpha \qquad \text{...(iv)}$$

and $$a_2 = a_1 - R\alpha \quad \text{or} \quad a_2 = a_1 - 0.1\alpha \qquad \text{...(v)}$$

Solving the above five equations, we get

$$a_1 = 0.27 \text{ m/s}^2$$

and $$a_2 = 0.135 \text{ m/s}^2$$

Example 4 *A solid sphere of radius r is gently placed on a rough horizontal ground with an initial angular speed ω_0 and no linear velocity. If the coefficient of friction is μ, find the time t when the slipping stops. In addition, state the linear velocity v and angular velocity ω at the end of slipping.*

Fig. 9.102

Solution Let m be the mass of the sphere.

Since, it is a case of backward slipping, force of friction is in forward direction. Limiting friction will act in this case.

Fig. 9.103

Linear acceleration $$a = \frac{f}{m} = \frac{\mu mg}{m} = \mu g$$

Angular retardation $$\alpha = \frac{\tau}{I} = \frac{f \cdot r}{\frac{2}{5}mr^2} = \frac{5}{2}\frac{\mu g}{r}$$

Slipping is ceased when. $$v = r\omega$$

or $$(at) = r(\omega_0 - \alpha t)$$

or $$\mu gt = r\left(\omega_0 - \frac{5}{2}\frac{\mu gt}{r}\right)$$

or
$$\frac{7}{2}\mu g t = r\omega_0$$

$\therefore$
$$t = \frac{2}{7}\frac{r\omega_0}{\mu g}$$

$$v = at = \mu g t = \frac{2}{7} r\omega_0$$

and
$$\omega = \frac{v}{r} = \frac{2}{7}\omega_0$$

Alternate Solution

Net torque on the sphere about the bottommost point is zero. Therefore, angular momentum of the sphere will remain conserved about the bottommost point.

$$L_i = L_f$$

$\therefore$
$$I\omega_0 = I\omega + mrv$$

or
$$\frac{2}{5}mr^2\omega_0 = \frac{2}{5}mr^2\omega + mr(\omega r)$$

$\therefore$
$$\omega = \frac{2}{7}\omega_0 \quad \text{and} \quad v = r\omega = \frac{2}{7}r\omega_0$$

Example 5 *A billiard ball, initially at rest, is given a sharp impulse by a cue. The cue is held horizontally a distance h above the centre line as shown in figure. The ball leaves the cue with a speed v_0 and because of its forward english (backward slipping) eventually acquires a final speed $\frac{9}{7}v_0$. Show that*

$$h = \frac{4}{5}R$$

where R is the radius of the ball.

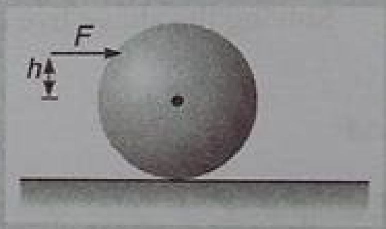


Fig. 9.104

Solution Let ω_0 be the angular speed of the ball just after it leaves the cue. The maximum friction acts in forward direction till the slipping continues. Let v be the linear speed and ω the angular speed when slipping is ceased.

Fig. 9.105

$\therefore$
$$v = R\omega \quad \text{or} \quad \omega = \frac{v}{R}$$

Given,
$$v = \frac{9}{7}v_0 \qquad \text{...(i)}$$

$\therefore$
$$\omega = \frac{9}{7}\frac{v_0}{R} \qquad \text{...(ii)}$$

Applying, Linear impulse = change in linear momentum

$$\therefore \quad F\,dt = mv_0 \qquad \text{...(iii)}$$

Angular impulse = change in angular momentum

$$\therefore \quad \tau\,dt = I\omega_0$$

or

$$Fh\,dt = \frac{2}{5}mR^2\,\omega_0 \qquad \text{...(iv)}$$

Angular momentum about bottommost point will remain conserved.

i.e.,

$$L_i = L_f$$

or

$$I\omega_0 + mRv_0 = I\omega + mRv$$

$$\therefore \quad \frac{2}{5}mR^2\,\omega_0 + mRv_0 = \frac{2}{5}mR^2\left(\frac{9}{7}\frac{v_0}{R}\right) + \frac{9}{7}mRv_0 \qquad \text{...(v)}$$

Solving Eqs. (iii), (iv) and (v), we get

$$h = \frac{4}{5}R$$

Proved.

Example 6 *Determine the maximum horizontal force F that may be applied to the plank of mass m for which the solid sphere does not slip as it begins to roll on the plank. The sphere has a mass M and radius R. The coefficient of static and kinetic friction between the sphere and the plank are* μ_s *and* μ_k *respectively.*

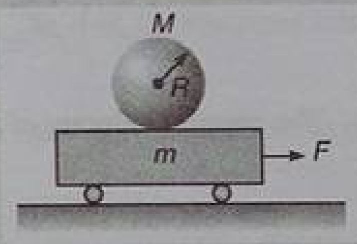


Fig. 9.106

Solution The free body diagrams of the sphere and the plank are as shown below:

Fig. 9.107

Writing equations of motion :

For sphere : Linear acceleration

$$a_1 = \frac{\mu_s Mg}{M} = \mu_s g \qquad \text{...(i)}$$

Angular acceleration

$$\alpha = \frac{(\mu_s Mg)R}{\frac{2}{5}MR^2} = \frac{5}{2}\frac{\mu_s g}{R} \qquad \text{...(ii)}$$

For plank : Linear acceleration

$$a_2 = \frac{F - \mu_s Mg}{m} \qquad \text{...(iii)}$$

For no slipping :

$$a_2 = a_1 + R\alpha \qquad \text{...(iv)}$$

Solving the above four equations, we get $F = \mu_s g\left(M + \frac{7}{2}m\right)$

Thus, maximum value of F can be $\mu_s g\left(M + \frac{7}{2}m\right)$

Example 7 *A uniform disc of radius r_0 lies on a smooth horizontal plane. A similar disc spinning with the angular velocity ω_0 is carefully lowered onto the first disc. How soon do both discs spin with the same angular-velocity if the friction coefficient between them is equal to μ ?*

Solution From the law of conservation of angular momentum.

$$I\omega_0 = 2I\omega$$

Here, I = moment of inertia of each disc relative to common rotation axis

$\therefore$
$$\omega = \frac{\omega_0}{2} = \text{steady state angular velocity}$$

The angular velocity of each disc varies due to the torque τ of the friction forces. To calculate τ, let us take an elementary ring with radii r and $r + dr$. The torque of the friction forces acting on the given ring is equal to.

$$d\tau = \mu r\left(\frac{mg}{\pi r_0^2}\right)2\pi r\,dr = \left(\frac{2\mu mg}{r_0^2}\right)r^2\,dr$$

where m is the mass of each disc. Integrating this with respect to r between 0 and r_0, we get

$$\tau = \frac{2}{3}\mu m g r_0$$

The angular velocity of the lower disc increases by $d\omega$ over the time interval

$$dt = \left(\frac{I}{\tau}\right)d\omega = \left(\frac{3r_0}{4\mu g}\right)d\omega$$

Integrating this equation with respect to ω between 0 and $\frac{\omega_0}{2}$, we find the desired time

$$t = \frac{3r_0\,\omega_0}{8\mu g}$$

EXERCISES

AIEEE Corner

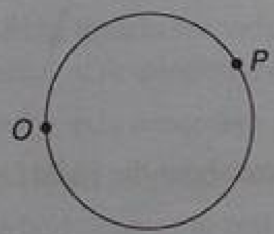
Subjective Questions (Level 1)

Moment of Inertia

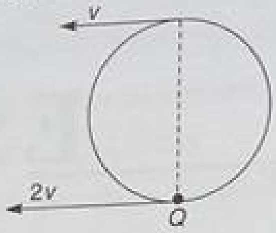
1. Four thin rods each of mass m and length l are joined to make a square. Find moment of inertia of all the four rods about any side of the square.
2. A mass of 1 kg is placed at (1 m, 2 m, 0). Another mass of 2 kg is placed at (3 m, 4 m, 0). Find moment of inertia of both the masses about z-axis.
3. Moment of inertia of a uniform rod of mass m and length l is $\frac{7}{12}ml^2$ about a line perpendicular to the rod. Find the distance of this line from the middle point of the rod.
4. Find the moment of inertia of a uniform square plate of mass M and edge a about one of its diagonals.
5. Radius of gyration of a body about an axis at a distance 6 cm from its centre of mass is 10 cm. Find its radius of gyration about a parallel axis through its centre of mass.
6. Two point masses m_1 and m_2 are joined by a weightless rod of length r. Calculate the moment of inertia of the system about an axis passing through its centre of mass and perpendicular to the rod.
7. Linear mass density (mass/length) of a rod depends on the distance from one end (say A) as $\lambda_x = (\alpha x + \beta)$. Here, α and β are constants. Find the moment of inertia of this rod about an axis passing through A and perpendicular to the rod. Length of the rod is l.

Angular Velocity

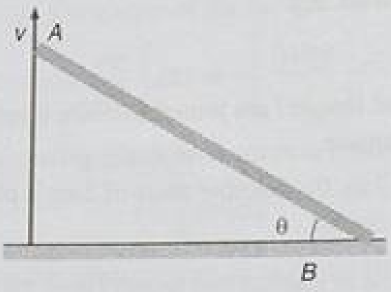
8. Find angular speed of second's clock.
9. A particle is located at (3 m, 4 m) and moving with $\vec{\mathbf{v}} = (4\hat{\mathbf{i}} - 3\hat{\mathbf{j}})$ m/s. Find its angular velocity about origin at this instant.
10. Particle P shown in figure is moving in a circle of radius $R = 10$ cm with linear speed $v = 2$ m/s. Find the angular speed of particle about point O.



11. Two points P and Q, diametrically opposite on a disc of radius R have linear velocities v and $2v$ as shown in figure. Find the angular speed of the disc.



12. Point A of rod AB ($l = 2$m) is moved upwards against a wall with velocity $v = 2$m/s. Find angular speed of the rod at an instant when $\theta = 60°$.

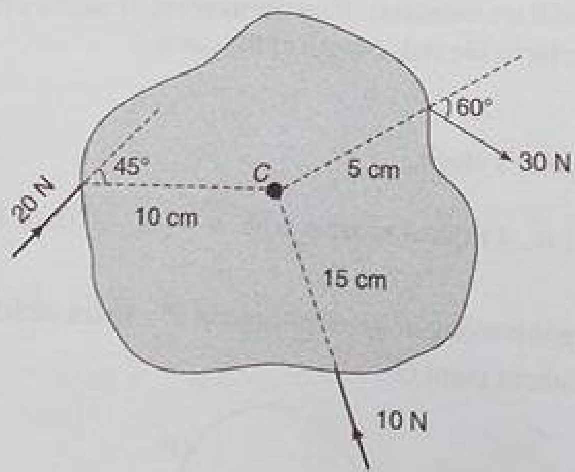


Torque

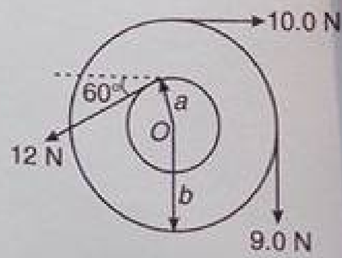
13. A force $\vec{\mathbf{F}} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$ N is acting on a body at point (2 m, 4 m, –2 m). Find torque of this force about origin.

14. A particle of mass $m = 1$kg is projected with speed $u = 20\sqrt{2}$ m/s at angle $\theta = 45°$ with horizontal. Find the torque of the weight of the particle about the point of projection when the particle is at the highest point.

15. Point C is the centre of mass of the rigid body shown in figure. Find the total torque acting on the body about point C.



16. Find the net torque on the wheel in figure about the point O if $a = 10$ cm and $b = 25$ cm.



Rotation of a Rigid Body About a Fixed Axis

Uniform angular acceleration

17. A wheel rotating with uniform angular acceleration covers 50 rev in the first five seconds after the start. Find the angular acceleration and the angular velocity at the end of five seconds.

18. A wheel starting from rest is uniformly accelerated with $\alpha = 2\,\text{rad/s}^2$ for 5 s. It is then allowed to rotate uniformly for the next two seconds and is finally brought to rest in the next 5 s. Find the total angle rotated by the wheel.

19. A wheel whose moment of inertia is $0.03\,\text{kg m}^2$, is accelerated from rest to 20 rad/s in 5 s. When the external torque is removed, the wheel stops in 1 min. Find :
(a) the frictional torque, (b) the external torque.

20. A body rotating at 20 rad/s is acted upon by a constant torque providing it a deceleration of $2\,\text{rad/s}^2$. At what time will the body have kinetic energy same as the initial value if the torque continues to act ?

21. A uniform disc of mass 20 kg and radius 0.5 m can turn about a smooth axis through its centre and perpendicular to the disc. A constant torque is applied to the disc for 3 s from rest and the angular velocity at the end of that time is $\frac{240}{\pi}$ rev/min. Find the magnitude of the torque. If the torque is then removed and the disc is brought to rest in t seconds by a constant force of 10 N applied tangentially at a point on the rim of the disc, find t.

22. A uniform disc of mass m and radius R is rotated about an axis passing through its centre and perpendicular to its plane with an angular velocity ω_0. It is placed on a rough horizontal plane with the axis of the disc keeping vertical. Coefficient of friction between the disc and the surface is μ. Find :
(a) the time when disc stops rotating,
(b) the angle rotated by the disc before stopping.

Non-uniform angular acceleration

23. A flywheel whose moment of inertia about its axis of rotation is 16 kg-m^2 is rotating freely in its own plane about a smooth axis through its centre. Its angular velocity is $9\,\text{rad s}^{-1}$ when a torque is applied to bring it to rest in t_0 seconds. Find t_0 if :
(a) the torque is constant and of magnitude of 4 Nm,
(b) the magnitude of the torque after t seconds is given by kt.

24. A shaft is turning at 65 rad/s at time zero. Thereafter, angular acceleration is given by
$$\alpha = -10\,\text{rad/s}^2 - 5t\,\text{rad/s}^2$$
where t is the elapsed time.
(a) Find its angular speed at $t = 3.0$ s.
(b) How far does it turn in these 3s ?

25. The angular velocity of a gear is controlled according to $\omega = 12 - 3t^2$ where ω, in radian per second, is positive in the clockwise sense and t is the time in seconds. Find the net angular displacement $\Delta\theta$ from the time $t = 0$ to $t = 3$s. Also, find the number of revolutions N through which the gear turns during the 3 s.

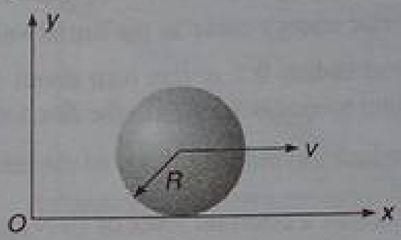
26. A solid body rotates about a stationary axis according to the law $\theta = at - bt^3$, where $a = 6$ rad/s and $b = 2\,\text{rad/s}^3$. Find the mean values of the angular velocity and acceleration over the time interval between $t = 0$ and the time, when the body comes to rest.

Angular Momentum

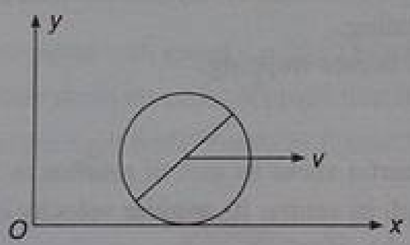
27. A particle of mass 1 kg is moving along a straight line $y = x + 4$. Both x and y are in metres. Velocity of the particle is 2 m/s. Find magnitude of angular momentum of the particle about origin.

28. A uniform rod of mass m is rotated about an axis passing through point O as shown. Find angular momentum of the rod about rotational axis.

29. A solid sphere of mass m and radius R is rolling without slipping as shown in figure. Find angular momentum of the sphere about z-axis.



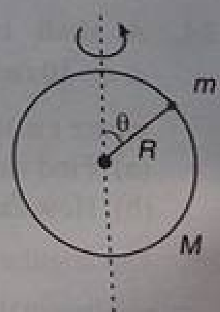
30. A rod of mass m and length $2R$ is fixed along the diameter of a ring of same mass m and radius R as shown in figure. The combined body is rolling without slipping along x-axis. Find the angular momentum about z-axis.



Conservation of Angular Momentum

31. If radius of earth is increased, without change in its mass, will the length of day increase, decrease or remain same?

32. The figure shows a thin ring of mass $M = 1$ kg and radius $R = 0.4$ m spinning about a vertical diameter. (Take $I = \frac{1}{2}MR^2$). A small bead of mass $m = 0.2$ kg can slide without friction along the ring. When the bead is at the top of the ring, the angular velocity is 5 rad/s. What is the angular velocity when the bead slips halfway to $\theta = 45°$.



33. A horizontal disc rotating freely about a vertical axis makes 100 rpm. A small piece of wax of mass 10 g falls vertically on the disc and adheres to it at a distance of 9 cm from the axis. If the number of revolutions per minute is thereby reduced to 90. Calculate the moment of inertia of disc.

34. A man stands at the centre of a circular platform holding his arms extended horizontally with 4 kg block in each hand. He is set rotating about a vertical axis at 0.5 rev/s. The moment of inertia of the man plus platform is 1.6 kg-m^2, assumed constant. The blocks are 90 cm from the axis of rotation. He now pulls the blocks in toward his body until they are 15 cm from the axis of rotation. Find (a) his new angular velocity and (b) the initial and final kinetic energy of the man and platform. (c) how much work must the man do to pull in the blocks ?

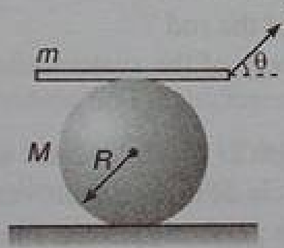
35. A horizontally oriented uniform disc of mass M and radius R rotates freely about a stationary vertical axis passing through its centre. The disc has a radial guide along which can slide without friction a small body of mass m. A light thread running down through the hollow axle of the disc is tied to the body. Initially the body was located at the edge of the disc and the whole system rotated with an angular velocity ω_0. Then, by means of a force F applied to the lower end of the thread the body was slowly pulled to the rotation axis. Find :

(a) the angular velocity of the system in its final state,

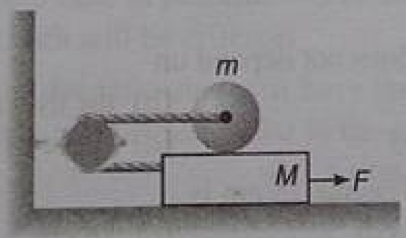
(b) the work performed by the force F.

Pure Rolling

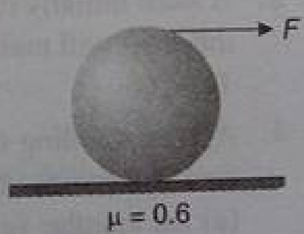
36. Consider a cylinder of mass M and radius R lying on a rough horizontal plane. It has a plank lying on its top as shown in figure. A force F is applied on the plank such that the plank moves and causes the cylinder to roll. The plank always remains horizontal. There is no slipping at any point of contact. Calculate the acceleration of the cylinder and the frictional forces at the two contacts.



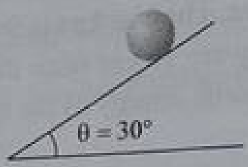
37. Find the acceleration of the cylinder of mass m and radius R and that of plank of mass M placed on smooth surface if pulled with a force F as shown in figure. Given that sufficient friction is present between cylinder and the plank surface to prevent sliding of cylinder.



38. In the figure shown a force F is applied at the top of a disc of mass 4 kg and radius 0.25 m. Find maximum value of F for no slipping



39. In the figure shown a solid sphere of mass 4 kg and radius 0.25 m is placed on a rough surface. Find : $(g = 10\ \text{m/s}^2)$



(a) minimum coefficient of friction for pure rolling to take place.
(b) If $\mu > \mu_{min}$, find linear acceleration of sphere.
(c) If $\mu = \frac{\mu_{min}}{2}$, find linear acceleration of cylinder.

Here, μ_{min} is the value obtained in part (a).

Angular Impulse

40. A uniform rod AB of length $2l$ and mass m is rotating in a horizontal plane about a vertical axis through A, with angular velocity ω, when the mid-point of the rod strikes a fixed nail and is brought immediately to rest. Find the impulse exerted by the nail.

41. A uniform rod of length L rests on a frictionless horizontal surface. The rod is pivoted about a fixed frictionless axis at one end. The rod is initially at rest. A bullet travelling parallel to the horizontal surface and perpendicular to the rod with speed v strikes the rod at its centre and becomes embedded in it. The mass of the bullet is one-sixth the mass of the rod.
(a) What is the final angular velocity of the rod ?
(b) What is the ratio of the kinetic energy of the system after the collision to the kinetic energy of the bullet before the collision ?

42. A uniform rod AB of mass $3m$ and length $2l$ is lying at rest on a smooth horizontal table with a smooth vertical axis through the end A. A particle of mass $2m$ moves with speed $2u$ across the table and strikes the rod at its mid-point C. If the impact is perfectly elastic. Find the speed of the particle after impact if :
(a) it strikes the rod normally,
(b) its path before impact was inclined at 60° to AC.

Objective Questions (Level 1)

Single Correct Option

1. The moment of inertia of a body does not depend on
(a) mass of the body
(b) the distribution of the mass in the body
(c) the axis of rotation of the body
(d) None of these

2. The radius of gyration of a disc of radius 25 cm is
(a) 18 cm
(b) 12.5 cm
(c) 36 cm
(d) 50 cm

3. A shaft initially rotating at 1725 rpm is brought to rest uniformly in 20s. The number of revolutions that the shaft will make during this time is
(a) 1680
(b) 575
(c) 287
(d) 627

4. A man standing on a platform holds weights in his outstretched arms. The system is rotated about a central vertical axis. If the man now pulls the weights inwards close to his body, then
(a) the angular velocity of the system will increase
(b) the angular momentum of the system will remain constant

(c) the kinetic energy of the system will increase
(d) All of the above

5. The moment of inertia of a uniform semicircular disc of mass M and radius r about a line perpendicular to the plane of the disc through the centre is
   (a) Mr^2 (b) $\frac{1}{2}Mr^2$ (c) $\frac{1}{4}Mr^2$ (d) $\frac{2}{5}Mr^2$

6. Two bodies A and B made of same material have the moment of inertial in the ratio $I_A : I_B = 16 : 18$. The ratio of the masses $m_A : m_B$ is given by
   (a) cannot be obtained (b) 2 : 3
   (c) 1 : 1 (d) 4 : 9

7. When a sphere rolls down an inclined plane, then identity the correct statement related to the work done by friction force
   (a) The friction force does positive translational work
   (b) The friction force does negative rotational work
   (c) The net work done by friction is zero
   (d) All of the above

8. A circular table rotates about a vertical axis with a constant angular speed ω. A circular pan rests on the turn table (with the centre coinciding with centre of table) and rotates with the table. The bottom of the pan is covered with a uniform thick layer of ice which also rotates with the pan. The ice starts melting. The angular speed of the turn table
   (a) remains the same
   (b) decreases
   (c) increases
   (d) may increase or decrease depending on the thickness of ice layer

9. If R is the radius of gyration of a body of mass M and radius r, then the ratio of its rotational to translational kinetic energy in the rolling condition is
   (a) $\frac{R^2}{R^2+r^2}$ (b) $\frac{R^2}{r^2}$ (c) $\frac{r^2}{R^2}$ (d) 1

10. A solid sphere rolls down two different inclined planes of the same height but of different inclinations
    (a) in both cases the speeds and time of descend will be same
    (b) the speeds will be same but time of descend will be different
    (c) the speeds will be different but time of descend will be same
    (d) speeds and time of descend both will be different

11. For the same total mass which of the following will have the largest moment of inertia about an axis passing through the centre of mass and perpendicular to the plane of the body
    (a) a disc of radius R (b) a ring of radius R
    (c) a square lamina of side $2R$ (d) four rods forming a square of side $2R$

12. A disc and a solid sphere of same mass and radius roll down an inclined plane. The ratio of the friction force acting on the disc and sphere is
    (a) $\frac{7}{6}$ (b) $\frac{5}{4}$
    (c) $\frac{3}{2}$ (d) depends on angle of inclination

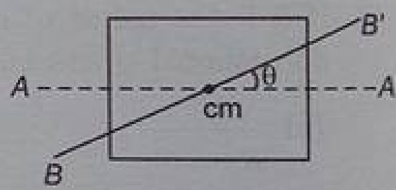
13. A horizontal disc rotates freely with angular velocity ω about a vertical axes through its centre. A ring, having the same mass and radius as the disc, is now gently placed coaxially on the disc. After some time, the two rotate with a common angular velocity. Then
(a) no friction exists between the disc and the ring
(b) the angular momentum of the system is conserved
(c) the final common angular velocity is $\frac{1}{2}\omega$
(d) All of the above

14. A solid homogeneous sphere is moving on a rough horizontal surface, partly rolling and partly sliding. During this kind of motion of the sphere
(a) total kinetic energy of the sphere is conserved
(b) angular momentum of the sphere about any point on the horizontal surface is conserved
(c) only the rotational kinetic energy about the centre of mass is conserved
(d) None of the above

15. A particle of mass $m = 3$ kg moves along a straight line $4y - 3x = 2$ where x and y are in metre, with constant velocity $v = 5\,\text{ms}^{-1}$. The magnitude of angular momentum about the origin is
(a) $12\,\text{kg m}^2\text{s}^{-1}$ (b) $6.0\,\text{kg m}^2\text{s}^{-1}$ (c) $4.5\,\text{kg m}^2\text{s}^{-1}$ (d) $8.0\,\text{kg m}^2\text{s}^{-1}$

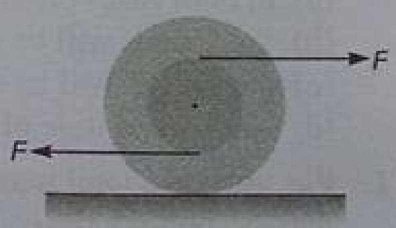
16. A solid sphere rolls without slipping on a rough horizontal floor, moving with a speed v. It makes an elastic collision with a smooth vertical wall. After impact,
(a) it will move with a speed v initially
(b) its motion will be rolling with slipping initially and its rotational motion will stop momentarily at some instant
(c) its motion will be rolling without slipping only after some time
(d) All of the above

17. The figure shows a square plate of uniform mass distribution. AA' and BB' are the two axes lying in the plane of the plate and passing through its centre of mass. It I_o is the moment of inertia of the plate about AA' then its moment of inertia about the BB' axis is



(a) I_o (b) $I_o \cos\theta$
(c) $I_o \cos^2\theta$ (d) None of these

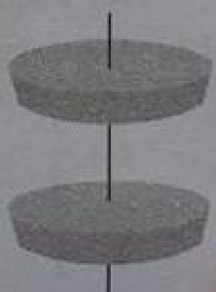
18. A spool is pulled horizontally on rough surface by two equal and opposite forces as shown in the figure. Which of the following statements are correct?



Rough

(a) The centre of mass moves towards left
(b) The centre of mass moves towards right
(c) The centre of mass remains stationary
(d) The net torque about the centre of mass of the spool is zero

19. Two identical discs are positioned on a vertical axis as shown in the figure. The bottom disc is rotating at angular velocity ω_0 and has rotational kinetic energy K_0. The top disc is initially at rest. It then falls and sticks to the bottom disc. The change in the rotational kinetic energy of the system is



(a) $K_0/2$ (b) $-K_0/2$
(c) $-K_0/4$ (d) $K_0/4$

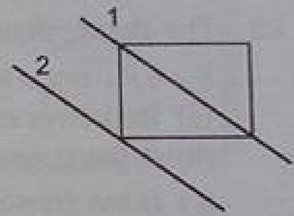
20. The moment of inertia of hollow sphere (mass M) of inner radius R and outer radius $2R$, having material of uniform density, about a diametric axis is

(a) $31\,MR^2/70$ (b) $43\,MR^2/90$ (c) $19\,MR^2/80$ (d) None of these

21. A rod of uniform cross-section of mass M and length L is hinged about an end to swing freely in a vertical plane. However, its density is non uniform and varies linearly from hinged end to the free end doubling its value. The moment of inertia of the rod, about the rotation axis passing through the hinge point is

(a) $\frac{2ML^2}{9}$ (b) $\frac{3ML^2}{16}$ (c) $\frac{7ML^2}{18}$ (d) None of these

22. Let I_1 and I_2 be the moment of inertia of a uniform square plate about axes shown in the figure. Then the ratio $I_1 : I_2$ is



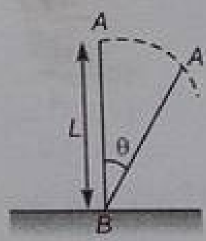
(a) $1:\frac{1}{7}$ (b) $1:\frac{12}{7}$

(c) $1:\frac{7}{12}$ (d) $1:7$

23. Moment of inertia of a uniform rod of length L and mass M, about an axis passing through $L/4$ from one end and perpendicular to its length is

(a) $\frac{7}{36}ML^2$ (b) $\frac{7}{48}ML^2$ (c) $\frac{11}{48}ML^2$ (d) $\frac{ML^2}{12}$

24. A uniform rod of length L is free to rorate in a vertical plane about a fixed horizontal axis through B. The rod begins rotating from rest. The angular velocity ω at angle θ is given as



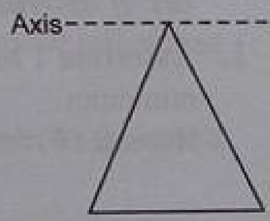
(a) $\sqrt{\left(\frac{6g}{L}\right)}\sin\frac{\theta}{2}$ (b) $\sqrt{\left(\frac{6g}{L}\right)}\cos\frac{\theta}{2}$

(c) $\sqrt{\left(\frac{6g}{L}\right)}\sin\theta$ (d) $\sqrt{\left(\frac{6g}{L}\right)}\cos\theta$

25. Two particles of masses m_1 and m_2 are connected by a light rod of length r to constitute a dumb-bell. The moment of inertia of the dumb-bell about an axis perpendicular to the rod passing through the centre of mass of the two particles is

(a) $\frac{m_1 m_2 r^2}{m_1+m_2}$ (b) $(m_1+m_2)r^2$ (c) $\frac{m_1 m_2 r^2}{m_1-m_2}$ (d) $(m_1-m_2)r^2$

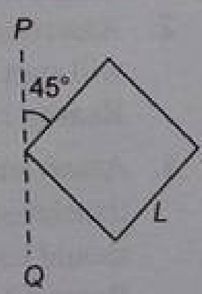
26. Find moment of inertia of a thin sheet of mass M in the shape of an equilateral triangle about an axis as shown in figure. The length of each side is L



(a) $ML^2/8$ (b) $3ML^2/8$

(c) $7ML^2/8$ (d) None of these

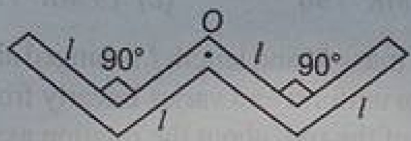
27. A square is made by joining four rods each of mass M and length L. Its moment of inertia about an axis PQ, in its plane and passing through one of its corner is



(a) $6\,ML^2$ (b) $\frac{4}{3}ML^2$

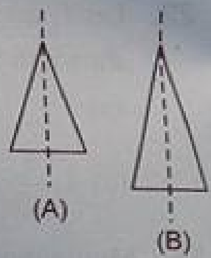
(c) $\frac{8}{3}ML^2$ (d) $\frac{10}{3}ML^2$

28. A thin rod of length $4l$, mass $4m$ is bent at the points as shown in the figure. What is the moment of inertia of the rod about the axis passing through O and perpendicular to the plane of the paper?



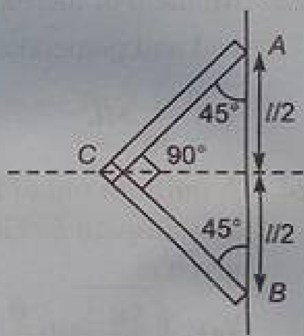
(a) $\frac{ml^2}{3}$ (b) $\frac{10ml^2}{3}$ (c) $\frac{ml^2}{12}$ (d) $\frac{ml^2}{24}$

29. The figure shows two cones A and B with the conditions : $h_A < h_B$; $\rho_A > \rho_B$; $R_A = R_B$ $m_A = m_B$. Identify the correct statement about their axis of symmetry.



(a) Both have same moment of inertia
(b) A has greater moment of inertia
(c) B has greater moment of inertia
(d) Nothing can be said

30. Linear mass density of the two rods system, AC and CB is x. Moment of inertia of two rods about an axis passing through AB is



(a) $\frac{xl^3}{4\sqrt{3}}$ (b) $\frac{xl^3}{\sqrt{2}}$

(c) $\frac{xl^3}{4}$ (d) $\frac{xl^3}{6\sqrt{2}}$

JEE Corner

Assertion and Reason

Directions : *Choose the correct option.*

(a) *If both **Assertion** and **Reason** are true and the **Reason** is correct explanation of the **Assertion**.*
(b) *If both **Assertion** and **Reason** are true but **Reason** is not the correct explanation of **Assertion**.*
(c) *If **Assertion** is true, but the **Reason** is false.*
(d) *If **Assertion** is false but the **Reason** is true.*

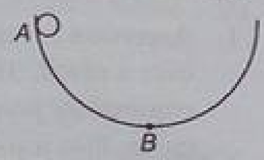
1. Assertion : Moment of inertia of a rigid body about any axis passing through its centre of mass is minimum.
Reason : From theorem of parallel axis,

$$I = I_{cm} + Mr^2$$

2. Assertion : A ball is released on a rough ground in the condition shown in figure. It will start pure rolling after some time towards left side.
Reason : Friction will convert the pure rotational motion of the ball into pure rolling.

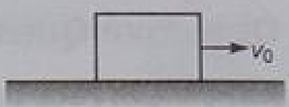
3. Assertion : A solid sphere and a hollow sphere are rolling on ground with same total kinetic energies. If translational kinetic energy of solid sphere is K, then translational kinetic energy of hollow sphere should be greater than K.
Reason : In case of hollow sphere rotational kinetic energy is less than its translational kinetic energy.

4. **Assertion :** A small ball is released from rest from point A as shown. If bowl is smooth than ball will exert more pressure at point B, compared to the situation if bowl is rough.



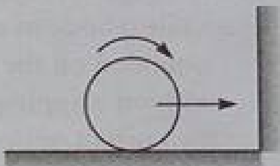
Reason : Linear velocity and hence, centripetal force in smooth situation is more.

5. **Assertion :** A cubical block is moving on a rough ground with velocity v_0. During motion net normal reaction on the block from ground will not pass through centre of cube. It will shift towards right.



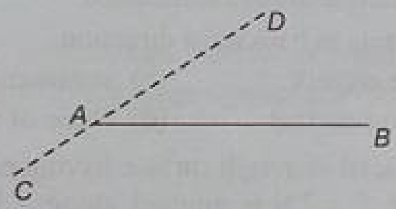
Reason : It is to keep the block in rotational equilibrium.

6. **Assertion :** A ring is rolling without slipping on a rough ground. It strikes elastically with a smooth wall as shown in figure. Ring will stop after some time while travelling in opposite direction.



Reason : Net angular momentum about an axis passing through bottommost point and perpendicular to plane of paper is zero.

7. **Assertion :** There is a thin rod AB and a dotted line CD. All the axes we are talking about are perpendicular to plane of paper. As we take different axes moving from A to D, moment of inertia of the rod may first decrease then increase.



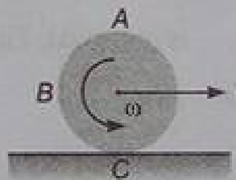
Reason : Theorem of perpendicular axis cannot be applied here.

8. **Assertion :** If linear momentum of a particle is constant, then its angular momentum about any axis will also remain constant.

Reason : Linear momentum remain constant, if $\vec{\mathbf{F}}_{\text{net}} = 0$ and angular momentum remains constant if $\vec{\tau}_{\text{net}} = 0$.

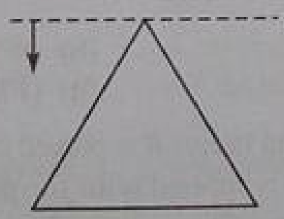
9. **Assertion :** In the figure shown, A, B and C are three points on the circumference of a disc. Let v_A, v_B and v_C are speeds of these three points, then

$$v_C > v_B > v_A$$



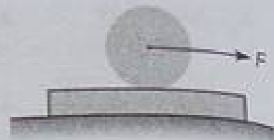
Reason : In case of rotational plus translational motion of a rigid body, net speed of any point (other than centre of mass) is greater than, less than or equal to the speed of centre of mass.

10. **Assertion :** There is a triangular plate as shown. A dotted axis is lying in the plane of slab. As the axis is moved downwards, moment of inertia of slab will first decrease then increase.



Reason : Axis is first moving towards its centre of mass and then it is receding from it.

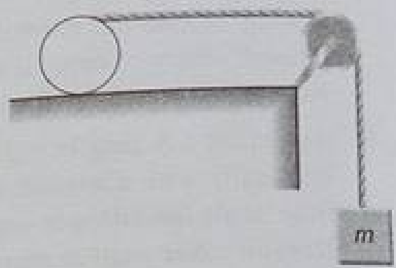
11. **Assertion :** A horizontal force F is applied at the centre of solid sphere placed over a plank. The minimum coefficient of friction between plank and sphere required for pure rolling is μ_1 when plank is kept at rest and μ_2 when plank can move, then $\mu_2 < \mu_1$.
Reason : Work done by frictional force on the sphere in both cases is zero.



Objective Questions (Level 2)

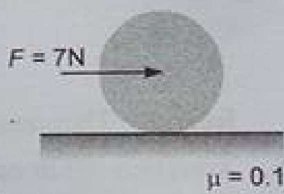
Single Correct Option

1. In the given figure a ring of mass m is kept on a horizontal surface while a body of equal mass m is attached through a string, which is wounded on the ring. When the system is released, the ring rolls without slipping. Consider the following statement and choose the correct option.



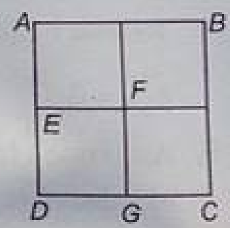
(i) acceleration of the centre of mass of ring is $\frac{2g}{3}$
(ii) acceleration of hanging particle is $\frac{4g}{3}$
(iii) frictional force (on the ring) acts in forward direction
(iv) frictional force (on the ring) acts in backward direction
(a) statement (i) and (ii) only are correct
(b) statement (ii) and (iii) only are correct
(c) statement (iii) and (iv) only are correct
(d) None of these

2. A solid sphere of mass 10 kg is placed on rough surface having coefficient of friction $\mu = 0.1$. A constant force $F = 7$ N is applied along a line passing through the centre of the sphere as shown in the figure. The value of frictional force on the sphere is



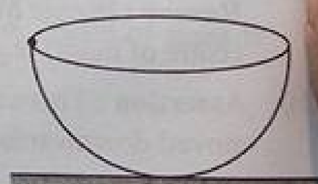
(a) 1 N
(b) 2 N
(c) 3 N
(d) 7 N

3. From a uniform square plate of side a and mass m, a square portion $DEFG$ of side $\frac{a}{2}$ is removed. Then, the moment of inertia of remaining portion about the axis AB is



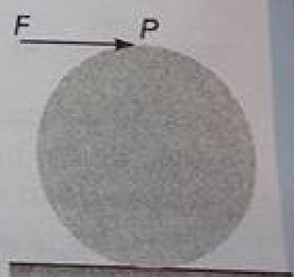
(a) $\frac{7ma^2}{16}$
(b) $\frac{3ma^2}{16}$
(c) $\frac{3ma^2}{4}$
(d) $\frac{9ma^2}{16}$

4. A small solid sphere of mass m and radius r starting from rest from the rim of a fixed hemispherical bowl of radius $R(>>r)$ rolls inside it without sliding. The normal reaction exerted by the sphere on the hemisphere when it reaches the bottom of hemisphere is



(a) $(3/7)\,mg$
(b) $(9/7)\,mg$
(c) $(13/7)\,mg$
(d) $(17/7)\,mg$

5. A uniform solid cylinder of mass m and radius R is placed on a rough horizontal surface. A horizontal constant force F is applied at the top point P of the cylinder so that it starts pure rolling. The acceleration of the cylinder is



(a) $F/3m$
(b) $2F/3m$
(c) $4F/3m$
(d) $5F/3m$

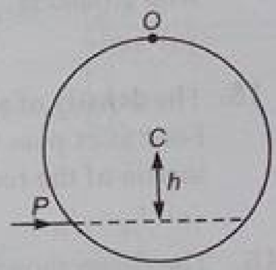
6. In the above question, the frictional force on the cylinder is

(a) $F/3$ towards right (b) $F/3$ towards left
(c) $2F/3$ towards right (d) $2F/3$ towards left

7. A small pulley of radius 20 cm and moment of inertia $0.32\ \text{kg-m}^2$ is used to hang a 2 kg mass with the help of massless string. If the block is released, for no slipping condition acceleration of the block will be

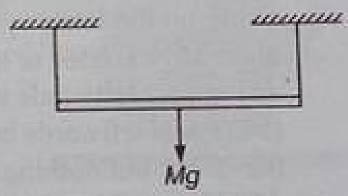
(a) $2\ \text{m/s}^2$
(b) $4\ \text{m/s}^2$
(c) $1\ \text{m/s}^2$
(d) $3\ \text{m/s}^2$

8. A uniform circular disc of radius R is placed on a smooth horizontal surface with its plane horizontal and hinged at circumference through point O as shown. An impulse P is applied at a perpendicular distance h from its centre C. The value of h so that the impulse due to hinge is zero, is



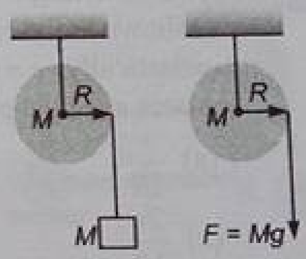
(a) R (b) $R/2$
(c) $R/3$ (d) $R/4$

9. A rod is supported horizontally by means of two strings of equal length as shown in figure. If one of the string is cut. Then tension in other string at the same instant will



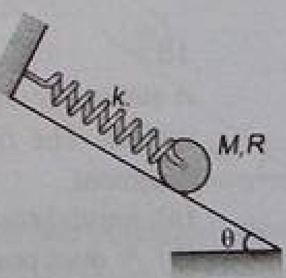
(a) remain unaffected
(b) increase
(c) decrease
(d) become equal to weight of the rod

10. The figure represents two cases. In first case a block of mass M is attached to a string which is tightly wound on a disc of mass M and radius R. In second case $F = Mg$. Initially, the disc is stationary in each case. If the same length of string is unwound from the disc, then



(a) same amount of work is done on both discs
(b) angular velocities of both the discs are equal
(c) both the discs have unequal angular accelerations
(d) All of the above

11. A uniform cylinder of mass M and radius R is released from rest on a rough inclined surface of inclination θ with the horizontal as shown in figure. As the cylinder rolls down the inclined surface, the maximum elongation in the spring of stiffness k is

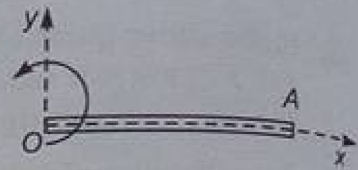


(a) $\frac{3}{4}\frac{Mg\sin\theta}{k}$ (b) $\frac{2\,Mg\sin\theta}{k}$
(c) $\frac{Mg\sin\theta}{k}$ (d) None of these

12. A uniform rod of mass m and length l rotates in a horizontal plane with an angular velocity ω about a vertical axis passing through one end. The tension in the rod at a distance x from the axis is

(a) $\frac{1}{2}m\omega^2 x$ (b) $\frac{1}{2}m\omega^2\left(1-\frac{x^2}{l}\right)$ (c) $\frac{1}{2}m\omega^2 l\left(1-\frac{x^2}{l^2}\right)$ (d) $\frac{1}{2}m\omega^2 l\left[1-\frac{x}{l}\right]$

13. A rod of length 1 m rotates in the xy plane about the fixed point O in the anticlockwise sense, as shown in figure with velocity $\omega = a + bt$ where $a = 10 \text{ rad s}^{-1}$ and $b = 5 \text{ rad s}^{-2}$. The velocity and acceleration of the point A at $t = 0$ is



(a) $+10\hat{i} \text{ ms}^{-1}$ and $+5\hat{i} \text{ ms}^{-2}$
(b) $+10\hat{j} \text{ ms}^{-1}$ and $(-100\hat{i} + 5\hat{j}) \text{ ms}^{-2}$
(c) $-10\hat{j} \text{ ms}^{-1}$ and $(100\hat{i} + 5\hat{j}) \text{ ms}^{-2}$
(d) $-10\hat{j} \text{ ms}^{-1}$ and $-5\hat{j} \text{ ms}^{-2}$

14. A ring of radius R rolls on a horizontal surface with constant acceleration a of the centre of mass as shown in figure. If ω is the instantaneous angular velocity of the ring, then the net acceleration of the point of contact of the ring with ground is

(a) zero
(b) $\omega^2 R$
(c) a
(d) $\sqrt{a^2 + (\omega^2 R)^2}$

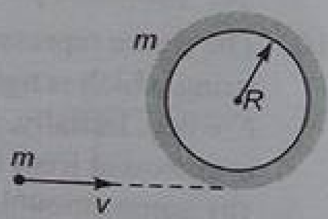
15. The density of a rod AB increases linearly from A to B. Its midpoint is O and its centre of mass is at C. Four axes pass through A, B, O and C, all perpendicular to the length of the rod. The moments of inertia of the rod about these axes are I_A, I_B, I_O and I_C respectively. Then

(a) $I_A > I_B$
(b) $I_C < I_B$
(c) $I_O > I_C$
(d) All of these

16. The figure shows a spool placed at rest on a horizontal rough surface. A tightly would string on the inner cylinder is pulled horizontally with a force F. Identity the correct alternative related to the friction force f acting on the spool

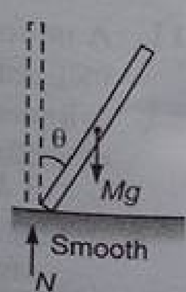
(a) f acts leftwards with $f < F$
(b) f acts leftwards but nothing can be said about its magnitude
(c) $f < F$ but nothing can be said about its magnitude
(d) None of the above

17. A circular ring of mass m and radius R rests flat on a horizontal smooth surface as shown in figure. A particle of mass m, and moving with a velocity v, collides inelastically $(e = 0)$ with the ring. The angular velocity with which the system rotates after the particle strikes the ring is



(a) $\dfrac{v}{2R}$
(b) $\dfrac{v}{3R}$
(c) $\dfrac{2v}{3R}$
(d) $\dfrac{3v}{4R}$

18. A stationary uniform rod in the upright position is allowed to fall on a smooth horizontal surface. The figure shows the instantaneous position of the rod. Identity the correct statement.

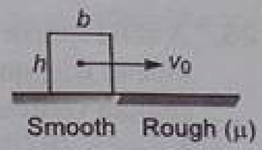


(a) normal reaction N is equal to Mg
(b) N does positive rotational work about the centre of mass
(c) a couple of equal and opposite forces acts on the rod
(d) All of the above

19. A thin uniform rod of mass m and length l is free to rotate about its upper end. When it is at rest. It receives an impulse J at its lowest point, normal to its length. Immediately after impact

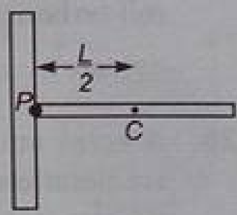
(a) the angular momentum of the rod is Jl
(b) the angular velocity of the rod is $3J/ml$
(c) the kinetic energy of the rod is $3J^2/2m$
(d) All of the above

20. A rectangular block of size $(b \times h)$ moving with velocity v_o enters on a rough surface where the coefficient of friction is μ as shown in figure. Identify the correct statement.



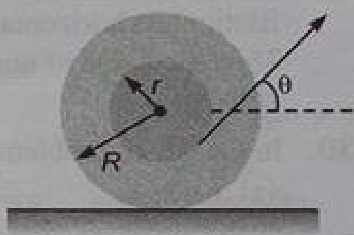
(a) The net torque acting on the block about its COM is $\mu\, mg\dfrac{h}{2}$ (clockwise)
(b) The net torque acting on the block about its COM is zero
(c) The net torque acting on the block about its COM is in the anticlockwise sense
(d) None of the above

21. A uniform rod of length L and mass m is free to rotate about a frictionless pivot at one end as shown in figure. The rod is held at rest in the horizontal position and a coin of mass m is placed at the free end. Now the rod is released. The reaction on the coin immediately after the rod starts falling is



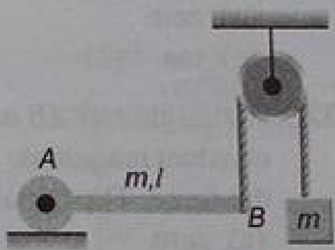
(a) $\dfrac{3mg}{2}$ (b) $2mg$
(c) zero (d) $\dfrac{mg}{2}$

22. A spool is pulled at an angle θ with the horizontal on a rough horizontal surface as shown in the figure. If the spool remains at rest, the angle θ is equal to



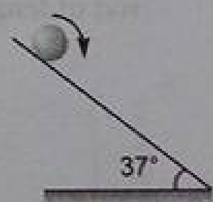
(a) $\cos^{-1}\left(\dfrac{R}{r}\right)$ (b) $\sin^{-1}\left(\sqrt{1-\dfrac{r^2}{R^2}}\right)$
(c) $\pi-\cos^{-1}\left(\dfrac{r}{R}\right)$ (d) $\sin^{-1}\left(\dfrac{r}{R}\right)$

23. Uniform rod AB is hinged at end A in horizontal position as shown in the figure. The other end is connected to a block through a massless string as shown. The pulley is smooth and massless. Mass of block and rod is same and is equal to m. Then acceleration of block just after release from this position is



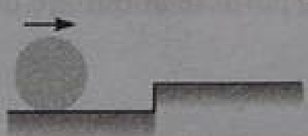
(a) $6g/13$ (b) $g/4$
(c) $3g/8$ (d) None of these

24. A cylinder having radius 0.4 m, initially rotating (at $t=0$) with $\omega_0 = 54$ rad/sec is placed on a rough inclined plane with $\theta = 37°$ having friction coefficient $\mu = 0.5$. The time taken by the cylinder to start pure rolling is ($g = 10\ \text{m/s}^2$)



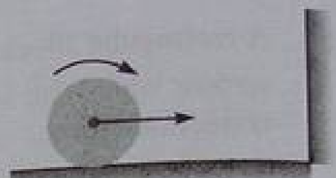
(a) 5.4 s (b) 1.2 s
(c) 1.4 s (d) 1.8 s

25. A disc of mass M and radius R is rolling purely with center's velocity v_0 on a flat horizontal floor when it hits a step in the floor of height $R/4$. The corner of the step is sufficiently rough to prevent any slipping of the disc against itself. What is the velocity of the centre of the disc just after impact?



(a) $4v_0/5$ (b) $4v_0/7$
(c) $5v_0/6$ (d) None of these

26. A solid sphere is rolling purely on a rough horizontal surface (coefficient of kinetic friction $=\mu$) with speed of centre $= u$. It collides inelastically with a smooth vertical wall at a certain moment, the coefficient of restitution being $\frac{1}{2}$. The sphere will begin pure rolling after a time



(a) $\frac{3u}{7\mu g}$ (b) $\frac{2u}{7\mu g}$ (c) $\frac{3u}{5\mu g}$ (d) $\frac{2u}{5\mu g}$

27. A thin hollow sphere of mass m is completely filled with non viscous liquid of mass m. When the sphere roll-on horizontal ground such that centre moves with velocity v, kinetic energy of the system is equal to

(a) mv^2 (b) $\frac{4}{3}mv^2$ (c) $\frac{4}{5}mv^2$ (d) None of these

28. A solid uniform disc of mass m rolls without slipping down a fixed inclined plank with an acceleration a. The frictional force on the disc due to surface of the plane is

(a) $\frac{1}{4}ma$ (b) $\frac{3}{2}ma$ (c) ma (d) $\frac{1}{2}ma$

29. A uniform slender rod of mass m and length L is released from rest, with its lower end touching a frictionless horizontal floor. At the initial moment, the rod is inclined at an angle $\theta = 30°$ with the vertical. Then the value of normal reaction from the floor just after release will be

(a) $4mg/7$ (b) $5mg/9$ (c) $2mg/5$ (d) None of these

30. In the above problem, the initial acceleration of the lower end of the rod will be

(a) $g\sqrt{3}/4$ (b) $g\sqrt{3}/5$ (c) $3g\sqrt{3}/7$ (d) None of these

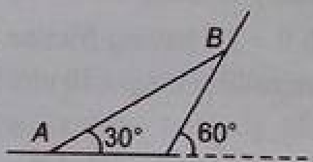
31. A disc of radius R is rolling purely on a flat horizontal surface, with a constant angular velocity. The angle between the velocity and acceleration vectors of point P is

(a) zero (b) 45°
(c) $\tan^{-1}(2)$ (d) $\tan^{-1}(1/2)$

32. A straight rod AB of mass M and length L is placed on a frictionless horizontal surface. A force having constant magnitude F and a fixed direction starts acting at the end A. The rod is initially perpendicular to the force. The initial acceleration of end B is

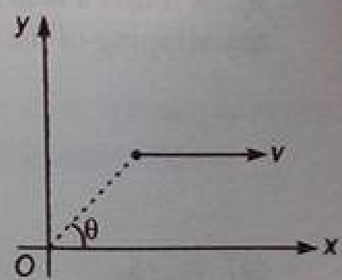
(a) zero (b) $2F/M$ (c) $4F/M$ (d) None of these

33. In the figure shown, the instantaneous speed of end A of the rod is v to the left. The angular velocity of the rod of length L, must be



(a) $v/2L$ (b) v/L (c) $\frac{\sqrt{3}v}{2L}$ (d) $\frac{2v}{L}$

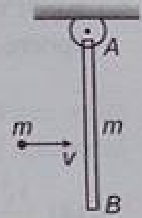
34. A particle moves parallel to x-axis with constant velocity v as shown in the figure. The angular velocity of the particle about the origin O



(a) remains constant
(b) continuously increases
(c) continuously decreases
(d) oscillates

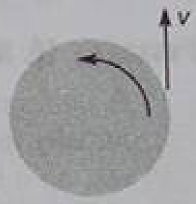
35. A thin uniform rod of mass M and length L is hinged at its upper end, and released from rest from a horizontal position. The tension at a point located at a distance $L/3$ from the hinge point, when the rod becomes vertical, will be
(a) $22\ Mg/27$ (b) $11\ Mg/13$ (c) $6\ Mg/11$ (d) $2\ Mg$

36. A uniform rod AB of length L and mass m is suspended freely at A and hangs vertically at rest when a particle of same mass m is fired horizontally with speed v to strike the rod at its mid point. If the particle is brought to rest after the impact. Then the impulsive reaction at A in horizontal direction is



(a) $mv/4$ (b) $mv/2$
(c) mv (d) $2\ mv$

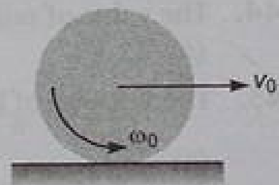
37. A child with mass m is standing at the edge of a merry go round having moment of inertia I, radius R and initial angular velocity ω as shown in the figure. The child jumps off the edge of the merry go round with tangential velocity v with respect to the ground. The new angular velocity of the merry go round is



(a) $\sqrt{\dfrac{I\omega^2 - mv^2}{I}}$ (b) $\sqrt{\dfrac{(I+mR^2)\omega^2 - mv^2}{I}}$

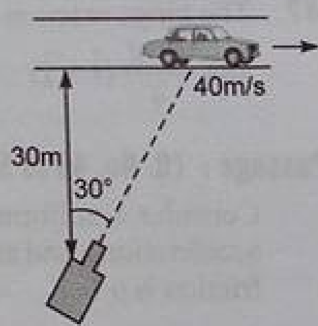
(c) $\dfrac{I\omega - mvR}{I}$ (d) $\dfrac{(I+mR^2)\omega - mvR}{I}$

38. A disc of radius R is spun to an angular speed ω_0 about its axis and then imparted a horizontal velocity of magnitude $\dfrac{\omega_0 R}{4}$. The coefficient of friction is μ. The sense of rotation and direction of linear velocity are shown in the figure. The disc will return to its initial position



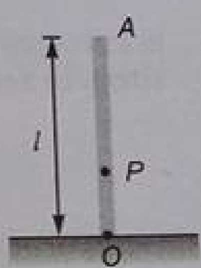
(a) if the value of $\mu < 0.5$ (b) irrespective of the value of μ
(c) if the value of $0.5 < \mu < 1$ (d) if $\mu > 1$

39. A racing car is travelling along a straight track at a constant velocity of 40 m/s. A fixed TV camera is recording the event as shown in figure. In order to keep the car in view, in the position shown, the angular velocity of camera should be



(a) 3 rad/s
(b) 2 rad/s
(c) 4 rad/s
(d) 1 rad/s

40. A uniform rod OA of length l, resting on smooth surface is slightly distributed from its vertical position. P is a point on the rod whose locus is a circle during the subsequent motion of the rod. Then the distance OP is equal to



(a) $l/2$
(b) $l/3$
(c) $l/4$
(d) there is no such point

41. In the above question, the velocity of end O when end A hits the ground is
(a) zero
(b) along the horizontal
(c) along the vertical
(d) at some inclination to the ground ($\neq 90°$)

42. In the above question, the velocity of end A at the instant it hits the ground is

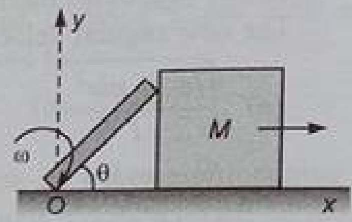
(a) $\sqrt{3gl}$ (b) $\sqrt{12gl}$ (c) $\sqrt{6gl}$ (d) None of these

43. A solid sphere of mass m and radius R is gently placed on a conveyer belt moving with constant velocity v_0. If coefficient of friction between belt and sphere is 2/7, the distance traveled by the centre of the sphere before it starts pure rolling is

(a) $\frac{v_0^2}{7g}$ (b) $\frac{2v_0^2}{49g}$ (c) $\frac{2v_0^2}{5g}$ (d) $\frac{2v_0^2}{7g}$

Passage : (Q. No. 44 to 47)

A uniform rod of mass m and length l is pivoted at point O. The rod is initially in vertical position and touching a block of mass M which is at rest on a horizontal surface. The rod is given a slight jerk and it starts rotating about point O. This causes the block to move forward as shown. The rod loses contact with the block at $\theta = 30°$. All surfaces are smooth. Now answer the following questions.



44. The value of ratio M/m is

(a) 2 : 3 (b) 3 : 2 (c) 4 : 3 (d) 3 : 4

45. The velocity of block when the rod loses contact with the block is

(a) $\frac{\sqrt{3gl}}{4}$ (b) $\frac{\sqrt{5gl}}{4}$ (c) $\frac{\sqrt{6gl}}{4}$ (d) $\frac{\sqrt{7gl}}{4}$

46. The acceleration of centre of mass of rod, when it loses contact with the block is

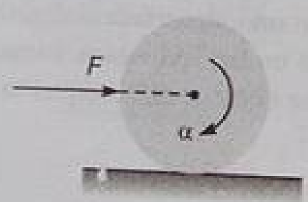
(a) $5g/4$ (b) $5g/2$ (c) $3g/2$ (d) $3g/4$

47. The hinge reaction at O on the rod when it loses contact with the block is

(a) $\frac{3mg}{4}(\hat{\mathbf{i}}+\hat{\mathbf{j}})$ (b) $\left(\frac{mg}{4}\right)\hat{\mathbf{j}}$ (c) $\left(\frac{mg}{4}\right)\hat{\mathbf{i}}$ (d) $\frac{mg}{4}(\hat{\mathbf{i}}+\hat{\mathbf{j}})$

Passage : (Q. No. 48 to 50)

Consider a uniform disc of mass m, radius r, rolling without slipping on a rough surface with linear acceleration a and angular acceleration α due to an external force F as shown in the figure. Coefficient of friction is μ



48. The work done by the frictional force at the instant of pure rolling is

(a) $\frac{\mu mgat^2}{2}$ (b) $\mu mgat^2$ (c) $\mu mg\frac{at^2}{\alpha}$ (d) zero

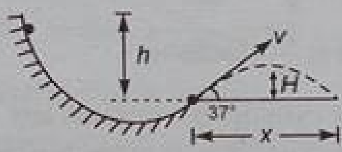
49. The magnitude of frictional force acting on the disc is

(a) ma (b) μmg (c) $\frac{ma}{2}$ (d) zero

50. Angular momentum of the disc will be conserved about
(a) centre of mass
(b) point of contact
(c) a point at a distance $3R/2$ vertically above the point of contact
(d) a point at a distance $4R/3$ vertically above the point of contact

Passage : (Q. No. 51 to 53)

A tennis ball, starting from rest, rolls down the hill in the drawing. At the end of the hill the ball becomes airborne, leaving at an angle of 37° with respect to the ground. Treat the ball as a thin-walled spherical shell.



51. The velocity of projection v is

(a) $\sqrt{2gh}$ (b) $\sqrt{\frac{10}{7}gh}$ (c) $\sqrt{\frac{5}{7}gh}$ (d) $\sqrt{\frac{6}{5}gh}$

52. Maximum height reached by ball H above ground is

(a) $\frac{9h}{35}$ (b) $\frac{18h}{35}$ (c) $\frac{18h}{25}$ (d) $\frac{27h}{125}$

53. Range x of the ball is

(a) $\frac{144}{125}h$ (b) $\frac{48}{25}h$ (c) $\frac{48}{35}h$ (d) $\frac{24}{7}h$

More than One Correct Options

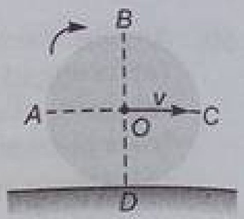
1. A mass m of radius r is rolling horizontally without any slip with a linear speed v. It then rolls up to a height given by $\frac{3}{4}\frac{v^2}{g}$

(a) the body is identified to be a disc or a solid cylinder
(b) the body is a solid sphere
(c) moment of inertia of the body about instantaneous axis of rotation is $\frac{3}{2}mr^2$
(d) moment of inertia of the body about instantaneous axis of rotation is $\frac{7}{5}mr^2$

2. Four identical rods each of mass m and length l are joined to form a rigid square frame. The frame lies in the xy plane, with its centre at the origin and the sides parallel to the x and y axes. Its moment of inertia about

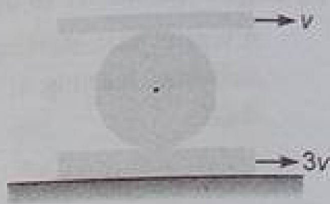
(a) the x-axis is $\frac{2}{3}ml^2$
(b) the z-axis is $\frac{4}{3}ml^2$
(c) an axis parallel to the z-axis and passing through a corner is $\frac{10}{3}ml^2$
(d) one side is $\frac{5}{3}ml^2$

3. A uniform circular ring rolls without slipping on a horizontal surface. At any instant, its position is as shown in the figure. Then



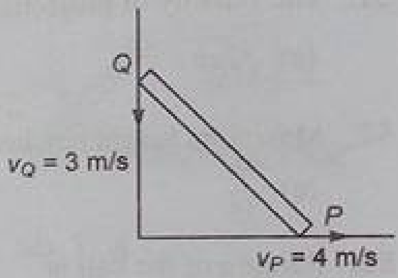
(a) section ABC has greater kinetic energy than section ADC
(b) section BC has greater kinetic energy than section CD
(c) section BC has the same kinetic energy as section DA
(d) the sections CD and DA have the same kinetic energy

4. A cylinder of radius R is to roll without slipping between two planks as shown in the figure. Then



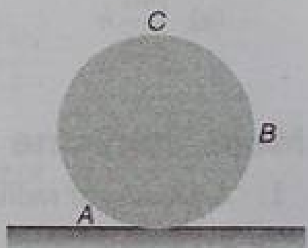
(a) angular velocity of the cylinder is $\frac{v}{R}$ counter clockwise
(b) angular velocity of the cylinder is $\frac{2v}{R}$ clockwise
(c) velocity of centre of mass of the cylinder is v towards left
(d) velocity of centre of mass of the cylinder is $2v$ towards right

5. A uniform rod of mass $m = 2$ kg and length $l = 0.5$ m is sliding along two mutually perpendicular smooth walls with the two ends P and Q having velocities $v_P = 4$ m/s and $v_Q = 3$ m/s as shown. Then



(a) The angular velocity of rod, $\omega = 10$ rad/s, counter clockwise
(b) The angular velocity of rod, $\omega = 5.0$ rad/s, counter clockwise
(c) The velocity of centre of mass of rod, $v_{cm} = 2.5$ m/s
(d) The total kinetic energy of rod, $K = \frac{25}{3}$ joule

6. A wheel is rolling without slipping on a horizontal plane with velocity v and acceleration a of centre of mass as shown in figure. Acceleration at



(a) A is vertically upwards
(b) B may be vertically downwards
(c) C cannot be horizontal
(d) a point on the rim may be horizontal leftwards

7. A uniform rod of length l and mass $2m$ rests on a smooth horizontal table. A point mass m moving horizontally at right angles to the rod with velocity v collides with one end of the rod and sticks it. Then

(a) angular velocity of the system after collision is $\frac{v}{l}$
(b) angular velocity of the system after collision is $\frac{v}{2l}$
(c) the loss in kinetic energy of the system as a whole as a result of the collision is $\frac{mv^2}{6}$
(d) the loss in kinetic energy of the system as a whole as a result of the collision is $\frac{7mv^2}{24}$

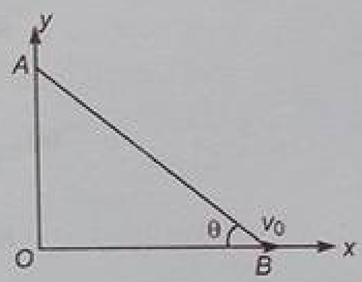
8. A nonuniform ball of radius R and radius of gyration about geometric centre $= R/2$, is kept on a frictionless surface. The geometric centre coincides with the centre of mass. The ball is struck horizontally with a sharp impulse $= J$. The point of application of the impulse is at a height h above the surface. Then

(a) the ball with slip on surface for all cases
(b) the ball will roll purely if $h = 5R/4$
(c) the ball will roll purely if $h = 3R/2$
(d) there will be no rotation if $h = R$

9. A hollow spherical ball is given an initial push up an incline of inclination angle α. The ball rolls purely. Coefficient of static friction between ball and incline $= \mu$. During its upwards journey
(a) friction acts up along the incline
(b) $\mu \geq 2 \tan \alpha / 5$
(c) friction acts down along the incline
(d) $\mu \geq 2 \tan \alpha / 7$

10. A uniform disc of mass m and radius R rotates about a fixed vertical axis passing through its centre with angular velocity ω. A particle of same mass m and having velocity of $2\omega R$ towards centre of the disc collides with the disc moving horizontally and sticks to its rim.
(a) The angular velocity of the disc will become $\omega/3$
(b) The angular velocity of the disc will become $5\omega/3$
(c) The impulse on the particle due to disc is $\frac{\sqrt{37}}{3} m\omega R$
(d) The impulse on the particle due to disc is $2m\omega R$

11. The end B of the rod AB which makes angle θ with the floor is being pulled with a constant velocity v_0 as shown. The length of the rod is l.



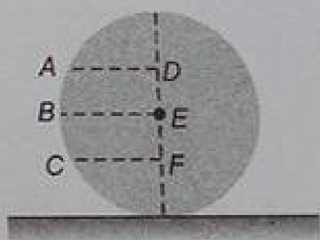
(a) At $\theta = 37°$ velocity of end A is $\frac{4}{3} v_0$ downwards
(b) At $\theta = 37°$ angular velocity of rod is $\frac{5v_0}{3l}$
(c) Angular velocity of rod is constant
(d) Velocity of end A is constant

Match the Columns

1. A solid sphere, a hollow sphere and a disc of same mass and same radius are released from a rough inclined plane. All of them rolls down without slipping. On reaching the bottom of the plane, match the two columns.

Column I	Column II
(a) time taken to reach the bottom	(p) maximum for solid sphere
(b) total kinetic energy	(q) maximum for hollow sphere
(c) rotational kinetic energy	(r) maximum for disc
(d) translational kinetic energy	(s) same for all

2. A solid sphere is placed on a rough ground as shown. E is the centre of sphere and $DE > EF$. We have to apply a linear impulse either at point A, B or C. Match the following two columns.

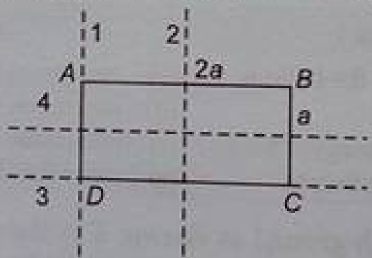


Column I	Column II
(a) Sphere will acquire maximum angular speed it impulse is applied at	(p) A
(b) Sphere will acquire maximum linear speed it impulse is applied at	(q) B
(c) Sphere can roll without slipping if impulse is applied at	(r) C
(d) Sphere can roll with forward slipping if impulse is applied at	(s) at any point A, B or C

3. The inclined surfaces shown in column I are sufficiently rough. In column II direction and magnitudes of frictional forces are mentioned. Match the two columns.

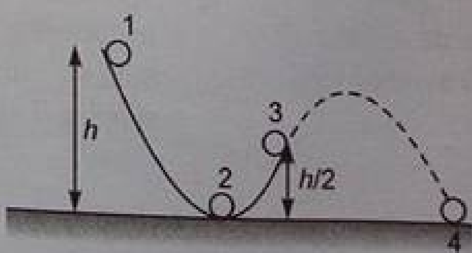
Column I	Column II
(a)  Rolling upwards	(p) upwards
(b)  Kept in rotating position	(q) downwards
(c)  Kept in translational position	(r) maximum friction will act
(d)  Kept in translational position	(s) required value of friction will act

4. A rectangular slab $ABCD$ have dimensions $a \times 2a$ as shown in figure. Match the following two columns.



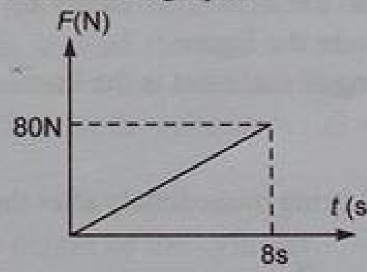
Column I	Column II
(a) Radius of gyration about axis-1	(p) $\frac{a}{\sqrt{12}}$
(b) Radius of gyration about axis-2	(q) $\frac{2a}{\sqrt{3}}$
(c) Radius of gyration about axis-3	(r) $\frac{a}{\sqrt{3}}$
(d) Radius of gyration about axis-4	(s) None

5. A small solid ball rolls down along sufficiently rough surface from 1 to 3 as shown in figure. From point-3 onwards it moves under gravity. Match the following two columns.



Column I	Column II
(a) Rotational kinetic energy of ball at point-2	(p) $\frac{1}{7}mgh$
(b) Translational kinetic energy of ball at point-3	(q) $\frac{2}{7}mgh$
(c) Rotational kinetic energy of ball at point-4	(r) $\frac{5}{7}mgh$
(d) Translational kinetic energy of ball at point-4	(s) None

6. A uniform disc of mass 10 kg, radius 1 m is placed on a rough horizontal surface. The co-efficient of friction between the disc and the surface is 0.2. A horizontal time varying force is applied on the centre of the disc whose variation with time is shown in graph.



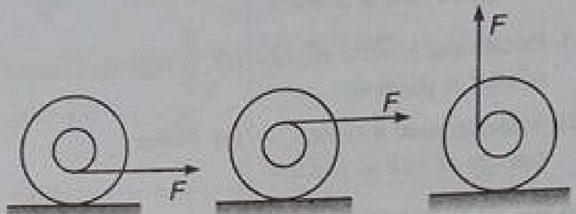
Column I	Column II
(a) Disc rolls without slipping	(p) at $t = 7$ s
(b) Disc rolls with slipping	(q) at $t = 3$ s
(c) Disc starts slipping at	(r) at $t = 4$ s
(d) Friction force is 10 N at	(s) None

7. Match the columns.

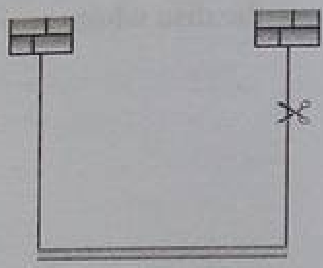
Column I	Column II
(a) Moment of inertia of a circular disc of mass M and radius R about a tangent parallel to plane of disc	(p) $\frac{MR^2}{2}$
(b) Moment of inertia of a solid sphere of mass M and radius R about a tangent	(q) $\frac{7}{5}MR^2$
(c) Moment of inertia of a circular disc of mass M and radius R about a tangent perpendicular to plane of disc	(r) $\frac{5}{4}MR^2$
(d) Moment of inertia of a cylinder of mass M and radius R about its axis	(s) $\frac{3}{2}MR^2$

Subjective Questions (Level 2)

1. Figure shows three identical yo-yos initially at rest on a horizontal surface. For each yo-yo the string is pulled in the direction shown. In each case there is sufficient friction for the yo-yo to roll without slipping. Draw the free-body diagram for each yo-yo. In what direction will each yo-yo rotate ?



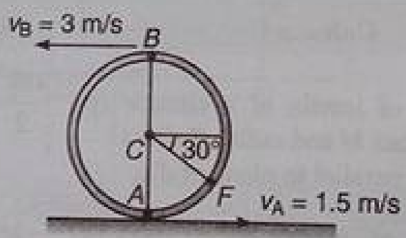
2. A uniform rod of mass m and length l is held horizontally by two vertical strings of negligible mass, as shown in the figure.
   (a) Immediately after the right string is cut, what is the linear acceleration of the free end of the rod ?
   (b) Of the middle of the rod ?
   (c) Determine the tension in the left string immediately after the right string is cut.



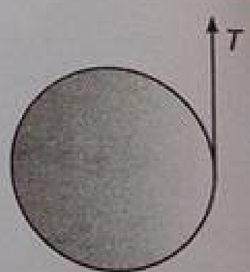
3. A solid disk is rolling without slipping on a level surface at a constant speed of 2.00 m/s. How far can it roll up a 30° ramp before it stops? (Take $g = 9.8\,\mathrm{m/s^2}$)

4. A lawn roller in the form of a thin-walled hollow cylinder of mass M is pulled horizontally with a constant horizontal force F applied by a handle attached to the axle. If it rolls without slipping, find the acceleration and the friction force.

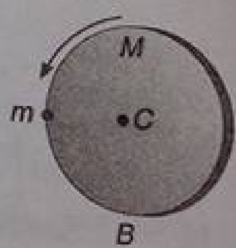
5. Due to slipping, points A and B on the rim of the disk have the velocities shown. Determine the velocities of the centre point C and point F at this instant.



6. A uniform cylinder of mass M and radius R has a string wrapped around it. The string is held fixed and the cylinder falls vertically, as in figure.
   (a) Show that the acceleration of the cylinder is downward with magnitude $a = \frac{2g}{3}$.
   (b) Find the tension in the string.

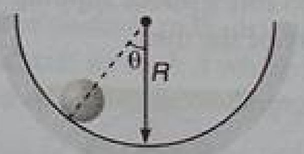


7. A uniform disc of mass M and radius R is pivoted about the horizontal axis through its centre C. A point mass m is glued to the disc at its rim, as shown in figure. If the system is released from rest, find the angular velocity of the disc when m reaches the bottom point B.

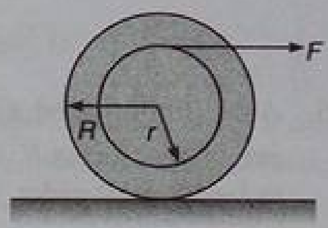


8. A disc of radius R and mass m is projected on to a horizontal floor with a backward spin such that its centre of mass speed is v_0 and angular velocity is ω_0. What must be the minimum value of ω_0 so that the disc eventually returns back?

9. A ball of mass m and radius r rolls along a circular path of radius R. Its speed at the bottom ($\theta = 0°$) of the path is v_0. Find the force of the path on the ball as a function of θ.

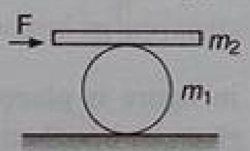


10. A heavy homogeneous cylinder has mass m and radius R. It is accelerated by a force F, which is applied through a rope wound around a light drum of radius r attached to the cylinder (figure). The coefficient of static friction is sufficient for the cylinder to roll without slipping.



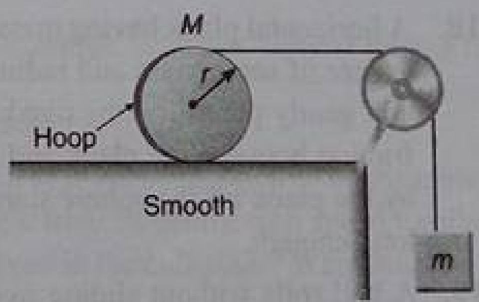
(a) Find the friction force.
(b) Find the acceleration a of the centre of the cylinder.
(c) Is it possible to choose r, so that a is greater than $\frac{F}{m}$? How ?
(d) What is the direction of the friction force in the circumstances of part (c) ?

11. A man pushes a cylinder of mass m_1 with the help of a plank of mass m_2 as shown. There is no slipping at any contact. The horizontal component of the force applied by the man is F. Find :



(a) the acceleration of the plank and the centre of mass of the cylinder and
(b) the magnitudes and directions of frictional forces at contact points.

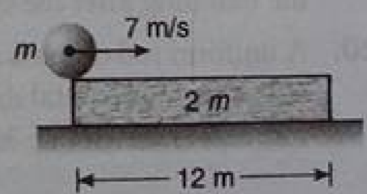
12. For the system shown in figure, $M = 1\,\text{kg}$, $m = 0.2\,\text{kg}$, $r = 0.2\,\text{m}$. Calculate : ($g = 10\,\text{m/s}^2$)



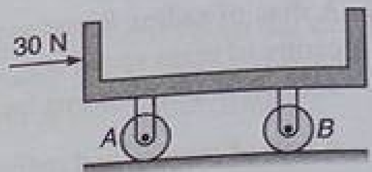
(a) the linear acceleration of hoop,
(b) the angular acceleration of the hoop of mass M and
(c) the tension in the rope.

Note *Treat hoop as the ring. Assume no slipping between string and hoop.*

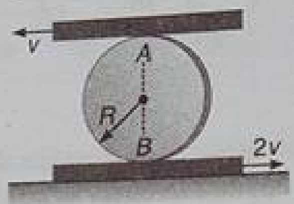
13. A cylinder of mass m is kept on the edge of a plank of mass $2m$ and length 12 m, which in turn is kept on smooth ground. Coefficient of friction between the plank and the cylinder is 0.1. The cylinder is given an impulse, which imparts it a velocity 7 m/s but no angular velocity. Find the time after which the cylinder falls off the plank. ($g = 10\,\text{m/s}^2$)



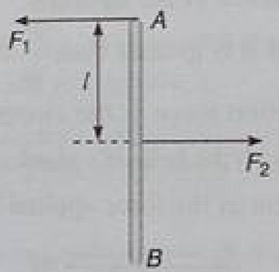
14. The 9 kg cradle is supported as shown by two uniform disks that roll without sliding at all surfaces of contact. The mass of each disk is $m = 6\,kg$ and the radius of each disk is $r = 80\,mm$. Knowing that the system is initially at rest, determine the velocity of the cradle after it has moved 250 mm.



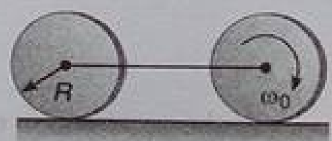
15. The disc of the radius R is confined to roll without slipping at A and B. If the plates have the velocities shown, determine the angular velocity of the disc.



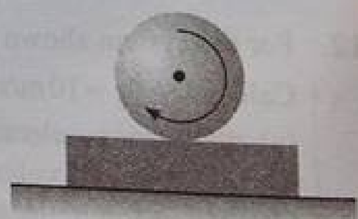
16. A thin uniform rod AB of mass $m = 1\,kg$ moves translationally with acceleration $a = 2\,m/s^2$ due to two antiparallel forces F_1 and F_2. The distance between the points at which these forces are applied is equal to $l = 20\,cm$. Besides, it is known that $F_2 = 5\,N$. Find the length of the rod.



17. The assembly of two discs as shown in figure is placed on a rough horizontal surface and the front disc is given an initial angular velocity ω_0. Determine the final linear and angular velocity when both the discs start rolling. It is given that friction is sufficient to sustain rolling in the rear wheel from the starting of motion.



18. A horizontal plank having mass m lies on a smooth horizontal surface. A sphere of same mass and radius r is spined to an angular frequency ω_0 and gently placed on the plank as shown in the figure. If coefficient of friction between the plank and the sphere is μ. Find the distance moved by the plank till the sphere starts pure rolling on the plank. The plank is long enough.



19. A ball rolls without sliding over a rough horizontal floor with velocity $v_0 = 7\,m/s$ towards a smooth vertical wall. If coefficient of restitution between the wall and the ball is $e = 07$. Calculate velocity v of the ball long after the collision.

20. A uniform rod of mass m and length l rests on a smooth horizontal surface. One of the ends of the rod is struck in a horizontal direction at right angles to the rod. As a result the rod obtains velocity v_0. Find the force with which one-half of the rod will act on the other in the process of motion.

21. A sphere, a disk and a hoop made of homogeneous materials have the same radius (10 cm) and mass (3 kg). They are released from rest at the top of a 30° incline and roll down without slipping through a vertical distance of 2 m. ($g = 9.8\,\text{m/s}^2$)

(a) What are their speeds at the bottom ?
(b) Find the frictional force f in each case
(c) If they start together at $t = 0$, at what time does each reach the bottom ?

22. ABC is a triangular framework of three uniform rods each of mass m and length $2l$. It is free to rotate in its own plane about a smooth horizontal axis through A which is perpendicular to ABC. If it is released from rest when AB is horizontal and C is above AB. Find the maximum velocity of C in the subsequent motion.

23. A uniform stick of length L and mass M hinged at one end is released from rest at an angle θ_0 with the vertical. Show that when the angle with the vertical is θ, the hinge exerts a force F_r along the stick and F_t perpendicular to the stick given by

$$F_r = \frac{1}{2} Mg\,(5\cos\theta - 3\cos\theta_0) \quad \text{and} \quad F_t = \frac{1}{4} Mg\sin\theta$$

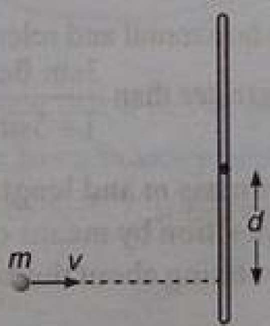
24. A uniform rod AB of mass $3m$ and length $4l$, which is free to turn in a vertical plane about a smooth horizontal axis through A, is released from rest when horizontal. When the rod first becomes vertical, a point C of the rod, where $AC = 3l$, strikes a fixed peg. Find the linear impulse exerted by the peg on the rod if :

(a) the rod is brought to rest by the peg,
(b) the rod rebounds and next comes to instantaneous rest inclined to the downward vertical at an angle $\frac{\pi}{3}$ radian.

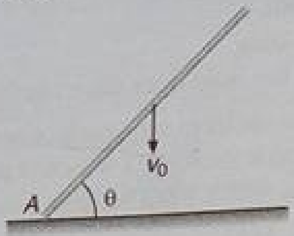
25. A uniform rod of length $4l$ and mass m is free to rotate about a horizontal axis passing through a point distant l from its one end. When the rod is horizontal, its angular velocity is ω as shown in figure. Calculate :

(a) reaction of axis at this instant,
(b) acceleration of centre of mass of the rod at this instant,
(c) reaction of axis and acceleration of centre mass of the rod when rod becomes vertical for the first time,
(d) minimum value of ω, so that centre of rod can complete circular motion.

26. A stick of length l lies on horizontal table. It has a mass M and is free to move in any way on the table. A ball of mass m, moving perpendicularly to the stick at a distance d from its centre with speed v collides elastically with it as shown in figure. What quantities are conserved in the collision ? What must be the mass of the ball, so that it remains at rest immediately after collision?



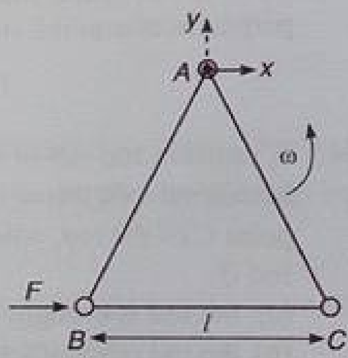
27. A rod of length l forming an angle θ with the horizontal strikes a frictionless floor at A with its centre of mass velocity v_0 and no angular velocity. Assuming that the impact at A is perfectly elastic. Find the angular velocity of the rod immediately after the impact.



28. Three particles A, B and C, each of mass m, are connected to each other by three massless rigid rods to form a rigid, equilateral triangular body of side l. This body is placed on a horizontal frictionless table (x-y plane) and is hinged to it at the point A, so that it can move without friction about the vertical axis through A (see figure). The body is set into rotational motion on the table about A with a constant angular velocity ω.

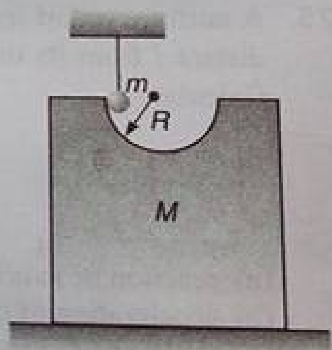
(a) Find the magnitude of the horizontal force exerted by the hinge on the body.

(b) At time T, when the side BC is parallel to the x-axis, a force F is applied on B along BC (as shown). Obtain the x-component and the y-component of the force exerted by the hinge on the body, immediately after time T.



29. A semicircular track of radius $R = 62.5\,\text{cm}$ is cut in a block. Mass of block, having track, is $M = 1$ kg and rests over a smooth horizontal floor. A cylinder of radius $r = 10\,\text{cm}$ and mass $m = 0.5\,\text{kg}$ is hanging by thread such that axes of cylinder and track are in same level and surface of cylinder is in contact with the track as shown in figure. When the thread is burnt, cylinder starts to move down the track. Sufficient friction exists between surface of cylinder and track, so that cylinder does not slip.

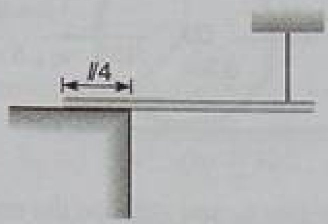
Calculate velocity of axis of cylinder and velocity of the block when it reaches bottom of the track. Also find force applied by block on the floor at that moment. ($g = 10\,\text{m/s}^2$)



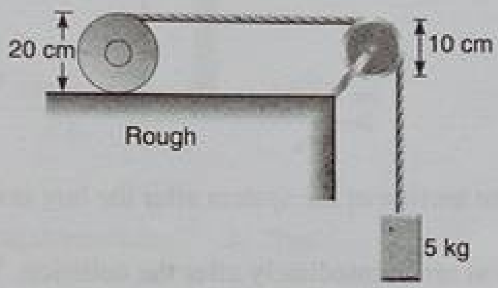
30. A uniform circular cylinder of mass m and radius r is given an initial angular velocity ω_0 and no initial translational velocity. It is placed in contact with a plane inclined at an angle α to the horizontal. If there is a coefficient of friction μ for sliding between the cylinder and plane. Find the distance the cylinder moves up before sliding stops. Also, calculate the maximum distance it travels up the plane. Assume $\mu > \tan\alpha$.

31. Show that if a rod held at angle θ to the horizontal and released, its lower end will not slip if the friction coefficient between rod and ground is greater than $\dfrac{3\sin\theta\cos\theta}{1+3\sin^2\theta}$.

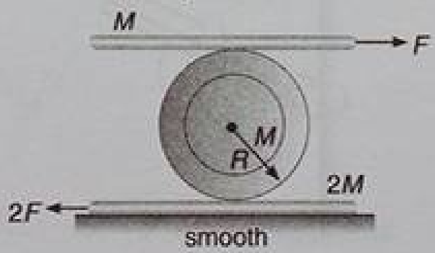
32. One-fourth length of a uniform rod of mass m and length l is placed on a rough horizontal surface and it is held stationary in horizontal position by means of a light thread as shown in the figure. The thread is then burnt and the rod starts rotating about the edge. Find the angle between the rod and the horizontal when it is about to slide on the edge. The coefficient of friction between the rod and the surface is μ.



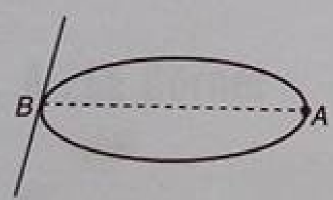
33. In figure the cylinder of mass 10 kg and radius 10 cm has a tape wrapped round it. The pulley weighs 100 N and has a radius 5 cm. When the system is released, the 5 kg mass comes down and the cylinder rolls without slipping. Calculate the acceleration and velocity of the mass as a function of time.



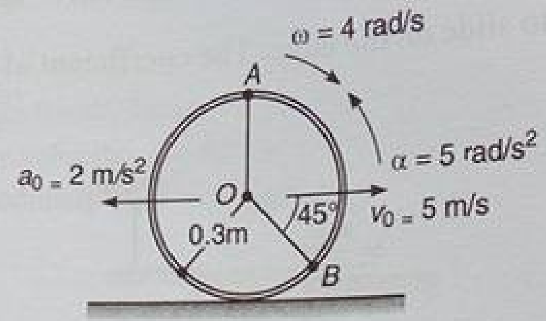
34. A cylinder is sandwiched between two planks. Two constant horizontal forces F and $2F$ are applied on the planks as shown. Determine the acceleration of the centre of mass of cylinder and the top plank, if there is no slipping at the top and bottom of cylinder.



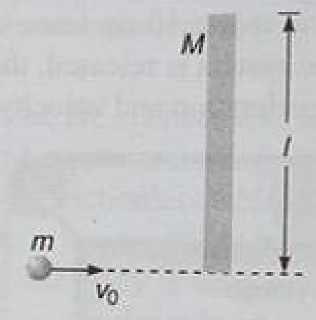
35. A ring of mass m and radius r has a particle of mass m attached to it at a point A. The ring can rotate about a smooth horizontal axis which is tangential to the ring at a point B diametrically opposite to A. The ring is released from rest when AB is horizontal. Find the angular velocity and the angular acceleration of the body when AB has turned through an angle $\frac{\pi}{3}$.



36. A hoop is placed on the rough surface such that it has an angular velocity $\omega = 4$ rad/s and an angular deceleration $\alpha = 5\text{ rad/s}^2$. Also, its centre has a velocity of $v_0 = 5$ m/s and a deceleration $a_0 = 2\text{ m/s}^2$. Determine the magnitude of acceleration of point B at this instant.



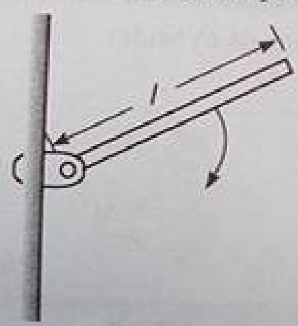
37. A boy of mass m runs on ice with velocity v_0 and steps on the end of a plank of length l and mass M whic
is perpendicular to his path.



(a) Describe quantitatively the motion of the system after the boy is on the plank. Neglect friction wit
the ice.

(b) One point on the plank is at rest immediately after the collision. Where is it?

38. A thin plank of mass M and length l is pivoted at one end. The plank is released at 60° from the vertical. What is the magnitude and direction of the force on the pivot when the plank is horizontal?



ANSWERS

Introductory Exercise 9.1

1. About a diagonal, because the mass is more concentrated about a diagonal 2. $\frac{\pi^2}{3}$ 3. $\frac{2}{\sqrt{3}} l$

4. (i) $\frac{8}{5} mr^2 + 2ma^2$ (ii) $\frac{8}{5} mr^2 + ma^2$ 5. (a) $2Ml^2$ (b) $\frac{1}{3} Ml^2$ 6. $\frac{2}{3} Ma^2$

7. 0.5 kg-m² 8. 0.43 kg-m² 9. $\frac{R}{\sqrt{2}}$ 10. The one having the smaller density

Introductory Exercise 9.2

1. 100 rad 2. 800 rad 3. 5 N-m 4. 0.87 N 5. (a) 4 rad-s^{-1}, – 6 rad-s^{-2} (b) – 12 rad-s^{-2}

6. 7 s

Introductory Exercise 9.3

2. $\frac{1}{2} mRv$ 3. $2\sqrt{2}\, mv$ 4. $\frac{mu^3 \cos\alpha \sin^2\alpha}{2g}$ 5. No

Introductory Exercise 9.4

1. $\frac{\omega_0 M}{M + 2m}$ 2. Duration of day-night incerease 3. True

Introductory Exercise 9.5

1. $y^2 = \left(\frac{2a_0}{\omega^2}\right) x$ 2. $\sqrt{\frac{3g}{l}(1 - \sin\theta)}$

Introductory Exercise 9.6

1. $\frac{2}{7} mgh$ 2. $\pm \cos^{-1}\left(\frac{v}{R\omega}\right)$ 3. $\frac{v_1 - v_2}{2R}$

Introductory Exercise 9.7

1. (a) $g \sin\theta - \mu g \cos\theta$ (b) $\frac{5}{2} \frac{\mu g \cos\theta}{R}$ 2. False 3. Leftwards 4. False 5. $\frac{I + 2Mr^2}{4Mr^2 - I}$

6. $\lim_{F \to 0}$ can make the body move 7. False

Introductory Exercise 9.8

1. (a) $\mu < 1$ (b) $\mu > 1$ 2. $\frac{2}{5} R$

AIEEE Corner

Subjective Questions (Level 1)

1. $\frac{5}{3} ml^2$ 2. 55 kg-m² 3. $\frac{l}{\sqrt{2}}$ 4. $\frac{Ma^2}{12}$ 5. 8 cm

6. $I = \mu r^2$, where $\mu = \frac{m_1 m_2}{m_1 + m_2}$ is called the reduced mass of two masses.

7. $I = \left(\frac{\alpha l^4}{4} + \frac{\beta l^3}{3}\right)$ 8. $\frac{\pi}{30}$ rad-s^{-1} 9. $(-\hat{k})$ rad-s^{-1} 10. 10 rad-s^{-1} 11. $\omega = \frac{v}{2R}$ 12. 2 rad/s

13. $(-2\hat{i} - 2\hat{k})$ N-m 14. 400 N-m (perpendicular to the plane of motion) 15. 2.71 N-m 16. $\frac{83}{20}$ N-m 17. (8π)rad-s^{-2}, (40π)rad-s^{-1} 18. 70 rad 19. (a) 0.01 N-m (b) 0.13 N-m 20. 20 s 21. $\frac{20}{3}$ N-m, 4 s 22. (a) $\frac{3\omega_0 R}{4\mu g}$ (b) $\frac{3\omega_0^2 R}{8\mu g}$ 23. (a) 36 s (b) $12\sqrt{\frac{2}{k}}$ 24. (a) $\omega = 12.5$ rad-s^{-1} (b) 127.5 rad 25. 9 rad, 1.43 26. $\omega_{av} = 4$ rad/s, $\alpha_{av} = -6.0$ rad-s^{-2} 27. $4\sqrt{2}$ kg-m^{2}s^{-1} 28. $ml^2\omega$ 29. $-\left(\frac{7}{5}mRv\right)\hat{k}$ 30. $-\left(\frac{10}{3}mRv\right)\hat{k}$ 31. Increase 32. $\frac{25}{6}$ rad-s^{-1} 33. 7.29×10^{-4} kg-m^2 34. (a) 14.3 rad-s^{-1} = 2.27 rev-s^{-1} (b) $E_i = 39.9$ J, $E_f = 181$ J (c) 141.1 J 35. (a) $\omega = \left(1 + \frac{2m}{M}\right)\omega_0$ (b) $\frac{1}{2}m\omega_0^2R^2\left(1 + \frac{2m}{M}\right)$ 36. $\frac{4F\cos\theta}{3M + 8m}, \frac{3MF\cos\theta}{3M + 8m}, \frac{MF\cos\theta}{3M + 8m}$ 37. $\frac{F}{M + 3m}$ 38. 72 N 39. (a) $\frac{2}{7\sqrt{3}}$ (b) $\frac{25}{7}$ ms^{-2} (c) $\frac{30}{7}$ ms^{-2} 40. $\frac{4}{3}ml\omega$ 41. (a) $\frac{2v}{9L}$ (b) $\frac{1}{9}$ 42. (a) $\frac{2u}{3}$ (b) $\frac{2u}{\sqrt{3}}$

Objective Questions (Level 1)

1. (d)	2. (a)	3. (c)	4. (d)	5. (b)	6. (a)	7. (c)	8. (b)	9. (b)	10. (b)
11. (d)	12. (a)	13. (b)	14. (b)	15. (b)	16. (d)	17. (a)	18. (b)	19. (b)	20. (d)
21. (c)	22. (d)	23. (b)	24. (a)	25. (a)	26. (b)	27. (c)	28. (b)	29. (a)	30. (d)

JEE Corner

Assertion and Reason

1. (d)	2. (b)	3. (d)	4. (a)	5. (a)	6. (a)	7. (c)	8. (b)	9. (b)	10. (a)
11. (c)									

Objective Questions (Level 2)

1. (d)	2. (b)	3. (b)	4. (d)	5. (c)	6. (a)	7. (a)	8. (b)	9. (c)	10. (c)
11. (b)	12. (c)	13. (b)	14. (b)	15. (d)	16. (a)	17. (b)	18. (b)	19. (d)	20. (b)
21. (c)	22. (b)	23. (c)	24. (d)	25. (c)	26. (a)	27. (b)	28. (d)	29. (a)	30. (c)
31. (b)	32. (b)	33. (b)	34. (c)	35. (d)	36. (a)	37. (d)	38. (b)	39. (d)	40. (c)
41. (a)	42. (a)	43. (a)	44. (c)	45. (a)	46. (d)	47. (b)	48. (d)	49. (c)	50. (c)
51. (d)	52. (d)	53. (a)							

More than One Correct Options

1. (a,c)	2. (all)	3. (a,b,d)	4. (a,b)	5. (a,c,d)	6. (all)	7. (a,c)
8. (b,d)	9. (a,b)	10. (a,c)	11. (a,b)			

Match the Columns

1. (a) → q	(b) → s	(c) → q	(d) → p
2. (a) → p	(b) → s	(c) → p	(d) → r
3. (a) → p,s	(b) → p,r	(c) → q,r	(d) → p,r
4. (a) → q	(b) → r	(c) → r	(d) → p,r
5. (a) → q	(b) → s	(c) → p	(d) → p
6. (a) → q,r	(b) → p	(c) → s	(d) → s
7. (a) → r	(b) → q	(c) → s	(d) → q

(d) → p

Subjective Questions (Level 2)

1. In each case in clockwise direction **2.** (a) $3g/2$, (b) $3g/4$ (c) $mg/4$ **3.** 0.612 m **4.** $\frac{F}{2M}$, $\frac{F}{2}$

5. 0.75 ms^{-1}, 1.98 ms^{-1} **6.** (b) $\frac{1}{3}Mg$ **7.** $\sqrt{\frac{4mg}{(2m+M)R}}$ **8.** $\frac{2v_0}{R}$

9. $f = \frac{2}{7} mg \sin\theta$, $N = \frac{mg}{7}(17\cos\theta - 10) + \frac{mv_0^2}{(R-r)}$

10. (a) $f = \frac{2}{3}\left(\frac{1}{2} - \frac{r}{R}\right)F$ assuming f opposite to F (b) $a = \left(\frac{2F}{3mR}\right)(R+r)$ (c) yes, if r is greater than $\frac{1}{2}R$.
(d) f in same direction as F.

11. (a) $\frac{8F}{3m_1 + 8m_2}$, $\frac{4F}{3m_1 + 8m_2}$
(b) $\frac{3m_1F}{3m_1 + 8m_2}$ (between plank and cylinder) $\frac{m_1F}{3m_1 + 8m_2}$ (between cylinder and ground)

12. (a) 1.43 ms^{-2} (b) 7.15 rad-s^{-2} (c) 1.43 N **13.** 2.25 s **14.** 0.745 ms^{-1} (rightwards)

15. $\frac{3}{2}\frac{v}{r}$ (anticlockwise) **16.** 1 m **17.** $\frac{\omega_0 R}{6}$; $\frac{\omega_0}{6}$ **18.** $S = \frac{2\omega_0^2 r^2}{81\mu g}$ **19.** $v = 1.5\ ms^{-1}$ **20.** $\frac{9}{2}\frac{mv_0^2}{l}$

21. (a) Sphere, 5.29 ms^{-1}, disk 5.11 ms^{-1}, hoop 4.43 ms^{-1} (b) Sphere 4.2 N, disk 4.9 N, hoop 7.36 N
(c) Sphere, 1.51 s disk 1.56 s hoop 1.81 s

22. $2l\sqrt{\frac{g\sqrt{3}}{l}}$ **24.** (a) $\left(\frac{8}{3}m\right)\sqrt{3gl}$, (b) $\frac{4}{3}m\sqrt{6gl}\,(\sqrt{2}+1)$

25. (a) $\frac{4}{7}mg\sqrt{1 + \left(\frac{7l\omega^2}{4g}\right)^2}$ (b) $\sqrt{\left(\frac{3g}{7}\right)^2 + (l\omega^2)^2}$ (c) $\left(\frac{13}{7}mg + ml\omega^2\right), \left(\frac{6g}{7} + l\omega^2\right)$ (d) $\sqrt{\frac{6g}{7l}}$

26. $\frac{Ml^2}{12d^2 + l^2}$ **27.** $\omega = \frac{6v_0}{l}\left(\frac{\cos\theta}{1 + 3\cos^2\theta}\right)$ **28.** (a) $\sqrt{3}\,ml\omega^2$ (b) $F_x = -\frac{F}{4}$, $F_y = \sqrt{3}\,ml\omega^2$

29. 2.0 ms^{-1}, 1.5 ms^{-1}, 16.67 N **30.** $d_1 = \frac{r^2\omega_0^2(\mu\cos\alpha - \sin\alpha)}{2g(3\mu\cos\alpha - \sin\alpha)^2}$, $d_{max} = \frac{r^2\omega_0^2(\mu\cos\alpha - \sin\alpha)}{4g\sin\alpha\,(3\mu\cos\alpha - \sin\alpha)}$

32. $\theta = \tan^{-1}\left(\frac{4\mu}{13}\right)$ **33.** 3.6 ms^{-2}, $\frac{4gt}{11}$ **34.** $\frac{F}{26M}$, $\frac{21F}{26M}$ **35.** $\sqrt{\frac{6g\sqrt{3}}{11r}}$, $\frac{3g}{11r}$ **36.** 6.21 ms^{-2}

37. (b) $\frac{2l}{3}$ from the boy **38.** $\frac{\sqrt{10}}{4}Mg$,  $\alpha = \tan^{-1}\left(\frac{1}{3}\right)$

10

Gravitation

Chapter Contents

Solved Examples

Level 1

Example 1 *If the radius of the earth contracts to half of its present value without change in its mass, what will be the new duration of the day?*

Solution Present angular momentum of earth

$$L_1 = I\omega = \frac{2}{5}MR^2\omega$$

New angular momentum because of change in radius

$$L_2 = \frac{2}{5}M\left(\frac{R}{2}\right)^2\omega'$$

If external torque is zero then angular momentum must be conserved

$$L_1 = L_2$$

$$\frac{2}{5}MR^2\omega = \frac{1}{4}\times\frac{2}{5}MR^2\omega' \quad i.e., \quad \omega' = 4\omega$$

$$T' = \frac{1}{4}T = \frac{1}{4}\times 24 = 6\,\text{h}$$

Example 2 *The minimum and maximum distances of a satellite from the centre of the earth are 2R and 4R respectively, where R is the radius of earth and M is the mass of the earth. Find :*

(a) its minimum and maximum speeds,

(b) radius of curvature at the point of minimum distance.

Solution (a) Applying conservation of angular momentum

$$mv_1(2R) = mv_2(4R)$$

$$v_1 = 2v_2 \quad \text{...(i)}$$

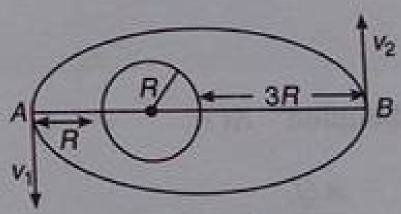


Fig. 10.38

From conservation of energy

$$\frac{1}{2}mv_1^2 - \frac{GMm}{2R} = \frac{1}{2}mv_2^2 - \frac{GMm}{4R} \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$v_2 = \sqrt{\frac{GM}{6R}}, \quad v_1 = \sqrt{\frac{2GM}{3R}}$$

(b) If r is the radius of curvature at point A

$$\frac{mv_1^2}{r} = \frac{GMm}{(2R)^2}$$

$$r = \frac{4v_1^2R^2}{GM} = \frac{8R}{3} \quad \text{(putting value of } v_1\text{)}$$

Example 3 *Three particles each of mass m are located at the vertices of an equilateral triangle of side a. At what speed must they move if they all revolve under the influence of their gravitational force of attraction in a circular orbit circumscribing the triangle while still preserving the equilateral triangle ?*

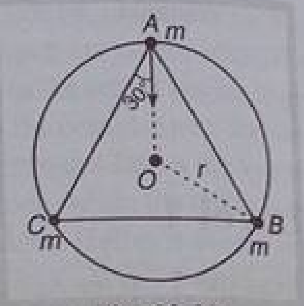


Fig. 10.39

Solution

$$\bar{F}_A = \bar{F}_{AB} + \bar{F}_{AC} = 2\left[\frac{GM^2}{a^2}\right]\cos 30° = \left[\frac{GM^2}{a^2}\cdot\sqrt{3}\right]$$

$$r = \frac{a}{\sqrt{3}}, \qquad \text{now } \frac{mv^2}{r} = F$$

or

$$\frac{mv^2\sqrt{3}}{a} = \frac{GM^2}{a^2}\sqrt{3}$$

$\therefore$

$$v = \sqrt{\frac{GM}{a}}$$

Example 4 *Two concentric shells of mass M_1 and M_2 are concentric as shown. Calculate the gravitational force on m due to M_1 and M_2 at points P, Q and R.*

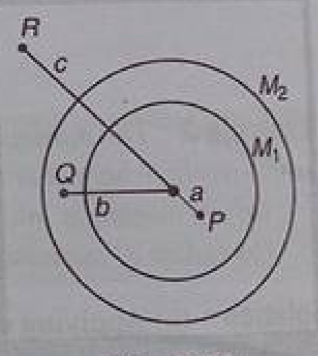


Fig. 10.40

Solution At P, $F = 0$

at Q,
$$F = \frac{GM_1 m}{b^2}$$

at R,
$$F = \frac{G(M_1 + M_2)m}{c^2}$$

Example 5 *What is the fractional decrease in the value of free-fall acceleration g for a particle when it is lifted from the surface to an elevation h ? ($h << R$)*

Solution

$$g = \frac{GM}{R^2}$$

$$\frac{dg}{dR} = \frac{-2GM}{R^3}$$

$$\Rightarrow \quad \frac{dg}{h} = \frac{-2GM}{R^2} \cdot \frac{1}{R}$$

$$\Rightarrow \quad \frac{dg}{g} = -2\left(\frac{h}{R}\right)$$

Example 6 *Three concentric shells of masses M_1, M_2 and M_3 having radii a, b and c respectively are situated as shown in figure. Find the force on a particle of mass m.*

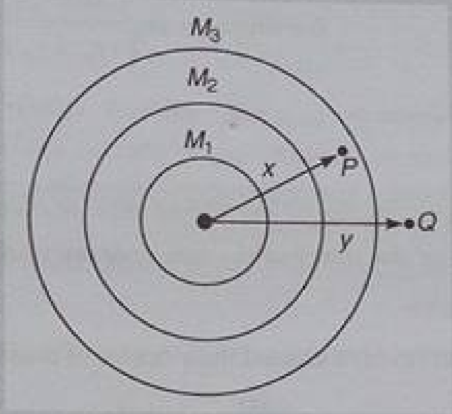


Fig. 10.41

(a) When the particle is located at Q.
(b) When the particle is located at P.

Solution Attraction at an external point due to spherical shell of mass M is $\left(\frac{GM}{r^2}\right)$ while at an internal point is zero.

(a) Point is external to shell M_1, M_2 and M_3.

So, force at Q will be

$$F_Q = \frac{GM_1 m}{y^2} + \frac{GM_2 m}{y^2} + \frac{GM_3 m}{y^2}$$

$$= \frac{Gm}{y^2}(M_1 + M_2 + M_3)$$

(b) Force at P will be

$$F_P = \frac{GM_1 m}{x^2} + \frac{GM_2 m}{x^2} + 0$$

$$= \frac{Gm}{x^2}(M_1 + M_2)$$

Example 7 *A planet of mass m revolves in elliptical orbit around the sun so that its maximum and minimum distances from the sun are equal to r_a and r_p respectively. Find the angular momentum of this planet relative to the sun.*

Solution Using conservation of angular momentum

$$mv_p r_p = mv_a r_a$$

As velocities are perpendicular to the radius vectors at apogee and perigee.

$$\Rightarrow \quad v_p r_p = v_a r_a$$

Using conservation of energy,

$$-\frac{GMm}{r_p}+\frac{1}{2}mv_p^2=\frac{-GMm}{r_a}+\frac{1}{2}mv_a^2$$

By solving, the above equations,

$$v_p=\sqrt{\frac{2GMr_a}{r_p(r_p+r_a)}}$$

$$L=mv_pr_p=m\sqrt{\frac{2GMr_pr_a}{(r_p+r_a)}}$$

Level 2

Example 1 *A planet of mass m_1 revolves round the sun of mass m_2. The distance between the sun and the planet is r. Considering the motion of the sun find the total energy of the system assuming the orbits to be circular.*

Solution Both the planet and the sun revolve around their centre of mass with same angular velocity (say ω)

$$r=r_1+r_2 \qquad \text{...(i)}$$

$$m_1r_1\omega^2=m_2r_2\omega^2=\frac{Gm_1m_2}{r^2} \qquad \text{...(ii)}$$

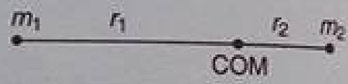


Fig. 10.42

Solving Eqs. (i) and (ii), we get

$$r_1=r\left(\frac{m_2}{m_1+m_2}\right)$$

$$r_2=r\left(\frac{m_1}{m_1+m_2}\right)$$

and

$$\omega^2=\frac{G(m_1+m_2)}{r^3}$$

Now, total energy of the system is

$$E=\text{PE}+\text{KE}$$

or

$$E=-\frac{Gm_1m_2}{r}+\frac{1}{2}m_1r_1^2\omega^2+\frac{1}{2}m_2r_2^2\omega^2$$

Substituting the values of r_1, r_2 and ω^2, we get

$$E=-\frac{Gm_1m_2}{2r}$$

Example 2 *Two masses m_1 and m_2 at an infinite distance from each other are initially at rest, start interacting gravitationally. Find their velocity of approach when they are at a distance r apart.*

Solution Let v_r be their velocity of approach. From conservation of energy:

Increase in kinetic energy = decrease in gravitational potential energy

$$\frac{1}{2}\mu v_r^2=\frac{Gm_1m_2}{r} \qquad \text{...(i)}$$

Here,

$$\mu = \text{reduced mass} = \frac{m_1 m_2}{m_1 + m_2}$$

Substituting in Eq. (i), we get

$$v_r = \sqrt{\frac{2G(m_1 + m_2)}{r}}$$

Example 3 *If a planet was suddenly stopped in its orbit supposed to be circular, show that it would fall onto the sun in a time $\frac{\sqrt{2}}{8}$ times the period of the planet's revolution.*

Solution Consider an imaginary planet moving along a strongly extended flat ellipse, the extreme points of which are located on the planet's orbit and at the centre of the sun. The semi-major axis of the orbit of such a planet would apparently be half the semimajor axis of the planet's orbit. So the time period of the imaginary planet T' according to Kepler's law will be given by :

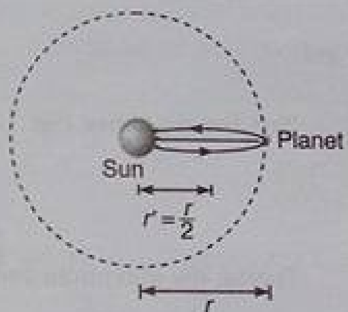


Fig. 10.43

$$\left(\frac{T'}{T}\right) = \left(\frac{r'}{r}\right)^{3/2}$$

or
$$T' = T\left(\frac{1}{2}\right)^{3/2} \qquad \left(\text{as} \quad r' = \frac{r}{2}\right)$$

$\therefore$ Time taken by the planet to fall onto the sun is

$$t = \frac{T'}{2} = \frac{T}{2}\left(\frac{1}{2}\right)^{3/2}$$

$$\Rightarrow \qquad t = \frac{\sqrt{2}}{8}\,T$$

Example 4 *A satellite is revolving round the earth in a circular orbit of radius r and velocity v_o. A particle is projected from the satellite in forward direction with relative velocity $v = (\sqrt{5/4} - 1)\,v_o$. Calculate its minimum and maximum distances from earth's centre during subsequent motion of the particle.*

Solution
$$v_o = \sqrt{\frac{GM}{r}} = \text{orbital speed of satellite} \qquad \text{...(i)}$$

where M = mass of earth

Absolute velocity of particle would be:

$$v_p = v + v_o = \sqrt{\frac{5}{4}}\,v_o = \sqrt{1.25}\,v_o \qquad \text{...(ii)}$$

Since, v_p lies between orbital velocity and escape velocity, path of the particle would be an ellipse with r being the minimum distance.

Let r' be the maximum distance and v'_p its velocity at that moment.

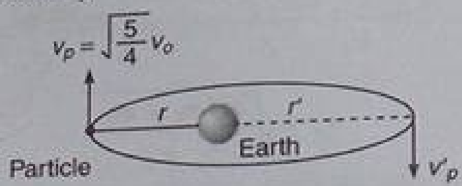


Fig. 10.44

Then from conservation of angular momentum and conservation of mechanical energy, we get

$$mv_p r = mv'_p r' \qquad \ldots\text{(iii)}$$

and
$$\frac{1}{2}mv_p^2 - \frac{GMm}{r} = \frac{1}{2}mv'^2_p - \frac{GMm}{r'} \qquad \ldots\text{(iv)}$$

Solving the above Eqs. (i), (ii), (iii) and (iv), we get

$$r' = \frac{5r}{3} \quad \text{and} \quad r$$

Hence, the maximum and minimum distance are $\frac{5r}{3}$ and r respectively.

Example 5 *An earth satellite is revolving in a circular orbit of radius a with velocity v_o. A gun is in the satellite and is aimed directly towards the earth. A bullet is fired from the gun with muzzle velocity $\frac{v_o}{2}$. Neglecting resistance offered by cosmic dust and recoil of gun, calculate maximum and minimum distance of bullet from the centre of earth during its subsequent motion.*

Solution Orbital speed of satellite is

$$v_o = \sqrt{\frac{GM}{a}} \qquad \ldots\text{(i)}$$

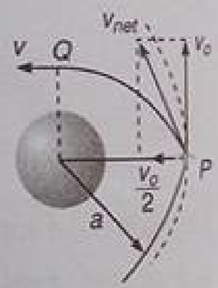


Fig. 10.45

From conservation of angular momentum at P and Q, we have

$$mav_o = mvr$$

or
$$v = \frac{av_o}{r} \qquad \ldots\text{(ii)}$$

From conservation of mechanical energy at P and Q, we have

$$\frac{1}{2}m\left(v_o^2 + \frac{v_o^2}{4}\right) - \frac{GMm}{a} = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

or
$$\frac{5}{8}v_o^2 - \frac{GM}{a} = \frac{v^2}{2} - \frac{GM}{r}$$

Substituting values of v and v_o from Eqs. (i) and (ii), we get

$$\frac{5}{8}\frac{GM}{a} - \frac{GM}{a} = \frac{a^2}{r^2}\cdot\left(\frac{GM}{2a}\right) - \frac{GM}{r}$$

or
$$-\frac{3}{8a} = \frac{a}{2r^2} - \frac{1}{r} \quad \text{or} \quad -3r^2 = 4a^2 - 8ar$$

or
$$3r^2 - 8ar + 4a^2 = 0$$

or
$$r = \frac{8a \pm \sqrt{64a^2 - 48a^2}}{6}$$

or
$$r = \frac{8a \pm 4a}{6} \quad \text{or} \quad r = 2a \quad \text{and} \quad \frac{2a}{3}$$

Hence, the maximum and minimum distances are $2a$ and $\frac{2a}{3}$ respectively.

Example 6 *Binary stars of comparable masses m_1 and m_2 rotate under the influence of each other's gravity with a time period T. If they are stopped suddenly in their motions, find their relative velocity when they collide with each other. The radii of the stars are R_1 and R_2 respectively. G is the universal constant of gravitation.*

Solution Both the stars rotate about their centre of mass (COM).

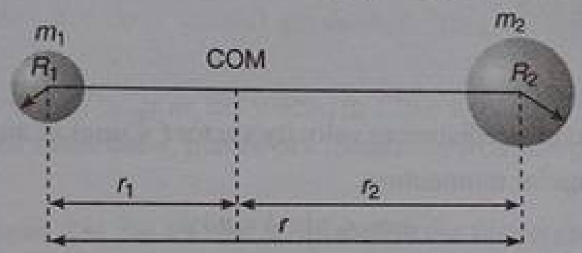


Fig. 10.46

For the position of COM
$$\frac{r_1}{m_2} = \frac{r_2}{m_1} = \frac{r_1 + r_2}{m_1 + m_2} = \frac{r}{m_1 + m_2} \qquad (r = r_1 + r_2)$$

Also,
$$m_1 r_1 \omega^2 = \frac{Gm_1 m_2}{r^2} \quad \text{or} \quad \omega^2 = \frac{Gm_2}{r_1 r^2} \qquad \left(\omega = \frac{2\pi}{T}\right)$$

But,
$$r_1 = \frac{m_2 r}{m_1 + m_2}$$

$\therefore$
$$\omega^2 = \frac{G(m_1 + m_2)}{r^3}$$

or
$$r = \left\{\frac{G(m_1 + m_2)}{\omega^2}\right\}^{1/3} \qquad \text{...(i)}$$

Applying conservation of mechanical energy we have
$$-\frac{Gm_1 m_2}{r} = -\frac{Gm_1 m_2}{(R_1 + R_2)} + \frac{1}{2}\mu v_r^2 \qquad \text{...(ii)}$$

Here,
$$\mu = \text{reduced mass} = \frac{m_1 m_2}{m_1 + m_2}$$

and v_r = relative velocity between the two stars.

From Eq. (ii), we find that
$$v_r^2 = \frac{2Gm_1 m_2}{\mu}\left(\frac{1}{R_1 + R_2} - \frac{1}{r}\right)$$

$$= \frac{2Gm_1 m_2}{\dfrac{m_1 m_2}{m_1 + m_2}} \left(\frac{1}{R_1 + R_2} - \frac{1}{r} \right)$$

$$= 2G(m_1 + m_2) \left(\frac{1}{R_1 + R_2} - \frac{1}{r} \right)$$

Substituting the value of r from Eq. (i), we get

$$v_r = \sqrt{2G(m_1 + m_2) \left[\frac{1}{R_1 + R_2} - \left\{ \frac{4\pi^2}{G(m_1 + m_2)T^2} \right\}^{1/3} \right]}$$

Example 7 *Find the maximum and minimum distances of the planet A from the sun S, if at a certain moment of time it was at a distance r_0 and travelling with the velocity v_0, with the angle between the radius vector and velocity vector being equal to ϕ.*

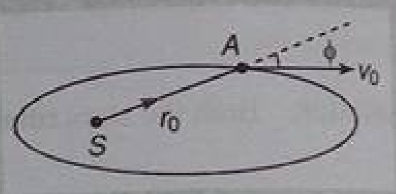


Fig. 10.47

Solution At minimum and maximum distances velocity vector ($\vec{v}$) makes an angle of 90° with radius vector. Hence, from conservation of angular momentum,

$$mv_0 r_0 \sin \phi = mrv \quad \text{...(i)}$$

Here, m is the mass of the planet.
From energy conservation law it follows that.

$$\frac{mv_0^2}{2} - \frac{GMm}{r_0} = \frac{mv^2}{2} - \frac{GMm}{r} \quad \text{...(ii)}$$

Here, M is the mass of the sun.
Solving Eqs. (i) and (ii) for r, we get two values of r, one is $r_{\max}$ and another is $r_{\min}$. So,

$$r_{\max} = \frac{r_0}{2-K} \left(1 + \sqrt{1 - K(2-K)\sin^2 \phi}\right)$$

and

$$r_{\min} = \frac{r_0}{2-K} \left(1 - \sqrt{1 - K(2-K)\sin^2 \phi}\right)$$

Here,

$$K = \frac{r_0^2 v_0^2}{GM}$$

EXERCISES

AIEEE Corner

Subjective Questions (Level 1)

Newton's Law of Gravitation

1. Two particles of masses 1.0 kg and 2.0 kg are placed at a separation of 50 cm. Assuming that the only forces acting on the particles are their mutual gravitation, find the initial accelerations of the two particles.
2. Three particles A, B and C, each of mass m, are placed in a line with $AB = BC = d$. Find the gravitational force on a fourth particle P of same mass, placed at a distance d from the particle B on the perpendicular bisector of the line AC.
3. Four particles having masses m, $2m$, $3m$ and $4m$ are placed at the four corners of a square of edge a. Find the gravitational force acting on a particle of mass m placed at the centre.
4. Three uniform spheres each having a mass M and radius a are kept in such a way that each touches the other two. Find the magnitude of the gravitational force on any of the spheres due to the other two.
5. The figure shows a uniform rod of length l whose mass per unit length is λ. What is the gravitational force of the rod on a particle of mass m located a distance d from one end of the rod?

Acceleration due to Gravity

6. Value of g on the surface of earth is 9.8 m/s^2. Find its value on the surface of a planet whose mass and radius both are two times that of earth.
7. Value of g on the surface of earth is 9.8 m/s^2. Find its value :
   (a) at height $h = R$ from the surface,
   (b) at depth $h = \dfrac{R}{2}$ from the surface. (R = radius of earth)
8. Calculate the distance from the surface of the earth at which the acceleration due to gravity is the same below and above the surface of the earth.
9. A body is weighed by a spring balance to be 1000 N at the north pole. How much will it weight at the equator. Account for the earth's rotation only.
10. At what rate should the earth rotate so that the apparent g at the equator becomes zero ? What will be the length of the day in this situation ?
11. Assuming earth to be spherical, at what height above the north pole, value of g is same as that on the earth's surface at equator ?

Gravitational Field Strength and Potential

12. Two concentric spherical shells have masses m_1, m_2 and radii $R_1, R_2 (R_1 < R_2)$. Calculate the force exerted by this system on a particle of mass m, if it is placed at a distance $\frac{(R_1 + R_2)}{2}$ from the centre.

13. Two spheres one of mass M has radius R. Another sphere has mass $4M$ and radius $2R$. The centre to centre distance between them is $12R$. Find the distance from the centre of smaller sphere where :
(a) net gravitational field is zero,
(b) net gravitational potential is half the potential on the surface of larger sphere.

14. A semicircular wire has a length L and mass M. Find the gravitational field at the centre of the circle.

15. A uniform solid sphere of mass M and radius a is surrounded symmetrically by a uniform thin spherical shell of equal mass and radius $2a$. Find the gravitational field at a distance (a) $\frac{3}{2}a$ from the centre, (b) $\frac{5}{2}a$ from the centre.

16. The density inside a solid sphere of radius a is given by $\rho = \rho_0 a/r$, where ρ_0 is the density at the surface and r denotes the distance from the centre. Find the gravitational field due to this sphere at a distance $2a$ from its centre.

17. A particle of mass m is placed at the centre of a uniform spherical shell of same mass and radius R. Find the gravitational potential at a distance $\frac{R}{2}$ from the centre.

Gravitational Potential Energy

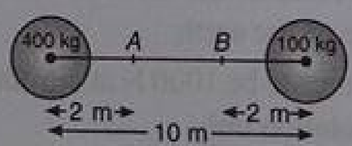
18. A rocket is accelerated to speed $v = 2\sqrt{gR}$ near earth's surface (R = radius of earth). Show that very far from earth its speed will be $v = \sqrt{2gR}$.

19. Two neutron stars are separated by a distance of 10^{10} m. They each have a mass of 10^{30} kg and a radius of 10^5 m. They are initially at rest with respect to each other.
As measured from the rest frame, how fast are they moving when :
(a) their separation has decreased to one-half its initial value,
(b) they are about to collide.

20. A projectile is fired vertically from earth's surface with an initial speed of 10 km/s. Neglecting air drag, how far above the surface of earth will it go ?

21. A mass m is taken to a height R from the surface of the earth and then is given a vertical velocity v. Find the minimum value of v, so that mass never returns to the surface of the earth.
(Radius of earth is R and mass of the earth M).

22. In the figure masses 400 kg and 100 kg are fixed.



(a) How much work must be done to move a 1 kg mass from point A to point B ?
(b) What is the minimum kinetic energy with which the 1 kg mass must be projected from A to the right to reach the point B ?

Planets and Satellites : Kepler's Law

23. A sky lab of mass 2×10^3 kg is first launched from the surface of earth in a circular orbit of radius $2R$ and then it is shifted from this circular orbit to another circular orbit of radius $3R$. Calculate the energy required :
    (a) to place the lab in the first orbit,
    (b) to shift the lab from first orbit to the second orbit. $(R = 6400\,\text{km}, g = 10\,\text{m/s}^2)$

24. Two identical stars of mass M orbit around their centre of mass. Each orbit is circular and has radius R, so that the two stars are always on opposite sides of the circle.
    (a) Find the gravitational force of one star on the other.
    (b) Find the orbital speed of each star and the period of the orbit.
    (c) What minimum energy would be required to separate the two stars to infinity ?

25. Consider two satellites A and B of equal mass, moving in the same circular orbit of radius r around the earth but in the opposite sense and therefore a collision occurs.
    (a) Find the total mechanical energy $E_A + E_B$ of the two satellite-plus-earth system before collision.
    (b) If the collision is completely inelastic, find the total mechanical energy immediately after collision. Describe the subsequent motion of the combined satellite.

26. Two satellites A and B revolve around a planet in two coplanar circular orbits in the same sense with radii 10^4 km and 2×10^4 km respectively. Time period of A is 28 hours. What is time period of another satellite. Find the speed of B with respect to A when A and B are at farthest distance from each other.

27. A satellite of mass 1000 kg is supposed to orbit the earth at a height of 2000 km above the earth's surface. Find (a) its speed in the orbit, (b) its kinetic energy, (c) the potential energy of the earth-satellite system and (d) its time period. Mass of the earth $= 6\times 10^{24}$ kg.

28. In a certain binary star system, each star has the same mass as our sun. They revolve about their centre of mass. The distance between them is the same as the distance between earth and the sun. What is their period of revolution in years ?

29. (a) Does it take more energy to get a satellite upto 1500 km above earth than to put it in circular orbit once it is there.
    (b) What about 3185 km?
    (c) What about 4500 km? (Take $R_e = 6370\,\text{km}$)

Objective Questions (Level 1)

Single Correct Option

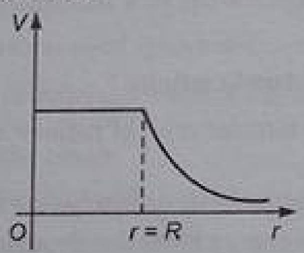
1. A satellite orbiting close to the surface of earth does not fall down because the gravitational pull of earth
   (a) is balanced by the gravitational pull of moon
   (b) is balanced by the gravitational pull of sun
   (c) provides the necessary acceleration for its motion along the circular path
   (d) makes it weightless

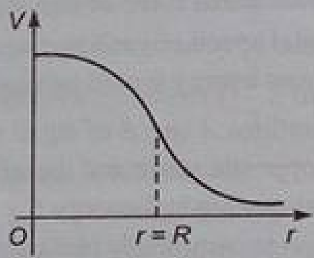
2. For the planet-sun system identify the correct statement
   (a) the angular momentum of the planet is conserved
   (b) the total energy of the system is conserved
   (c) the momentum of the planet is conserved
   (d) All of the above

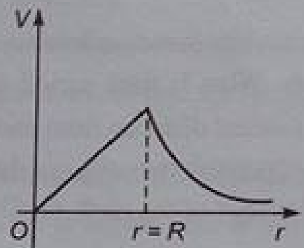
3. If the earth stops rotating about its axis, then the magnitude of gravity
   (a) increases everywhere on the surface of earth
   (b) will increase only at the poles
   (c) will not change at the poles
   (d) All of the above

4. For a body to escape from earth, angle from horizontal at which it should be fired is
   (a) 45° (b) 0° (c) 90° (d) any angle

5. The correct variation of gravitational potential V with radius r measured from the centre of earth of radius R is given by

(a) 

(b) 

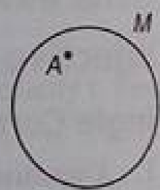
(c) 

(d) None of the above

6. The Gauss' theorem for gravitational field may be written as
   (a) $\oint \vec{g}\cdot d\vec{S} = \frac{m}{G}$ (b) $-\oint \vec{g}\cdot d\vec{S} = 4\pi mG$ (c) $\oint \vec{g}\cdot d\vec{S} = \frac{m}{4\pi G}$ (d) $-\oint \vec{g}\cdot d\vec{S} = \frac{m}{G}$

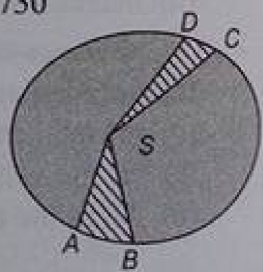
7. In the earth-moon system, if T_1 and T_2 are period of revolution of earth and moon respectively about the centre of mass of the system then
   (a) $T_1 > T_2$ (b) $T_1 = T_2$ (c) $T_1 < T_2$ (d) Insufficient data

8. The figure shows a spherical shell of mass M. The point A is not at the centre but away from the centre of the shell. If a particle of mass m is placed at A, then
   (a) it remains at rest
   (b) it experiences a net force towards the centre
   (c) it experiences a net force away from the centre
   (d) None of the above



9. If the distance between the earth and the sun were reduced to half its present value, then the number of days in one year would have been
   (a) 65 (b) 129 (c) 183 (d) 730

10. The figure represents an elliptical orbit of a planet around sun. The planet takes time T_1 to travel from A to B and it takes time T_2 to travel from C to D. If the area CSD is double that of area ASB, then
   (a) $T_1 = T_2$ (b) $T_1 = 2T_2$
   (c) $T_1 = 0.5T_2$ (d) Data insufficient



11. At what depth from the surface of earth the time period of a simple pendulum is 0.5% more than that on the surface of the Earth? (Radius of earth is 6400 km)
(a) 32 km (b) 64 km (c) 96 km (d) 128 km

12. If M is the mass of the earth and R its radius, the ratio of the gravitational acceleration and the gravitational constant is
(a) $\frac{R^2}{M}$ (b) $\frac{M}{R^2}$ (c) MR^2 (d) $\frac{M}{R}$

13. The height above the surface of earth at which the gravitational field intensity is reduced to 1% of its value on the surface of earth is
(a) $100R_e$ (b) $10R_e$ (c) $99R_e$ (d) $9R_e$

14. For a satellite orbiting close to the surface of earth the period of revolution is 84 minute. The time period of another satellite orbiting at a height three times the radius of earth from its surface will be
(a) $(84)2\sqrt{2}$ minute (b) (84) 8 minute (c) $(84)3\sqrt{3}$ minute (d) (84) 3 minute

15. The angular speed of rotation of earth about its axis at which the weight of man standing on the equator becomes half of its weight at the poles is given by
(a) 0.034 rad s^{-1} (b) 8.75×10^{-4} rad s^{-1}
(c) 1.23×10^{-2} rad s^{-1} (d) 7.65×10^{-7} rad s^{-1}

16. The height from the surface of earth at which the gravitational potential energy of a ball of mass m is half of that at the centre of earth is (where R is the radius of earth)
(a) $\frac{R}{4}$ (b) $\frac{R}{3}$ (c) $\frac{3R}{4}$ (d) $\frac{4R}{3}$

17. A body of mass m is lifted up from the surface of earth to a height three times the radius of the earth R. The change in potential energy of the body is
(a) $3mgR$ (b) $\frac{5}{4}mgR$ (c) $\frac{3}{4}mgR$ (d) $2mgR$

18. A satellite is revolving around earth in its equatorial plane with a period T. If the radius of earth suddenly shrinks to half its radius without change in the mass. Then, the new period of revolution will be
(a) $8T$ (b) $2\sqrt{2}\,T$ (c) $2T$ (d) T

19. If the radius of moon is 1.7×10^6 m and its mass is 7.34×10^{22} kg. Then its escape velocity is
(a) 2.4×10^3 ms^{-1} (b) 2.4×10^2 ms^{-1} (c) 3.4×10^3 ms^{-1} (d) 3.4×10^2 ms^{-1}

20. A planet has twice the density of earth but the acceleration due to gravity on its surface is exactly the same as on the surface of earth. Its radius in terms of earth's radius R will be
(a) $R/4$ (b) $R/2$ (c) $R/3$ (d) $R/8$

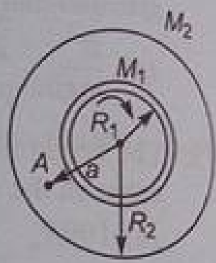
21. The speed of earth's rotation about its axis is ω. Its speed is increased to x times to make the effective acceleration due to gravity equal to zero at the equator, then x is around ($g = 10\,ms^{-2}$; $R = 6400$ km)
(a) 1 (b) 8.5 (c) 17 (d) 34

22. A satellite is seen every 6 hours over the equator. It is known that it rotates opposite to that of earth's direction. Then the angular velocity (in radian per hour) of satellite about the centre of earth will be
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{8}$

23. For a planet revolving around sun, if a and b are the respective semi-major and semi-minor axes, then the square of its time period is proportional to

(a) $\left(\frac{a+b}{2}\right)^3$ (b) $\left(\frac{a-b}{2}\right)^3$ (c) b^3 (d) a^3

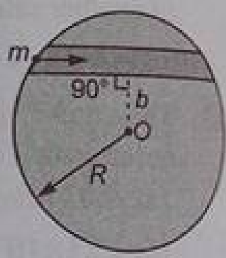
24. The figure represents two concentric shells of radii R_1 and R_2 and masses M_1 and M_2 respectively. The gravitational field intensity at the point A at distance a $(R_1 < a < R_2)$ is



(a) $\frac{G(M_1+M_2)}{a^2}$ (b) $\frac{GM_1}{a^2}+\frac{GM_2}{R_2^2}$

(c) $\frac{GM_1}{a^2}$ (d) Zero

25. A straight tunnel is dug into the earth as shown in figure at a distance b from its centre. A ball of mass m is dopped from one of its ends. The time it takes to reach the other end is approximately



(a) 42 min (b) 84 min

(c) $84\left(\frac{b}{R}\right)$ min (d) $42\left(\frac{b}{R}\right)$ min

26. Three identical particles each of mass m are placed at the corners of an equilateral triangle of side l. The work done by external force to increase the side of triangle from l to $2l$ is

(a) $-\frac{3}{2}\frac{GM^2}{l}$ (b) $\frac{-3GM^2}{l}$ (c) $\frac{3}{2}\frac{GM^2}{l}$ (d) $\frac{3GM^2}{l}$

27. A particle is thrown vertically upwards from the surface of earth and it reaches to a maximum height equal to the radius of earth. The ratio of the velocity of projection to the escape velocity on the surface of earth is

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2\sqrt{2}}$

28. The gravitational potential energy of a body at a distance r from the centre of earth is U. Its weight at a distance $2r$ from the centre of earth is

(a) $\frac{U}{r}$ (b) $\frac{U}{2r}$ (c) $\frac{U}{4r}$ (d) $\frac{U}{\sqrt{2}r}$

JEE Corner

Assertion and Reason

Directions : *Choose the correct option.*

(a) *If both **Assertion** and **Reason** are true and the **Reason** is correct explanation of the **Assertion**.*

(b) *If both **Assertion** and **Reason** are true but **Reason** is not the correct explanation of **Assertion**.*

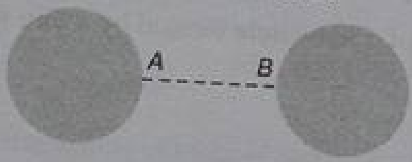
(c) *If **Assertion** is true, but the **Reason** is false.*

(d) *If **Assertion** is false but the **Reason** is true.*

1. **Assertion :** When two masses come closer, their gravitational potential energy decreases.
Reason : Two masses attract each other.

2. **Assertion :** In moving from centre of a solid sphere to its surface, gravitational potential increases.
Reason : Gravitational field strength increases.

3. **Assertion :** There are two identical spherical bodies fixed in two positions as shown. While moving from A to B gravitational potential first increases then decreases.



Reason : At centre of A and B field strength will be zero.

4. **Assertion :** If we plot potential versus x-coordinate graph along the x-axis, then field strength is zero where slope of V-x graph is zero.
Reason : If potential is function of x-only then

$$E = -\frac{dV}{dx}$$

5. **Assertion :** A particle is projected upwards with speed v and it goes to a height h. If we double the speed then it will move to height $4h$.
Reason : In case of earth, acceleration due to gravity g varies as

$$g \propto \frac{1}{r^2} \qquad (\text{for } r \geq R)$$

6. **Assertion :** In planetary motion angular momentum of system remains constant. But linear momentum does not remain constant.
Reason : Net torque on a system about any point is zero.

7. **Assertion :** Plane of space satellite is always equitorial plane.
Reason : On the equator value of g is minimum.

8. **Assertion :** On satellites we feel weightlessness. Moon is also a satellite of earth. But we do not feel weightlessness on moon.
Reason : Mass of moon is considerable.

9. **Assertion :** Plane of geostationary satellites always passes through equator.
Reason : Geostationary satellites always lies above Moscow.

10. **Assertion :** It we double the circular radius of a satellite, then its potential energy, kinetic energy and total mechanical energy will become half.
Reason : Orbital speed of a satellite.

$$v \propto \frac{1}{\sqrt{r}}$$

where r is its radius of orbit.

11. **Assertion :** If the radius of earth is decreased keeping its mass constant, effective value of g may increase or decrease at pole.
Reason : Value of g on the surface of earth is given by $g = \frac{GM}{R^2}$.

Objective Questions (Level 2)

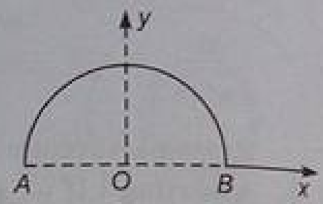
Single Correct Option

1. An artificial satellite of mass m is moving in a circular orbit at a height equal to the radius R of the earth. Suddenly due to internal explosion the satellite breaks into two parts of equal pieces. One part of the

satellite stops just after the explosion. The increase in the mechanical energy of the system due to explosion will be (Given : acceleration due to gravity on the surface of earth is g)

(a) mgR (b) $\frac{mgR}{2}$ (c) $\frac{mgR}{4}$ (d) $\frac{3mgR}{4}$

2. Gravitational field at the centre of a semicircle formed by a thin wire AB of mass M and length l is



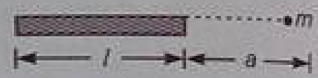
(a) $\frac{GM}{l^2}$ along x-axis

(b) $\frac{GM}{\pi l^2}$ along y-axis

(c) $\frac{2\pi GM}{l^2}$ along x-axis

(d) $\frac{2\pi GM}{l^2}$ along y-axis

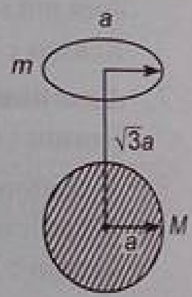
3. A mass m is at a distance a from one end of a uniform rod of length l and mass M. The gravitational force on the mass due to the rod is



(a) $\frac{GMm}{(a+l)^2}$ (b) $\frac{GmM}{a(l+a)}$

(c) $\frac{GMm}{a^2}$ (d) $\frac{GmM}{2(l+a)^2}$

4. A uniform ring of mass m is lying at a distance $\sqrt{3}\,a$ from the centre of a sphere of mass M just over the sphere (where a is the radius of the ring as well as that of the sphere). Then magnitude of gravitational force between them is



(a) $\frac{GMm}{8a^2}$ (b) $\frac{GMm}{\sqrt{3}a^2}$

(c) $\sqrt{3}\frac{GMm}{a^2}$ (d) $\sqrt{3}\frac{GMm}{8a^2}$

5. Four particles, each of mass M, move along a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is

(a) $\frac{GM}{R}$ (b) $\sqrt{2\sqrt{2}\frac{GM}{R}}$ (c) $\sqrt{\frac{GM}{R}(2\sqrt{2}+1)}$ (d) $\sqrt{\frac{GM}{R}\frac{2\sqrt{2}+1}{4}}$

6. A projectile is fired from the surface of earth of radius R with a velocity kv_e (where v_e is the escape velocity from surface of earth and $k<1$). Neglecting air resistance, the maximum height of rise from the centre of earth is

(a) $\frac{R}{k^2-1}$ (b) k^2R (c) $\frac{R}{1-k^2}$ (d) kR

7. Suppose a vertical tunnel is along the diameter of earth, assumed to be a sphere of uniform mass density ρ. If a body of mass m is thrown in this tunnel, its acceleration at a distance y from the centre is given by

(a) $\frac{4\pi}{3}G\rho ym$ (b) $\frac{3}{4}\pi\rho y$

(c) $\frac{4}{3}\pi\rho y$ (d) $\frac{4}{3}\pi G\rho y$

8. A train of mass m moves with a velocity v on the equator from east to west. If ω is the angular speed of earth about its axis and R is the radius of the earth then the normal reaction acting on the train is

(a) $mg\left[1-\frac{(\omega R-2v)\,\omega}{g}-\frac{v^2}{Rg}\right]$

(b) $mg\left[1-2\frac{(\omega R-v)\,\omega}{g}-\frac{v^2}{Rg}\right]$

(c) $mg\left[1-\frac{(\omega R+2v)\,\omega}{g}-\frac{v^2}{Rg}\right]$

(d) $mg\left[1-2\frac{(\omega R-v)\,\omega}{g}-\frac{v^2}{Rg}\right]$

9. The figure represents a solid uniform sphere of mass M and radius R. A spherical cavity of radius r is at a distance a from the centre of the sphere. The gravitational field inside the cavity is

(a) non-uniform
(b) towards the centre of the cavity
(c) directly proportional to a
(d) All of the above

10. If v_e is the escape velocity for earth when a projectile is fired from the surface of earth. Then the escape velocity if the same projectile is fired from its centre is

(a) $\sqrt{\frac{3}{2}}\,v_e$ (b) $\frac{3}{2}\,v_e$ (c) $\sqrt{\frac{2}{3}}\,v_e$ (d) $\frac{2}{3}\,v_e$

11. If the gravitational field intensity at a point is given by $g=\frac{GM}{r^{2.5}}$. Then the potential at distance r is

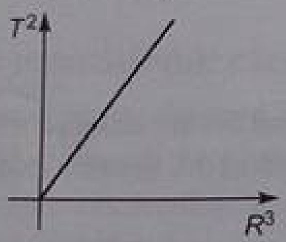
(a) $\frac{-2GM}{3r^{1.5}}$ (b) $\frac{-GM}{r^{3.5}}$ (c) $\frac{2GM}{3r^{1.5}}$ (d) $\frac{GM}{r^{3.5}}$

12. Three identical particles each of mass M move along a common circular path of radius R under the mutual interaction of each other. The velocity of each particle is

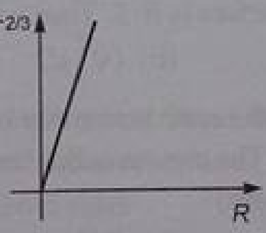
(a) $\sqrt{\frac{GM}{R}\sqrt{\frac{2}{3}}}$ (b) $\sqrt{\frac{GM}{\sqrt{3}R}}$ (c) $\sqrt{\frac{GM}{3R}}$ (d) $\sqrt{\frac{2}{3}\frac{GM}{R}}$

13. If T be the period of revolution of a planet revolving around sun in an orbit of mean radius R, then identify the **incorrect** graph.

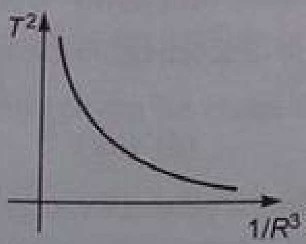
(a)



(b)



(c)

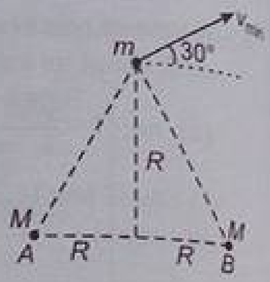


(d) None of these

14. A person brings a mass of 1 kg from infinity to a point A. Initially, the mass was at rest but it moves at a speed of 3 m/s as it reaches A. The work done by the person on the mass is – 5.5 J. The gravitational potential at A is

(a) – 1 J/kg (b) – 4.5 J/kg (c) – 5.5 J/kg (d) – 10 J/kg

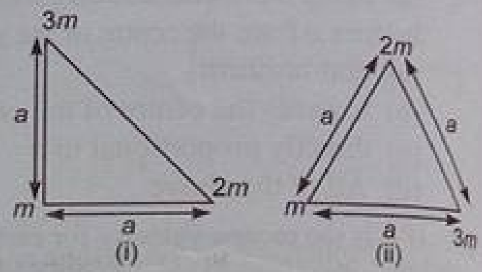
15. With what minimum speed should m be projected from point C in presence of two fixed masses M each at A and B as shown in the figure such that mass m should escape the gravitational attraction of A and B?



(a) $\sqrt{\frac{2GM}{R}}$ (b) $\sqrt{\frac{2\sqrt{2}GM}{R}}$

(c) $2\sqrt{\frac{GM}{R}}$ (d) $2\sqrt{2}\sqrt{\frac{GM}{R}}$

16. Consider two configurations of a system of three particles of masses m, $2m$ and $3m$. The work done by gravity in changing the configuration of the system from figure (i) to figure (ii) is



(a) zero (b) $\frac{6Gm^2}{a}\left\{1+\frac{1}{\sqrt{2}}\right\}$

(c) $\frac{6Gm^2}{a}\left\{1-\frac{1}{\sqrt{2}}\right\}$ (d) $\frac{6Gm^2}{a}\left\{2-\frac{1}{\sqrt{2}}\right\}$

17. A tunnel is dug along the diameter of the earth. There is a particle of mass m at the centre of the tunnel. Find the minimum velocity given to the particle so that is just reaches to the surface of the earth. (R = radius of earth)

(a) $\sqrt{\frac{GM}{R}}$

(b) $\sqrt{\frac{GM}{2R}}$

(c) $\sqrt{\frac{2GM}{R}}$

(d) it will reach with the help of negligible velocity

18. A body is projected horizontally from the surface of the Earth (radius = R) with a velocity equal to n times the escape velocity. Neglect rotational effects of the earth. The maximum height attained by the body from the earth's surface is $R/2$. Then n must be

(a) $\sqrt{0.6}$ (b) $(\sqrt{3})/2$ (c) $\sqrt{0.4}$ (d) $1/2$

19. A tunnel is dug in the earth across one of its diameter. Two masses m and $2m$ are dropped from the two ends of the tunnel. The masses collide and stick each other. They perform SHM, the amplitude of which is (R = radius of earth)

(a) R (b) $R/2$ (c) $R/3$ (d) $2R/3$

20. There are two planets. The ratio of radius of the two planets is k but ratio of acceleration due to gravity of both planets is g. What will be the ratio of their escape velocity?

(a) $(kg)^{1/2}$ (b) $(kg)^{-1/2}$ (c) $(kg)^2$ (d) $(kg)^{-2}$

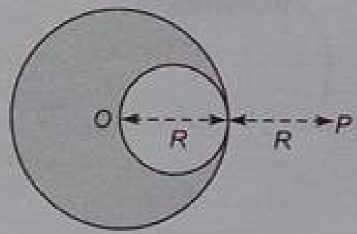
21. A body of mass 2 kg is moving under the influence of a central force whose potential energy is given by $U = 2r^3$ J. If the body is moving in a circular orbit of 5 m, its energy will be

(a) 625 J (b) 250 J (c) 500 J (d) 125 J

22. A research satellite of mass 200 kg circles the earth in an orbit of average radius $3R/2$ where R is the radius of the earth. Assuming the gravitational pull on the mass of 1 kg on the earth's surface to be 10 N, the pull on the satellite will be
(a) 1212 N (b) 889 N (c) 1280 N (d) 960 N

23. A satellite of mass m revolves around the earth of radius R at a height x from its surface. If g is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite is
(a) $\sqrt{gx}$ (b) $\sqrt{\frac{gR}{R-x}}$ (c) $\sqrt{\frac{gR^2}{R-x}}$ (d) $\sqrt{\frac{gR^2}{R+x}}$

24. A solid sphere of uniform density and radius R applies a gravitational force of attraction equal to F_1 on a particle placed at P, distance $2R$ from the centre O of the sphere. A spherical cavity of radius $R/2$ is now made in the sphere as shown in figure. The sphere with cavity now applies a gravitational force F_2 on same particle placed at P. The ratio F_2/F_1 will be



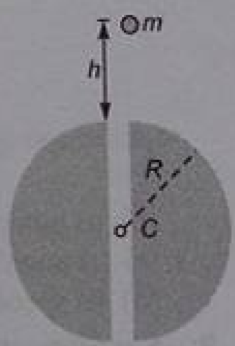
(a) 1/2 (b) 7/9
(c) 3 (d) 7

More than One Correct Options

1. Three planets of same density have radii R_1, R_2 and R_3 such that $R_1 = 2R_2 = 3R_3$. The gravitational field at their respective surfaces are g_1, g_2 and g_3 and escape velocities from their surfaces are v_1, v_2 and v_3, then
(a) $g_1/g_2 = 2$ (b) $g_1/g_3 = 3$ (c) $v_1/v_2 = 1/4$ (d) $v_1/v_3 = 3$

2. For a geostationary satellite orbiting around the earth identify the necessary condition.
(a) it must lie in the equatorial plane of earth
(b) its height from the surface of earth must be 36000 km
(c) it period of revolution must be $2\pi\sqrt{\frac{R}{g}}$ where R is the radius of earth
(d) its period of revolution must be 24 hrs

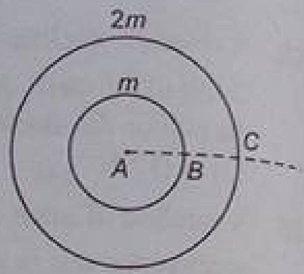
3. A ball of mass m is dropped from a height h equal to the radius of the earth above the tunnel dug through the earth as shown in the figure. Choose the correct options.



(a) Particle will oscillate through the earth to a height h on both sides
(b) Particle will execute simple harmonic motion
(c) Motion of the particle is periodic
(d) Particle passes the centre of earth with a speed $v = \sqrt{\frac{2GM}{R}}$

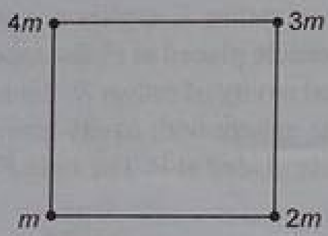
4. Two point masses m and $2m$ are kept at points A and B as shown. E represents magnitude of gravitational field strength and V the gravitational potential. As we move from A to B
(a) E will first decrease then increases
(b) E will first increase then decrease
(c) V will first decrease then increase
(d) V will first increase then decrease

5. Two spherical shells have mass m and $2m$ as shown. Choose the correct options.



(a) Between A and B gravitational field strength is zero
(b) Between A and B gravitational potential is constant
(c) There will be two points one lying between B and C and other lying between C and infinity where gravitational field strength are same
(d) There will be a point between B and C where gravitational potential will be zero

6. Four point masses are placed at four corners of a square as shown. When positions of m and $2m$ are interchanged

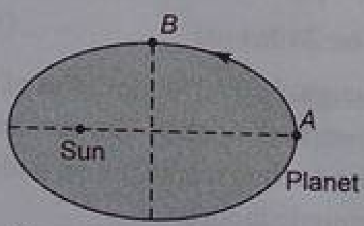


(a) gravitational field strength at centre will increase
(b) gravitational field strength at centre will decrease
(c) gravitational potential at centre will remain unchanged
(d) gravitational potential at centre will decrease

7. Two identical particles 1 and 2 are projected from surface of earth with same velocities in the directions shown in figure.

(a) Both the particles will stop momentarily (before striking with ground) at different times
(b) Particle-2 will rise upto lesser height compared to particle-2
(c) Minimum speed of particle-2 is more than that of particle-1
(d) Particle-1 will strike the ground earlier

8. A planet is moving round the sun in an elliptical orbit as shown. As the planet moves from A to B
(a) its kinetic energy will decrease



(b) its potential energy will remain unchanged
(c) its angular momentum about centre of sun will remain unchanged
(d) its speed is minimum at A

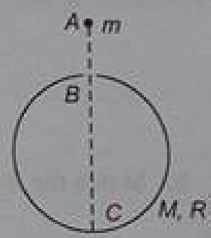
9. A satellite of mass m is just placed over the surface of earth. In this position mechanical energy of satellite is E_1. Now it starts orbiting round the earth in a circular path at height h = radius of earth. In this position, kinetic energy, potential energy and total mechanical energy of satellite are K_2, U_2 and E_2 respectively. Then

(a) $U_2 = \dfrac{E_1}{2}$ (b) $E_2 = \dfrac{E_1}{4}$ (c) $K_2 = -E_2$ (d) $K_2 = -\dfrac{U_2}{2}$

10. A satellite is revolving round the earth in circular orbit
(a) if mass of earth is made four times, keeping other factors constant, orbital speed of satellite will become two times
(b) corresponding to change in part (a), times period of satellite will remain half
(c) when value of G is made two times orbital speed increases and time period decreases
(d) G has no effect on orbital speed and time period

Match the Columns

1. There is a small hole in a spherical shell of mass M and radius R. A particle of mass m is dropped from point A as shown. Match the two columns for the situation shown in figure.



Column I	Column II
(a) Potential energy from A to B	(p) Continuously increases
(b) Potential energy from B to C	(q) Continuously decreases
(c) Speed of particle from B to C	(r) First increases then remains constant
(d) Acceleration of particle from A to C	(s) None of these

2. Five point masses m each are placed at five corners of a regular pentagon. Distance of any corner from centre is r. Match the following two columns.

Column I	Column II
(a) Gravitational field strength at centre	(p) Gm/r^2
(b) Gravitational potential at centre	(q) $4Gm/r$
(c) When one mass is removed gravitational field strength at centre	(r) zero
(d) When one mass is removed gravitational potential at centre	(s) None of these

3. | Potential | on the surface of a solid sphere is x and radius is y. Match the following two columns.

Column I	Column II
(a) Field strength at distance $2y$ from centre	(p) $\frac{x}{2y}$
(b) | Potential | at distance $\frac{y}{2}$ from centre	(q) $\frac{x}{2}$
(c) Field strength at distance $y/2$ from centre	(r) $\frac{x}{4y}$
(d) | Potential | at distance $2y$ from centre	(s) None

4. Match the following two columns.

Column I	Column II
(a) Work done is raising a mass m to a height $h = R$	(p) $\frac{1}{4}mgR$
(b) Kinetic energy of a satellite of mass m at height $h = R$	(q) mgR
(c) Difference in energies of two satellites each of mass m but one at height $h_1 = R$ and another of height $h_2 = 2R$	(r) $\frac{1}{2}mgR$
(d) Kinetic energy required to raise a particle of mass m to a height $h = R$ if projected vertically from surface of earth.	(s) None

5. Match the following two columns.

Column I	Column II
(a) Gravitational field strength is maximum at	(p) $r = 0$
(b) Gravitational field strength is zero at	(q) $r = R$
(c) Gravitational potential is minimum at	(r) $r = \frac{R}{\sqrt{2}}$
(d) Gravitational potential is zero at	(s) None of these

Here r is distance from centre of a solid sphere or distance from centre of a ring along its axis.

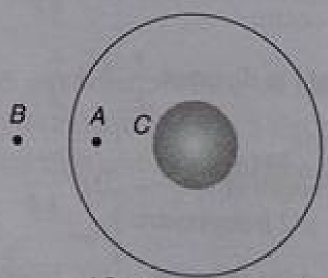
Subjective Questions (Level 2)

1. Three particles of mass m each are placed at the three corners of an equilateral triangle of side a. Find the work which should be done on this system to increase the side of the triangle to $2a$.

2. A man can jump vertically to a height of 1.5 m on the earth. Calculate the radius of a planet of the same mean density as that of the earth from whose gravitational field he could escape by jumping. Radius of earth is 6.41×10^6 m.

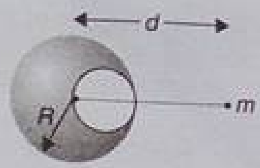
3. An artificial satellite is moving in a circular orbit around the earth with a speed equal to half the magnitude of escape velocity from the surface of earth. (Radius of earth = 6400 km)
   (a) Determine the height of the satellite above the earth's surface.
   (b) If the satellite is stopped suddenly in its orbit and allowed to fall freely on the earth, find the speed with which it hits the surface of earth.

4. A uniform metal sphere of radius R and mass m is surrounded by a thin uniform spherical shell of same mass and radius $4R$. The centre of the shell C falls on the surface of the inner sphere. Find the gravitational fields at points A and B.



$AC = 3R$, $BC = 5R$

5. Figure shows a spherical cavity inside a lead sphere. The surface of the cavity passes through the centre of the sphere and touches the right side of the sphere. The mass of the sphere before hollowing was M. With what gravitational force does the hollowed out lead sphere attract a particle of mass m that lies at a distance d from the centre of the lead sphere on the straight line connecting the centres of the spheres and of the cavity.



6. The density of the core of a planet is ρ_1 and that of the outer shell is ρ_2, the radii of the core and that of the planet are R and $2R$ respectively. The acceleration due to gravity at the surface of the planet is same as at a depth R. Find the ratio of $\frac{\rho_1}{\rho_2}$.

7. If a satellite is revolving around a planet of mass M in an elliptical orbit of semi-major axis a. Show that the orbital speed of the satellite when it is at a distance r from the focus will be given by

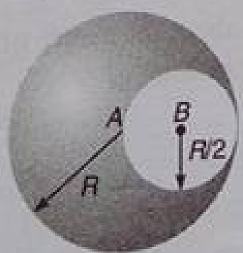
$$v^2 = GM\left[\frac{2}{r} - \frac{1}{a}\right]$$

8. A uniform ring of mass m and radius a is placed directly above a uniform sphere of mass M and of equal radius. The centre of the ring is at a distance $\sqrt{3}a$ from the centre of the sphere. Find the gravitational force exerted by the sphere on the ring.

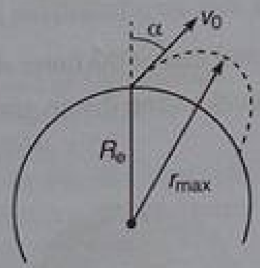
9. Distance between the centres of two stars is $10a$. The masses of these stars are M and $16M$ and their radii a and $2a$ respectively. A body of mass m is fired straight from the surface of the larger star towards the smaller star. What should be its minimum initial speed to reach the surface of the smaller star? Obtain the expression in terms of G, M and a.

10. A smooth tunnel is dug along the radius of earth that ends at centre. A ball is released from the surface of earth along tunnel. Coefficient of restitution for collision between soil at centre and ball is 0.5. Calculate the distance travelled by ball just before second collision at centre. Given mass of the earth is M and radius of the earth is R.

11. Inside a fixed sphere of radius R and uniform density ρ, there is spherical cavity of radius $\frac{R}{2}$ such that surface of the cavity passes through the centre of the sphere as shown in figure. A particle of mass m_0 is released from rest at centre B of the cavity. Calculate velocity with which particle strikes the centre A of the sphere. Neglect earth's gravity. Initially sphere and particle are at rest.

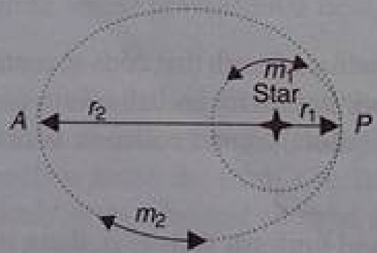


12. A projectile of mass m is fired from the surface of the earth at an angle $\alpha = 60°$ from the vertical. The initial speed v_0 is equal to $\sqrt{\frac{GM_e}{R_e}}$. How high does the projectile rise ? Neglect air resistance and the earth's rotation.



13. A ring of radius $R = 4$ m is made of a highly dense material. Mass of the ring is $m_1 = 5.4 \times 10^9$ kg distributed uniformly over its circumference. A highly dense particle of mass $m_2 = 6 \times 10^8$ kg is placed on the axis of the ring at a distance $x_0 = 3$ m from the centre. Neglecting all other forces, except mutual gravitational interaction of the two. Calculate :
    (i) displacement of the ring when particle is at the centre of ring, and
    (ii) speed of the particle at that instant.

14. Two planets of equal mass orbit a much more massive star (figure). Planet m_1 moves in a circular orbit of radius 1×10^8 km with period 2 yr. Planet m_2 moves in an elliptical orbit with closest distance $r_1 = 1 \times 10^8$ km and farthest distance $r_2 = 1.8 \times 10^8$ km, as shown.
    (a) Using the fact that the mean radius of an elliptical orbit is the length of the semi-major axis, find the period of m_2's orbit.
    (b) Which planet has the greater speed at point P ? Which has the greater total energy ?
    (c) Compare the speed of planet m_2 at P with that at A.



15. In a double star, two stars one of mass m_1 and another of mass m_2, with a separation d, rotate about their common centre of mass. Find :
    (a) an expression for their time period of revolution,
    (b) the ratio of their kinetic energies,
    (c) the ratio of their angular momenta about the centre of mass, and
    (d) the total angular momentum of the system,
    (e) the kinetic energy of the system

ANSWERS

Introductory Exercise 10.1

1. $-0.0168\ ms^{-2}$ 2. 7.8×10^{-4} rad/s 3. 1600 km 4. $\frac{a}{r^2}$ 5. $\frac{2\pi GMm}{L^2}$

Introductory Exercise 10.2

1. $-\frac{3Gm}{R}$ 2. 200 N/kg along positive x-direction 3. $-[6xy\hat{i} + (3x^2 + 3y^2z)\hat{j} + y^3\hat{k}]$ 4. False
5. $10\sqrt{2}$ N 6. Zero

Introductory Exercise 10.3

1. True 2. mgR 3. $2.1 \times 10^{-5}\ ms^{-1}$, $4.2 \times 10^{-5}\ ms^{-1}$ 4. $10\ kms^{-1}$

Introductory Exercise 10.4

1. $11.2\ kms^{-1}$ 2. Angular momentum, mechanical energy 3. 2 : 1, 2 : 1
5. (a) Orbit will become elliptical (b) The satellite will escape

AIEEE Corner

Subjective Questions (Level 1)

1. $a_1 = 5.3 \times 10^{-10}\ ms^{-2}$, $a_2 = 2.65 \times 10^{-10}\ ms^{-2}$ 2. $\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right)\frac{Gm^2}{d^2}$ (along PB) 3. $\frac{4\sqrt{2}Gm^2}{a^2}$

4. $\frac{\sqrt{3}GM^2}{4a^2}$ 5. $\frac{Gm\lambda l}{d(l+d)}$ 6. $4.9\ ms^{-2}$ 7. (a) $2.45\ ms^{-2}$ (b) $4.9\ ms^{-2}$

8. $\frac{(\sqrt{5}-1)R}{2}$, where R is the radius of earth 9. 997 N 10. $1.237 \times 10^{-3}\ rads^{-1}$, 84.6 min

11. approximately 10 km 12. $F = \frac{4Gm_1m}{(R_1+R_2)^2}$ 13. (a) $4R$ (b) 7.65 R and 1.49 R 14. $E = \frac{2\pi GM}{L^2}$

15. (a) $\frac{4GM}{9a^2}$ (towards the centre) (b) $\frac{8GM}{25a^2}$ (towards the centre) 16. $\frac{\pi G\rho_0 a}{2}$ 17. $\frac{-3Gm}{R}$

19. (a) 82 km/s (b) $1.8 \times 10^4\ kms^{-1}$ 20. 2.5×10^4 km 21. $v = \sqrt{\frac{GM}{R}}$

22. (a) 7.5×10^{-9} J (b) 8.17×10^{-9} J 23. (a) 9.6×10^{10} J (b) 1.07×10^{10} J

24. (a) $F = \frac{GM^2}{4R^2}$ (b) $v = \sqrt{\frac{GM}{4R}}$, $T = \frac{4\pi R^{3/2}}{\sqrt{GM}}$ (c) $\frac{GM^2}{4R}$ 25. (a) $\frac{-GMm}{r}$ (b) $\frac{-2GMm}{r}$

26. $56\sqrt{2}$ h, $38844\ kmh^{-1}$

27. (a) $6.90\ kms^{-1}$ (b) 2.38×10^{10} J (c) -4.76×10^{10} J with usual reference (d) 2.1 h

28. 0.71 yr 29. (a) No (b) Same (c) Yes

Objective Questions (Level 1)

1. (c)	2. (b)	3. (c)	4. (d)	5. (d)	6. (b)	7. (b)	8. (a)	9. (b)	10. (c)
11. (b)	12. (b)	13. (d)	14. (b)	15. (b)	16. (b)	17. (c)	18. (d)	19. (a)	20. (b)
21. (c)	22. (c)	23. (d)	24. (c)	25. (a)	26. (c)	27. (a)	28. (c)		

JEE Corner

Assertion and Reason

1. (b) 2. (b) 3. (b) 4. (d) 5. (d) 6. (d) 7. (d) 8. (a) 9. (c) 10. (b)
11. (d)

Objective Questions (Level 2)

1. (c) 2. (d) 3. (b) 4. (d) 5. (d) 6. (c) 7. (d) 8. (a) 9. (c) 10. (a)
11. (a) 12. (b) 13. (d) 14. (d) 15. (b) 16. (c) 17. (a) 18. (a) 19. (c) 20. (a)
21. (a) 22. (b) 23. (d) 24. (b)

More than One Correct Options

1. (a,b,d) 2. (a,b,d) 3. (a,c,d) 4. (a,d) 5. (a,b,c) 6. (a,c) 7. (b,c,d)
8. (c,d) 9. (all) 10. (a,b,c)

Match the Columns

1. (a) → q (b) → s (c) → s (d) → r
2. (a) → r (b) → s (c) → p (d) → s
3. (a) → r (b) → s (c) → p (d) → q
4. (a) → r (b) → p (c) → s (d) → r
5. (a) → q,r (b) → p (c) → p (d) → s

Subjective Questions (Level 2)

1. $\frac{3Gm^2}{2a}$ 2. 3.1×10^3 m 3. (a) 6400 km (b) 7.92 kms^{-1} 4. $\frac{Gm}{16R^2}, \frac{61Gm}{900R^2}$

5. $\frac{GMm}{d^2}\left[1 - \frac{1}{8(1 - R/2d)^2}\right]$ 6. 7/3 8. $\frac{\sqrt{3}GMm}{8a^2}$ 9. $\frac{3}{2}\sqrt{\frac{5GM}{a}}$ 10. $d = 2R$ 11. $\sqrt{\frac{2}{3}\pi G\rho R^2}$

12. $\frac{R_e}{2}$ 13. (i) 0.3 m (ii) 18 cms^{-1}

14. (a) 3.31 yr (b) m_2 has greater speed and greater total energy (c) $v_P = 1.8v_A$

15. (a) $2\pi\sqrt{\frac{d^3}{G(m_1 + m_2)}}$ (b) $\frac{m_2}{m_1}$ (c) $\frac{m_2}{m_1}$ (d) $\mu\omega d^2$

(e) $\frac{1}{2}\mu\omega^2 d^2$, where μ is the reduced mass and ω the angular velocity.

11

Simple Harmonic Motion

Chapter Contents

Solved Examples

Level 1

Example 1 *If a SHM is represented by the equation* $x = 10\sin\left(\pi t + \frac{\pi}{6}\right)$ *in SI units determine its amplitude, time period and maximum velocity* v_{max} ?

Solution Comparing the above equation with

$$x = A\sin(\omega t + \phi), \text{ we get}$$

$$A = 10\text{ m} \qquad \textbf{Ans.}$$

$$\omega = \pi\text{s}^{-1} \quad \text{and} \quad \phi = \frac{\pi}{6}$$

$$\because \quad T = \frac{2\pi}{\omega} \Rightarrow T = 2\text{s} \qquad \textbf{Ans.}$$

$$v_{max} = \omega A = 10\pi\text{ m/s} \qquad \textbf{Ans.}$$

Example 2 *A particle executes SHM with a time period of 4 s. Find the time taken by the particle to go directly from its mean position to half of its amplitude.*

Solution $x = A\sin(\omega t + \phi)$

At $t = 0,\ x = 0 \Rightarrow A\sin\phi = 0$ or $\phi = 0$

Hence, $$x = A\sin(\omega t)$$

or $$\frac{A}{2} = A\sin(\omega t)$$

or $$\frac{1}{2} = \sin(\omega t)$$

$$\omega t = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

$$t = \frac{\pi}{6\omega} = \frac{\pi T}{6(2\pi)}$$

as $$\omega = \frac{2\pi}{T} \Rightarrow t = \frac{T}{12} = \frac{1}{3}\text{s} \qquad \textbf{Ans.}$$

Example 3 *A particle executes SHM.*

(a) *What fraction of total energy is kinetic and what fraction is potential when displacement is one half of the amplitude?*

(b) *At what value of displacement are the kinetic and potential energies equal?*

Solution We know that
$$E_{\text{total}} = \frac{1}{2} m\omega^2 A^2$$
$$\text{KE} = \frac{1}{2} m\omega^2 (A^2 - x^2)$$
and
$$U = \frac{1}{2} m\omega^2 x^2$$
(a) When
$$x = \frac{A}{2}$$
$$\text{KE} = \frac{1}{2} m\omega^2 \frac{3A^2}{4} \quad \Rightarrow \quad \frac{\text{KE}}{E_{\text{total}}} = \frac{3}{4} \qquad \textbf{Ans.}$$
At $x = \frac{A}{2}$,
$$U = \frac{1}{2} m\omega^2 \frac{A^2}{4}$$
$\Rightarrow$
$$\frac{\text{PE}}{E_{\text{total}}} = \frac{1}{4} \qquad \textbf{Ans.}$$
(b) Since,
$$K = U$$
$$\frac{1}{2} m\omega^2 (A^2 - x^2) = \frac{1}{2} m\omega^2 x^2$$
or
$$2x^2 = A^2 \quad \text{or} \quad x = \frac{A}{\sqrt{2}} = 0.707A \qquad \textbf{Ans.}$$

Example 4 *Two particles move parallel to x-axis about the origin with the same amplitude and frequency. At a certain instant they are found at distance $\frac{A}{3}$ from the origin on opposite sides but their velocities are found to be in the same direction. What is the phase difference between the two ?*

Solution Let equations of two SHM be
$$x_1 = A \sin \omega t \qquad \text{...(i)}$$
$$x_2 = A \sin (\omega t + \phi) \qquad \text{...(ii)}$$
Give that
$$\frac{A}{3} = A \sin \omega t$$
and
$$-\frac{A}{3} = A \sin (\omega t + \phi)$$
Which gives
$$\sin \omega t = \frac{1}{3} \qquad \text{...(iii)}$$
$$\sin (\omega t + \phi) = -\frac{1}{3} \qquad \text{...(iv)}$$
From Eq. (iv),
$$\sin \omega t \cos \phi + \cos \omega t \sin \phi = -\frac{1}{3}$$

$$\Rightarrow \quad \frac{1}{3}\cos\phi + \sqrt{1-\frac{1}{9}}\sin\phi = -\frac{1}{3}$$

Solving this equation, we get

$$\text{or} \quad \cos\phi = -1, \frac{7}{9}$$

$$\Rightarrow \quad \phi = \pi \quad \text{or} \quad \cos^{-1}\left(\frac{7}{9}\right)$$

Differentiating Eqs. (i) and (ii), we obtain

$$v_1 = A\omega\cos\omega t \quad \text{and} \quad v_2 = A\omega\cos(\omega t + \phi)$$

If we put $\phi = \pi$, we find v_1 and v_2 are of opposite signs. Hence, $\phi = \pi$ is not acceptable.

$$\therefore \quad \phi = \cos^{-1}\left(\frac{7}{9}\right)$$ **Ans.**

Example 5 *A particle executes simple harmonic motion about the point $x = 0$. At time $t = 0$ it has displacement $x = 2$ cm and zero velocity. If the frequency of motion is $0.25\ s^{-1}$, find (a) the period, (b) angular frequency, (c) the amplitude, (d) maximum speed, (e) the displacement at $t = 3$ s and (f) the velocity at $t = 3$ s.*

Solution (a) Period $$T = \frac{1}{f} = \frac{1}{0.25\ \text{s}^{-1}} = 4\ \text{s}$$ **Ans.**

(b) Angular frequency $$\omega = \frac{2\pi}{T} = \frac{2\pi}{4} = \frac{\pi}{2}\ \text{rad/s} = 1.57\ \text{rad/s}$$ **Ans.**

(c) Amplitude is the maximum displacement from mean position. Hence, $A = 2 - 0 = 2$ cm.

(d) Maximum speed $$v_{max} = A\omega = 2\cdot\frac{\pi}{2} = \pi\ \text{cm/s} = 3.14\ \text{cm/s}$$ **Ans.**

(e) The displacement is given by $$x = A\sin(\omega t + \phi)$$

Initially at $t = 0$, $x = 2$ cm, then

$$2 = 2\sin\phi$$

or $$\sin\phi = 1 = \sin 90°$$

or $$\phi = 90°$$ **Ans.**

Now, at $t = 3$ s $$x = 2\sin\left(\frac{\pi}{2}\times 3 + \frac{\pi}{2}\right) = 0$$ **Ans.**

(f) Velocity at $x = 0$ is v_{max} *i.e.*, 3.14 cm/s.

Example 6 *Show that the period of oscillation of simple pendulum at depth h below earth's surface is inversely proportional to $\sqrt{R-h}$, where R is the radius of earth. Find out the time period of a second pendulum at a depth R/2 from the earth's surface?*

Solution At earth's surface the value of time period is given by

$$T = 2\pi\sqrt{\frac{L}{g}} \quad \text{or} \quad T \propto \frac{1}{\sqrt{g}}$$

At a depth h below the surface, $$g' = g\left(1 - \frac{h}{R}\right)$$

$$\therefore \quad \frac{T'}{T} = \sqrt{\frac{g}{g'}} = \sqrt{\frac{1}{\left(1 - \frac{h}{R}\right)}} = \sqrt{\frac{R}{R-h}}$$

$$\therefore \quad T' = T\sqrt{\frac{R}{R-h}}$$

or $$T' \propto \frac{1}{\sqrt{R-h}}$$ **Hence proved.**

Further, $$T_{R/2} = 2\sqrt{\frac{R}{R - R/2}} = 2\sqrt{2}\ \text{sec}$$ **Ans.**

Example 7 *A spring mass system is hanging from the ceiling of an elevator in equilibrium. The elevator suddenly starts accelerating upwards with acceleration a. Find :*

(a) the frequency and

(b) the amplitude of the resulting SHM.

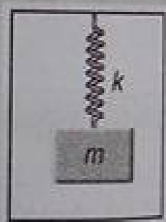


Fig. 11.29

Solution (a) Frequency $= 2\pi\sqrt{\frac{m}{k}}$ (Frequency is independent of g in spring)

(b) Extension in spring in equilibrium

$$\text{initial} = \frac{mg}{k}$$

Extension in spring in equilibrium in accelerating lift $= \dfrac{m(g+a)}{k}$

$$\therefore \quad \text{Amplitude} = \frac{m(g+a)}{k} - \frac{mg}{k} = \frac{ma}{k}$$ **Ans.**

Example 8 *A ring of radius r is suspended from a point on its circumference. Determine its angular frequency of small oscillations.*

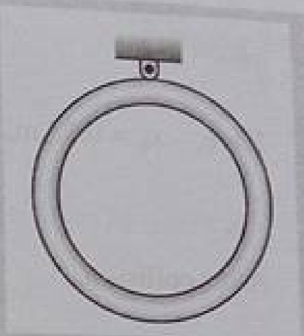


Fig. 11.30

Solution It is a physical pendulum, the time period of which is,

$$T = 2\pi\sqrt{\frac{I}{mgl}}$$

Here, I = moment of inertia of the ring about point of suspension

$$= mr^2 + mr^2$$

$$= 2mr^2$$

and l = distance of point of suspension from centre of gravity

$$= r$$

$\therefore$
$$T = 2\pi\sqrt{\frac{2mr^2}{mgr}} = 2\pi\sqrt{\frac{2r}{g}}$$

$\therefore$ Angular frequency
$$\omega = \frac{2\pi}{T}$$

or
$$\omega = \sqrt{\frac{g}{2r}}$$ **Ans.**

Level 2

Example 1 *For the arrangement shown in figure, the spring is initially compressed by 3 cm. When the spring is released the block collides with the wall and rebounds to compress the spring again.*

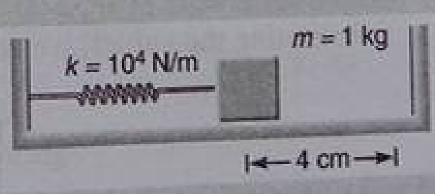


Fig. 11.31

(a) If the coefficient of restitution is $\frac{1}{\sqrt{2}}$, find the maximum compression in the spring after collision.

(b) If the time starts at the instant when spring is released, find the minimum time after which the block becomes stationary.

Solution (a) Velocity of the block just before collision,

$$\frac{1}{2}mv_0^2 + \frac{1}{2}kx^2 = \frac{1}{2}kx_0^2$$

or $$v_0 = \sqrt{\frac{k}{m}(x_0^2 - x^2)}$$

Here, $x_0 = 0.03$ m, $x = 0.01$ m, $k = 10^4$ N/m, $m = 1$ kg

$\therefore$ $$v_0 = 2\sqrt{2} \text{ m/s}$$

After collision, $$v = ev_0 = \frac{1}{\sqrt{2}} 2\sqrt{2} = 2 \text{ m/s}$$

Maximum compression in the spring is

$$\frac{1}{2}kx_m^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

or $$x_m = \sqrt{x^2 + \frac{m}{k}v^2} = \sqrt{(0.01)^2 + \frac{1(2)^2}{10^4}} \text{ m}$$

$$= 2.23 \text{ cm}$$ **Ans.**

(b) In the case of spring-mass system, since the time period is independent of the amplitude of oscillation.

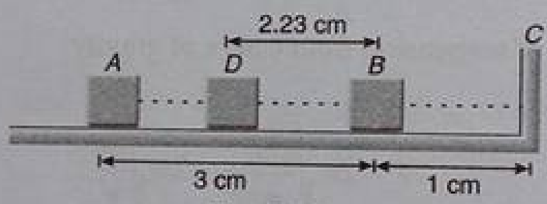


Fig. 11.32

$$\text{Time} = t_{AB} + 2t_{BC} + t_{BD}$$

$$= \frac{T_0}{4} + 2\left(\frac{T_0}{2\pi}\right)\sin^{-1}\left(\frac{1}{3}\right) + \frac{T_0}{4}$$

Here, $$T_0 = 2\pi\sqrt{\frac{m}{k}}$$

Substituting the values, we get

$$\text{Total time} = \sqrt{\frac{m}{k}}\left[\pi + 2\sin^{-1}\left(\frac{1}{3}\right)\right]$$ **Ans.**

Example 2 *With the assumption of no slipping, determine the mass m of the block which must be placed on the top of a 6 kg cart in order that the system period is 0.75 s. What is the minimum coefficient of static friction μ_s for which the block will not slip relative to the cart if the cart is displaced 50 mm from the equilibrium position and released? Take ($g = 9.8$ m/s^2).*

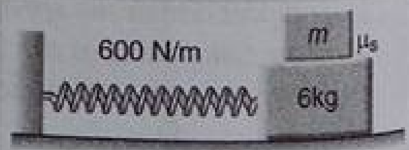


Fig. 11.33

Solution (a)
$$T = 2\pi\sqrt{\frac{m+6}{600}} \qquad \left(T = 2\pi\sqrt{\frac{m}{k}}\right)$$

or
$$0.75 = 2\pi\sqrt{\frac{m+6}{600}}$$

$\therefore$
$$m = \frac{(0.75)^2 \times 600}{(2\pi)^2} - 6$$
$$= 2.55 \text{ kg}$$
Ans.

(b) Maximum acceleration of SHM is,
$$a_{max} = \omega^2 A \qquad (A = \text{amplitude})$$

i.e., maximum force on mass 'm' is $m\omega^2 A$ which is being provided by the force of friction between the mass and the cart. Therefore,
$$\mu_s mg \geq m\omega^2 A$$

or
$$\mu_s \geq \frac{\omega^2 A}{g}$$

or
$$\mu_s \geq \left(\frac{2\pi}{T}\right)^2 \cdot \frac{A}{g}$$

or
$$\mu_s \geq \left(\frac{2\pi}{0.75}\right)^2 \left(\frac{0.05}{9.8}\right) \qquad (A = 50 \text{ mm})$$

or
$$\mu_s \geq 0.358$$

Thus, the minimum value of μ_s should be 0.358. **Ans.**

Example 3 *A long uniform rod of length L and mass M is free to rotate in a vertical plane about a horizontal axis through its one end 'O'. A spring of force constant k is connected vertically between one end of the rod and ground. When the rod is in equilibrium it is parallel to the ground.*

(a) What is the period of small oscillation that result when the rod is rotated slightly and released?

(b) What will be the maximum speed of the displaced end of the rod, if the amplitude of motion is θ_0?

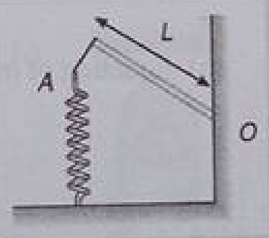


Fig. 11.34

Solution (a) Restoring torque about 'O' due to elastic force of the spring
$$\tau = -FL = -kyL \qquad (F = ky)$$
$$\tau = -kL^2\theta \qquad (\text{as } y = L\theta)$$
$$\tau = I\alpha = \frac{1}{3}ML^2\frac{d^2\theta}{dt^2}$$
$$\frac{1}{3}ML^2\frac{d^2\theta}{dt^2} = -kL^2\theta$$

$$\frac{d^2\theta}{dt^2} = -\frac{3k}{M}\theta$$

$$\omega = \sqrt{\frac{3k}{M}} \Rightarrow T = 2\pi\sqrt{\frac{M}{3k}}$$ **Ans.**

(b) In angular SHM maximum angular velocity

$$\left(\frac{d\theta}{dt}\right)_{\max} = \theta_0\omega = \theta_0\sqrt{\frac{3k}{M}}$$

$$v = r\left(\frac{d\theta}{dt}\right)$$

So,
$$v_{\max} = L\left(\frac{d\theta}{dt}\right)_{\max} = L\theta_0\sqrt{\frac{3k}{M}}$$ **Ans.**

Example 4 *A block with a mass of 2 kg hangs without vibrating at the end of a spring of spring constant 500 N/m, which is attached to the ceiling of an elevator. The elevator is moving upwards with an acceleration $\frac{g}{3}$. At time $t = 0$, the acceleration suddenly ceases.*

(a) What is the angular frequency of oscillation of the block after the acceleration ceases?
(b) By what amount is the spring stretched during the time when the elevator is accelerating?
(c) What is the amplitude of oscillation and initial phase angle observed by a rider in the elevator? Take the upward direction to be positive. Take $g = 10.0\ m/s^2$.

Solution (a) Angular frequency

$$\omega = \sqrt{\frac{k}{m}} \quad \text{or} \quad \omega = \sqrt{\frac{500}{2}}$$

or
$$\omega = 15.81\ \text{rad/s}$$

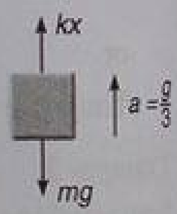


Fig. 11.35

(b) Equation of motion of the block (while elevator is accelerating) is,

$$kx - mg = ma = m\frac{g}{3}$$

$\therefore$
$$x = \frac{4mg}{3k} = \frac{(4)(2)(10)}{(3)(500)} = 0.053\ \text{m}$$

or
$$x = 5.3\ \text{cm}$$

(c) (i) In equilibrium when the elevator has zero acceleration, the equation of motion is,

$$kx_0 = mg$$

or
$$x_0 = \frac{mg}{k} = \frac{(2)(10)}{500} = 0.04\ \text{m}$$
$$= 4\ \text{cm}$$

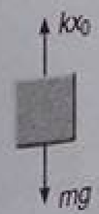


Fig. 11.36

$\therefore$ Amplitude
$$A = x - x_0 = 5.3 - 4.0$$
$$= 1.3\ \text{cm}$$ **Ans.**

(ii) At time $t = 0$, block is at $x = -A$. Therefore, substituting $x = -A$ and $t = 0$ in equation,

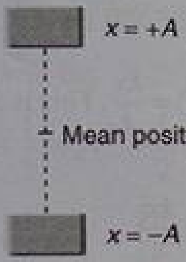


Fig. 11.37

$$x = A \sin(\omega t + \phi)$$

We get initial phase $\phi = \dfrac{3\pi}{2}$ **Ans.**

Example 5 *Figure shows a system consisting of a massless pulley, a spring of force constant k and a block of mass m. If the block is slightly displaced vertically down from its equilibrium position and released, find the period of its vertical oscillation in cases (a), (b) and (c).*

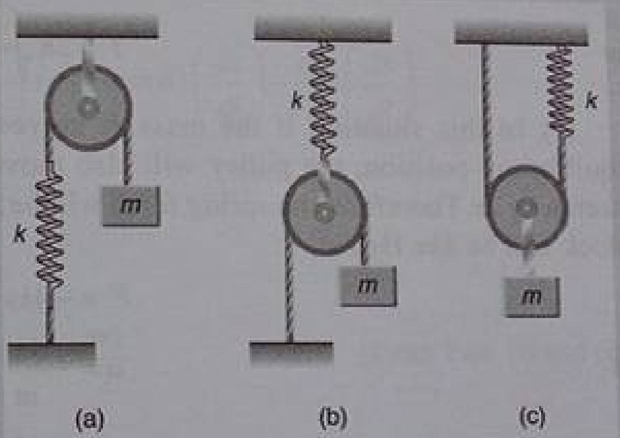


Fig. 11.38

Solution (a) In equilibrium, $kx_0 = mg$ ...(i)

When further depressed by an amount x, net restoring force (upwards) is,

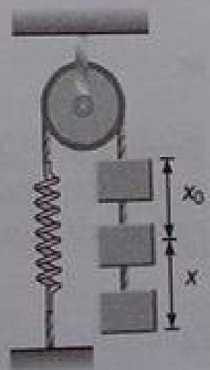


Fig. 11.39

$$F = -\{k(x + x_0) - mg\}$$

$$F = -kx \quad (\text{as } kx_0 = mg)$$

or $$a = -\frac{k}{m}x$$

$\therefore$ $$T = 2\pi\sqrt{\left|\frac{x}{a}\right|}$$

or $$T = 2\pi\sqrt{\frac{m}{k}}$$ **Ans.**

(b) In this case if the mass m moves down a distance x from its equilibrium position, then pulley will move down by $\frac{x}{2}$. So, the extra force in spring will be $k\frac{x}{2}$. Now, as the pulley is massless, this force $\frac{kx}{2}$ is equal to extra $2T$ or $T = \frac{kx}{4}$. This is also the restoring force of the mass. Hence,

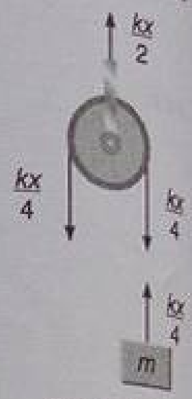


Fig. 11.40

$$F = -\frac{kx}{4}$$

or $$a = -\frac{k}{4m}x$$

or $$T = 2\pi\sqrt{\left|\frac{x}{a}\right|}$$

or $$T = 2\pi\sqrt{\frac{4m}{k}}$$ **Ans.**

(c) In this situation if the mass m moves down a distance x from its equilibrium position, the pulley will also move by x and so the spring will stretch by $2x$. Therefore, the spring force will be $2kx$. The restoring force on the block will be $4kx$. Hence,

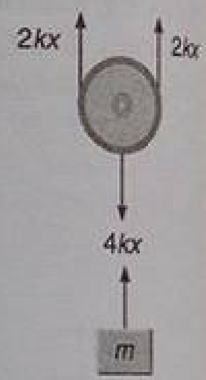


Fig. 11.41

$$F = -4kx$$

or $$a = -\frac{4k}{m}.x$$

$\therefore$ $$T = 2\pi\sqrt{\left|\frac{x}{a}\right|}$$

or $$T = 2\pi\sqrt{\frac{m}{4k}}$$ **Ans.**

Example 6 *Calculate the angular frequency of the system shown in figure. Friction is absent everywhere and the threads, spring and pulleys are massless. Given that* $m_A = m_B = m$.

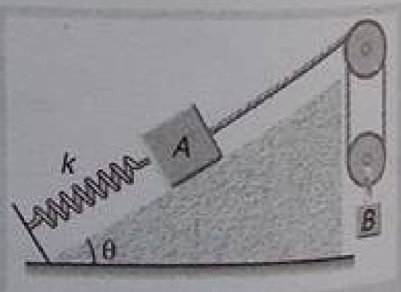


Fig. 11.42

Solution Let x_0 be the extension in the spring in equilibrium. Then equilibrium of A and B give,

$$T = kx_0 + mg\sin\theta \quad \text{...(i)}$$

and $$2T = mg \quad \text{...(ii)}$$

Here, T is the tension in the string. Now, suppose A is further displaced by a distance x from its mean position and v be its speed at this moment. Then B lowers by $\frac{x}{2}$ and speed of B at this instant will be $\frac{v}{2}$. Total energy of the system in this position will be,

$$E = \frac{1}{2}k(x+x_0)^2 + \frac{1}{2}m_A v^2 + \frac{1}{2}m_B\left(\frac{v}{2}\right)^2 + m_A g h_A - m_B g\, h_B$$

or $$E = \frac{1}{2}k(x+x_0)^2 + \frac{1}{2}mv^2 + \frac{1}{8}mv^2 + mgx\sin\theta - mg\frac{x}{2}$$

or $$E = \frac{1}{2}k(x+x_0)^2 + \frac{5}{8}mv^2 + mgx\sin\theta - mg\frac{x}{2}$$

Since, E is constant,

$$\frac{dE}{dt} = 0$$

or $$0 = k(x+x_0)\frac{dx}{dt} + \frac{5}{4}mv\left(\frac{dv}{dt}\right) + mg(\sin\theta)\left(\frac{dx}{dt}\right) - \frac{mg}{2}\left(\frac{dx}{dt}\right)$$

Substituting, $$\frac{dx}{dt} = v$$

$$\frac{dv}{dt} = a$$

and $$kx_0 + mg\sin\theta = \frac{mg}{2} \qquad \text{[From Eqs. (i) and (ii)]}$$

We get, $$\frac{5}{4}m\,a = -kx$$

Since, $$a \propto -x$$

Motion is simple harmonic, time period of which is,

$$T = 2\pi\sqrt{\left|\frac{x}{a}\right|} = 2\pi\sqrt{\frac{5m}{4k}}$$

$$\therefore \qquad \omega = \frac{2\pi}{T} = \sqrt{\frac{4k}{5m}} \qquad \textbf{Ans.}$$

Example 7 *A solid sphere (radius = R) rolls without slipping in a cylindrical through (radius = 5R). Find the time period of small oscillations.*

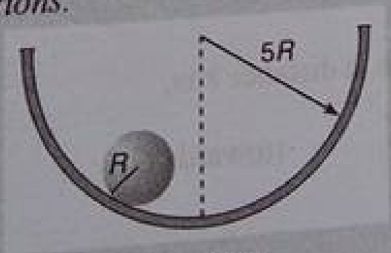


Fig. 11.43

Solution For pure rolling to take place,

$$v = R\omega$$

$$\omega' = \text{angular velocity of COM of sphere } C \text{ about } O$$

$$= \frac{v}{4R} = \frac{R\omega}{4R} = \frac{\omega}{4}$$

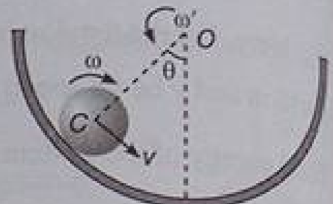


Fig. 11.44

$$\therefore \qquad \frac{d\omega'}{dt} = \frac{1}{4}\frac{d\omega}{dt}$$

or $$\alpha' = \frac{\alpha}{4}$$

$$\alpha = \frac{a}{R} \text{ for pure rolling}$$

where, $$a = \frac{g \sin\theta}{1 + \frac{I}{mR^2}} = \frac{5g\sin\theta}{7}$$

as, $$I = \frac{2}{5}mR^2$$

$$\therefore \qquad \alpha' = \frac{5g\sin\theta}{28R}$$

For small θ, $\sin\theta \approx \theta$, being restoring in nature,

$$\alpha' = -\frac{5g}{28R}\theta$$

$$\therefore \qquad T = 2\pi\sqrt{\left|\frac{\theta}{\alpha'}\right|} = 2\pi\sqrt{\frac{28R}{5g}}$$ **Ans.**

Example 8 *Consider the earth as a uniform sphere of mass M and radius R. Imagine a straight smooth tunnel made through the earth which connects any two points on its surface. Show that the motion of a particle of mass m along this tunnel under the action of gravitation would be simple harmonic. Hence, determine the time that a particle would take to go from one end to the other through the tunnel.*

Solution Suppose at some instant the particle is at radial distance r from centre of earth O. Since, the particle is constrained to move along the tunnel, we define its position as distance x from C. Hence, equation of motion of the particle is,

$$ma_x = F_x$$

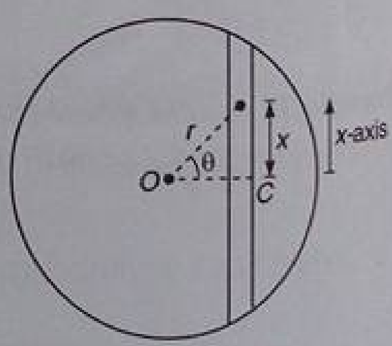


Fig. 11.45

The gravitational force on mass m at distance r is,

$$F = \frac{GMmr}{R^3} \qquad \text{(towards } O\text{)}$$

Therefore,
$$F_x = -F\sin\theta = -\frac{GMmr}{R^3}\left(\frac{x}{r}\right)$$
$$= -\frac{GMm}{R^3}\cdot x$$

Since, $F_x \propto -x$, motion is simple harmonic in nature. Further,
$$ma_x = -\frac{GMm}{R^3}\cdot x \quad \text{or} \quad a_x = -\frac{GM}{R^3}\cdot x$$

$\therefore$ Time period of oscillation is,
$$T = 2\pi\sqrt{\left|\frac{x}{a_x}\right|} = 2\pi\sqrt{\frac{R^3}{GM}}$$

The time taken by particle to go from one end to the other is $\frac{T}{2}$.

$$\therefore \quad t = \frac{T}{2} = \pi\sqrt{\frac{R^3}{GM}}$$ **Ans.**

EXERCISES

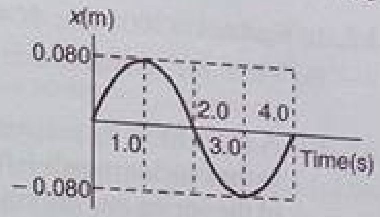
AIEEE Corner

Subjective Questions (Level 1)

Kinematics of SHM

1. A 50 g mass hangs at the end of a massless spring. When 20 g more are added to the end of the spring, it stretches 7.0 cm more.
   (a) Find the spring constant.
   (b) If the 20 g are now removed, what will be the period of the motion ?
2. A body of weight 27 N hangs on a long spring of such stiffness that an extra force of 9 N stretches the spring by 0.05 m. If the body is pulled downward and released, what is the period ?
3. A 0.5 kg body performs simple harmonic motion with a frequency of 2 Hz and an amplitude of 8 mm. Find the maximum velocity of the body, its maximum acceleration and the maximum restoring force to which the body is subjected.
4. A body describing SHM has a maximum acceleration of $8\pi\,\text{m/s}^2$ and a maximum speed of 1.6 m/s. Find the period T and the amplitude A.
5. Given that the equation of motion of a mass is $x = 0.20 \sin(2.0t)$ m. Find the velocity and acceleration of the mass when the object is 5 cm from its equilibrium position. Repeat for $x = 0$.
6. A particle executes simple harmonic motion of amplitude A along the x-axis. At $t = 0$, the position of the particle is $x = A/2$ and it moves along the positive x-direction. Find the phase constant δ, if the equation is written as $x = A \sin(\omega t + \delta)$.
7. A body makes angular simple harmonic motion of amplitude $\pi/10$ rad and time period 0.05 s. If the body is at a displacement $\theta = \pi/10$ rad at $t = 0$, write the equation giving angular displacement as a function of time.
8. The equation of motion of a particle started at $t = 0$ is given by $x = 5\sin\left(20t + \frac{\pi}{3}\right)$, where x is in cm and t in sec. When does the particle
   (a) first come to rest,
   (b) first have zero acceleration,
   (c) first have maximum speed.
9. A particle executes simple harmonic motion of period 16 s. Two seconds later after it passes through the centre of oscillation its velocity is found to be 2 m/s. Find the amplitude.
10. The period of a particle in SHM is 8 s. At $t = 0$ it is in its equilibrium position.
    (a) How much the distance it travels in the first 4 s. Compare with the distance it travels in the second 4 s?
    (b) Compare the distance travelled in the first 2 s and the second 2 s.

11. An object of mass 0.8 kg is attached to one end of a spring and the system is set into simple harmonic motion. The displacement x of the object as a function of time is shown in the figure. With the aid of the data, determine
(a) the amplitude A of the motion,
(b) the angular frequency ω,
(c) the spring constant k,
(d) the speed of the object at $t = 1.0$ s and
(e) the magnitude of the object's acceleration at $t = 1.0$ s.



12. (a) The motion of the particle in simple harmonic motion is given by $x = a \sin \omega t$. If its speed is u, when the displacement is x_1 and speed is v, when the displacement is x_2, show that the amplitude of the motion is

$$a = \left[\frac{v^2 x_1^2 - u^2 x_2^2}{v^2 - u^2}\right]^{1/2}$$

(b) A particle is moving with simple harmonic motion in a straight line. When the distance of the particle from the equilibrium position has the values x_1 and x_2, the corresponding values of velocity are u_1 and u_2, show that the period is

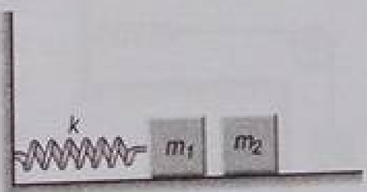
$$T = 2\pi\left[\frac{x_2^2 - x_1^2}{u_1^2 - u_2^2}\right]^{1/2}$$

Energy in SHM

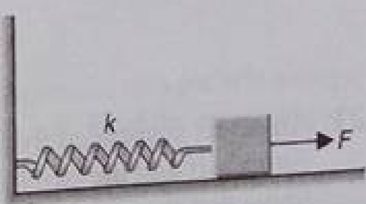
13. A spring-mass oscillator has a total energy E_0 and an amplitude x_0.
(a) How large will K and U be for it when $x = \frac{1}{2}x_0$?
(b) For what value of x will $K = U$?

14. Show that the combined spring energy and gravitational energy for a mass m hanging from a light spring of force constant k can be expressed as $\frac{1}{2}ky^2$, where y is the distance above or below the equilibrium position.

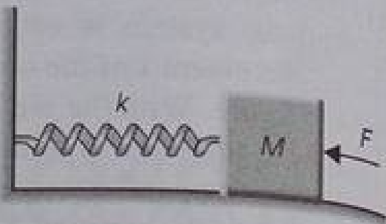
15. The masses in figure slide on a frictionless table. m_1 but not m_2, is fastened to the spring. If now m_1 and m_2 are pushed to the left, so that the spring is compressed a distance d, what will be the amplitude of the oscillation of m_1 after the spring system is released ?



16. The spring shown in figure is unstretched when a man starts pulling on the cord. The mass of the block is M. If the man exerts a constant force F, find
(a) the amplitude and the time period of the motion of the block,
(b) the energy stored in the spring when the block passes through the equilibrium position and
(c) the kinetic energy of the block at this position.



17. In figure, $k = 100$ N/m, $M = 1$ kg and $F = 10$ N.



(a) Find the compression of the spring in the equilibrium position.
(b) A sharp blow by some external agent imparts a speed of 2 m/s to the block towards left. Find the sum of the potential energy of the spring and the kinetic energy of the block at this instant.
(c) Find the time period of the resulting simple harmonic motion.
(d) Find the amplitude.
(e) Write the potential energy of the spring when the block is at the left extreme.
(f) Write the potential energy of the spring when the block is at the right extreme.

The answers of (b), (e) and (f) are different. Explain why this does not violate the principle of conservation of energy?

18. A point particle of mass 0.1 kg is executing SHM of amplitude 0.1 m. When the particle passes through the mean position, its kinetic energy is 8×10^{-3} J. Write down the equation of motion of this particle when the initial phase of oscillation is 45°.

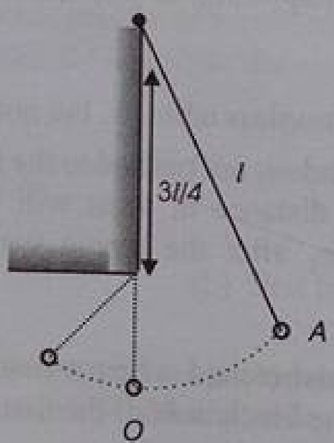
19. Potential energy of a particle in SHM along x-axis is given by :

$$U = 10 + (x - 2)^2$$

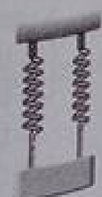
Here, U is in joule and x in metre. Total mechanical energy of the particle is 26 J. Mass of the particle is 2 kg. Find :
(a) angular frequency of SHM,
(b) potential energy and kinetic energy at mean position and extreme position,
(c) amplitude of oscillation,
(d) x-coordinates between which particle oscillates.

Time period of SHM

20. A pendulum has a period T for small oscillations. An obstacle is placed directly beneath the pivot, so that only the lowest one-quarter of the string can follow the pendulum bob when it swings to the left of its resting position. The pendulum is released from rest at a certain point. How long will it take to return to that point ? In answering this question, you may assume that the angle between the moving string and the vertical stays small throughout the motion.



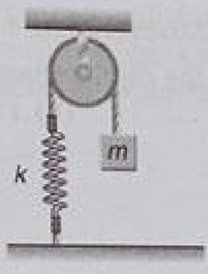
21. An object suspended from a spring exhibits oscillations of period T. Now, the spring is cut in half and the two halves are used to support the same object, as shown in figure. Show that the new period of oscillation is $T/2$.



22. Assume that a narrow tunnel is dug between two diametrically opposite points of the earth. Treat the earth as a solid sphere of uniform density. Show that if a particle is released in this tunnel, it will execute a simple harmonic motion. Calculate the time period of this motion.

23. A simple pendulum is taken at a place where its separation from the earth's surface is equal to the radius of the earth. Calculate the time period of small oscillations if the length of the string is 1.0 m. Take $g = \pi^2$ m/s^2 at the surface of the earth.

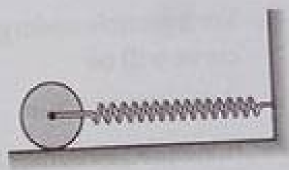
24. The string, the spring and the pulley shown in figure are light. Find the time period of the mass m.



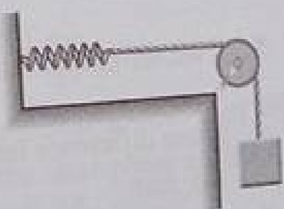
25. A solid cylinder of mass $M = 10$ kg and cross-sectional area $A = 20$ cm^2 is suspended by a spring of force constant $k = 100$ N/m and hangs partially immersed in water. Calculate the period of small oscillations of the cylinder.

26. Pendulum A is a physical pendulum made from a thin, rigid and uniform rod whose length is d. One end of this rod is attached to the ceiling by a frictionless hinge, so that the rod is free to swing back and forth. Pendulum B is a simple pendulum whose length is also d. Obtain the ratio $\frac{T_A}{T_B}$ of their periods for small angle oscillations.

27. A solid cylinder of mass m is attached to a horizontal spring with force constant k. The cylinder can roll without slipping along the horizontal plane. (See the accompanying figure.) Show that the centre of mass of the cylinder executes simple harmonic motion with a period $T = 2\pi\sqrt{\frac{3m}{2k}}$, if displaced from mean position.



28. A cord is attached between a 0.50 kg block and a spring with force constant $k = 20$ N/m. The other end of the spring is attached to the wall and the cord is placed over a pulley ($I = 0.60\ MR^2$) of mass 5.0 kg and radius 0.50 m. (See the accompanying figure). Assuming no slipping occurs, what is the frequency of the oscillations when the body is set into motion ?



Combination of two or more simple harmonic motions

29. Two linear SHM of equal amplitudes A and frequencies ω and 2ω are impressed on a particle along x and y-axes respectively. If the initial phase difference between them is $\pi/2$. Find the resultant path followed by the particle.

30. Three simple harmonic motions of equal amplitudes A and equal time periods in the same direction combine. The phase of the second motion is 60° ahead of the first and the phase of the third motion is 60° ahead of the second. Find the amplitude of the resultant motion.

31. A particle is subjected to two simple harmonic motions in the same direction having equal amplitudes and equal frequency. If the resultant amplitude is equal to the amplitude of the individual motions. Find the phase difference between the individual motions.

32. A particle is subjected to two simple harmonic motions given by

$$x_1 = 2.0 \sin(100\,\pi t)$$

and

$$x_2 = 2.0 \sin(120\,\pi t + \pi/3)$$

where x is in cm and t in second. Find the displacement of the particle at

(a) $t = 0.0125$, (b) $t = 0.025$.

Objective Questions (Level 1)

Single Correct Option

1. A simple harmonic oscillation has an amplitude A and time period T. The time required to travel from $x = A$ to $x = \frac{A}{2}$ is

(a) $\frac{T}{6}$ (b) $\frac{T}{4}$ (c) $\frac{T}{3}$ (d) $\frac{T}{12}$

2. The potential energy of a particle executing SHM varies sinusoidally with frequency f. The frequency of oscillation of the particle will be

(a) $\frac{f}{2}$ (b) $\frac{f}{\sqrt{2}}$ (c) f (d) $2f$

3. For a particle undergoing simple harmonic motion, the velocity is plotted against displacement. The curve will be

(a) a straight line (b) a parabola (c) a circle (d) an ellipse

4. A simple pendulum is made of bob which is a hollow sphere full of sand suspended by means of a wire. If all the sand is drained out, the period of the pendulum will

(a) increase (b) decrease (c) remain same (d) become erratic

5. Two simple harmonic motions are given by $y_1 = a \sin\left[\left(\frac{\pi}{2}\right)t + \phi\right]$ and $y_2 = b \sin\left[\left(\frac{2\pi}{3}\right)t + \phi\right]$. The phase difference between these after 1 s is

(a) zero (b) $\pi/2$ (c) $\pi/4$ (d) $\pi/6$

6. A particle starts performing simple harmonic motion. Its amplitude is A. At one time its speed is half that of the maximum speed. At this moment the displacement is

(a) $\frac{\sqrt{2}A}{3}$ (b) $\frac{\sqrt{3}A}{2}$ (c) $\frac{2A}{\sqrt{3}}$ (d) $\frac{3A}{\sqrt{2}}$

7. The length of a simple pendulum is decreased by 21%. The percentage change in its time period is

(a) 11% decrease (b) 11% increase (c) 21% decrease (d) 21% increase

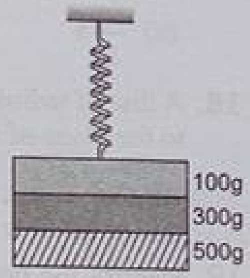
8. Which of the following is not simple harmonic function?

(a) $y = a\sin 2\omega t + b\cos^2 \omega t$
(b) $y = a\sin \omega t + b\cos 2\omega t$
(c) $y = 1 - 2\sin^2 \omega t$
(d) $y = (\sqrt{a^2 + b^2})\sin \omega t \cos \omega t$

9. A mass M, attached to a spring, oscillates with a period of 2 s. If the mass is increased by 4 kg, the time period increases by one second. Assuming that Hooke's law is obeyed, the initial mass M was

(a) 3.2 kg
(b) 1 kg
(c) 2 kg
(d) 8 kg

10. Three masses of 500 g, 300 g and 100 g are suspended at the end of a spring as shown and are in equilibrium. When the 500 g mass is removed suddenly, the system oscillates with a period of 2 seconds. When the 300 g mass is also removed, it will oscillate with period

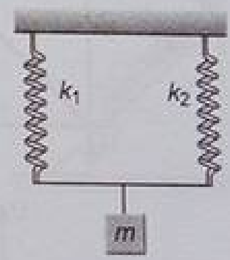


(a) 2 s
(b) 1.25 s
(c) 1.5 s
(d) 1 s

11. The displacement of a particle varies according to the relation $y = 4(\cos \pi t + \sin \pi t)$. The amplitude of the particle is

(a) 8 units
(b) 2 units
(c) 4 units
(d) $4\sqrt{2}$ units

12. A mass is suspended separately by two springs of spring constant k_1 and k_2 in successive order. The time periods of oscillations in the two cases are T_1 and T_2 respectively. If the same mass be suspended by connecting the two springs in parallel (as shown in figure) then the time period of oscillations is T.



The correct relation is

(a) $T^2 = T_1^2 + T_2^2$
(b) $T^{-2} = T_1^{-2} + T_2^{-2}$
(c) $T^{-1} = T_1^{-1} + T_2^{-1}$
(d) $T = T_1 + T_2$

13. A particle executes simple harmonic motion. Its instantaneous acceleration is given by $a = -px$ where p is a positive constant and x is the displacement from the mean position. Angular frequency of the particle is given by

(a) $\frac{1}{p}$
(b) p
(c) $\sqrt{p}$
(d) $\frac{1}{\sqrt{p}}$

14. Two pendulums X and Y of time periods 4 s and 4.2 s are made to vibrate simultaneously. They are initially in same phase. After how many vibrations of X, they will be in the same phase again?

(a) 30
(b) 25
(c) 21
(d) 26

15. A mass M is suspended from a massless spring. An additional mass m stretches the spring further by a distance x. The combined mass will oscillate with a period

(a) $2\pi\sqrt{\left\{\frac{(M+m)x}{mg}\right\}}$
(b) $2\pi\sqrt{\left\{\frac{mg}{(M+m)x}\right\}}$
(c) $2\pi\sqrt{\left\{\frac{(M+m)}{mgx}\right\}}$
(d) $\frac{\pi}{2}\sqrt{\left\{\frac{mg}{(M+m)x}\right\}}$

16. Two bodies P and Q of equal masses are suspended from two separate massless springs of force constants k_1 and k_2 respectively. If the two bodies oscillate vertically such that their maximum velocities are equal. The ratio of the amplitude of P to that of Q is

(a) $\sqrt{\frac{k_1}{k_2}}$ (b) $\frac{k_1}{k_2}$ (c) $\sqrt{\frac{k_2}{k_1}}$ (d) $\frac{k_2}{k_1}$

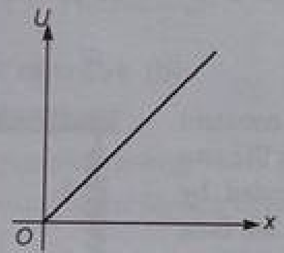
17. A simple harmonic oscillator has an acceleration of 1.25 m/s^2 at 5 cm from the equilibrium. Its period of oscillation is

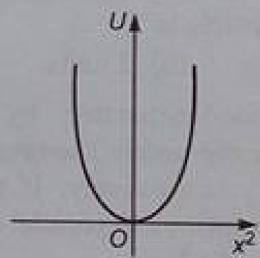
(a) $\frac{4\pi}{5}$ s (b) $\frac{5\pi}{2}$ s (c) $\frac{2\pi}{5}$ s (d) $\frac{2\pi}{25}$ s

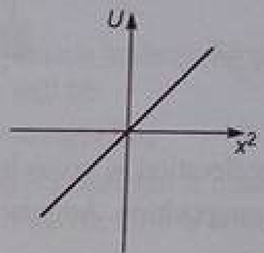
18. A disc of radius R is pivoted at its rim. The period for small oscillations about an axis perpendicular to the plane of disc is

(a) $2\pi\sqrt{\frac{R}{g}}$ (b) $2\pi\sqrt{\frac{2R}{g}}$ (c) $2\pi\sqrt{\frac{2R}{3g}}$ (d) $2\pi\sqrt{\frac{3R}{2g}}$

19. Identify the correct variation of potential energy U as a function of displacement x from mean positioin of a harmonic oscillator (U at mean position $= 0$)

(a) 

(b) 

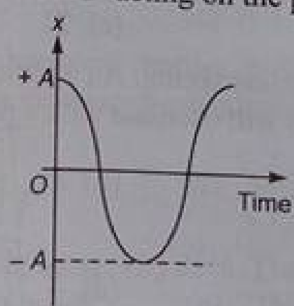
(c) 

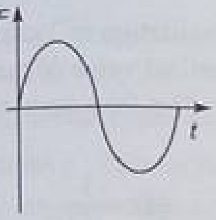
(d) None of these

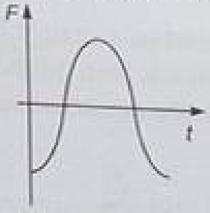
20. If the length of a simple pendulum is equal to the radius of the earth, its time period will be

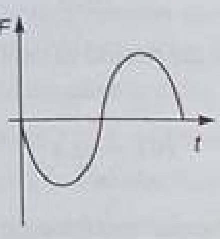
(a) $2\pi\sqrt{R/g}$ (b) $2\pi\sqrt{R/2g}$ (c) $2\pi\sqrt{2R/g}$ (d) infinite

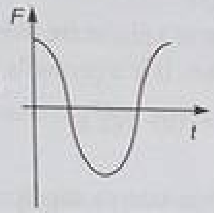
21. The displacement-time (x-t) graph of a particle executing simple harmonic motion is shown in figure. The correct variation of net force F acting on the particle as a function of time is



(a)


(b)


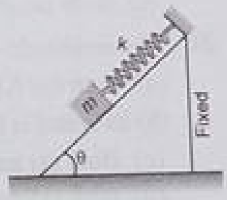
(c)


(d)


22. A mass M is suspended from a spring of negligible mass. The spring is pulled a little then released, so that the mass executes simple harmonic motion of time period T. If the mass is increased by m, the time period becomes $\frac{5T}{3}$. The ratio of m/M is

(a) $\frac{5}{3}$ (b) $\frac{3}{5}$ (c) $\frac{16}{9}$ (d) $\frac{25}{9}$

23. In the figure shown the time period and the amplitude respectively, when m is left from rest when spring is relaxed are (the inclined plane is smooth)



(a) $2\pi\sqrt{\frac{m}{k}}, \frac{mg\sin\theta}{k}$

(b) $2\pi\sqrt{\frac{m\sin\theta}{k}}, \frac{2\,mg\sin\theta}{k}$

(c) $2\pi\sqrt{\frac{m}{k}}, \frac{mg\cos\theta}{k}$

(d) None of the above

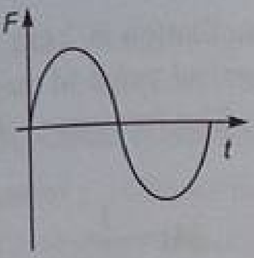
24. The equation of motion of a particle of mass 1 g is $\frac{d^2x}{dt^2} + \pi^2 x = 0$, where x is displacement (in m) from mean position. The frequency of oscillation is (in Hz)

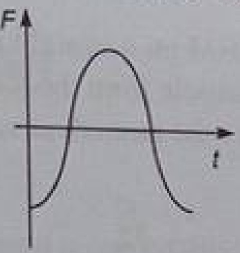
(a) 1/2 (b) 2 (c) $5\sqrt{10}$ (d) $1/5\sqrt{10}$

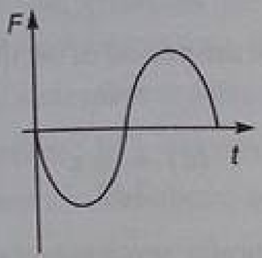
25. The spring as shown in figure is kept in a stretched position with extension x when the system is released. Assuming the horizontal surface to be frictionless, the frequency of oscillation is

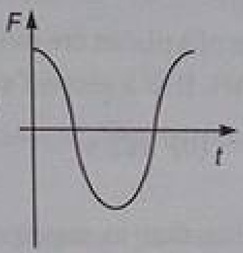
(a) $\frac{1}{2\pi}\sqrt{\left[\frac{k(M+m)}{Mm}\right]}$ (b) $\frac{1}{2\pi}\sqrt{\left[\frac{mM}{k(M+m)}\right]}$

(c) $\frac{1}{2\pi}\sqrt{\left[\frac{kM}{m+M}\right]}$ (d) $\frac{1}{2\pi}\sqrt{\left[\frac{km}{M+m}\right]}$

(a)


(b)


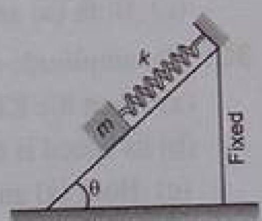
(c)


(d)


22. A mass M is suspended from a spring of negligible mass. The spring is pulled a little then released, so that the mass executes simple harmonic motion of time period T. If the mass is increased by m, the time period becomes $\frac{5T}{3}$. The ratio of m/M is

(a) $\frac{5}{3}$ (b) $\frac{3}{5}$ (c) $\frac{16}{9}$ (d) $\frac{25}{9}$

23. In the figure shown the time period and the amplitude respectively, when m is left from rest when spring is relaxed are (the inclined plane is smooth)



(a) $2\pi\sqrt{\frac{m}{k}}, \frac{mg\sin\theta}{k}$

(b) $2\pi\sqrt{\frac{m\sin\theta}{k}}, \frac{2\,mg\sin\theta}{k}$

(c) $2\pi\sqrt{\frac{m}{k}}, \frac{mg\cos\theta}{k}$

(d) None of the above

24. The equation of motion of a particle of mass 1 g is $\frac{d^2x}{dt^2} + \pi^2 x = 0$, where x is displacement (in m) from mean position. The frequency of oscillation is (in Hz)

(a) 1/2 (b) 2 (c) $5\sqrt{10}$ (d) $1/5\sqrt{10}$

25. The spring as shown in figure is kept in a stretched position with extension x when the system is released. Assuming the horizontal surface to be frictionless, the frequency of oscillation is

(a) $\frac{1}{2\pi}\sqrt{\left[\frac{k(M+m)}{Mm}\right]}$ (b) $\frac{1}{2\pi}\sqrt{\left[\frac{mM}{k(M+m)}\right]}$

(c) $\frac{1}{2\pi}\sqrt{\left[\frac{kM}{m+M}\right]}$ (d) $\frac{1}{2\pi}\sqrt{\left[\frac{km}{M+m}\right]}$

26. A particle executes SHM on a straight line path. The amplitude of oscillation is 2 cm. When the displacement of the particle from the mean position is 1 cm, the numerical value of magnitude of acceleration is equal to the numerical value of magnitude of velocity. The frequency of SHM (in Hz) is

(a) $\frac{1}{\pi}$ (b) $\frac{\sqrt{2}}{\pi}$ (c) $\frac{\sqrt{3}}{2\pi}$ (d) $\frac{1}{2\pi\sqrt{3}}$

27. The mass and diameter of a planet are twice those of earth. What will be the period of oscillation of a pendulum on this planet. It is a second's pendulum on earth?

(a) $\sqrt{2}$ s (b) $2\sqrt{2}$ s (c) $\frac{1}{\sqrt{2}}$ s (d) $\frac{1}{2\sqrt{2}}$ s

28. The resultant amplitude due to superposition of three simple harmonic motions $x_1 = 3\sin\omega t$, $x_2 = 5\sin(\omega t + 37°)$ and $x_3 = -15\cos\omega t$ is

(a) 18 (b) 10 (c) 12 (d) None of these

29. Two SHMs $s_1 = a\sin\omega t$ and $s_2 = b\sin\omega t$ are superimposed on a particle. The s_1 and s_2 are along the directions which makes 37° to each other

(a) the particle will perform SHM (b) the path of particle is straight line
(c) Both (a) and (b) are correct (d) Both (a) and (b) are wrong

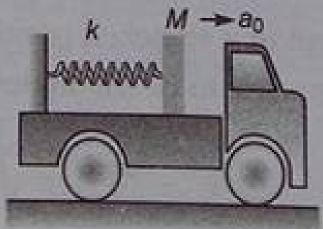
30. The amplitude of a particle executing SHM about O is 10 cm. Then

(a) when the KE is 0.64 times of its maximum KE, its displacement is 6 cm from O
(b) its speed is half the maximum speed when its displacement is half the maximum displacement
(c) Both (a) and (b) are correct
(d) Both (a) and (b) are wrong

31. A particle is attached to a vertical spring and is pulled down a distance 4 cm below its equilibrium and is released from rest. The initial upward acceleration is 0.5 ms^{-2}. The angular frequency of oscillation is

(a) 3.53 rad/s (b) 0.28 rad/s (c) 1.25 rad/s (d) 0.08 rad/s

32. A block of mass 1 kg is kept on smooth floor of a truck. One end of a spring of force constant 100 N/m is attached to the block and other end is attached to the body of truck as shown in the figure. At $t = 0$, truck begins to move with constant acceleration 2 m/s^2. The amplitude of oscillation of block relative to the floor of truck is



(a) 0.06 m (b) 0.02 m (c) 0.04 m (d) 0.03 m

JEE Corner

Assertion and Reason

Directions : *Choose the correct option.*

(a) *If both **Assertion** and **Reason** are true and the **Reason** is correct explanation of the **Assertion.***

(b) *If both **Assertion** and **Reason** are true but **Reason** is not the correct explanation of **Assertion.***

(c) *If **Assertion** is true, but the **Reason** is false.*

(d) *If **Assertion** is false but the **Reason** is true.*

1. **Assertion :** In $x = A\cos\omega t$, x is the displacement measured from extreme position.
Reason : In the above equation $x = A$ at time $t = 0$.

2. **Assertion :** A particle is under SHM along the x-axis. Its mean position is $x = 2$, amplitude is $A = 2$ and angular frequency ω. At $t = 0$, particle is at origin, then x-co-ordinate versus time equation of the particle will be $x = -2\cos\omega t + 2$.
Reason : At $t = 0$, particle is at rest.

3. **Assertion :** A spring block system is kept over a smooth surface as shown in figure. If a constant horizontal force F is applied on the block it will start oscillating simple harmonically.

Reason : Time period of oscillation is less than $2\pi\sqrt{\frac{m}{k}}$.

4. **Assertion :** Time taken by a particle in SHM to move from $x = A$ to $x = \frac{\sqrt{3}A}{2}$ is same as the time taken by the particle to move from $x = \frac{\sqrt{3}A}{2}$ to $x = \frac{A}{2}$.
Reason : Corresponding angles rotated in the reference circle are same in the given time intervals.

5. **Assertion :** Path of a particle in SHM is always a straight line.
Reason : All straight line motions are not simple harmonic.

6. **Assertion :** In spring block system if length of spring and mass of block both are halved, then angular frequency of oscillations will remain unchanged.
Reason : Angular frequency is given by $\omega = \sqrt{\frac{k}{m}}$

7. **Assertion :** All small oscillations are simple harmonic in nature.
Reason : Oscillations of spring block system are always simple harmonic whether amplitude is small or large.

8. **Assertion :** In $x = A\cos\omega t$, the dot product of acceleration and velocity is positive for time interval $0 < t < \frac{\pi}{2\omega}$.
Reason : Angle between them is $0°$.

9. **Assertion :** In simple harmonic motion displacement and acceleration always have a constant ratio.

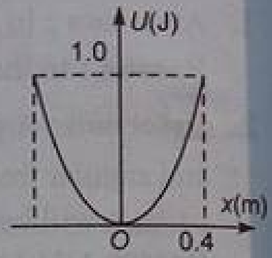
Reason : $T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}$

10. **Assertion :** We can call circular motion also as simple harmonic motion.
Reason : Angular velocity in uniform circular motion and angular frequency in simple harmonic motion have the same meanings.

Objective Questions (Level 2)

Single Correct Option

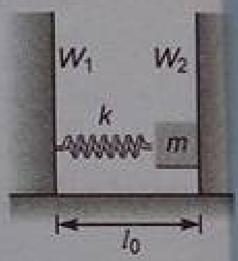
1. A particle of mass 2 kg moves in simple harmonic motion and its potential energy U varies with position x as shown. The period of oscillation of the particle is



(a) $\frac{2\pi}{5}$ s (b) $\frac{2\sqrt{2}\pi}{5}$ s

(c) $\frac{\sqrt{2}\pi}{5}$ s (d) $\frac{4\pi}{5}$ s

2. In the figure shown, a spring mass system is placed on a horizontal smooth surface in between two vertical rigid walls W_1 and W_2. One end of spring is fixed with wall W_1 and other end is attached with mass m which is free to move. Initially, spring is tension free and having natural length l_0. Mass m is compressed through a distance a and released. Taking the collision between wall W_2 and mass m as elastic and K as spring constant, the average force exerted by mass m on wall W_2 in one oscillation of block is



(a) $\frac{2aK}{\pi}$ (b) $\frac{2ma}{\pi}$ (c) $\frac{aK}{\pi}$ (d) $\frac{2aK}{m}$

3. Two simple harmonic motions are represented by the following equations $y = 40 \sin \omega t$ and $y_2 = 10 (\sin \omega t + c \cos \omega t)$. If their displacement amplitudes are equal, then the value of c (in appropriate units) is

(a) $\sqrt{13}$ (b) $\sqrt{15}$ (c) $\sqrt{17}$ (d) 4

4. A particle executes simple harmonic motion with frequency 2.5 Hz and amplitude 2 m. The speed of the particle 0.3 s after crossing, the equilibrium position is

(a) zero (b) 2π m/s (c) 4π m/s (d) π m/s

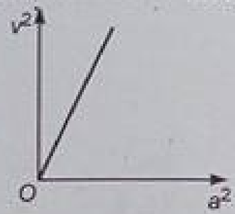
5. A particle oscillates simple harmonically with a period of 16 s. Two second after crossing the equilibrium position its velocity becomes 1 m/s. The amplitude is

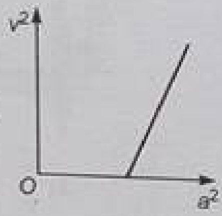
(a) $\frac{\pi}{4}$ m (b) $\frac{8\sqrt{2}}{\pi}$ m (c) $\frac{8}{\pi}$ m (d) $\frac{4\sqrt{2}}{\pi}$ m

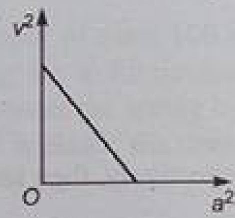
6. A seconds pendulum is suspended from the ceiling of a trolley moving horizontally with an acceleration of 4 m/s^2. Its period of oscillation is

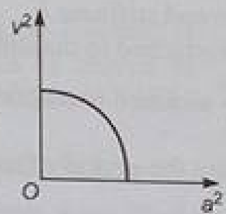
(a) 1.90 s (b) 1.70 s (c) 2.30 s (d) 1.40 s

7. A particle is performing a linear simple harmonic motion. If the instantaneous acceleration and velocity of the particle are a and v respectively, identify the graph which correctly represents the relation between a and v

(a) 

(b) 

(c) 

(d) 

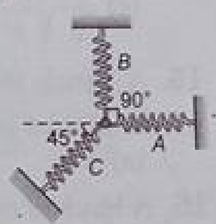
8. In a vertical U-tube a column of mercury oscillates simple harmonically. If the tube contains 1 kg of mercury and 1 cm of mercury column weighs 20 g, then the period of oscillation is

(a) 1 s (b) 2 s (c) $\sqrt{2}$ s (d) Insufficient data

9. A solid cube of side a and density ρ_0 floats on the surface of a liquid of density ρ. If the cube is slightly pushed downward, then it oscillates simple harmonically with a period of

(a) $2\pi\sqrt{\frac{\rho_0}{\rho}\frac{a}{g}}$ (b) $2\pi\sqrt{\frac{\rho}{\rho_0}\frac{a}{g}}$ (c) $2\pi\sqrt{\frac{a}{\left(1-\frac{\rho}{\rho_0}\right)g}}$ (d) $2\pi\sqrt{\frac{a}{\left(1+\frac{\rho}{\rho_0}\right)g}}$

10. A particle of mass m is attached with three springs A, B and C of equal force constants k as shown in figure. The particle is pushed slightly against the spring C and released, the time period of oscillation will be



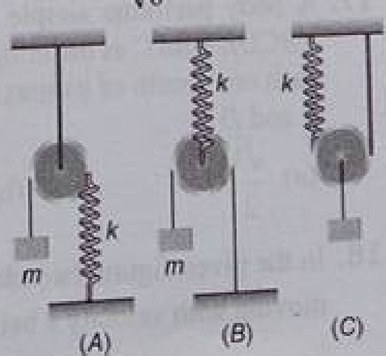
(a) $2\pi\sqrt{\frac{m}{k}}$ (b) $2\pi\sqrt{\frac{m}{2k}}$

(c) $2\pi\sqrt{\frac{m}{3k}}$ (d) $2\pi\sqrt{\frac{m}{5k}}$

11. A uniform stick of length l is mounted so as to rotate about a horizontal axis perpendicular to the stick and at a distance d from the centre of mass. The time period of small oscillations has a minimum value when d/l is

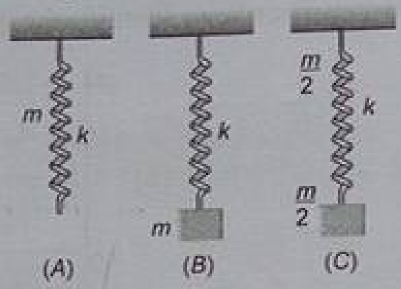
(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{12}}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{\sqrt{6}}$

12. Three arrangements of spring-mass system are shown in figures (A), (B) and (C). If T_1, T_2 and T_3 represent the respective periods of oscillation, then correct relation is



(a) $T_1 > T_2 > T_3$
(b) $T_3 > T_2 > T_1$
(c) $T_2 > T_1 > T_3$
(d) $T_2 > T_3 > T_1$

13. Three arrangements are shown in figure.



(A) A spring of mass m and stiffness k

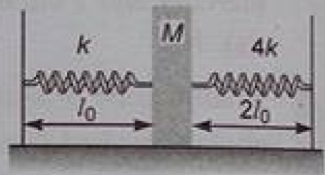
(B) A block of mass m attached to massless spring of stiffness k

(C) A block of mass $\frac{m}{2}$ attached to a spring of mass $\frac{m}{2}$ and stiffness k

If T_1, T_2 and T_3 represent the period of oscillation in the three cases respectively, then identify the correct relation

(a) $T_1 < T_2 < T_3$ (b) $T_1 < T_3 < T_2$ (c) $T_1 > T_3 > T_2$ (d) $T_3 < T_1 < T_2$

14. A block of mass M is kept on a smooth surface and touches the two springs as shown in the figure but not attached to the springs. Initially springs are in their natural length. Now, the block is shifted $(l_0/2)$ from the given position in such a way that it compresses a spring and released. The time-period of oscillation of mass will be



(a) $\frac{\pi}{2}\sqrt{\frac{M}{k}}$ (b) $2\pi\sqrt{\frac{M}{5k}}$

(c) $\frac{3\pi}{2}\sqrt{\frac{M}{k}}$ (d) $\pi\sqrt{\frac{M}{2k}}$

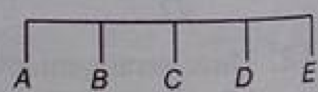
15. A particle moving on x-axis has potential energy $U = 2 - 20x + 5x^2$ Joule along x-axis. The particle is released at $x = -3$. The maximum value of x will be (x is in metre)

(a) 5 m (b) 3 m (c) 7 m (d) 8 m

16. A block of mass m, when attached to a uniform ideal spring with force constant k and free length L executes SHM. The spring is then cut in two pieces, one with free length nL and other with free length $(1-n)L$. The block is also divided in the same fraction. The smaller part of the block attached to longer part of the spring executes SHM with frequency f_1. The bigger part of the block attached to smaller part of the spring executes SHM with frequency f_2. The ratio f_1/f_2 is

(a) 1 (b) $\frac{n}{1-n}$ (c) $\frac{1+n}{n}$ (d) $\frac{n}{1+n}$

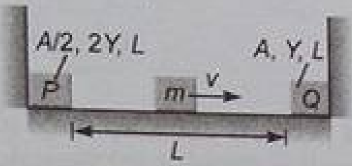
17. A body performs simple harmonic oscillations along the straight line $ABCDE$ with C as the midpoint of AE. Its kinetic energies at B and D are each one fourth of its maximum value. If $AE = 2R$, the distance between B and D is



(a) $\frac{\sqrt{3}}{2}R$ (b) $\frac{R}{\sqrt{2}}$ (c) $\sqrt{3}R$ (d) $\sqrt{2}R$

18. In the given figure, two elastic rods P and Q are rigidly joined to end supports. A small mass m is moving with velocity v between the rods. All collisions are assumed to be elastic and the surface is

given to be smooth. The time period of small mass m will be; (A = area of cross section, Y = Young's modulus, L = length of each rod)



(a) $\frac{2L}{v}+2\pi\sqrt{\frac{mL}{AY}}$ (b) $\frac{2L}{v}+2\pi\sqrt{\frac{2mL}{AY}}$ (c) $\frac{2L}{v}+\pi\sqrt{\frac{mL}{AY}}$ (d) $\frac{2L}{v}$

19. A block of mass 100 g attached to a spring of stiffness 100 N/m is lying on a frictionless floor as shown. The block is moved to compress the spring by 10 cm and released. If the collision with the wall is elastic the time period of motion is

(a) 0.2 s (b) 0.166 s (c) 0.155 s (d) 0.133 s

20. A particle executes SHM of period 1.2 s and amplitude 8 cm. Find the time it takes to travel 3 cm from the positive extremity of its oscillation. [$\cos^{-1}(5/8)=0.9$ rad]

(a) 0.28 s (b) 0.32 s (c) 0.17 s (d) 0.42 s

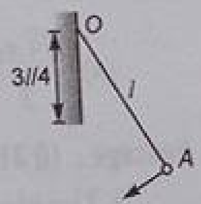
21. A wire frame in the shape of an equilateral triangle is hinged at one vertex so that it can swing freely in a vertical plane, with the plane of the triangle always remaining vertical. The side of the frame is $1/\sqrt{3}$ m. The time period in seconds of small oscillations of the frame will be ($g = 10$ m/s^2)

(a) $\pi/\sqrt{2}$ (b) $\pi/\sqrt{3}$ (c) $\pi/\sqrt{6}$ (d) $\pi/\sqrt{5}$

22. A particle moves along the x-axis according to $x = A[1+\sin\omega t]$. What distance does is travel in time interval from $t=0$ to $t=2.5\,\pi/\omega$?

(a) $4A$ (b) $6A$ (c) $5A$ (d) $3A$

23. A small bob attached to a light inextensible thread of length l has a periodic time T when allowed to vibrate as a simple pendulum. The thread is now suspended from a fixed end O of a vertical rigid rod of length $3l/4$. If now the pendulum performs periodic oscillations in this arrangement, the periodic time will be

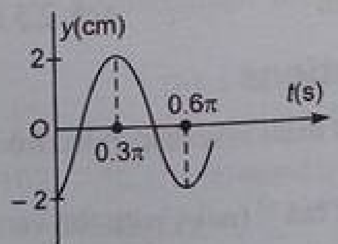


(a) $3T/4$ (b) $4T/5$
(c) $2T/3$ (d) $5T/6$

24. A stone is swinging in a horizontal circle of diameter 0.8 m at 30 rev/min. A distant light causes a shadow of the stone on a nearly wall. The amplitude and period of the SHM for the shadow of the stone are

(a) 0.4 m, 4s (b) 0.2 m, 2s (c) 0.4 m, 2s (d) 0.8 m, 2s

25. Part of SHM is graphed in the figure. Here y is displacement from mean position. The correct equation describing the SHM is



(a) $y = 4\cos(0.6\,t)$ (b) $y = 2\sin\left(\frac{10}{3}t - \frac{\pi}{2}\right)$

(c) $y = 2\sin\left(\frac{\pi}{2} - \frac{10}{3}t\right)$ (d) $y = 2\cos\left(0.6\,t + \frac{\pi}{2}\right)$

26. A particle performs SHM with a period T and amplitude a. The mean velocity of particle over the time interval during which it travels a distance $a/2$ from the extreme position is

(a) $6\,a/T$ (b) $2\,a/T$ (c) $3\,a/T$ (d) $a/2T$

27. A man of mass 60 kg is standing on a platform executing SHM in the vertical plane. The displacement from the mean position varies as $y = 0.5\sin(2\pi ft)$. The value of f, for which the man will feel weightlessness at the highest point, is (y in metre)

(a) $g/4\pi$ (b) $4\pi g$ (c) $\frac{\sqrt{2g}}{2\pi}$ (d) $2\pi\sqrt{2g}$

28. A particle performs SHM on a straight line with time period T and amplitude A. The average speed of the particle between two successive instants, when potential energy and kinetic energy become same is

(a) $\frac{A}{T}$ (b) $\frac{4\sqrt{2}A}{T}$ (c) $\frac{2A}{T}$ (d) $\frac{2\sqrt{2}A}{T}$

29. The time taken by a particle performing SHM to pass from point A to B where its velocities are same is 2 s. After another 2 s it returns to B. The ratio of distance OB to its amplitude (where O is the mean position) is

(a) $1:\sqrt{2}$ (b) $(\sqrt{2}-1):1$ (c) $1:2$ (d) $1:2\sqrt{2}$

30. A particle is executing SHM according to the equation $x = A\cos\omega t$. Average speed of the particle during the interval $0 \le t \le \frac{\pi}{6\omega}$ is

(a) $\frac{\sqrt{3}A\omega}{2}$ (b) $\frac{\sqrt{3}A\omega}{4}$ (c) $\frac{3\,A\omega}{\pi}$ (d) $\frac{3\,A\omega}{\pi}(2-\sqrt{3})$

Passage : (Q 31 to Q 32)

A 2 kg block hangs without vibrating at the bottom end of a spring with a force constant of 400 N/m. The top end of the spring is attached to the ceiling of an elevator car. The car is rising with an upward acceleration of 5 m/s^2 when the acceleration suddenly ceases at time $t = 0$ and the car moves upward with constant speed ($g = 10$ m/s^2)

31. What is the angular frequency of oscillation of the block after the acceleration ceases?

(a) $10\sqrt{2}$ rad/s (b) 20 rad/s (c) $20\sqrt{2}$ rad/s (d) 32 rad/s

32. The amplitude of the oscillation is

(a) 7.5 cm (b) 5 cm (c) 2.5 cm (d) 1 cm

More than One Correct Options

1. A simple pendulum with a bob of mass m is suspended from the roof of a car moving with horizontal acceleration a

(a) The string makes an angle of $\tan^{-1}(a/g)$ with the vertical

(b) The string makes an angle of $\sin^{-1}\left(\frac{a}{g}\right)$ with the vertical

(c) The tension in the string is $m\sqrt{a^2+g^2}$

(d) The tension in the string is $m\sqrt{g^2-a^2}$

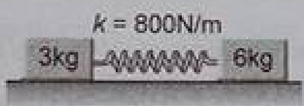
2. A particle starts from a point P at a distance of $A/2$ from the mean position O and travels towards left as shown in the figure. If the time period of SHM, executed about O is T and amplitude A then the equation of the motion of particle is

(a) $x = A\sin\left(\frac{2\pi}{T}t+\frac{\pi}{6}\right)$
(b) $x = A\sin\left(\frac{2\pi}{T}t+\frac{5\pi}{6}\right)$
(c) $x = A\cos\left(\frac{2\pi}{T}t+\frac{\pi}{6}\right)$
(d) $x = A\cos\left(\frac{2\pi}{T}t+\frac{\pi}{3}\right)$

3. A spring has natural length 40 cm and spring constant 500 N/m. A block of mass 1 kg is attached at one end of the spring and other end of the spring is attached to a ceiling. The block is released from the position, where the spring has length 45 cm

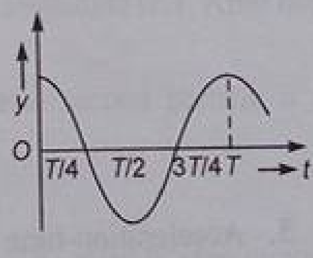
(a) the block will perform SHM of amplitude 5 cm
(b) the block will have maximum velocity $30\sqrt{5}$ cm/s
(c) the block will have maximum acceleration $15\ \text{m/s}^2$
(d) the minimum elastic potential energy of the spring will be zero

4. The system shown in the figure can move on a smooth surface. They are initially compressed by 6 cm and then released



(a) The system performs, SHM with time period $\frac{\pi}{10}$ s
(b) The block of mass 3 kg perform SHM with amplitude 4 cm
(c) The block of mass 6 kg will have maximum momentum of 2.40 kg m/s
(d) The time periods of two blocks are in the ratio of $1:\sqrt{2}$

5. The displacement-time graph of a particle executing SHM is shown in figure. Which of the following statements is/are true?



(a) The velocity is maximum at $t = T/2$
(b) The acceleration is maximum at $t = T$
(c) The force is zero at $t = 3T/4$
(d) The kinetic energy equals the total oscillation energy at $t = T/2$

6. For a particle executing SHM, x = displacement from mean position, v = velocity and a = acceleration at any instant, then

(a) v-x graph is a circle
(b) v-x graph is an ellipse
(c) a-x graph is a straight line
(d) a-x graph is a circle

7. The acceleration of a particle is $a = -100x + 50$. It is released from $x = 2$. Here a and x are in SI units

(a) the particle will perform SHM of amplitude 2 m
(b) the particle will perform SHM of amplitude 1.5 m

(c) the particle will perform SHM of time period 0.63 s
(d) the particle will have a maximum velocity of 15 m/s

8. Two particles are performing SHM in same phase. It means that
(a) the two particles must have same distance from the mean position simultaneously
(b) two particles may have same distance from the mean position simultaneously
(c) the two particles must have maximum speed simultaneously
(d) the two particles may have maximum speed simultaneously

9. A particle moves along y-axis according to the equation

$$y\,(\text{in cm}) = 3\sin 100\pi t + 8\sin^2 50\pi t - 6$$

(a) the particle performs SHM
(b) the amplitude of the particle's oscillation is 5 cm
(c) the mean position of the particle is at $y = -2$ cm
(d) the particle does not perform SHM

Match the Columns

1. For the x-t equation of a particle in SHM along x-axis, match the following two columns.

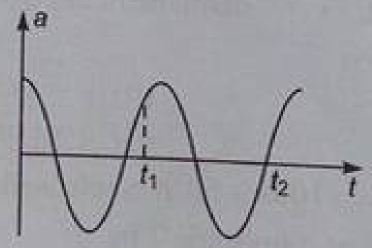
$$x = 2 + 2\cos\omega t$$

Column I	Column II
(a) Mean position	(p) $x = 0$
(b) Extreme position	(q) $x = 2$
(c) Maximum potential energy at	(r) $x = 4$
(d) Zero potential energy at	(s) Can't tell

2. Potential energy of a particle at mean position is 4 J and at extreme position is 20 J. Given that amplitude of oscillation is A. Match the following two columns.

Column I	Column II
(a) Potential energy at $x = \frac{A}{2}$	(p) 18 J
(b) Kinetic energy at $x = \frac{A}{4}$	(q) 16 J
(c) Kinetic energy at $x = 0$	(r) 8 J
(d) Kinetic energy at $x = \frac{A}{2}$	(s) None

3. Acceleration-time graph of a particle in SHM is as shown in figure. Match the following two columns.



Column I	Column II
(a) Displacement of particle at t_1	(p) zero
(b) Displacement of particle at t_2	(q) positive
(c) Velocity of particle at t_1	(r) negative
(d) Velocity of particle at t_2	(s) maximum

4. Mass of a particle is 2 kg. Its displacement-time equation in SHM is

$$x = 2\sin(4\pi t) \qquad \text{(SI Units)}$$

Match the following two columns for 1 second time interval.

Column I	Column II
(a) Speed becomes 30 m/s	(p) two times
(b) Velocity becomes + 10 m/s	(q) four times
(c) Kinetic energy becomes 400 J	(r) one time
(d) Acceleration becomes $-100\ \text{m/s}^2$	(s) None

5. x-t equation of a particle in SHM is,

$$x = 4 + 6\sin \pi t$$

Match the following tables corresponding to time taken in moving from

Column I	Column II
(a) $x = 10$ m to $x = 4$ m	(p) $\frac{1}{3}$ second
(b) $x = 10$ m to $x = 7$ m	(q) $\frac{1}{2}$ second
(c) $x = 7$ m to $x = 1$ m	(r) 1 second
(d) $x = 10$ m to $x = -2$ m	(s) None

Subjective Questions (Level 2)

1. A 1 kg block is executing simple harmonic motion of amplitude 0.1 m on a smooth horizontal surface under the restoring force of a spring of spring constant 100 N/m. A block of mass 3 kg is gently placed on it at the instant it passes through the mean position. Assuming that the two blocks move together. Find the frequency and the amplitude of the motion.

2. Two particles are in SHM along same line. Time period of each is T and amplitude is A. After how much time will they collide if at time $t = 0$.

(a) first particle is at $x_1 = +\frac{A}{2}$ and moving towards positive x-axis and second particle is at $x_2 = -\frac{A}{\sqrt{2}}$ and moving towards negative x-axis,

(b) rest information are same as mentioned in part (a) except that particle first is also moving towards negative x-axis.

3. A particle that hangs from a spring oscillates with an angular frequency of 2 rad/s. The spring particle system is suspended from the ceiling of an elevator car and hangs motionless (relative to the elevator car) as the car descends at a constant speed of 1.5 m/s. The car then stops suddenly.

(a) With what amplitude does the particle oscillate ?

(b) What is the equation of motion for the particle ?

(Choose upward as the positive direction)

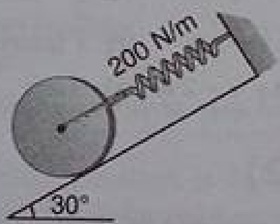
4. A 2 kg mass is attached to a spring of force constant 600 N/m and rests on a smooth horizontal surface. A second mass of 1 kg slides along the surface toward the first at 6 m/s.
   (a) Find the amplitude of oscillation if the masses make a perfectly inelastic collision and remain together on the spring. What is the period of oscillation ?
   (b) Find the amplitude and period of oscillation if the collision is perfectly elastic.
   (c) For each case, write down the position x as a function of time t for the mass attached to the spring, assuming that the collision occurs at time $t = 0$. What is the impulse given to the 2 kg mass in each case ?

5. A block of mass 4 kg hangs from a spring of force constant $k = 400$ N/m. The block is pulled down 15 cm below equilibrium and released. How long does it take the block to go from 12 cm below equilibrium (on the way up) to 9 cm above equilibrium?

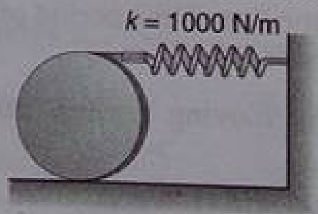
6. A plank with a body of mass m placed on it starts moving straight up according to the law $y = a(1 - \cos \omega t)$, where y is the displacement from the initial position, $\omega = 11$ rad/s. Find :
   (a) The time dependence of the force that the body exerts on the plank.
   (b) The minimum amplitude of oscillation of the plank at which the body starts falling behind the plank.

7. A particle of mass m free to move in the x-y plane is subjected to a force whose components are $F_x = -kx$ and $F_y = -ky$, where k is a constant. The particle is released when $t = 0$ at the point (2, 3). Prove that the subsequent motion is simple harmonic along the straight line $2y - 3x = 0$.

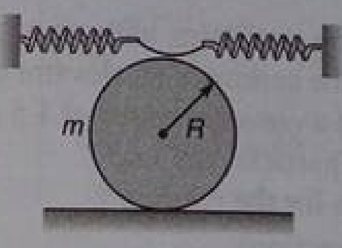
8. Determine the natural frequency of vibration of the 100 N disk. Assume the disk does not slip on the inclined surface.



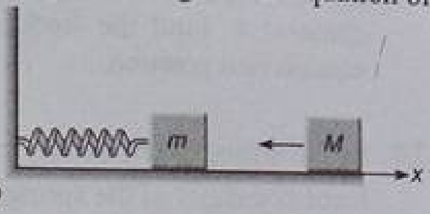
9. The disk has a weight of 100 N and rolls without slipping on the horizontal surface as it oscillates about its equilibrium position. If the disk is displaced, by rolling it counterclockwise 0.4 rad, determine the equation which describes its oscillatory motion when it is released.



10. A solid uniform cylinder of mass m performs small oscillations due to the action of two springs of stiffness k each (figure). Find the period of these oscillations in the absence of sliding.

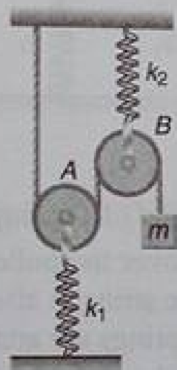


11. One end of an ideal spring is fixed to a wall at origin O and axis of spring is parallel to x-axis. A block of mass $m=1$ kg is attached to free end of the spring and it is performing SHM. Equation of position of the block in co-ordinate system shown in figure is $x=10+3\sin(10t)$. Here, t is in second and x in cm. Another block of mass $M=3$ kg, moving towards the origin with velocity 30 cm/s collides with the block performing SHM at $t=0$ and gets stuck to it. Calculate :

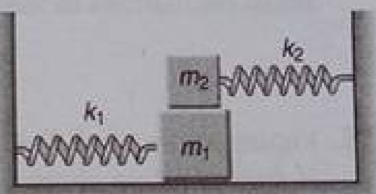


(a) new amplitude of oscillations,
(b) new equation for position of the combined body,
(c) loss of energy during collision. Neglect friction.

12. A block of mass m is attached to one end of a light inextensible string passing over a smooth light pulley B and under another smooth light pulley A as shown in the figure. The other end of the string is fixed to a ceiling. A and B are held by springs of spring constants k_1 and k_2. Find angular frequency of small oscillations of the system.



13. In the shown arrangement, both the springs are in their natural lengths. The coefficient of friction between m_2 and m_1 is μ. There is no friction between m_1 and the surface. If the blocks are displaced slightly, they together perform simple harmonic motion. Obtain

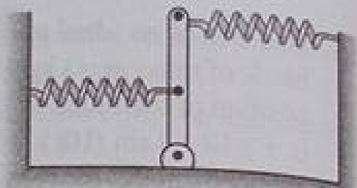


(a) Frequency of such oscillations.
(b) The condition if the frictional force on block m_2 is to act in the direction of its displacement from mean position.
(c) If the condition obtained in (b) is met, what can be maximum amplitude of their oscillations?

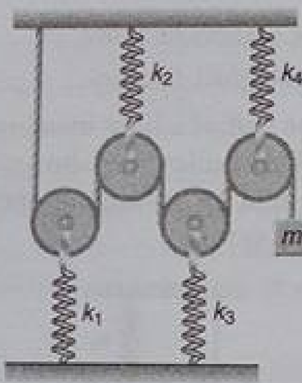
14. Two blocks A and B of masses $m_1=3$ kg and $m_2=6$ kg respectively are connected with each other by a spring of force constant $k=200$ N/m as shown in figure. Blocks are pulled away from each other by $x_0=3$ cm and then released. When spring is in its natural length and blocks are moving towards each other, another block C of mass $m=3$ kg moving with velocity $v_0=0.4$ m/s (towards right) collides with A and gets stuck to it. Neglecting friction, calculate

(a) velocities v_1 and v_2 of the blocks A and B respectively just before collision and their angular frequency.
(b) velocity of centre of mass of the system, after collision,
(c) amplitude of oscillations of combined body,
(d) loss of energy during collision.

15. A rod of length l and mass m, pivoted at one end, is held by a spring at its mid-point and a spring at far end. The springs have spring constant k. Find the frequency of small oscillations about the equilibrium position.

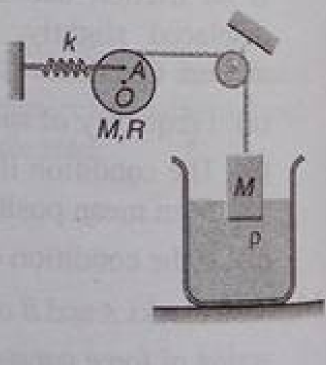


16. In the arrangement shown in figure, pulleys are light and springs are ideal. k_1, k_2, k_3 and k_4 are force constants of the springs. Calculate period of small vertical oscillations of block of mass m.

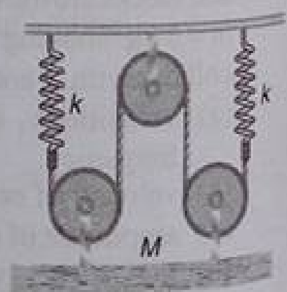


17. A light pulley is suspended at the lower end of a spring of constant k_1, as shown in figure. An inextensible string passes over the pulley. At one end of string a mass m is suspended, the other end of the string is attached to another spring of constant k_2. The other ends of both the springs are attached to rigid supports, as shown. Neglecting masses of springs and any friction, find the time period of small oscillations of mass m about equilibrium position.

18. Figure shows a solid uniform cylinder of radius R and mass M, which is free to rotate about a fixed horizontal axis O and passes through centre of the cylinder. One end of an ideal spring of force constant k is fixed and the other end is higned to the cylinder at A. Distance OA is equal to $\frac{R}{2}$. An inextensible thread is wrapped round the cylinder and passes over a smooth, small pulley. A block of equal mass M and having cross sectional area A is suspended from free end of the thread. The block is partially immersed in a non-viscous liquid of density ρ. If in equilibrium, spring is horizontal and line OA is vertical, calculate frequency of small oscillations of the system.



19. Find the natural frequency of the system shown in figure. The pulleys are smooth and massless.



Answers

Introductory Exercise 11.1

1. 6.0 N/m 2. $\frac{1}{4}, \frac{3}{4}$ 3. 10.0 cm, $\frac{\pi}{6}$ rad
4. (a) 15.0 cm (b) 0.726 s (c) 1.38 Hz (d) 1.69 J (e) 1.30 m/s 5. 1.4×10^{-3} s
6. (a) 1.39 J (b) 1.1 s 7. (a) 0.28 s (b) 0.4%

Introductory Exercise 11.2

1. $2\pi \sqrt{\frac{l}{\left\{g^2 + \left(\frac{v^2}{r}\right)^2\right\}^{1/2}}}$ and inclined to the vertical at an angle $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$ away from the centre
2. (a) $2\pi\sqrt{\frac{l}{(g+a)}}$ (b) $2\pi\sqrt{\frac{l}{(g-a)}}$ (c) Infinite (d) $2\pi\sqrt{\frac{l}{(g^2+a^2)^{1/2}}}$
3. The clock will lose 10.37 s 4. $\left(\sqrt{\frac{10}{9}}\right)T$

Introductory Exercise 11.3

1. $T = 2\pi\sqrt{\frac{m}{k}}$ 2. 0.314 s 3. $\frac{mv_0}{\sqrt{k(M+m)}}$ 4. $\frac{1}{\sqrt{2}}$ times

Introductory Exercise 11.4

1. $T = 2\pi\sqrt{\frac{\frac{3R}{2} + \frac{r^2}{2R}}{g}}$ 2. 1.4×10^5 g-cm^2

Introductory Exercise 11.5

1. (a) 7.0 cm (b) 6.1 cm (c) 5.0 cm (d) 1.0 cm
2. (a) $\frac{3\sqrt{3}}{2}$ unit (b) $100\pi\sqrt{37}$ units (c) $(100\pi)^2\sqrt{37}$ units.

AIEEE Corner

Subjective Questions (Level 1)

1. (a) 2.8 N/m (b) 0.84 s 2. 0.78 s 3. 0.101 m/s, 1.264 m/s^2, 0.632 N
4. 0.4 s, 0.102 m 5. 0.58 m/s, -0.45 m/s^{-2}, 0.60 m/s^2, zero 6. $\frac{\pi}{6}$ 7. $\theta = \left(\frac{\pi}{10}\text{ rad}\right)\cos[(40\pi s^{-1})t]$
8. (a) $\frac{\pi}{120}$ sec (b) $\frac{\pi}{30}$ sec (c) $\frac{\pi}{30}$ sec 9. 7.2 m 10. (a) equal (b) equal
11. (a) 0.08 m (b) 1.57 rad/s (c) 1.97 N/s (d) zero (e) 0.197 m/s^2
13. (a) $U = E_0/4, K = 3E_0/4$ (b) $x = \frac{x_0}{\sqrt{2}}$ 15. $A = d\sqrt{\frac{m_1}{m_1+m_2}}$ 16. (a) $\frac{F}{k}, 2\pi\sqrt{\frac{M}{k}}$ (b) $\frac{F^2}{2k}$ (c) $\frac{F^2}{2k}$
17. (a) 10 cm (b) 2.5 J (c) $\frac{\pi}{5}$ sec (d) 20 cm (e) 4.5 J (f) 0.5 J 18. $y = (0.1\text{ m})\sin\left[(4s^{-1})t + \frac{\pi}{4}\right]$
19. (a) 1 rad/s (b) $U_{mean} = 10$ J, $K_{mean} = 16$ J, $U_{extreme} = 26$ J, $K_{extreme} = 0$ (c) 4 m (d) $x = 6$ m and $x = -2$ m
20. $3T/4$ 22. $T = 2\pi\sqrt{\frac{R^3}{GM}}$ 23. 4 s 24. $T = 2\pi\sqrt{\frac{m}{k}}$ 25. 1.8 s 26. 0.816 28. 0.38 Hz

29. Parabola, $y = A\left(1 - \frac{2x^2}{A^2}\right)$ 30. 2 A 31. $\frac{2\pi}{3}$ 32. (a) –2.41 cm (b) 0.27 cm

Objective Questions (Level 1)

1. (a)	2. (a)	3. (d)	4. (c)	5. (d)	6. (b)	7. (a)	8. (b)	9. (a)	10. (d)
11. (d)	12. (b)	13. (c)	14. (c)	15. (a)	16. (c)	17. (c)	18. (d)	19. (c)	20. (b)
21. (b)	22. (c)	23. (a)	24. (a)	25. (a)	26. (c)	27. (b)	28. (d)	29. (c)	30. (a)
31. (a)	32. (b)								

JEE Corner

Assertion and Reason

1. (d)	2. (b)	3. (c)	4. (a)	5. (d)	6. (d)	7. (d)	8. (a)	9. (a)	10. (d)

Objective Questions (Level 2)

1. (d)	2. (a)	3. (b)	4. (a)	5. (b)	6. (a)	7. (c)	8. (a)	9. (a)	10. (b)
11. (b)	12. (c)	13. (b)	14. (c)	15. (c)	16. (a)	17. (c)	18. (a)	19. (d)	20. (c)
21. (d)	22. (c)	23. (a)	24. (c)	25. (b)	26. (c)	27. (c)	28. (b)	29. (a)	30. (d)
31. (a)	32. (c)								

More than One Correct Options

1. (a,c)	2. (b,d)	3. (b,c,d)	4. (a,b,c)	5. (b,c)	6. (b,c)	7. (b,c,d)
8. (b,c)	9. (a,b,c)					

Match the Columns

1.	(a) → q	(b) → p,r	(c) → p,r	(d) → s
2.	(a) → r	(b) → s	(c) → q	(d) → s
3.	(a) → r	(b) → p	(c) → r	(d) → r,s
4.	(a) → s	(b) → p	(c) → s	(d) → p
5.	(a) → q	(b) → p	(c) → p	(d) → r

Subjective Questions (Level 2)

1. 0.8 Hz, 0.05 m 2. (a) $\frac{19}{48}$ T (b) $\frac{11}{48}$ T 3. (a) $A = 0.75$ m (b) $x = -0.75 \sin 2t$

4. (a) 14.1 cm, 0.44 s (b) 23 cm, 0.36 s (c) $x = \pm(14.1 \text{ cm}) \sin(10\sqrt{2}\,t)$, $x = \pm(23 \text{ cm}) \sin(10\sqrt{3}\,t)$, 4 N-s, 8 N-s

5. $\frac{\pi}{20}$ s = 0.157 s 6. (a) $N = m(g + a\omega^2 \cos \omega t)$ (b) 8.1 cm 8. 0.56 Hz 9. $\theta = 0.4 \cos(16.16\,t)$

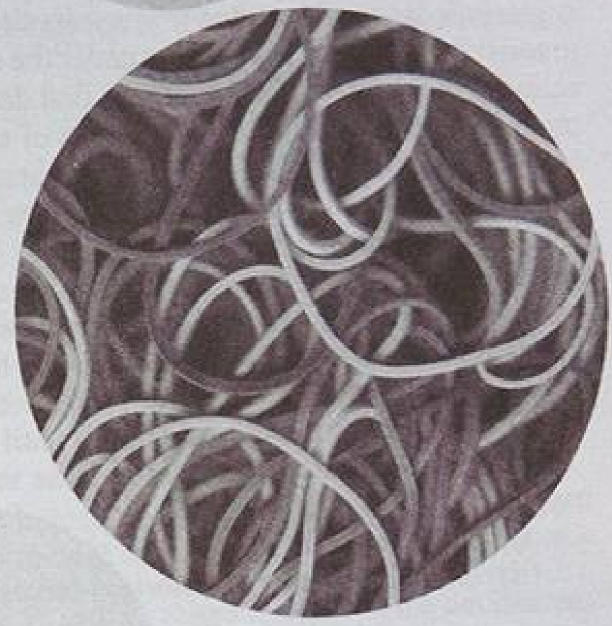
10. $T = \frac{\pi}{2}\sqrt{\frac{3m}{k}}$ 11. (a) 3 cm (b) $x = 10 - 3 \sin 5t$ (c) 0.135 J 12. $\sqrt{\frac{k_1 k_2}{4m(k_1 + k_2)}}$

13. (a) $\frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m_1 + m_2}}$ (b) $\frac{k_1}{k_2} < \frac{m_1}{m_2}$ (c) $\frac{\mu(m_1 + m_2)m_2 g}{m_1 k_2 - m_2 k_1}$

14. (a) 0.2 m/s, 0.1 m/s, 10 rad/s (b) 0.1 m/s (towards right) (c) 4.8 cm (d) 0.03 J

15. $\frac{1}{2\pi}\sqrt{\frac{15k}{4m}}$ 16. $T = 4\pi\sqrt{m\left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \frac{1}{k_4}\right)}$ 17. $T = 2\pi\sqrt{\frac{m(4k_2 + k_1)}{k_1 k_2}}$ 18. $f = \frac{1}{2\pi}\sqrt{\frac{k + 4A\rho g}{6M}}$

19. $\frac{1}{\pi}\sqrt{\frac{2k}{M}}$



12

Elasticity

Chapter Contents

Solved Examples

Level 1

Example 1 *A steel wire of length 4 m and diameter 5 mm is stretched by 5 kg-wt. Find the increase in its length, if the Young's modulus of steel is* 2.4×10^{12} *dyne/cm*^{2}.

Solution Here, $l = 4\text{ m} = 400\text{ cm}$, $2r = 5\text{ mm}$

or
$$r = 2.5\text{ mm} = 0.25\text{ cm}$$
$$f = 5\text{ kg-wt} = 5000\text{ g-wt} = 5000 \times 980\text{ dyne}$$
$$\Delta l = ?, \quad Y = 2.4 \times 10^{12}\text{ dyne/cm}^2$$

As
$$Y = \frac{F}{\pi r^2} \times \frac{l}{\Delta l}$$

or
$$\Delta l = \frac{Fl}{\pi r^2 Y} = \frac{(5000 \times 980) \times 400}{(22/7) \times (0.25)^2 \times 2.4 \times 10^{12}}$$
$$= 0.0041\text{ cm}$$ **Ans.**

Example 2 *The bulk modulus of water is* 2.3×10^9 *N/m*^{2}.

(a) Find its compressibility.
(b) How much pressure in atmospheres is needed to compress a sample of water by 0.1%?

Solution Here, $B = 2.3 \times 10^9\text{ N/m}^2$
$$= \frac{2.3 \times 10^9}{1.01 \times 10^5} = 2.27 \times 10^4\text{ atm}$$

(a) Compressibility $= \dfrac{1}{B} = \dfrac{1}{2.27 \times 10^4} = 4.4 \times 10^{-5}\text{ atm}^{-1}$ **Ans.**

(b) Here, $\dfrac{\Delta V}{V} = -0.1\% = -0.001$

Required increase in pressure,
$$\Delta P = B \times \left(-\frac{\Delta V}{V}\right) = 2.27 \times 10^4 \times 0.001 = 22.7\text{ atm}$$ **Ans.**

Example 3 *A steel wire 4.0 m in length is stretched through 2.0 mm. The cross-sectional area of the wire is* 2.0 mm^2. *If Young's modulus of steel is* 2.0×10^{11} *N/m*^{2}. *Find :*

(a) the energy density of wire.
(b) the elastic potential energy stored in the wire.

Solution Here, $l = 4.0\text{ m}$, $\Delta l = 2 \times 10^{-3}\text{ m}$, $A = 2.0 \times 10^{-6}\text{ m}^2$, $Y = 2.0 \times 10^{11}\text{ N/m}^2$

(a) The energy density of stretched wire

$$U = \frac{1}{2} \times \text{stress} \times \text{strain} = \frac{1}{2} \times Y \times (\text{strain})^2$$

$$= \frac{1}{2} \times 2.0 \times 10^{11} \times \left(\frac{(2 \times 10^{-3})}{4} \right)^2$$

$$= 0.25 \times 10^5 = 2.5 \times 10^4 \text{ J/m}^3$$ **Ans.**

(b) Elastic potential energy = energy density × volume

$$= 2.5 \times 10^4 \times (2.0 \times 10^{-6}) \times 4.0 \text{ J}$$

$$= 20 \times 10^{-2} = 0.20 \text{ J}$$ **Ans.**

Example 4 *Find the greatest length of steel wire that can hang vertically without breaking. Breaking stress of steel* $= 8.0 \times 10^8 \ N/m^2$. *Density of steel* $= 8.0 \times 10^3 \ kg/m^3$. *Take* $g = 10 \ m/s^2$.

Solution Let l be the length of the wire that can hang vertically without breaking. Then the stretching force on it is equal to its own weight. If therefore, A is the area of cross-section and ρ the density, then

$$\text{Maximum stress } (\sigma_m) = \frac{\text{weight}}{A} \qquad \left(\text{Stress} = \frac{\text{force}}{\text{area}} \right)$$

or

$$\sigma_m = \frac{(Al\rho)g}{A}$$

$\therefore$

$$l = \frac{\sigma_m}{\rho g}$$

Substituting the values

$$l = \frac{8.0 \times 10^8}{(8.0 \times 10^3)(10)} = 10^4 \text{ m}$$ **Ans.**

Example 5 *(a) A wire* 4 *m long and* 0.3 *mm in diameter is stretched by a force of* 100 *N. If extension in the wire is* 0.3 *mm, calculate the potential energy stored in the wire.*

(b) Find the work done in stretching a wire of cross-section 1 mm^2 *and length* 2 *m through* 0.1 *mm. Young's modulus for the material of wire is* $2.0 \times 10^{11} \ N/m^2$.

Solution (a) Energy stored

$$U = \frac{1}{2}(\text{stress})(\text{strain})(\text{volume})$$

or

$$U = \frac{1}{2}\left(\frac{F}{A}\right)\left(\frac{\Delta l}{l}\right)(Al)$$

$$= \frac{1}{2} F \cdot \Delta l$$

$$= \frac{1}{2}(100)(0.3 \times 10^{-3})$$

$$= 0.015 \text{ J}$$ **Ans.**

(b) Work done = Potential energy stored

$$= \frac{1}{2} k(\Delta l)^2$$

$$= \frac{1}{2}\left(\frac{YA}{l}\right)(\Delta l)^2 \qquad \left(\text{as } k = \frac{YA}{l}\right)$$

Substituting the values, we have

$$W = \frac{1}{2} \frac{(2.0 \times 10^{11})(10^{-6})}{(2)} (0.1 \times 10^{-3})^2$$

$$= 5.0 \times 10^{-4} \text{ J}$$ **Ans.**

Example 6 *A rubber cord has a cross-sectional area 1 mm^2 and total unstretched length 10.0 cm. It is stretched to 12.0 cm and then released to project a missile of mass 5.0 g. Taking Young's modulus Y for rubber as 5.0×10^8 N/m^2. Calculate the velocity of projection.*

Solution Equivalent force constant of rubber cord.

$$k = \frac{YA}{l} = \frac{(5.0 \times 10^8)(1.0 \times 10^{-6})}{(0.1)}$$

$$= 5.0 \times 10^3 \text{ N/m}$$

Now, from conservation of mechanical energy, elastic potential energy of cord

= kinetic energy of missile

$$\therefore \quad \frac{1}{2} k (\Delta l)^2 = \frac{1}{2} mv^2$$

$$\therefore \quad v = \left(\sqrt{\frac{k}{m}}\right) \Delta l$$

$$= \left(\sqrt{\frac{5.0 \times 10^3}{5.0 \times 10^{-3}}}\right)(12.0 - 10.0) \times 10^{-2}$$

$$= 20 \text{ m/s}$$ **Ans.**

Note *Following assumptions have been made in this Problem :*

(i) *k has been assumed constant, even though it depends on the length (l).*

(ii) *The whole of the elastic potential energy is converting into kinetic energy of missile.*

Example 7 *What is the density of lead under a pressure of 2.0×10^8 N/m^2, if the bulk modulus of lead is 8.0×10^9 N/m^2 and initially the density of lead is 11.4 g/cm^3?*

Solution The changed density, $\rho' = \dfrac{\rho}{1 - \dfrac{dP}{B}}$

Substituting the value, we have

$$\rho' = \frac{11.4}{1 - \dfrac{2.0 \times 10^8}{8.0 \times 10^9}}$$

or $$\rho' = 11.69 \text{ g/cm}^3$$ **Ans.**

Level 2

Example 1 *A light rod of length 2.00 m is suspended from the ceiling horizontally by means of two vertical wires of equal length tied to its ends. One of the wires is made of steel and is of cross-section* $10^{-3}\ m^2$ *and the other is of brass of cross-section* $2 \times 10^{-3}\ m^2$. *Find out the position along the rod at which a weight may be hung to produce,*

(a) equal stresses in both wires

(b) equal strains on both wires.

Young's modulus for steel is $2 \times 10^{11}\ N/m^2$ *and for brass is* $10^{11}\ N/m^2$.

Solution (a) Given,

Stress in steel = stress in brass

$$\therefore \quad \frac{T_S}{A_S} = \frac{T_B}{A_B}$$

$$\therefore \quad \frac{T_S}{T_B} = \frac{A_S}{A_B}$$

$$= \frac{10^{-3}}{2 \times 10^{-3}} = \frac{1}{2} \quad \ldots\text{(i)}$$

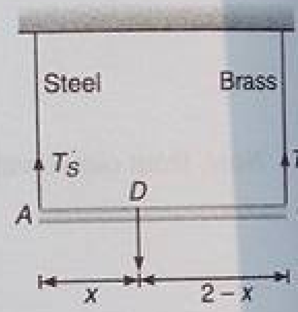


Fig. 12.18

As the system is in equilibrium, taking moments about D, we have

$$T_S \cdot x = T_B (2 - x)$$

$$\therefore \quad \frac{T_S}{T_B} = \frac{2 - x}{x} \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get $$x = 1.33 \text{ m}$$ **Ans.**

(b) $$\text{Strain} = \frac{\text{stress}}{Y}$$

Given, strain in steel = strain in brass

$$\therefore \quad \frac{T_S / A_S}{Y_S} = \frac{T_B / A_B}{Y_B}$$

$$\therefore \quad \frac{T_S}{T_B} = \frac{A_S Y_S}{A_B Y_B} = \frac{(1 \times 10^{-3})(2 \times 10^{11})}{(2 \times 10^{-3})(10^{11})} = 1 \quad \ldots\text{(iii)}$$

From Eqs. (ii) and (iii), we have $$x = 1.0 \text{ m}$$ **Ans.**

Example 2 *A steel rod of length 6.0 m and diameter 20 mm is fixed between two rigid supports. Determine the stress in the rod, when the temperature increases by 80°C if*

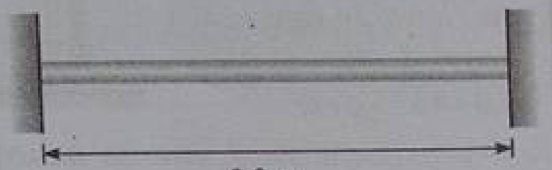


Fig. 12.19

(a) the ends do not yield
(b) the ends yield by 1 mm.
Take $Y = 2.0 \times 10^6$ kg/cm^2 and $\alpha = 12 \times 10^{-6}$ per°C.

Solution Given, length of the rod $l = 6\text{ m} = 600\text{ cm}$

Diameter of the rod $d = 20\text{ mm} = 2\text{ cm}$

Increase in temperature $t = 80°\text{C}$

Young's modulus $Y = 2.0 \times 10^6\ \text{kg/cm}^2$

and thermal coefficient of linear expansion

$$\alpha = 12 \times 10^{-6}\ \text{per°C}$$

When the ends do not yield

Let, σ_1 = stress in the rod

Using the relation $\sigma = \alpha t Y$

$$\therefore \quad \sigma_1 = (12 \times 10^{-6})(80)(2 \times 10^6)$$

$$= 1920\ \text{kg/cm}^2$$ **Ans.**

When the ends yield by 1 mm

Increase in length due to increase in temperature

$$\Delta l = l\alpha t$$

of this 1 mm or 0.1 cm is allowed to expand. Therefore, net compression in the rod

$$\Delta l_{\text{net}} = (l\alpha t - 0.1)$$

or compressive strain in the rod,

$$\varepsilon = \frac{\Delta l_{\text{net}}}{l} = \left(\alpha t - \frac{0.1}{l}\right)$$

$$\therefore \quad \text{stress } \sigma_2 = Y\varepsilon = Y\left(\alpha t - \frac{0.1}{l}\right)$$

Substituting the values,

$$\sigma_2 = 2 \times 10^6 \left(12 \times 10^{-6} \times 80 - \frac{0.1}{600}\right)$$ **An**

$$= 1587\ \text{kg/cm}^2$$

Example 3 *A steel rod of cross-sectional area* $16\ cm^2$ *and two brass rods each of cross-sectional area* $10\ cm^2$ *together support a load of 5000 kg as shown in figure. Find the stress in the rods. Take Y for steel* $= 2.0 \times 10^6\ kg/cm^2$ *and for brass* $= 1.0 \times 10^6\ kg/cm^2$.

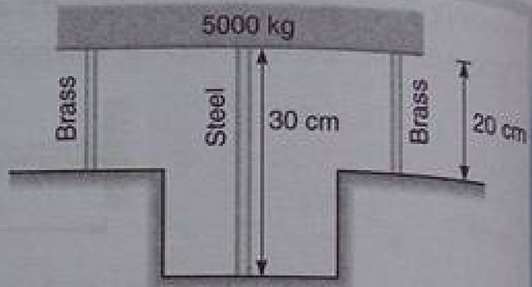


Fig. 12.20

Solution Given area of steel rod

$$A_S = 16\,\text{cm}^2$$

Area of two brass rods $\quad A_B = 2 \times 10 = 20\,\text{cm}^2$

Load, $\quad F = 5000\,\text{kg}$

Y for steel $\quad Y_S = 2.0 \times 10^6\ \text{kg/cm}^2$

Y for brass $\quad Y_B = 1.0 \times 10^6\ \text{kg/cm}^2$

Length of steel rod $l_S = 30\,\text{cm}$

Length of brass rod $l_B = 20\,\text{cm}$

Let $\quad \sigma_S = \text{stress in steel}$

and $\quad \sigma_B = \text{stress in brass}$

Decrease in length of steel rod = decrease in length of brass rod

or
$$\frac{\sigma_S}{Y_S} \times l_S = \frac{\sigma_B}{Y_B} \times l_B$$

or
$$\sigma_S = \frac{Y_S}{Y_B} \times \frac{l_B}{l_S} \times \sigma_B$$

$$= \frac{2.0 \times 10^6}{1.0 \times 10^6} \times \frac{20}{30} \times \sigma_B$$

$\therefore$
$$\sigma_S = \frac{4}{3}\sigma_B \qquad \text{...(i)}$$

Now, using the relation,

$$F = \sigma_S A_S + \sigma_B A_B$$

or
$$5000 = \sigma_S \times 16 + \sigma_B \times 20 \qquad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$\sigma_B = 120.9\ \text{kg/cm}^2$$

and
$$\sigma_S = 161.2\ \text{kg/cm}^2$$

Ans.

Example 4 *A sphere of radius 0.1 m and mass 8π kg is attached to the lower end of a steel wire of length 5.0 m and diameter 10^{-3} m. The wire is suspended from 5.22 m high ceiling of a room. When the sphere is made to swing as a simple pendulum, it just grazes the floor at its lowest point. Calculate the velocity of the sphere at the lowest position. Young's modulus of steel is 1.994×10^{11} N/m^2.*

Solution Let Δl be the extension of wire when the sphere is at mean position. Then, we have

$$l + \Delta l + 2r = 5.22$$

or

$$\Delta l = 5.22 - l - 2r$$

$$= 5.22 - 5 - 2 \times 0.1$$

$$= 0.02 \text{ m}$$

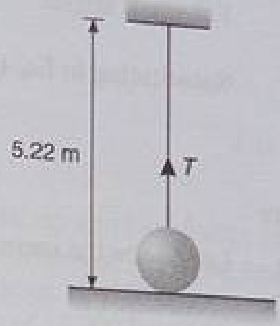


Fig. 12.21

Let T be the tension in the wire at mean position during oscillations, then

$$Y = \frac{T/A}{\Delta l / l}$$

$$\therefore \quad T = \frac{YA\Delta l}{l} = \frac{Y\pi r^2 \Delta l}{l}$$

Substituting the values, we have

$$T = \frac{(1.994 \times 10^{11}) \times \pi \times (0.5 \times 10^{-3})^2 \times 0.02}{5}$$

$$= 626.43 \text{ N}$$

The equation of motion at mean position is,

$$T - mg = \frac{mv^2}{R} \quad \text{...(i)}$$

Here, $\quad R = 5.22 - r = 5.22 - 0.1 = 5.12$ m

and $\quad m = 8\pi \text{ kg} = 25.13$ kg

Substituting the proper values in Eq. (i), we have

$$(626.43) - (25.13 \times 9.8) = \frac{(25.13)v^2}{5.12}$$

Solving this equation, we get $\quad v = 8.8$ m/s **Ans.**

Example 5 *A thin ring of radius R is made of a material of density ρ and Young's modulus Y. If the ring is rotated about its centre in its own plane with angular velocity ω, find the small increase in its radius.*

Solution Consider an element PQ of length dl. Let T be the tension and A the area of cross-section of the wire.

$$\text{Mass of element } dm = \text{volume} \times \text{density}$$

$$= A(dl)\rho$$

The component of T, towards the centre provides the necessary centripetal force

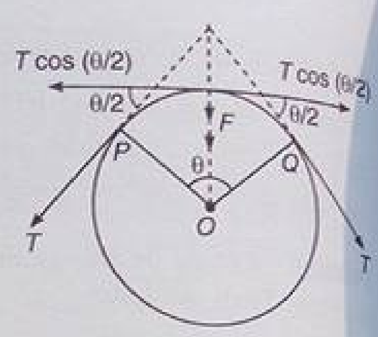


Fig. 12.22

$$\therefore \qquad 2T\sin\left(\frac{\theta}{2}\right) = (dm)R\omega^2 \qquad \ldots\text{(i)}$$

For small angles $\qquad \sin\frac{\theta}{2} \approx \frac{\theta}{2} = \frac{(dl/R)}{2}$

Substituting in Eq. (i), we have

$$T\cdot\frac{dl}{R} = A(dl)\rho R\omega^2$$

or $$T = A\rho\omega^2 R^2$$

Let ΔR be the increase in radius,

$$\text{Longitudinal strain} = \frac{\Delta l}{l} = \frac{\Delta(2\pi R)}{2\pi R} = \frac{\Delta R}{R}$$

Now, $$Y = \frac{T/A}{\Delta R/R}$$

$$\therefore \qquad \Delta R = \frac{TR}{AY} = \frac{(A\rho\omega^2 R^2)R}{AY}$$

or $$\Delta R = \frac{\rho\omega^2 R^3}{Y}$$ **Ans.**

Example 6 *A member ABCD is subjected to point loads F_1, F_2, F_3 and F_4 as shown in figure. Calculate the force F_2 for equilibrium if $F_1 = 4500$ kg, $F_3 = 45000$ kg and $F_4 = 13000$ kg. Determine the total elongation of the member, assuming modulus of elasticity to be 2.1×10^6 kg/cm^2.*

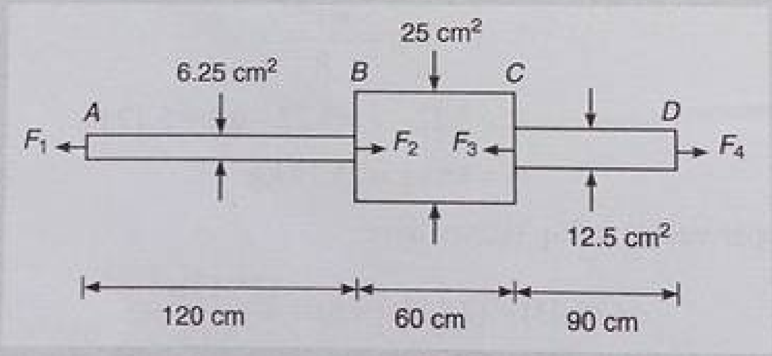


Fig. 12.23

Solution Given

Area of part AB, $\quad A_1 = 6.25\,\text{cm}^2$

Area of part BC, $\quad A_2 = 25\,\text{cm}^2$

Area of part CD, $\quad A_3 = 12.5\,\text{cm}^2$

Length of part AB, $\quad l_1 = 120\,\text{cm}$

Length of part BC, $\quad l_2 = 60\,\text{cm}$

Length of part CD, $l_3 = 90$ cm

Young's modulus of elasticity $Y = 2.1 \times 10^6$ kg/cm^2

Magnitude of the force F_2 for Equilibrium

The magnitude of force F_2 may be found by equating the forces acting towards right to those acting towards left,

$$F_2 + F_4 = F_1 + F_3$$

$$F_2 + 13000 = 4500 + 45000$$

$$\therefore \quad F_2 = 36500 \text{ kg}$$ **Ans.**

Total Elongation of the Member

For the sake of simplicity, the force of 36500 kg (acting at B) may be split up into two forces of 4500 kg and 32000 kg. The force of 45000 kg acting at C may be split into two forces of 32000 kg and 13000 kg. Now, it will be seen that the part AB of the member is subjected to a tension of 4500 kg, part BC is subjected to a compression of 32000 kg and part CD is subjected to a tension of 13,000 kg. Using the relation,

$$\Delta l = \frac{1}{Y}\left(\frac{F_1 l_1}{A_1} - \frac{F_2 l_2}{A_2} + \frac{F_3 l_3}{A_3}\right) \quad \text{with usual notations}$$

$$\Delta l = \frac{1}{2.1 \times 10^6}\left(\frac{4500 \times 120}{6.25} - \frac{32000 \times 60}{25} + \frac{13000 \times 90}{12.5}\right) \text{cm}$$

$$= 0.049 \text{ cm}$$

or $$\Delta l = 0.49 \text{ mm}$$ **Ans.**

EXERCISES

AIEEE Corner

Subjective Questions

1. A cylindrical steel wire of 3 m length is to stretch no more than 0.2 cm when a tensile force of 400 N is applied to each end of the wire. What minimum diameter is required for the wire ?
$Y_{steel} = 2.1 \times 10^{11}\ N/m^2$

2. The elastic limit of a steel cable is $3.0 \times 10^8\ N/m^2$ and the cross-section area is $4\ cm^2$. Find the maximum upward acceleration that can be given to a 900 kg elevator supported by the cable if the stress is not to exceed one-third of the elastic limit.

3. If the elastic limit of copper is $1.5 \times 10^8\ N/m^2$, determine the minimum diameter a copper wire can have under a load of 10.0 kg, if its elastic limit is not to be exceeded.

4. Find the increment in the length of a steel wire of length 5 m and radius 6 mm under its own weight. Density of steel $= 8000\ kg/m^3$ and Young's modulus of steel $= 2 \times 10^{11}\ N/m^2$. What is the energy stored in the wire ? (Take $g = 9.8\ m/s^2$)

5. Two wires shown in figure are made of the same material which has a breaking stress of $8 \times 10^8\ N/m^2$. The area of cross-section of the upper wire is $0.006\ cm^2$ and that of the lower wire is $0.003\ cm^2$. The mass $m_1 = 10\ kg$, $m_2 = 20$ kg and the hanger is light. Find the maximum load that can be put on the hanger without breaking a wire. Which wire will break first if the load is increased ? (Take $g = 10\ m/s^2$)

6. A steel wire and a copper wire of equal length and equal cross-sectional area are joined end to end and the combination is subjected to a tension. Find the ratio of :

(a) the stresses developed in the two wires,
(b) the strains developed. (Y of steel $= 2 \times 10^{11}$ N/m^2 and Y of copper $= 1.3 \times 10^{11}$ N/m^2)

7. Calculate the approximate change in density of water in a lake at a depth of 400 m below the surface. The density of water at the surface is 1030 kg/m^3 and bulk modulus of water is 2×10^9 N/m^2.

8. A wire of length 3 m, diameter 0.4 mm and Young's modulus 8×10^{10} N/m^2 is suspended from a point and supports a heavy cylinder of volume 10^{-3} m^3 at its lower end. Find the decrease in length when the metal cylinder is immersed in a liquid of density 800 kg/m^3.

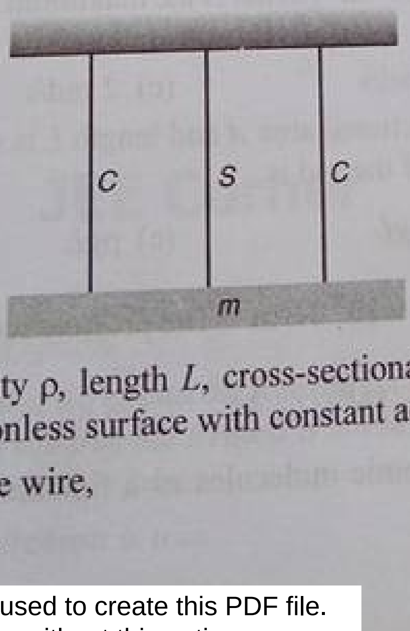
9. In taking a solid ball of rubber from the surface to the bottom of a lake of 180 m depth, reduction in the volume of the ball is 0.1%. The density of water of the lake is 1×10^3 kg/m^3. Determine the value of the bulk modulus of elasticity of rubber. ($g = 9.8$ m/s^2)

10. A sphere of radius 10 cm and mass 25 kg is attached to the lower end of a steel wire of length 5 m and diameter 4 mm which is suspended from the ceiling of a room. The point of support is 521 cm above the floor. When the sphere is set swinging as a simple pendulum, its lowest point just grazes the floor. Calculate the velocity of the ball at its lowest position. ($Y_{steel} = 2 \times 10^{11}$ N/m^2)

11. A uniform ring of radius R and made up of a wire of cross-sectional radius r is rotated about its axis with a frequency ν. If density of the wire is ρ and Young's modulus is Y. Find the fractional change in radius of the ring.

12. A 6 kg weight is fastened to the end of a steel wire of unstretched length 60 cm. It is whirled in a vertical circle and has an angular velocity of 2 rev/s at the bottom of the circle. The area of cross-section of the wire is 0.05 cm^2. Calculate the elongation of the wire when the weight is at the lowest point of the path. Young's modulus of steel $= 2 \times 10^{11}$ N/m^2.

13. A homogeneous block with a mass m hangs on three vertical wires of equal length arranged symmetrically. Find the tension of the wires if the middle wire is of steel and the other two are of copper. All the wires have the same cross-section. Consider the modulus of elasticity of steel to be double than that of copper.



14. A uniform copper bar of density ρ, length L, cross-sectional area S and Young's modulus Y is moving horizontally on a frictionless surface with constant acceleration a_0. Find :

(a) the stress at the centre of the wire,
(b) total elongation of the wire.

15. A 5 m long cylindrical steel wire with radius 2×10^{-3} m is suspended vertically from a rigid support and carries a bob of mass 100 kg at the other end. If the bob gets snapped, calculate the change in temperature of the wire ignoring radiation losses. (Take $g = 10\,m/s^2$)
(For the steel wire : Young's modulus $= 2.1 \times 10^{11}\ N/m^2$; Density $= 7860\,kg/m^3$; Specific heat $= 420\,J/kg\text{-}^\circ C$).

Objective Questions (Level 1)

Single Correct Option

1. The bulk modulus for an incompressible liquid is
(a) zero (b) unity (c) infinity (d) between 0 and 1

2. The Young's modulus of a wire of length (L) and radius (r) is Y. If the length is reduced to $\frac{L}{2}$ and radius $\frac{r}{2}$, then its Young's modulus will be
(a) $\frac{Y}{2}$ (b) Y (c) $2Y$ (d) $4Y$

3. The maximum load that a wire can sustain is W. If the wire is cut to half its value, the maximum load it can sustain is
(a) W (b) $\frac{W}{2}$ (c) $\frac{W}{4}$ (d) $2W$

4. Identify the case when an elastic metal rod does not undergo elongation
(a) it is pulled with a constant acceleration on a smooth horizontal surface
(b) it is pulled with constant velocity on a rough horizontal surface
(c) it is allowed to fall freely
(d) All of the above

5. Vessel of $1 \times 10^{-3}\,m^3$ volume contains an oil. If a pressure of $1.2 \times 10^5\ N/m^2$ is applied on it, then volume decreases by $0.3 \times 10^{-3}\,m^3$. The bulk modulus of oil is
(a) $6 \times 10^{10}\ N/m^2$ (b) $4 \times 10^5\ N/m^2$ (c) $2 \times 10^7\ N/m^2$ (d) $1 \times 10^6\ N/m^2$

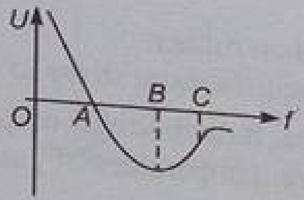
6. A bob of mass 10 kg is attached to a wire 0.3 m long. Its breaking stress is $4.8 \times 10^7\ N/m^2$. The area of cross-section of the wire is $10^{-6}\ m^2$. What is the maximum angular velocity with which it can be rotated in a horizontal circle?
(a) 8 rad/s (b) 4 rad/s (c) 2 rad/s (d) 1 rad/s

7. A uniform steel rod of cross-sectional area A and length L is suspended so that it hangs vertically. The stress at the middle point of the rod is
(a) $\frac{1}{2}\rho g L$ (b) $\frac{1}{4}\rho g L$ (c) $\rho g L$ (d) None of these

8. The bulk modulus of water is $2.0 \times 10^9\ N/m^2$. The pressure required to increase the density of water by 0.1% is
(a) $2.0 \times 10^3\ N/m^2$ (b) $2.0 \times 10^6 N/m^2$ (c) $2.0 \times 10^5 N/m^2$ (d) $2.0 \times 10^7 N/m^2$

9. The potential energy U of diatomic molecules as a function of separation r is shown in figure. Identify the correct statement.

10.
11.
12.
13.
14.
A



(a) The atoms are in equilibrium if $r = OA$
(b) The force is repulsive only if r lies between A and B
(c) The force is attractive if r lies between A and B
(d) The atoms are in equilibrium if $r = OB$

10. The length of a steel wire is l_1 when the stretching force is T_1 and l_2 when the stretching force is T_2. The natural length of the wire is

(a) $\frac{l_1T_1 + l_2T_2}{T_1 + T_2}$ (b) $\frac{l_2T_1 + l_1T_2}{T_1 + T_2}$ (c) $\frac{l_2T_1 - l_1T_2}{T_1 - T_2}$ (d) $\frac{l_1T_1 - l_2T_2}{T_1 - T_2}$

11. A mass m is suspended from a wire. Change in length of the wire is Δl. Now the same wire is stretched to double its length and the same mass is suspended from the wire. The change in length in this case will become (it is assumed that elongation in the wire is within the proportional limit)

(a) Δl (b) $2\Delta l$ (c) $4\Delta l$ (d) $8\Delta l$

12. A uniform metal rod fixed at its ends of $2\ \text{mm}^2$ cross-section is cooled from 40°C to 20°C. The coefficient of the linear expansion of the rod is 12×10^{-6} per degree celsius and its Young's modulus of elasticity is $10^{11}\ \text{N/m}^2$. The energy stored per unit volume of the rod is

(a) $2880\ \text{J/m}^3$ (b) $1500\ \text{J/m}^3$ (c) $5760\ \text{J/m}^3$ (d) $1440\ \text{J/m}^3$

13. A rod of length 1000 mm and coefficient of linear expansion $\alpha = 10^{-4}$ per degree celsius is placed in horizontal smooth surface symmetrically between fixed walls separated by 1001 mm. The Young's modulus of rod is $10^{11}\ \text{N/m}^2$. If the temperature is increased by 20°C, then the stress developed in the rod is (in N/m^2)

(a) 10^5 (b) 10^8 (c) 10^7 (d) 10^6

14. A uniform elastic plank moves due to a constant force F_0 distributed uniformly over the end face whose area is S. The Young's modulus of the plank is Y. The strain produced in the direction of force is

(a) $\frac{F_0}{2SY}$ (b) $\frac{F_0}{SY}$ (c) $\frac{2F_0}{SY}$ (d) $\frac{\sqrt{2}F_0}{SY}$

JEE Corner

Assertion and Reason

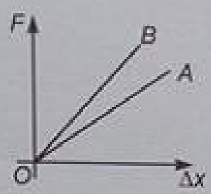
Directions : *Choose the correct option.*

(a) *If both **Assertion** and **Reason** are true and the **Reason** is correct explanation of the **Assertion.***
(b) *If both **Assertion** and **Reason** are true but **Reason** is not the correct explanation of **Assertion.***
(c) *If **Assertion** is true, but the **Reason** is false.*
(d) *If **Assertion** is false but the **Reason** is true.*

1. **Assertion :** Steel is more elastic than rubber.
   Reason : For same strain, steel requires more stress to be produced in it.
2. **Assertion :** If pressure is increased, bulk modulus of gases will increase.
   Reason : With increase in pressure, temperature of gas also increases.
3. **Assertion :** From the relation $Y = \frac{Fl}{A\Delta l}$, we can say that, if length of a wire is doubled, its Young's modulus of elasticity will also becomes two times.
   Reason : Modulus of elasticity is a material property.
4. **Assertion :** Bulk modulus of elasticity can be defined for all three states of matter, solid liquid and gas.
   Reason : Young's modulus is not defined for liquids and gases.
5. **Assertion :** Every wire is like a spring, whose spring constant,
$$K \propto \frac{1}{l}$$
   where l is length of wire.
   Reason : It follows from the relation
$$K = \frac{YA}{l}$$
6. **Assertion :** Ratio of stress and strain is always constant for a substance.
   Reason : This ratio is called modulus of elasticity.
7. **Assertion :** Ratio of isothermal bulk modulus and adiabatic bulk modulus for a monoatomic gas at a given pressure is $\frac{3}{5}$.
   Reason : This ratio is equal to $\gamma = \frac{C_p}{C_V}$.

More than One Correct Options

1. A metal wire of length L, area of cross-section A and Young's modulus Y is stretched by a variable force F such that F is always slightly greater than the elastic forces of resistance in the wire. When the elongation of the wire is l
   (a) the work done by F is $\frac{YAl^2}{2L}$
   (b) the work done by F is $\frac{YAl^2}{L}$
   (c) the elastic potential energy stored in the wire is $\frac{YAl^2}{2L}$
   (d) the elastic potential energy stored in the wire is $\frac{YAl^2}{4L}$
2. Two wires A and B of same length are made of same material. The figure represents the load F *versus* extension Δx graph for the two wires. Then :



(a) The cross sectional area of A is greater than that of B
(b) The elasticity of B is greater than that of A
(c) The cross-sectional area of B is greater than that of A
(d) The elasticity of A is greater than that of B

3. A body of mass M is attached to the lower end of a metal wire, whose upper end is fixed. The elongation of the wire is l.
(a) Loss in gravitational potential energy of M is Mgl
(b) The elastic potential energy stored in the wire is Mgl
(c) The elastic potential energy stored in the wire is $\frac{1}{2}Mgl$
(d) Heat produced is $\frac{1}{2}Mgl$

Match the Columns

1. Match the following two columns. (dimension wise)

Column I	Column II
(a) Stress	(p) coefficient of friction
(b) Strain	(q) relative density
(c) Modulus of elasticity	(r) energy density
(d) Force constant of a wire	(s) None

2. A wire of length l, area of cross section A and Young's modulus of elasticity Y is stretched by a longitudinal force F. The change in length is Δl. Match the following two columns.

Note *In column I, corresponding to every option, other factors remain constant.*

Column I	Column II
(a) F is increased	(p) Δl will increase
(b) l is increased	(q) stress will increase
(c) A is increased	(r) Δl will decrease
(d) Y is increased	(s) stress will decrease

ANSWERS

Introductory Exercise 12.1

1. Wire B 2. 0.0125 cm 3. 250.2 mm

AIEEE Corner

Subjective Questions

1. 1.91 mm 2. 34.64 m/s^2 3. Diameter, $d = 0.912$ mm 4. $\Delta l = 4.9\times 10^{-6}$ m, $U = 5.43\times 10^{-5}$ J
5. Load = 14 kg, lower string 6. (a) 1 (b) Ratio = $\frac{13}{20}$ 7. 2.0 kg/m^3 8. $\Delta l \approx 2.34\times 10^{-3}$m
9. $B = 1.76\times 10^9$N/m^2 10. $v = 31.23$ m/s 11. $\frac{4\pi^2 v^2 \rho R^3}{Y}$ 12. $\Delta l = 3.8\times 10^{-4}$ m
13. $T_C = \frac{mg}{4}, T_S = 2T_C$ 14. (a) $\frac{1}{2}L\rho a_0$ (b) $\frac{1}{2}\frac{\rho a_0 L^2}{Y}$ 15. 4.568×10^{-3}°C

Objective Questions

1. (c) 2. (b) 3. (a) 4. (c) 5. (b) 6. (b) 7. (a) 8. (b) 9. (d) 10. (c)
11. (c) 12. (a) 13. (b) 14. (a)

JEE Corner

Assertion and Reason

1. (a) 2. (c) 3. (d) 4. (b) 5. (a) 6. (d) 7. (c)

More than One Correct Options

1. (a,c) 2. (c) 3. (a,c,d)

Match the Columns

1. (a) → r (b) → p,q (c) → r (d) → s
2. (a) → pq (b) → p (c) → rs (d) → r

13

Fluid Mechanics

Chapter Contents

Solved Examples

Level 1

Example 1 *For the arrangement shown in the figure, what is the density of oil ?*

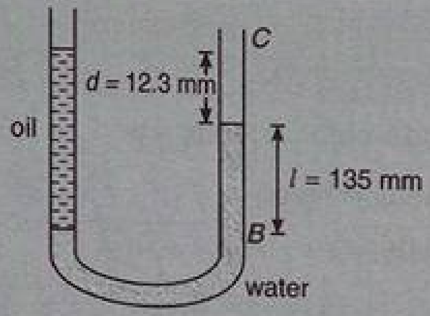


Fig. 13.63

Solution $P_0 + \rho_w gl = P_0 + \rho_{oil}(l+d)g$

$$\Rightarrow \qquad \rho_{oil} = \frac{\rho_w l}{l+d} = \frac{1000.(135)}{(135+12.3)} = 916 \text{ kg/m}^3$$ **Ans.**

Example 2 *A solid floats in a liquid of different material. Carry out an analysis to see whether the level of liquid in the container will rise or fall when the solid melts.*

Solution Let M = Mass of the floating solid

ρ_1 = density of liquid formed by the melting of the solid

ρ_2 = density of the liquid in which the solid is floating

The mass of liquid displaced by the solid is M. Hence, the volume of liquid displaced is $\frac{M}{\rho_2}$.

When the solid melts, the volume occupied by it is $\frac{M}{\rho_1}$. Hence, the level of liquid in container will rise or fall according as

$$\frac{M}{\rho_1} > \text{ or } < \frac{M}{\rho_2} \qquad \textit{i.e.,} \qquad \rho_1 < \text{ or } > \rho_2$$

There will be no change in the level if $\rho_1 = \rho_2$. In case of ice floating in water $\rho_1 = \rho_2$ and hence, the level of water remains unchanged when ice melts.

Example 3 *An iron casting containing a number of cavities weighs 6000 N in air and 4000 N in water. What is the volume of the cavities in the casting? Density of iron is 7.87 g/cm^3. Take $g = 9.8\ m/s^2$ and density of water $= 10^3\ kg/m^3$.*

Solution Let v be the volume of cavities and V the volume of solid iron. Then,

$$V = \frac{\text{mass}}{\text{density}} = \left(\frac{6000/9.8}{7.87 \times 10^3}\right) = 0.078 \text{ m}^3$$

Further, decrease in weight = upthrust

$\therefore$ $$(6000-4000)=(V+v)\rho_w g$$

or $$2000=(0.078+v)\times 10^3\times 9.8$$

or $$0.078+v\approx 0.2$$

$\therefore$ $$v=0.12\text{ m}^3$$ **Ans.**

Example 4 *A boat floating in a water tank is carrying a number of stones. If the stones were unloaded into water, what will happen to the water level?*

Solution Let weight of boat $=W$ and weight of stone $=w$.

Assuming density of water $=1$ g/cc

Volume of water displaced initially $=(w+W)$

Later, volume displaced $=\left(W+\dfrac{w}{\rho}\right)$ (ρ = density of stones)

$\Rightarrow$ *Water level comes down.*

Example 5 *Water rises in a capillary tube to a height of* 2.0 *cm. In another capillary tube whose radius is one third of it, how much the water will rise?*

Solution $$h=\frac{2T\cos\theta}{r\rho g}$$

$\therefore$ $$hr=\frac{2T\cos\theta}{\rho g}=\text{constant}$$

$\therefore$ $$h_1r_1=h_2r_2 \quad \text{or} \quad h_2=\frac{h_1r_1}{r_2}$$

Substituting the values $$h_2=(2.0)(3) \qquad \left(\frac{r_2}{r_1}=\frac{1}{3}\right)$$

$$=6.0\text{ cm}$$ **Ans.**

Example 6 *Mercury has an angle of contact of* $120°$ *with glass. A narrow tube of radius* 1.0 *mm made of this glass is dipped in a trough containing mercury. By what amount does the mercury dip down in the tube relative to the liquid surface outside. Surface tension of mercury at the temperature of the experiment is* 0.5 *N/m and density of mercury is* 13.6×10^3 *kg/m*^{3}. *(Take g* $=9.8$ *m/s*^{2}*).*

Solution $$h=\frac{2T\cos\theta}{r\rho g}$$

Substituting the values, we get $$h=\frac{2\times 0.5\times\cos 120°}{10^{-3}\times 13.6\times 10^3\times 9.8}=-3.75\times 10^{-3}\text{ m}$$

or $$h=-3.75\text{ mm}$$ **Ans.**

Note *Here, negative sign implies that mercury suffers capillary depression.*

Example 7 *Two narrow bores of radius 3.0 mm and 6.0 mm are joined together to form a U-shaped tube open at both ends. If the U-tube contains water, what is the difference in its levels in the two limbs of the tube? Surface tension of water is 7.3×10^{-2} N/m. Take the angle of contact to be zero and density of water to be 10^3 kg/m^3. ($g = 9.8$ m/s^2)*

Solution

$$h\rho g = \Delta P = \frac{2T\cos\theta}{r_1} - \frac{2T\cos\theta}{r_2}$$

or

$$h = \frac{2T\cos\theta}{\rho g}\left(\frac{r_2 - r_1}{r_1 r_2}\right)$$

Substituting the values, we have

$$h = \frac{2 \times 7.3 \times 10^{-2} \times \cos 0°}{10^3 \times 9.8}\left(\frac{6.0 - 3.0}{6.0 \times 3.0}\right) \times \frac{1}{10^{-3}}$$

$$= 2.48 \times 10^{-3} \text{ m}$$

$$= 2.48 \text{ mm}$$

Ans.

Example 8 *With what terminal velocity will an air bubble 0.8 mm in diameter rise in a liquid of viscosity 0.15 N-s/m^2 and specific gravity 0.9? Density of air is 1.293 kg/m^3.*

Solution The terminal velocity of the bubble is given by,

$$v_T = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$$

Here, $r = 0.4 \times 10^{-3}$ m, $\sigma = 0.9 \times 10^3$ kg/m^3, $\rho = 1.293$ kg/m^3, $\eta = 0.15$, N-s/m^2

and $g = 9.8$ m/s^2

Substituting the values, we have

$$v_T = \frac{2}{9} \times \frac{(0.4 \times 10^{-3})^2(1.293 - 0.9 \times 10^3) \times 9.8}{0.15}$$

$$= -0.0021 \text{ m/s}$$

or

$$v_T = -0.21 \text{ cm/s}$$

Ans.

te *Here negative sign implies that the bubble will rise up.*

ample 9 *A spherical ball of radius 3.0×10^{-4} m and density 10^4 kg/m^3 falls freely under gravity rough a distance h before entering a tank of water. If after entering the water the velocity of the ball es not change, find h. Viscosity of water is 9.8×10^{-6} N-s/m^2.*

ution Before entering the water the velocity of ball is $\sqrt{2gh}$. If after entering the water this velocity s not change then this value should be equal to the terminal velocity. Therefore,

$$\sqrt{2gh} = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$$

$$\therefore \quad h=\frac{\left\{\frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}\right\}^2}{2g}$$

$$=\frac{2}{81}\times\frac{r^4(\rho-\sigma)^2 g}{\eta^2}$$

$$=\frac{2}{81}\times\frac{(3\times10^{-4})^4(10^4-10^3)^2\times9.8}{(9.8\times10^{-6})^2}$$

$$=1.65\times10^3\ \text{m}$$ **Ans.**

Level 2

Example 1 *A solid ball of density half that of water falls freely under gravity from a height of 19.6 m and then enters water. Upto what depth will the ball go. How much time will it take to come again to the water surface? Neglect air resistance and viscosity effects in water. (Take $g=9.8\ m/s^2$)*

Solution $v=\sqrt{2gh}=\sqrt{2\times9.8\times19.6}=19.6\ \text{m/s}$

Let ρ be the density of ball and 2ρ the density of water. Net retardation inside the water,

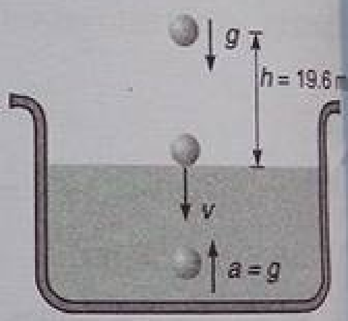


Fig. 13.64

$$a=\frac{\text{upthrust}-\text{weight}}{\text{mass}}$$

$$=\frac{V(2\rho)g-V(\rho)(g)}{V(\rho)g} \quad (V=\text{volume of ball})$$

$$=g=9.8\ \text{m/s}^2$$

Hence, the ball will go upto the same depth 19.6 m below the water surface. **Ans.**

Further, time taken by the ball to come back to water surface is,

$$t=2\left(\frac{v}{a}\right)=2\left(\frac{19.6}{9.8}\right)=4\ \text{s}$$ **Ans.**

Example 2 *A block of mass 1 kg and density $0.8\ g/cm^3$ is held stationary with the help of a string as shown in figure. The tank is accelerating vertically upwards with an acceleration $a=1.0\ m/s^2$. Find :*

(a) the tension in the string,
(b) if the string is now cut find the acceleration of block.
(Take $g=10\ m/s^2$ and density of water $=10^3\ kg/m^3$).

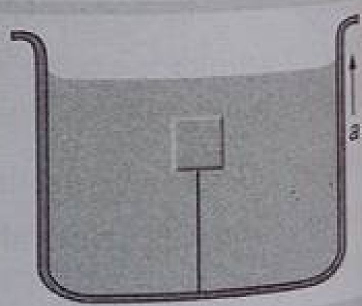


Fig. 13.65

Solution (a) Free body diagram of the block is shown in Fig. 13.66

In the figure,

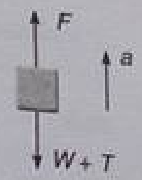


Fig. 13.66

$$F = \text{upthrust force}$$
$$= V\rho_\omega (g+a)$$
$$= \left(\frac{\text{mass of block}}{\text{density of block}}\right)\rho_\omega (g+a)$$
$$= \left(\frac{1}{800}\right)(1000)(10+1) = 13.75 \text{ N}$$
$$W = mg = 10 \text{ N}$$

Equation of motion of the block is,

$$F - T - W = ma$$

$\therefore$ $$13.75 - T - 10 = 1\times 1$$

$\therefore$ $$T = 2.75 \text{ N}$$

Ans.

(b) When the string is cut $T = 0$

$\therefore$ $$a = \frac{F-W}{m}$$
$$= \frac{13.75-10}{1}$$
$$= 3.75 \text{ m/s}^2$$

Ans.

Example 3 *A fresh water on a reservoir is* 10 *m deep. A horizontal pipe* 4.0 *cm in diameter passes through the reservoir* 6.0 *m below the water surface as shown in figure. A plug secures the pipe opening.*

(a) Find the friction force between the plug and pipe wall.

(b) The plug is removed. What volume of water flows out of the pipe in 1 *h? Assume area of reservoir to be too large.*

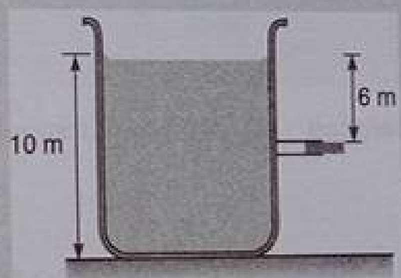


Fig. 13.67

Solution (a) Force of friction

$$= \text{pressure difference on the sides of the plug} \times \text{area of cross section of the plug}$$
$$= (\rho g h)A = (10^3)(9.8)(6.0)(\pi)(2\times 10^{-2})^2$$
$$= 73.9 \text{ N}$$

Ans.

(b) Assuming the area of the reservoir to be too large,

Velocity of efflux $$v = \sqrt{2gh} = \text{constant}$$

$\therefore$ $$v = \sqrt{2\times 9.8\times 6} = 10.84 \text{ m/s}$$

Volume of water coming out per sec,

$$\frac{dV}{dt} = Av$$
$$= \pi(2\times10^{-2})^2(10.84)$$
$$= 1.36\times10^{-2}\ \text{m}^3/\text{s}$$

$\therefore$ The volume of water flowing through the pipe in 1 h

$$V = \left(\frac{dV}{dt}\right)t$$
$$= (1.36\times10^{-2})(3600)$$
$$= 48.96\ \text{m}^3$$

Ans.

Example 4 *The U-tube acts as a water siphon. The bend in the tube is 1 m above the water surface. The tube outlet is 7 m below the water surface. The water issues from the bottom of the siphon as a free jet at atmospheric pressure. Determine the speed of the free jet and the minimum absolute pressure of the water in the bend. Given atmospheric pressure $= 1.01\times10^5\ N/m^2$, $g = 9.8\ m/s^2$ and density of water $= 10^3\ kg/m^3$.*

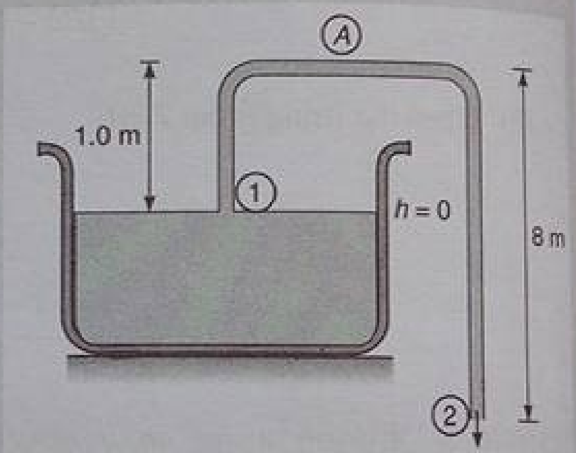


Fig. 13.68

Solution (a) Applying Bernoulli's equation between points (1) and (2)

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

Since, area of reservoir >> area of pipe

$$v_1 \approx 0, \quad \text{also} \quad P_1 = P_2 = \text{atmospheric pressure}$$

So, $$v_2 = \sqrt{2g(h_1 - h_2)} = \sqrt{2\times9.8\times7} = 11.7\ \text{m/s}$$ **Ans.**

(b) The minimum pressure in the bend will be at A. Therefore, applying Bernoulli's equation between (1) and (A)

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_A + \frac{1}{2}\rho v_A^2 + \rho g h_A$$

Again, $v_1 \approx 0$ and from conservation of mass $v_A = v_2$

or $$P_A = P_1 + \rho g(h_1 - h_A) - \frac{1}{2}\rho v_2^2$$

Therefore, substituting the values, we have

$$P_A = (1.01\times10^5)+(1000)(9.8)(-1)-\frac{1}{2}\times(1000)(11.7)^2 2$$

$$= 2.27\times10^4 \text{ N/m}^2$$

Ans.

Example 5 *A wooden rod weighing 25 N is mounted on a hinge below the free surface of water as shown. The rod is 3 m long and uniform in cross section and the support is 1.6 m below the free surface. At what angle α will it come to rest when allowed to drop from a vertical position. The cross-section of the rod is 9.5×10^{-4} m^2 in area. Density of water is 1000 kg/m^3. Assume buoyancy to act at centre of immersion. $g = 9.8\, m/s^2$. Also find the reaction on the hinge in this position.*

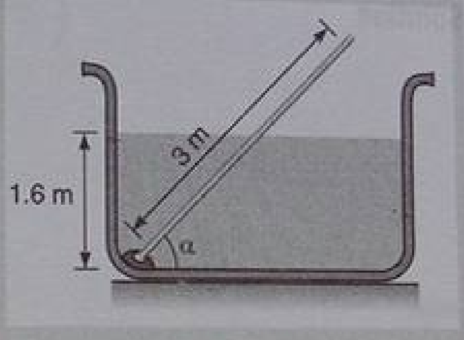


Fig. 13.69

Solution Let G be the mid-point of AB and E the mid point of AC (*i.e.*, the centre of buoyancy)

$$AC = 1.6 \operatorname{cosec}\alpha$$

Volume of $AC = (1.6\times9.5\times10^{-4})\operatorname{cosec}\alpha$

Weight of water displaced by AC

$$= (1.6\times9.5\times10^{-4}\times10^3\times9.8)\operatorname{cosec}\alpha$$

$$= 14.896 \operatorname{cosec}\alpha$$

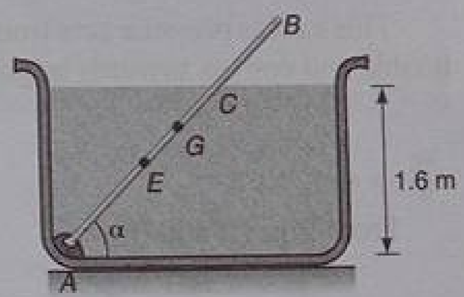


Fig. 13.70

Hence, the buoyant force is $14.896 \operatorname{cosec}\alpha$ acting vertically upwards at E. While the weight of the rod is 25 N acting vertically downwards at G. Taking moments about A,

$$(14.896 \operatorname{cosec}\alpha)(AE\cos\alpha) = (25)(AG\cos\alpha)$$

or $$(14.896 \operatorname{cosec}\alpha)\left(\frac{1.6 \operatorname{cosec}\alpha}{2}\right) = 25\times\frac{3}{2}$$

or $$\sin^2\alpha = 0.32$$

$\therefore$ $$\sin\alpha = 0.56$$

or $$\alpha = 34.3°$$ **Ans.**

Further, let F be the reaction at hinge in vertically downward direction. Then, considering the translatory equilibrium of rod in vertical direction we have,

$$F + \text{weight of the rod} = \text{upthrust}$$

$\therefore$ $$F = \text{upthrust} - \text{weight of the rod}$$

$$= 14.896 \operatorname{cosec}(34.3°) - 25$$

$$= 26.6 - 25$$

Ans.

$\therefore$ $$F = 1.6 \text{ N} \quad \text{(downwards)}$$

Example 6 *Two separate air bubbles (radii 0.004 m and 0.002 m) formed of the same liquid (surface tension 0.07 N/m) come together to form a double bubble. Find the radius and the sense of curvature of the internal film surface common to both the bubbles.*

Solution

$$P_1 = P_0 + \frac{4T}{r_1}$$

$$P_2 = P_0 + \frac{4T}{r_2}$$

$$r_2 < r_1$$

$$\therefore \quad P_2 > P_1$$

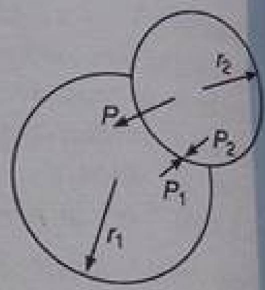


Fig. 13.71

i.e., pressure inside the smaller bubble will be more. The excess pressure

$$P = P_2 - P_1 = 4T\left(\frac{r_1 - r_2}{r_1 r_2}\right) \quad \text{...(i)}$$

This excess pressure acts from concave to convex side, the interface will be concave towards smaller bubble and convex towards larger bubble. Let R be the radius of interface then,

$$P = \frac{4T}{R} \quad \text{...(ii)}$$

From Eqs. (i) and (ii)

$$R = \frac{r_1 r_2}{r_1 - r_2} = \frac{(0.004)(0.002)}{(0.004 - 0.002)}$$

$$= 0.004 \text{ m} \quad \textbf{Ans.}$$

Example 7 *Under isothermal condition two soap bubbles of radii r_1 and r_2 coalesce to form a single bubble of radius r. The external pressure is P_0. Find the surface tension of the soap in terms of the given parameters.*

Solution As mass of the air is conserved,

$$\therefore \quad n_1 + n_2 = n \quad (\text{as } PV = nRT)$$

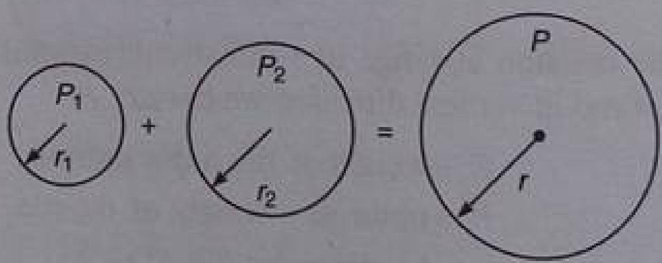


Fig. 13.72

$$\therefore \quad \frac{P_1 V_1}{RT_1} + \frac{P_2 V_2}{RT_2} = \frac{PV}{RT}$$

As temperature is constant, $T_1 = T_2 = T$

$\therefore$ $P_1V_1 + P_2V_2 = PV$

$\therefore$
$$\left(P_0 + \frac{4S}{r_1}\right)\left(\frac{4}{3}\pi r_1^3\right) + \left(P_0 + \frac{4S}{r_2}\right)\left(\frac{4}{3}\pi r_2^3\right) = \left(P_0 + \frac{4S}{r}\right)\left(\frac{4}{3}\pi r^3\right)$$

Solving, this we get

$$S = \frac{P_0(r^3 - r_1^3 - r_2^3)}{4(r_1^2 + r_2^2 - r^2)}$$ **Ans.**

Note *To avoid confusion with the temperature surface tension here is represented by S.*

Example 8 *A cylindrical tank of base area A has a small hole of area 'a' at the bottom. At time $t = 0$, a tap starts to supply water into the tank at a constant rate α m^3/s.*
(a) what is the maximum level of water h_{max} in the tank?
(b) find the time when level of water becomes $h (< h_{max})$.

Solution (a) Level will be maximum when

Rate of inflow of water = rate of outflow of water

i.e., $$\alpha = av$$

or $$\alpha = a\sqrt{2gh_{max}}$$

$\Rightarrow$ $$h_{max} = \frac{\alpha^2}{2ga^2}$$ **Ans.**

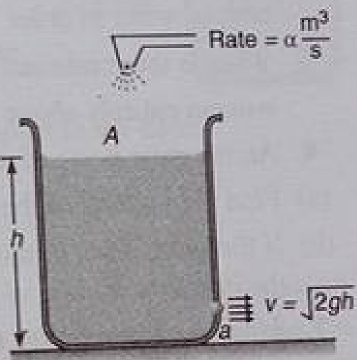


Fig. 13.73

(b) Let at time t, the level of water be h. Then,

$$A\left(\frac{dh}{dt}\right) = \alpha - a\sqrt{2gh}$$

or $$\int_0^h \frac{dh}{\alpha - a\sqrt{2gh}} = \int_0^t \frac{dt}{A}$$

Solving this, we get

$$t = \frac{A}{ag}\left[\frac{\alpha}{a}\ln\left\{\frac{\alpha - a\sqrt{2gh}}{\alpha}\right\} - \sqrt{2gh}\right]$$ **Ans.**

EXERCISES

AIEEE Corner

Subjective Questions (Level 1)

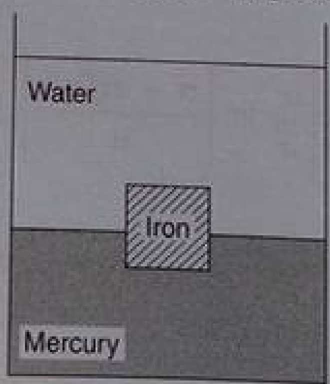
Upthrust

1. A block of material has a density ρ_1 and floats three-fourth submerged in a liquid of unknown density. Show that the density ρ_2 of the unknown liquid is given by $\rho_2 = \frac{4}{3}\rho_1$.
2. A metal ball weighs 0.096 N. When suspended in water it has an apparent weight of 0.071 N. Find the density of the metal.
3. A block of wood has a mass of 25 g. When a 5 g metal piece with a volume of 2 cm^3 is attached to the bottom of the block, the wood barely floats in water. What is the volume V of the wood?
4. A block of wood weighing 71.2 N and of specific gravity 0.75 is tied by a string to the bottom of a tank of water in order to have the block totally immersed. What is the tension in the string?
5. What is the minimum volume of a block of wood (density $= 850 \, kg/m^3$) if it is to hold a 50 kg woman entirely above the water when she stands on it ?
6. An irregular piece of metal weighs 10.00 g in air and 8.00 g when submerged in water.

(a) Find the volume of the metal and its density.

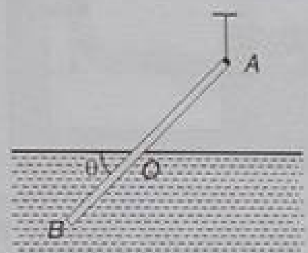
(b) If the same piece of metal weighs 8.50 g when immersed in a particular oil, what is the density of the oil ?

7. A beaker when partly filled with water has total mass 20.00 g. If a piece of metal with density 3.00 g/cm^3 and volume 1.00 cm^3 is suspended by a thin string, so that it is submerged in the water but does not rest on the bottom of the beaker, how much does the beaker then appear to weigh if it is resting on a scale?
8. A tank contains water on top of mercury. A cube of iron, 60 mm along each edge, is sitting upright in equilibrium in the liquids. Find how much of it is in each liquid. The densities of iron and mercury are $7.7 \times 10^3 \, kg/m^3$ and $13.6 \times 10^3 \, kg/m^3$ respectively.

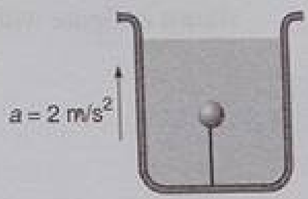


9. A small block of wood, of density 0.4×10^3 kg/m^3, is submerged in water at a depth of 2.9 m. Find :
(a) the acceleration of the block toward the surface when the block is released and
(b) the time for the block to reach the surface. Ignore viscosity.

10. A uniform rod AB, 4 m long and weighing 12 kg, is supported at end A, with a 6 kg lead weight at B. The rod floats as shown in figure with one-half of its length submerged. The buoyant force on the lead mass is negligible as it is of negligible volume. Find the tension in the cord and the total volume of the rod.

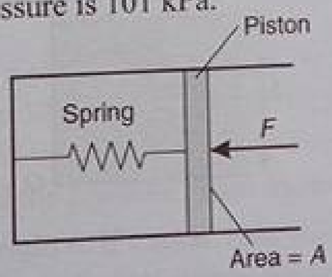


11. A solid sphere of mass $m = 2$ kg and density $\rho = 500\,\text{kg/m}^3$ is held stationary relative to a tank filled with water. The tank is accelerating upward with acceleration $2\,\text{m/s}^2$. Calculate :
(a) Tension in the thread connected between the sphere and the bottom of the tank.
(b) If the thread snaps, calculate the acceleration of sphere with respect to the tank.
[Density of water $= 1000\,\text{kg/m}^3$, $g = 10\,\text{m/s}^2$]

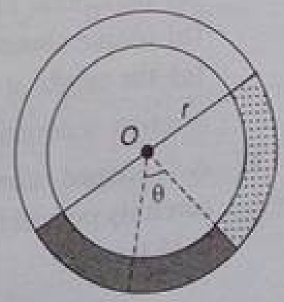


Pressure and Pascal's Law

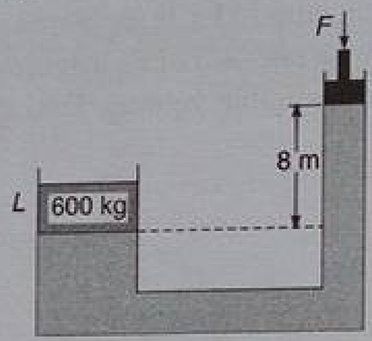
12. The pressure gauge shown in figure has a spring for which $k = 60$ N/m and the area of the piston is $0.50\,\text{cm}^2$. Its right end is connected to a closed container of gas at a gauge pressure of 30 kPa. How far will the spring be compressed if the region containing the spring is (a) in vacuum and (b) open to the atmosphere ? Atmospheric pressure is 101 kPa.



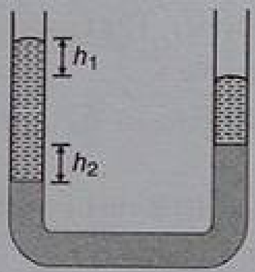
13. A small uniform tube is bent into a circle of radius r whose plane is vertical. Equal volumes of two fluids whose densities are ρ and σ $(\rho > \sigma)$ fill half the circle. Find the angle that the radius passing through the interface makes with the vertical.



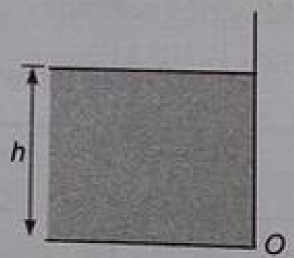
14. For the system shown in figure, the cylinder on the left, at L, has a mass of 600 kg and a cross-sectional area of 800 cm^2. The piston on the right, at S, has cross-sectional area 25 cm^2 and negligible weight. If the apparatus is filled with oil ($\rho = 0.78$ g/cm^3), what is the force F required to hold the system in equilibrium ?



15. A U-tube of uniform cross-sectional area and open to the atmosphere is partially filled with mercury. Water is then poured into both arms. If the equilibrium configuration of the tube is as shown in figure with $h_2 = 1.0$ cm, determine the value of h_1.



16. Water stands at a depth h behind the vertical face of a dam. It exerts a resultant horizontal force on the dam tending to slide it along its foundation and a torque tending to overturn the dam about the point O. Find :



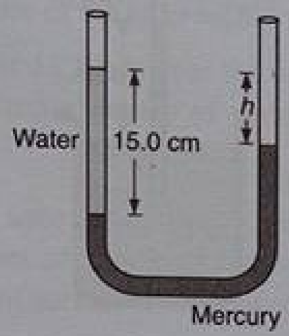
(a) horizontal force,
(b) torque about O,
(c) the height at which the resultant force would have to act to produce the same torque,

l = cross-sectional length and ρ = density of water.

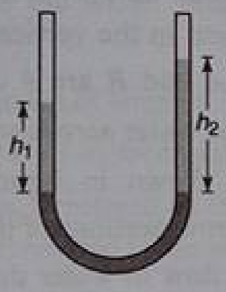
17. A U-shaped tube open to the air at both ends contains some mercury. A quantity of water is carefully poured into the left arm of the U-shaped tube until the vertical height of the water column is 15.0 cm.

(a) What is the gauge pressure at the water mercury interface ?

(b) Calculate the vertical distance h from the top of the mercury in the right hand arm of the tube to the top of the water in the left-hand arm.



18. Water and oil are poured into the two limbs of a U-tube containing mercury. The interfaces of the mercury and the liquids are at the same height in both limbs.
Determine the height of the water column h_1 if that of the oil $h_2 = 20$ cm. The density of the oil is 0.9.



19. Mercury is poured into a U-tube in which the cross-sectional area of the left-hand limb is three times smaller than that of the right one. The level of the mercury in the narrow limb is a distance $l = 30$ cm from the upper end of the tube. How much will the mercury level rise in the right-hand limb if the left one is filled to the top with water ?

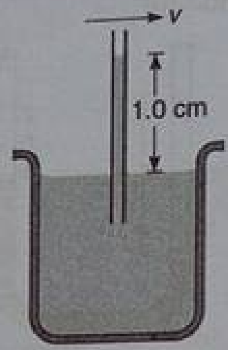
Fluids in Motion

20. Water is flowing smoothly through a closed-pipe system. At one point the speed of the water is 3.0 m/s, while at another point 1.0 m higher the speed is 4.0 m/s. If the pressure is 20 kPa at the lower point, what is the pressure at the upper point ? What would the pressure at the upper point be if the water were to stop flowing and the pressure at the lower point were 18 kPa ?

21. A water barrel stands on a table of height h. If a small hole is punched in the side of the barrel at its base, it is found that the resultant stream of water strikes the ground at a horizontal distance R from the barrel. What is the depth of water in the barrel ?

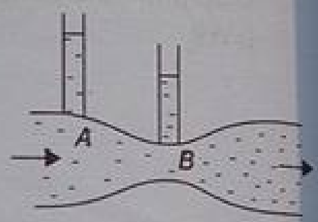
22. A pump is designed as a horizontal cylinder with a piston of area A and an outlet orifice of area a arranged near the cylinder axis. Find the velocity of out flow of the liquid from the pump if the piston moves with a constant velocity under the action of a constant force F. The density of the liquid is ρ.

23. When air of density $1.3\ \text{kg/m}^3$ flows across the top of the tube shown in the accompanying figure, water rises in the tube to a height of 1.0 cm. What is the speed of the air ?



24. The area of cross-section of a large tank is $0.5\ \text{m}^2$. It has an opening near the bottom having area of cross-section $1\ \text{cm}^2$. A load of 20 kg is applied on the water at the top. Find the velocity of the water coming out of the opening at the time when the height of water level is 50 cm above the bottom. (Take $g = 10\ \text{m/s}^2$)

25. Water flows through a horizontal tube as shown in figure . If the difference of heights of water column in the vertical tubes is 2 cm and the areas of cross-section at A and B are $4\ \text{cm}^2$ and $2\ \text{cm}^2$ respectively. Find the rate of flow of water across any section.



26. Water flows through the tube as shown in figure. The areas of cross-section of the wide and the narrow portions of the tube are $5\ \text{cm}^2$ and 2cm^2 respectively. The rate of flow of water through the tube is $500\ \text{cm}^3/\text{s}$. Find the difference of mercury levels in the U-tube.

Viscosity and Surface Tension

Note 1 Poise = $0.1\ \text{N-s/m}^2$ and 1 Pl = $1.0\ \text{N-s/m}^2$.

27. A typical riverborne silt particle has a radius of $20\ \mu\text{m}$ and a density of $2\times10^3\ \text{kg/m}^3$. The viscosity of water is 1.0 mPl. Find the terminal speed with which such a particle will settle to the bottom of a motionless volume of water.

28. What is the pressure drop (in mm Hg) in the blood as it passes through a capillary 1 mm long and 2μ m in radius if the speed of the blood through the centre of the capillary is 0.66 mm/s ? (The viscosity of whole blood is 4×10^{-3} Pl).

29. Two equal drops of water are falling through air with a steady velocity v. If the drops coalesced, what will be the new velocity ?

30. A long cylinder of radius R is displaced along its axis with a constant velocity u_0 inside a stationary co-axial cylinder of radius R_2. The space between the cylinders is filled with viscous liquid. Find the velocity of the liquid as a function of the distance r from the axis of the cylinders. The flow is laminar.

31. A large wooden plate of area $10\ m^2$ floating on the surface of a river is made to move horizontally with a speed of 2 m/s by applying a tangential force. If the river is 1 m deep and the water in contact with the bed is stationary, find the tangential force needed to keep the plate moving. Coefficient of viscosity of water at the temperature of the river $= 10^{-2}$ poise.

32. The velocity of water in a river is 18 km/h near the surface. If the river is 5 m deep, find the shearing stress between the horizontal layers of water. The coefficient of viscosity of water $= 10^{-2}$ poise.

33. Water rises up in a glass capillary upto a height of 9.0 cm, while mercury falls down by 3.4 cm in the same capillary. Assume angles of contact for water glass and mercury glass 0° and 135° respectively. Determine the ratio of surface tension of mercury and water ($\cos 135° = -0.71$).

34. Two equal spherical bubbles of radius a each coalesce to form a spherical bubble of radius b. If P is the atmospheric pressure, show that the surface tension of the bubble is $\dfrac{P(2a^3 - b^3)}{4(b^2 - 2a^2)}$.

35. A tube of insufficient length is immersed in water (surface tension $= 0.07$ N/m) with 1 cm of it projecting vertically upwards outside the water. What is the radius of meniscus ? Radius of tube $= 1$ mm.

36. A glass capillary sealed at the upper end is of length 0.11 m and internal diameter 2×10^{-5} m. The tube is immersed vertically into a liquid of surface tension 5.06×10^{-2} N/m.

To what length has the capillary to be immersed so that the liquid levels inside and outside the capillary become the same ? What will happen to the water levels inside the capillary if the seal is now broken ?

Objective Questions (Level 1)

Single Correct Option

1. When a sphere falling in a viscous fluid attains a terminal velocity, then
(a) the net force acting on the sphere is zero
(b) the drag force balances the buoyant force
(c) the drag force balances the weight of the sphere
(d) the buoyant force balances the weight and drag force

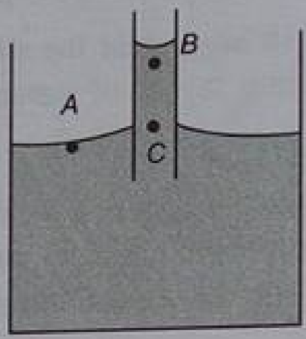
2. Which one of the following represents the correct dimensions of the quantity : $x = \dfrac{\eta}{\rho}$, where η = coefficient of viscosity and ρ = the density of a liquid?
(a) $[ML^{-2}T^{-1}]$ (b) $[ML^{-4}T^{-2}]$ (c) $[ML^{-5}T^{-2}]$ (d) $[M^0L^2T^{-1}]$

3. Viscosity of liquids
(a) increases with increase in temperature (b) is independent of temperature
(c) decreases with decrease in temperature (d) decreases with increase in temperature

4. Two soap bubbles in vacuum of radius 3 cm and 4 cm coalesce to form a single bubble under isothermal conditions. Then the radius of bigger bubble is

(a) 7 cm (b) $\frac{12}{7}$ cm (c) 12 cm (d) 5 cm

5. A capillary tube is dipped in a liquid. Let pressures at points A, B and C be P_A, P_B and P_C respectively then

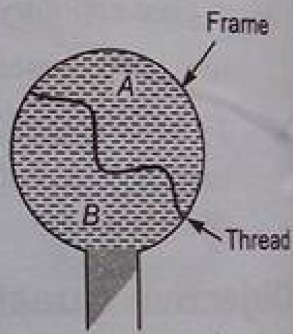


(a) $P_A = P_B = P_C$ (b) $P_A = P_B < P_C$ (c) $P_A = P_C < P_B$ (d) $P_A = P_C > P_B$

6. A small ball (mass m) falling under gravity in a viscous medium experiences a drag force proportional to the instantaneous speed u such that $F_{drag} = ku$. Then the terminal speed of ball within viscous medium is

(a) $\frac{k}{mg}$ (b) $\frac{mg}{k}$ (c) $\sqrt{\frac{mg}{k}}$ (d) None of these

7. A thread is tied slightly loose to a wire frame as in figure and the frame is dipped into a soap solution and taken out. The frame is completely covered with the film. When the portion A is punctured with a pin, the thread



(a) becomes concave towards A
(b) becomes convex towards A
(c) either (a) or (b) depending on the size of A with respect to B
(d) remains in the initial position

8. There is a small hole at the bottom of tank filled with water. If total pressure at the bottom is 3 atm ($1 \text{ atm} = 10^5 \text{ Nm}^{-2}$), then velocity of water flowing from hole is

(a) $\sqrt{400}$ ms^{-1} (b) $\sqrt{600}$ ms^{-1} (c) $\sqrt{60}$ ms^{-1} (d) None of these

9. A steel ball of mass m falls in a viscous liquid with terminal velocity v, then the steel ball of mass 8 m will fall in the same liquid with terminal velocity

(a) v (b) $4v$ (c) $8v$ (d) $16\sqrt{2}v$

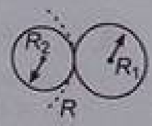
10. A liquid flows between two parallel plates along the x-axis. The difference between the velocity of two layers separated by the distance dy is dv. If A is the area of each plate, then Newton's law of viscosity may be written as

(a) $F = -\eta A \frac{dv}{dx}$ (b) $F = +\eta A \frac{dv}{dx}$ (c) $F = -\eta A \frac{dv}{dy}$ (d) $F = +\eta A \frac{dv}{dy}$

11. The work done to split a liquid drop of radius R into N identical drops is (take σ as the surface tension of the liquid)

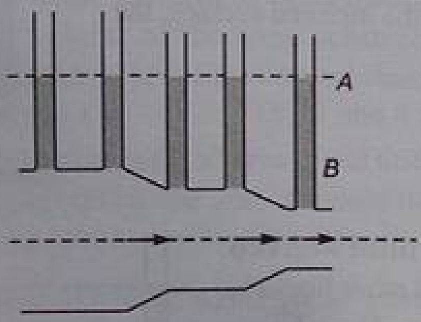
(a) $4\pi R^2 (N^{1/3} - 1)\sigma$ (b) $4\pi R^2 N\sigma$ (c) $4\pi R^2 (N^{1/2} - 1)$ (d) None of these

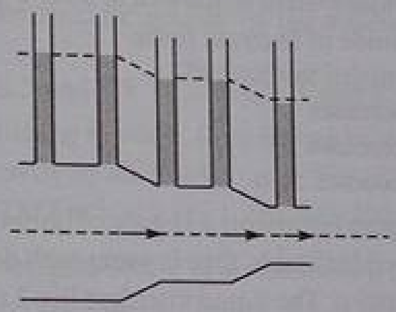
12. Two soap bubbles of different radii R_1 and $R_2 (< R_1)$ coalesce to form an interface of radius R as shown in figure. The correct value of R is

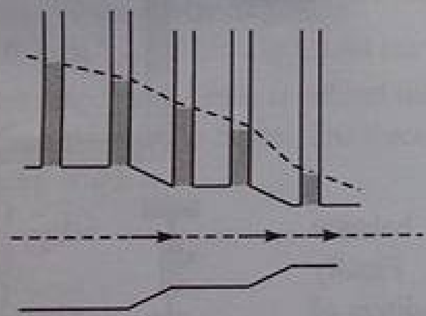


(a) $R = R_1 - R_2$
(b) $R = \frac{R_1 + R_2}{2}$
(c) $\frac{1}{R} = \frac{1}{R_2} - \frac{1}{R_1}$
(d) $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

13. A viscous liquid flows through a horizontal pipe of varying cross-sectional area. Identify the option which correctly represents the variation of height of rise of liquid in each vertical tube

(a) 

(b) 

(c) 

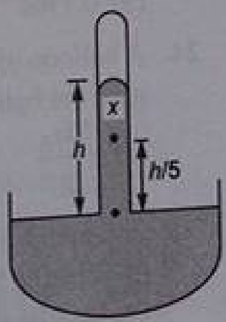
(d) None of these

14. The terminal velocity of a rain drop is 30 cm/s. If the viscosity of air is 1.8×10^{-5} Nsm^{-2}. The radius of rain drop is
(a) $1\,\mu$m
(b) 0.5 mm
(c) 0.05 mm
(d) 1 mm

15. If a capillary tube is dipped and the liquid levels inside and outside the tube are same, then the angle of contact is
(a) zero
(b) 90°
(c) 45°
(d) Cannot be obtained

16. Uniform speed of 2 cm diameter ball is 20 cm/s in a viscous liquid. Then, the speed of 1 cm diameter ball in the same liquid is
(a) 5 cms^{-1}
(b) 10 cms^{-1}
(c) 40 cms^{-1}
(d) 80 cms^{-1}

17. The height of mercury barometer is h when the atmospheric pressure is 10^5 Pa. The pressure at x in the shown diagram is

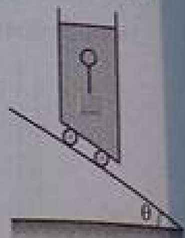


(a) 10^5 Pa
(b) 0.8×10^5 Pa
(c) 0.2×10^5 Pa
(d) 120×10^5 Pa

18. A body floats in water with its one-third volume above the surface. The same body floats in a liquid with one third volume immersed. The density of the liquid is
(a) 9 times more than that of water
(b) 2 times more than that of water
(c) 3 times more than that of water
(d) 1.5 times more than that of water

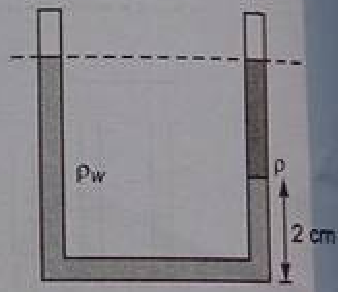
19. A piece of ice is floating in a beaker containing thick sugar solution of water. As the ice melts, the total level of the liquid
(a) increases
(b) decreases
(c) remains unchanged
(d) insufficient data

20. A body floats in completely immersed condition in water as shown in figure. As the whole system is allowed to slide down freely along the inclined surface, the magnitude of buoyant force



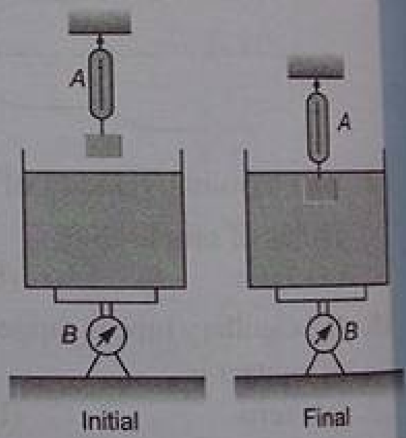
(a) remains unchanged
(b) increases
(c) decreases
(d) becomes zero

21. The figure represents a U-tube of uniform cross-section filled with two immiscible liquids. One is water with density ρ_w and the other liquid is of density ρ. The liquid interface lies 2 cm above the base. The relation between ρ and ρ_w is



(a) $\rho = \rho_w$
(b) $\rho = 1.02\rho_w$
(c) $\rho = 1.2\rho_w$
(d) None of the above

22. For the arrangement shown in figure, initially the balance A and B reads F_1 and F_2 respectively and $F_1 > F_2$. Finally when the block is immersed in the liquid then the readings of balance A and B are f_1 and f_2 respectively. Identify the statement which is not always (where F is some force) correct statement

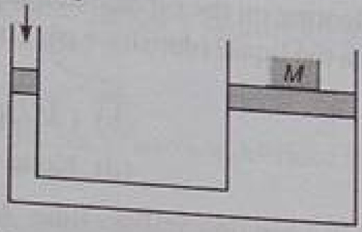


(a) $f_1 > f_2$
(b) $F_1 - F > F_2 + F$
(c) $f_1 + f_2 = F_1 + F_2$
(d) None of the above

23. When a tap is closed, the manometer attached to the pipe reads 3.5×10^5 Nm^{-2}. When the tap is opened, the reading of manometer falls to 3.0×10^5 Nm^{-2}. The velocity of water in the pipe is
(a) 0.1 ms^{-1}
(b) 1 ms^{-1}
(c) 5 ms^{-1}
(d) 10 ms^{-1}

24. A balloon of mass M descends with an acceleration a_0. The mass that must be thrown out in order to give the balloon an equal upward acceleration will be
(a) $\dfrac{Ma_0}{g}$
(b) $\dfrac{2Ma_0}{g}$
(c) $\dfrac{2Ma_0}{g + a_0}$
(d) $\dfrac{M(g + a_0)}{a_0}$

25. The hydraulic press shown in the figure is used to raise the mass M through a height of 0.5 cm be performing 500 J of work at the small piston. The diameter of the large piston is 10 cm, while that of the smaller one is 2 cm. The mass M is



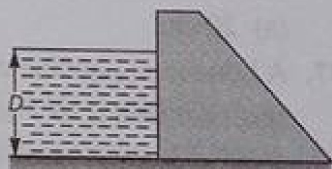
(a) 100 kg (b) 10^4 kg (c) 10^3 kg (d) 10^5 kg

26. When equal volumes of two substances are mixed, the specific gravity of the mixture is 4. When equal weights of the same substances are mixed, the specific gravity of the mixture is 3. The specific gravities of the two substances could be
(a) 6 and 2 (b) 3 and 4 (c) 2.5 and 3.5 (d) 5 and 3

27. A block of ice of total area A and thickness 0.5 m is floating in water. In order to just support a man of mass 100 kg, the area A should be (the specific gravity of ice is 0.9)
(a) 2.2 m^2 (b) 20 m^2 (c) 2222 m^2 (d) 20×10^4 m^2

28. A piece of gold ($\rho = 19.3\ g/cm^3$) has a cavity in it. It weights 38.2 g in air and 36.2 g in water. The volume of the cavity in gold is
(a) 0.2 cm^3 (b) 0.04 cm^3 (c) 0.02 cm^3 (d) 0.01 cm^3

29. Water stands at a depth D behind the vertical upstream face of a dam as shown in the figure. The force exerted on the dam by water per unit width is

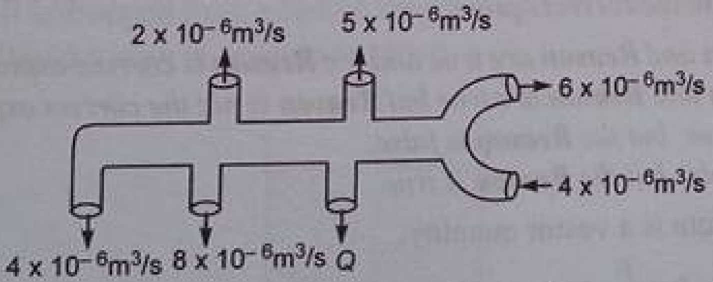


(a) $\frac{1}{3}\rho g D^2$ (b) $\frac{1}{2}\rho g D^2$
(c) $\frac{1}{3}\rho g D$ (d) $\frac{1}{2}(\rho g D)^2$

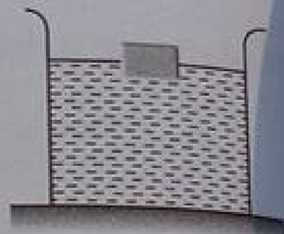
30. The volume of a liquid flowing per second out of an orifice at the bottom of a tank does not depend upon
(a) the height of the liquid above the orifice (b) the acceleration due to gravity
(c) the density of the liquid (d) the area of the orifice

31. A human heart pumps out 60 mL of blood at an average pressure of 100 mm of mercury and makes 72 heart beats per minute. Its pumping power is
(a) 0.98 W (b) 1.96 W (c) 0.49 W (d) 1.47 W

32. The pipe shows the volume flow rate of an ideal liquid at certain time and its direction. What is the value of Q in m^3/s (Assume steady state and equal area of cross section at each opening)



(a) 10×10^{-6} (b) 11×10^{-6} (c) 13×10^{-6} (d) 18×10^{-6}



33. A uniform cube of mass M is floating on the surface of a liquid with three fourth of its volume immersed in the liquid (density = ρ). The length of the side of the cube is equal to

(a) $(4M/3\rho)^{2/3}$ (b) $(M/3\rho)^{2/3}$
(c) $(M/4\rho)^{2/3}$ (d) None of these

34. Water rises to a height of 10 cm in a certain capillary tube. An another identical tube when dipped in mercury the level of mercury is depressed by 3.42 cm. Density of mercury is 13.6 g/cc. The angle of contact for water in contact with glass is $0°$ and mercury in contact with glass is $135°$. The ratio of surface tension of water to that of Hg is

(a) 1 : 3 (b) 1 : 4 (c) 1 : 5.5 (d) 1 : 6.5

35. A capillary glass tube records a rise of 20 cm when dipped in water. When the area of cross-section of the tube is reduced to half of the former value, water will rise to a height of

(a) $10\sqrt{2}$ cm (b) 10 cm (c) 20 cm (d) $20\sqrt{2}$ cm

36. A cylindrical vessel open at the top is 20 cm high and 10 cm in diameter. A circular hole of cross sectional area $1\ \text{cm}^2$ is cut at the centre of the bottom of the vessel. Water flows from a tube above it into the vessel at the rate of $10\ \text{cm}^3/\text{s}$. The height of water in the vessel under steady state is

(Take $g = 10\ \text{m/s}^2$)

(a) 20 cm (b) 15 cm (c) 10 cm (d) 5 cm

37. A horizontal pipeline carries water in a streamline flow. At a point along the tube where the cross sectional area is $10^{-2}\ \text{m}^2$, the water velocity is 2 m/s and the pressure is 8000 Pa. The pressure of water at another point where cross sectional area is $0.5 \times 10^{-2}\ \text{m}^2$ is

(a) 4000 Pa (b) 1000 Pa (c) 2000 Pa (d) 3000 Pa

38. Eight spherical rain drops of the same mass and radius are falling down with a terminal speed of $6\ \text{cms}^{-1}$. If they coalesce to form one big drop, what will be its terminal speed? Neglect the buoyancy due to air

(a) $1.5\ \text{cms}^{-1}$ (b) $6\ \text{cms}^{-1}$ (c) $24\ \text{cms}^{-1}$ (d) $32\ \text{cms}^{-1}$

JEE Corner

Assertion and Reason

Directions : *Choose the correct option.*

*(a) If both **Assertion** and **Reason** are true and the **Reason** is correct explanation of the **Assertion**.*
*(b) If both **Assertion** and **Reason** are true but **Reason** is not the correct explanation of **Assertion**.*
*(c) If **Assertion** is true, but the **Reason** is false.*
*(d) If **Assertion** is false but the **Reason** is true.*

1. Assertion : Pressure is a vector quantity.

Reason : Pressure $P = \dfrac{F}{A}$. Here F, the force is a vector quantity.

2. **Assertion :** Surface tension $\left(T=\frac{F}{l}\right)$ is not a vector quantity.

Reason : Direction of force is specified.

3. **Assertion :** At depth 'h' below the water surface pressure is P. Then at depth '$2h$' pressure will be $2P$. (Ignore density variation).

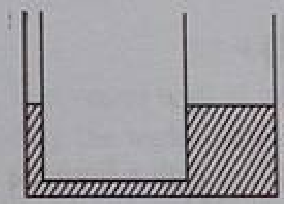
Reason : With depth pressure increases linearly.

4. **Assertion :** Weight of solid in air is W and in water is $\frac{2W}{3}$. Then relative density of solid is 3.0.

Reason : Relative density of any solid is given by :

$$\text{RD}=\frac{\text{Weight in air}}{\text{Change in weight in water}}$$

5. **Assertion :** Water is filled in a U-tube of different cross-sectional area on two sides as shown in figure. Now equal amount of oil (RD = 0.5) is poured on two sides. Level of water on both sides will remain unchanged.

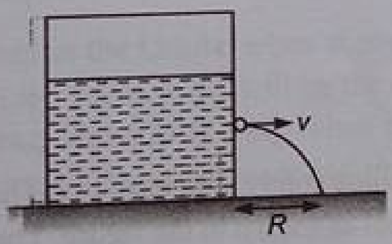


Reason : Same weight of oil poured on two sides will produce different pressures.

6. **Assertion :** An ideal fluid is flowing through a pipe. Speed of fluid particles is more at places where pressure is low.

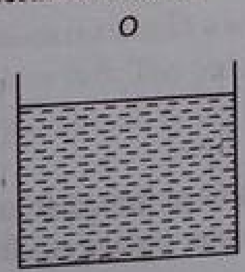
Reason : Bernoulli's theorem can be derived from work-energy theorem.

7. **Assertion :** In the figure shown v and R will increase if pressures above the liquid surface inside the chamber is increased.



Reason : Value of v or R is independent of density of liquid.

8. **Assertion :** A ball is dropped from a certain height above the free surface of an ideal fluid. When the ball enters the liquid it may accelerate or retard.

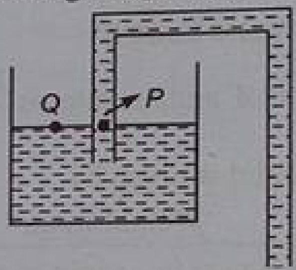


Reason : Ball accelerates or retards it all depends on the density of ball and the density of liquid.

9. **Assertion :** On moon barometer height will be six times compared to the height or earth.

Reason : Value of 'g' on moon's surface is $\frac{1}{6}$ the value of 'g' on earth's surface.

10. **Assertion :** In the siphon shown in figure, pressure at P is equal to atmospheric pressure.



Reason : Pressure at Q is atmospheric pressure and points P and Q are at same levels.

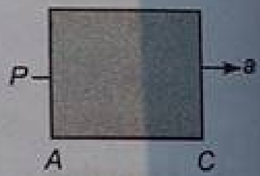
11. **Assertion :** Force of buoyancy due to atmosphere on a small body is almost zero (or negligible).

Reason : If a body is completely submerged in a fluid, then buoyant force is zero.

Objective Questions (Level 2)

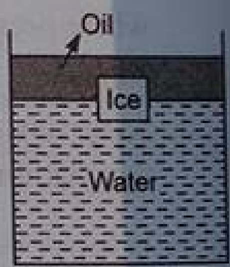
Single Correct Option

1. A cubical open vessel of side 5 m filled with a liquid is accelerated with an acceleration a. The value of a so that pressure at mid point of AC is equal to pressure at point p is



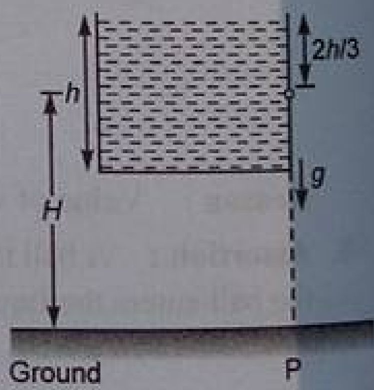
(a) g (b) $2g$
(c) $g/2$ (d) $2g/5$

2. An ice cube is floating in water above which a layer of lighter oil is poured. As the ice melts completely, the level of interface and the upper most level of oil will respectively



(a) rise and fall
(b) fall and rise
(c) not change and no change
(d) not change and fall

3. An open vessel full of water is falling freely under gravity. There is a small hole in one face of the vessel as shown in the figure. The water which comes out from the hole at the instant when hole is at height H above the ground, strikes the ground at a distance of x from P. Which of the following is correct for the situation described?



(a) The value of x is $2\sqrt{\frac{2hH}{3}}$

(b) The value of x is $\sqrt{\frac{4hH}{3}}$

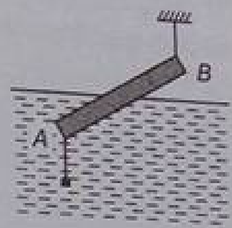
(c) The value of x can't be computed from information provided

(d) The question is irrevalent as no water comes out from the hole

4. A uniform rod AB, 12 m long weighing 24 kg, is supported at end B by a flexible light string and a lead weight (of very small size) of 12 kg attached at end A.

The rod floats in water with one-half of its length submerged. For this situation, mark out the correct statement.

[Take $g = 10\ \text{m/s}^2$, density of water $= 1000\ \text{kg/m}^3$]



(a) The tension in the string is 36 g

(b) The tension in the string is 12 g

(c) The volume of the rod is $6.4 \times 10^{-2}\ \text{m}^3$

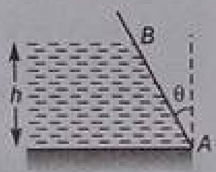
(d) The point of application of the buoyancy force is passing through C (centre of mass of rod)

5. A water hose pipe of cross-section area $5\ \text{cm}^2$ is used to fill a tank of 120 L. It has been observed that it takes 2 min to fill the tank. Now, a nozzle with an opening of cross-section area $1\ \text{cm}^2$ is attached to the hose. The nozzle is held so that water is projected horizontally from a point 1 m above the ground. The horizontal distance over which the water can be projected is

{Take $g = 10\ \text{m/s}^2$}

(a) 3 m (b) 8 m (c) 4.47 m (d) 8.64 m

6. The height of water in a vessel is h. The vessel wall of width b is at an angle θ to the vertical. The net force exerted by the water on the wall is



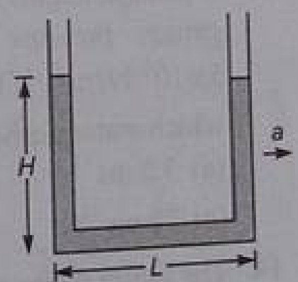
(a) $\frac{1}{3}\rho b h^2 g \cos\theta$ (b) $\frac{1}{2} b h^2 \rho g$

(c) $\frac{1}{2}\rho b h^2 g \sec\theta$ (d) zero

7. A body of density ρ is dropped from rest from a height h into a lake of density σ ($\sigma > \rho$). The maximum depth the body sinks inside the liquid is (neglect viscous effect of liquid)

(a) $\frac{h\rho}{\sigma - \rho}$ (b) $\frac{h\sigma}{\sigma - \rho}$ (c) $\frac{h\rho}{\sigma}$ (d) $\frac{h\sigma}{\rho}$

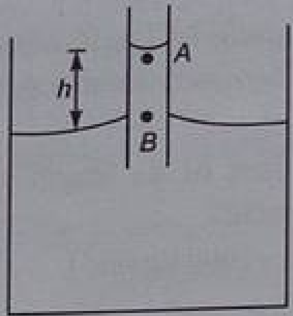
8. A liquid stands at the plane level in the U-tube when at rest. If areas of cross-section of both the limbs are equal, what will be the difference in heights h of the liquid in the two limbs of U-tube, when the system is given an acceleration a in horizontal direction towards right as shown?



(a) $\frac{g}{a}\frac{L^2}{H}$ (b) $\frac{La}{g}$

(c) $\frac{L^2}{H}\frac{a}{g}$ (d) $\frac{Lg}{a}$

9. A liquid of density ρ and surface tension σ rises in a capillary tube of inner radius R. The angle of contact between the liquid and the glass is θ. The point A lies just below the meniscus in the tube and the point B lies at the outside level of liquid in the beaker as shown in figure. The pressure at A is



(a) $p_B - \rho g h$ (b) $p_B - \dfrac{2\sigma\cos\theta}{R}$ (c) $p_{atm} - \dfrac{2\sigma\cos\theta}{R}$ (d) All of these

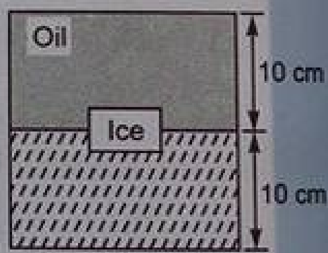
10. A large open tank has two holes in the wall. One is a square hole of side L at a depth h from the top and the other is a circular hole of radius R at a depth $4h$ from the top. When the tank is completely filled with water, quantities of water flowing out per second from both holes are the same. Then R is equal to

(a) $\dfrac{L}{\sqrt{2\pi}}$ (b) $2\pi L$ (c) L (d) $\dfrac{L}{2\pi}$

11. Two identical cylindrical vessels with their bases at the same level each contain a liquid of density ρ. The area of either base is A but in one vessel the liquid height is h_1 and in the other liquid height is $h_2 (h_2 < h_1)$. If the two vessels are connected, the work done by gravity in equalizing the levels is

(a) $\dfrac{1}{2}(h_1 - h_2)^2 A\rho g$ (b) $\dfrac{1}{2}(h_1 + h_2)A\rho g$ (c) $\dfrac{1}{2}(h_1^2 - h_2^2)A\rho g$ (d) $\dfrac{1}{4}(h_1 - h_2)^2 A\rho g$

12. A cubical block of side 10 cm floats at the interface of an oil and water as shown in the figure. The density of oil is 0.6 g cm^{-3} and the lower face of ice cube is 2 cm below the interface. The pressure above that of the atmosphere at the lower face of the block is

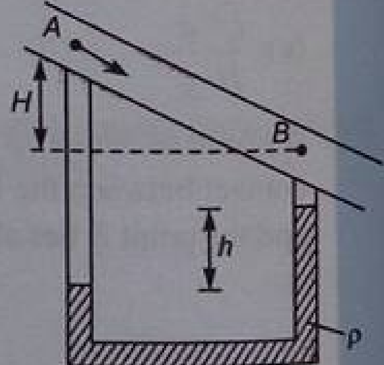


(a) 200 Pa (b) 620 Pa
(c) 900 Pa (d) 800 Pa

13. A leakage begins in water tank at position P as shown in the figure. The initial gauage pressure (pressure above that of the atmosphere) at P was $5 \times 10^5 \text{ N/m}^2$. If the density of water is 1000 kg/m^3 the initial velocity with which water gushes out is

(a) 3.2 ms^{-1} (b) 32 ms^{-1}
(c) 28 ms^{-1} (d) 2.8 ms^{-1}

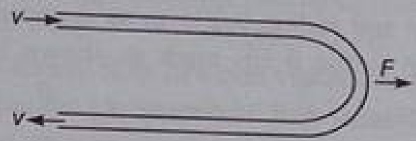
14. The figure shows a pipe of uniform cross-section inclined in a vertical plane. A U-tube manometer is connected between the points A and B. If the liquid of density ρ_0 flows with velocity v_0 in the pipe. Then the reading h of the manometer is



(a) $h = 0$ (b) $h = \dfrac{v_0^2}{2g}$

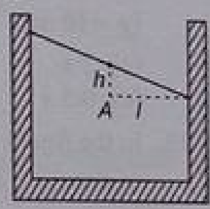
(c) $h = \dfrac{\rho_0}{\rho}\left(\dfrac{v_0^2}{2g}\right)$ (d) $h = \dfrac{\rho_0 H}{\rho - \rho_0}$

15. A horizontal tube of uniform cross-sectional area A is bent in the form of U as shown in figure. If the liquid of density ρ enters and leaves the tube with velocity v then the external force F required to hold the bend stationary is



(a) $F = 0$ (b) $\rho A v^2$ (c) $2\rho A v^2$ (d) $\frac{1}{2}\rho A v^2$

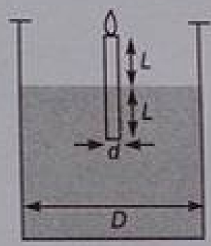
16. A rectangular container moves with an acceleration a along the positive direction as shown in figure. The pressure at the point A in excess of the atmospheric pressure p_0 is (take ρ as the density of liquid)



(a) ρgh (b) ρal

(c) $\rho\,(gh + al)$ (d) Both (a) and (c)

17. A candle of diameter d is floating on a liquid in a cylindrical container of diameter D ($D >> d$) as shown in figure. It it is burning at the rate of 2 cm/h. Then the top of the candle will



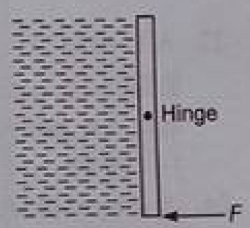
(a) remain at the same height

(b) fall at the rate of 1 cm/h

(c) fall at the rate of 2 cm/h

(d) go up at the rate of 1 cm/h

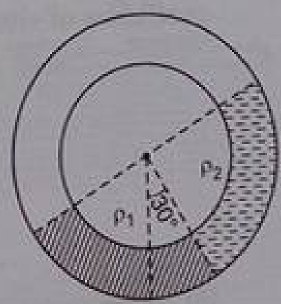
18. A square gate of size 1 m × 1 m is hinged at its mid point. A fluid of density ρ fills the space to the left of the gate. The force F required to hold the gate stationary is



(a) $\frac{\rho g}{3}$ (b) $\frac{1}{2}\rho g$

(c) $\frac{\rho g}{6}$ (d) None of these

19. A thin uniform circular tube is kept in a vertical plane. Equal volumes of two immiscible liquids whose densities are ρ_1 and ρ_2 fill half of the tube as shown. In equilibrium the radius passing through the interface makes an angle of 30° with vertical. The ratio of densities (ρ_1/ρ_2) is equal to



(a) $\frac{\sqrt{3}-1}{2-\sqrt{3}}$ (b) $\frac{\sqrt{3}+1}{2+\sqrt{3}}$

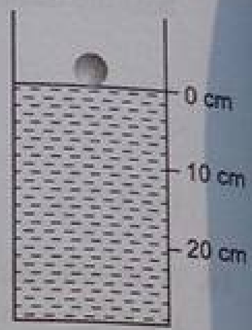
(c) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ (d) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$

20. A plate moves normally with the speed v_1 towards a horizontal jet of water of uniform area of cross-section. The jet discharges water at the rate of volume V per second at a speed of v_2. The density of water is ρ. Assume that water splashes along the surface of the plate ar right angles to the original motion. The magnitude of the force acting on the plate due to the jet of water is

(a) $\rho V v_1$ (b) $\rho\left(\frac{V}{v_2}\right)(v_1+v_2)^2$ (c) $\frac{\rho V}{v_1+v_2}(v_1)^2$ (d) $\rho V(v_1+v_2)$

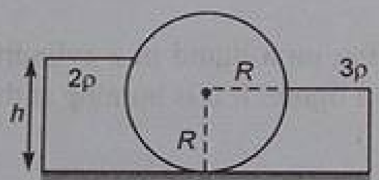
21. A spherical ball of density ρ and radius 0.003 m is dropped into a tube containing a viscous fluid up to the 0 cm mark as shown in the figure. Viscosity of the fluid = 1.26 N-s/m^2 and its density $\rho_L = \frac{\rho}{2} = 1260\ \text{kg/m}^3$.

Assume the ball reaches a terminal speed at 10 cm mark. The time taken by the ball to travel the distance between the 10 cm and 20 cm mark is ($g = 10\ \text{m/s}^2$)



(a) 2 s (b) 1 s
(c) 0.5 s (d) 5 s

22. In the figure shown, the heavy cylinder (radius R) resting on a smooth surface separates two liquids of densities 2ρ and 3ρ. The height h for the equilibrium of cylinder must be

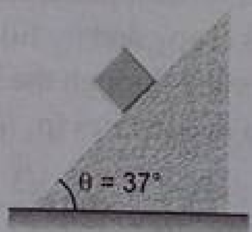


(a) $3R/2$ (b) $R\sqrt{\frac{3}{2}}$ (c) $R\sqrt{2}$ (d) None of these

23. A U-tube having horizontal arm of length 20 cm, has uniform cross-sectional area $= 1\ \text{cm}^2$. It is filled with water of volume 60 cc. What volume of a liquid of density 4g/cc should be poured from one side into the U-tube so that no water is left in the horizontal arm of the tube?

(a) 60 cc (b) 45 cc (c) 50 cc (d) 35 cc

24. A cubical block of side a and density ρ slides over a fixed inclined plane with constant velocity v. There is a thin film of viscous fluid of thickness t between the plane and the block. Then the coefficient of viscosity of the film will be



(a) $\frac{3\rho agt}{5v}$ b) $\frac{4\rho agt}{5v}$ (c) $\frac{\rho agt}{v}$ (d) None of these

25. A water barrel stands on a table of height h. If a small hole is punched in the side of the barrel at its base, it is found that the stream of water strikes the ground at a horizontal distance R from the barrel. The depth of water in the barrel is

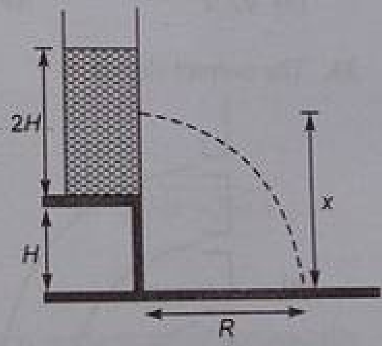
(a) $\frac{R}{2}$ (b) $\frac{R^2}{4h}$ (c) $\frac{R^2}{h}$ (d) $\frac{h}{2}$

26. A spring balance reads 10 kg when a bucket of water is suspended from it. What will be the reading of the balance when an iron piece of mass 7.2 kg suspended by a string is immersed with half its volume inside the water in the bucket? Relative density of iron is 7.2?
(a) 10 kg (b) 10.5 kg (c) 13.6 kg (d) 17.2 kg

27. Three points A, B and C on a steady flow of a non viscous and incompressible fluid are observed. The pressure, velocity and height of the points A, B and C are (2, 3, 1), (1, 2, 2) and (4, 1, 2) respectively. Density of the fluid is 1 kgm^{-3} and all other parameters are given in SI units. Then which of the following is correct? ($g = 10\ \text{ms}^{-2}$).
(a) points A and B lie on the same stream line
(b) point B and C lie on same stream line
(c) point C and A lie on same stream line
(d) None of the above

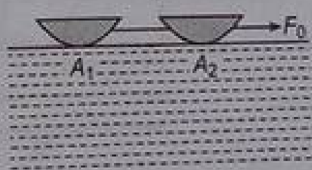
28. A body of density ρ is dropped from rest from height 'h' (from the surface of water) into a lake of density of water σ ($\sigma > \rho$). Neglecting all dissipative effects, the acceleration of body while it is in the lake is
(a) $g\left(\frac{\sigma}{\rho}-1\right)$ upwards (b) $g\left(\frac{\sigma}{\rho}-1\right)$ downwards
(c) $g\left(\frac{\sigma}{\rho}\right)$ upwards (d) $g\left(\frac{\sigma}{\rho}\right)$ downwards

29. A tank is filed up to a height $2H$ with a liquid and is placed on a platform of height H from the ground. The distance x from the ground where a small hole is punched to get the maximum range R is



(a) H
(b) $1.25\ H$
(c) $1.5\ H$
(d) $2\ H$

30. Two boats of base areas A_1 and A_2, connected by a string are being pulled by an external force F_0. The viscosity of water is η and depth of the water body is H. When the system attains a constant speed, the tension in the thread will be

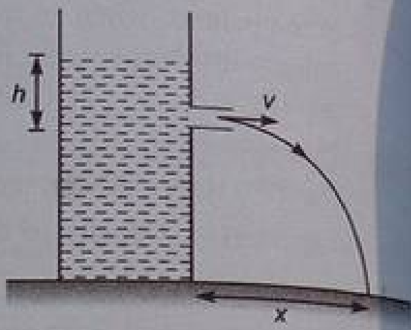


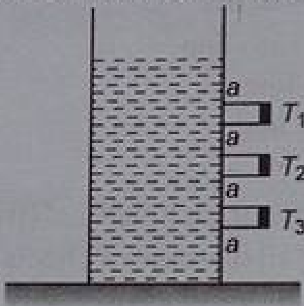
(a) zero (b) $F_0 \dfrac{A_2}{(A_1 + A_2)}$
(c) $F_0 \dfrac{A_1}{(A_1 + A_2)}$ (d) None of these

31. A U-tube is partially filled with water. Oil which does not mix with water is next poured into one side, until water rises by 25 cm on the other side. If the density of oil is $0.8\ \text{g/cm}^3$, the oil level will stand higher than the water level by
(a) 6.25 cm (b) 12.50 cm (c) 31.75 cm (d) 25 cm

Passage : (Q 32 to Q 34)

The spouting can is something used to demonstrate the variation of pressure with depth. When the corks are removed from the tubes in the side of the can, water flows out with a speed that depends on the depth. In a certain can, three tubes T_1, T_2 and T_3 are set at equal distances a above the base of the can. When water contained in this can is allowed to come out of the tubes the distances on the horizontal surface are measured as x_1, x_2 and x_3.





32. Speed of efflux is

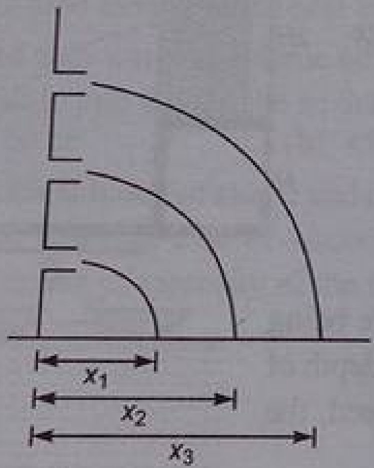
(a) $\sqrt{3gh}$ (b) $\sqrt{2gh}$ (c) $\sqrt{gh}$ (d) $\frac{1}{2}\sqrt{2gh}$

33. Distance x_3 is given by

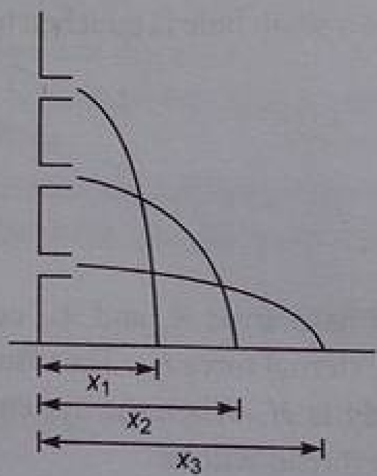
(a) $\sqrt{3}\,a$ (b) $\sqrt{2}\,a$ (c) $\frac{1}{2}\sqrt{3}\,a$ (d) $2\sqrt{3}\,a$

34. The correct sketch is

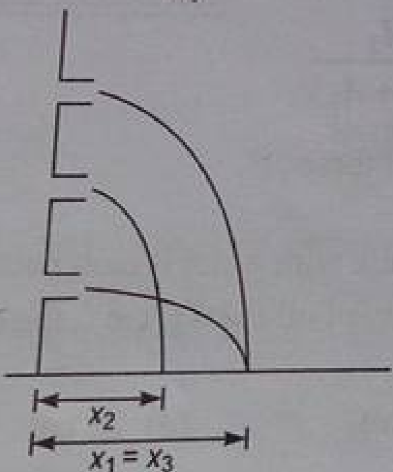
(a)



(b)



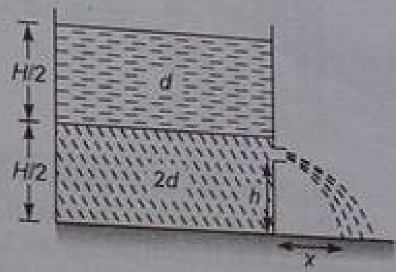
(c)



(d) None of these

Passage : (Q 35 to Q 38)

A container of large uniform cross-sectional area A resting on a horizontal surface, holds, two immiscible, non-viscous and incompressible liquids of densities d and $2d$ each of height $H/2$ as shown in the figure. The lower density liquid is open to the atmosphere having pressure P_0. A homogeneous solid cylinder of length $L(L< H/2)$, cross-sectional area $A/5$ is immersed such that it floats with its axis vertical at the liquid-liquid interface with length $L/4$ in the denser liquid. The cylinder is then removed and the original arrangement is restored. A tiny hole of area $s(s<< A)$ is punched on the vertical side of the container at a height $h(h< H/2)$. As a result of this, liquid starts flowing out of the hole with a range x on the horizontal surface.



35. The density D of the material of the floating cylinder is
(a) $5d/4$ (b) $3d/4$ (c) $4d/5$ (d) $4d/3$

36. The total pressure with cylinder, at the bottom of the container is
(a) $P_0+\frac{(6L+H)}{4}dg$ (b) $P_0+\frac{(L+6H)}{4}dg$ (c) $P_0+\frac{(L+3H)}{4}dg$ (d) $P_0+\frac{(L+2H)}{4}dg$

37. The initial speed of efflux without cylinder is
(a) $v=\sqrt{\frac{g}{3}[3H+4h]}$ (b) $v=\sqrt{\frac{g}{2}[4H-3h]}$
(c) $v=\sqrt{\frac{g}{2}[3H-4h]}$ (d) None of these

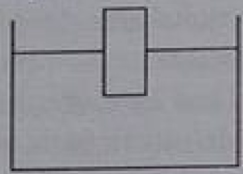
38. The horizontal distance traveled by the liquid, initially, is
(a) $\sqrt{(3H+4h)\,h}$ (b) $\sqrt{(3h+4H)h}$ (c) $\sqrt{(3H-4h)\,h}$ (d) $\sqrt{(3H-3h)\,h}$

More than One Correct Options

1. A large wooden plate of area $10\ m^2$ floating on the surface of a river is made to move horizontally with a speed of 2 m/s by applying a tangential force. River is 1 m deep and the water in contact with the bed is stationary. Then choose correct statement.
(coefficient of viscosity of water $=10^{-3}$ N-s/m^2)
(a) velocity gradient is $2\ s^{-1}$
(b) velocity gradient is $1\ s^{-1}$
(c) force required to keep the plate moving with constant speed is 0.02 N
(d) force required to keep the plate moving with constant speed is 0.01 N

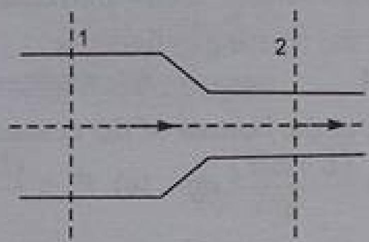
2. Choose the correct options
(a) viscosity of liquids increases with temperature
(b) viscosity of gases increases with temperature
(c) surface tension of liquids decreases with temperature
(d) for angle of contact $\theta=0°$, liquid neither rises nor falls on capillary

3. A plank is floating in a non-viscous liquid as shown. Choose the correct options.



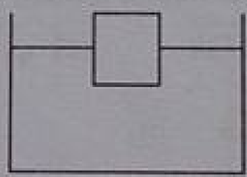
(a) Equilibrium of plank is stable in vertical direction
(b) For small oscillations of plank in vertical direction motion is simple harmonic
(c) Even if oscillations are large, motion is simple harmonic
(d) On vertical displacement motion is periodic but not simple harmonic

4. A non-viscous incompressible liquid is flowing from a horizontal pipe of non-uniform cross section as shown. Choose the correct options.



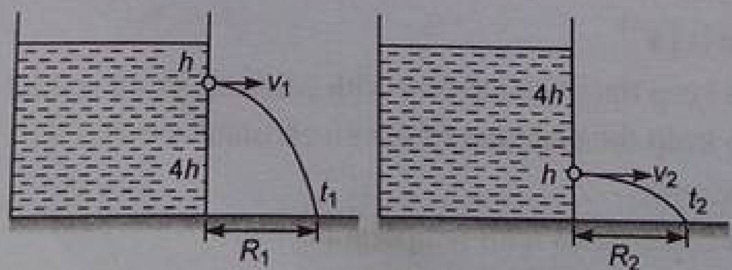
(a) speed of liquid at section-2 is more
(b) volume of liquid flowing per second from section-2 is more
(c) mass of liquid flowing per second at both the sections is same
(d) pressure at section-2 is less

5. A plank is floating in a liquid as shown. Fraction f of its volume is immersed. Choose the correct options.



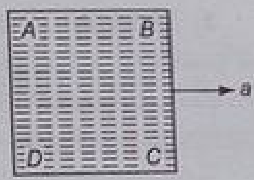
(a) If the system is taken to a place where atmospheric pressure is more, 'f' will increase
(b) In above condition f will remain unchanged
(c) If temperature is increased and expansion of only liquid is considered f will increase
(d) If temperature is increased and expansion of only plank is considered f will decrease

6. In two figures



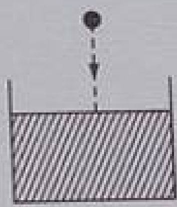
(a) $v_1/v_2 = 1/2$ (b) $t_1/t_2 = 2/1$ (c) $R_1/R_2 = 1$ (d) $v_1/v_2 = 1/4$

7. A liquid is filled in a container as shown in figure. Container is accelerated towards right. There are four points A, B, C and D in the liquid. Choose the correct options.



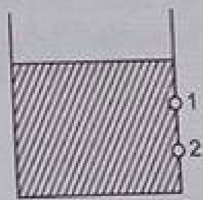
(a) $P_A > P_B$
(b) $P_C > P_A$
(c) $P_D > P_B$
(d) $P_A > P_C$

8. A ball of density ρ is dropped from a height on the surface of a non-viscous liquid of density 2ρ. Choose the correct options.



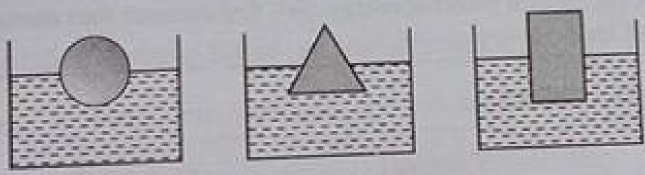
(a) Motion of ball is periodic but not simple harmonic
(b) Acceleration of ball in air and in liquid are equal
(c) Magnitude of upthrust in the liquid is two times the weight of ball
(d) Net force on ball in air and in liquid are equal and opposite

9. Two holes 1 and 2 are made at depths h and $16h$ respectively. Both the holes are circular but radius of hole-1 is two times.



(a) Initially equal volumes of liquid will flow from both the holes in unit time
(b) Initially more volume of liquid will flow from hole-2 per unit time
(c) After some time more volume of liquid will flow from hole-1 per unit time
(d) After some time more volume of liquid will flow from hole-2 per unit time

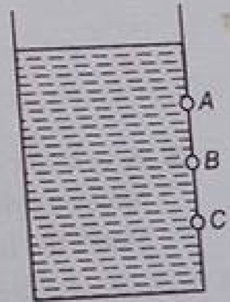
10. A solid sphere, a cone and a cylinder are floating in water. All have same mass, density and radius. Let f_1, f_2 and f_3 are the fraction of their volumes inside the water and h_1, h_2 and h_3 depths inside water. Then



(a) $f_1 = f_2 = f_3$
(b) $f_3 > f_2 > f_1$
(c) $h_3 < h_1$
(d) $h_3 < h_2$

Match the Columns

1. Three holes A, B and C are made at depths 1m, 2 m and 5 m as shown. Total height of liquid in the container is 8 m. Let v is the speed with which liquid comes out of the hole and R the range on ground. Match the following two columns.



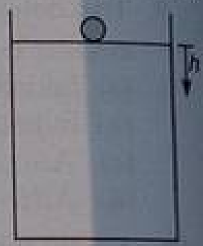
Column I	Column II
(a) v is maximum for	(p) hole A
(b) v is minimum for	(q) hole B
(c) R is maximum for	(r) hole C
(d) R is minimum for	(s) will depend on density of liquid

2. Match the following two columns.

Column I	Column II
(a) When temperature is increased	(p) Upthrust on a floating solid of constant volume will increase
(b) When density of liquid is increased	(q) Upthrust on a floating solid of constant volume will decrease
(c) When density of solid is increased	(r) Viscosity of gas will decrease
(d) When atmospheric pressure is increased	(s) None

3. A ball of density ρ is released from the surface of a liquid whose density varies with depth h as, $\rho_l = \alpha h$

Here α is a positive constant. Match the following two columns. (liquid is ideal)

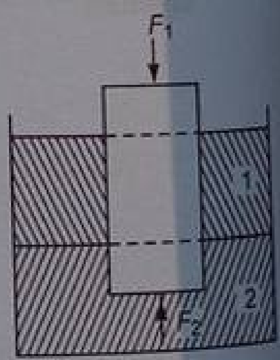


Column I	Column II
(a) Upthrust on ball	(p) will continuously decrease
(b) Speed of ball	(q) will continuously increase
(c) Net force on ball	(r) first increase then decrease
(d) Gravitational potential energy of ball	(s) first decrease then increase

4. Match the following two columns.

Column I	Column II
(a) Surface tension	(p) $[ML^{-1}T^{-2}]$
(b) Coefficient of viscosity	(q) $[L^3T^{-1}]$
(c) Energy density	(r) $[MT^{-2}]$
(d) Volume flow rate	(s) $[ML^{-1}T^{-1}]$

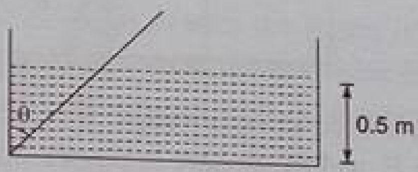
5. A cylinder of weight W is floating in two liquids as shown in figure. Net force on cylinder from top is F_1 and force on cylinder from the bottom is F_2. Match the following two columns.



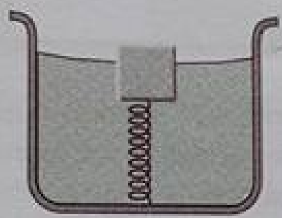
Column I	Column II
(a) Net force on cylinder from liquid-1	(p) Zero
(b) $F_2 - F_1$	(q) W
(c) Net force on cylinder from liquid-2	(r) Net upthrust
(d) Net force on cylinder from atmosphere	(s) None of these

Subjective Questions (Level 2)

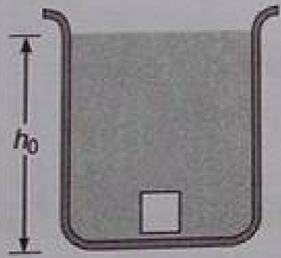
1. A wooden plank of length 1 m and uniform cross-section is hinged at one end to the bottom of a tank as shown in figure. The tank is filled with water upto a height of 0.5 m. The specific gravity of the plank is 0.5. Find the angle θ that the plank makes with the vertical in the equilibrium position. (Exclude the case $\theta = 0$).



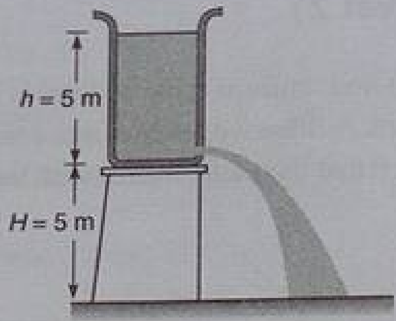
2. A cubical block of wood of edge 3 cm floats in water. The lower surface of the cube just touches the free end of a vertical spring fixed at the bottom of the pot. Find the maximum weight that can be put on the block without wetting the new weight. Density of wood $= 800\,\text{kg/m}^3$ and spring constant of the spring $= 50\,\text{N/m}$. (Take $g = 10\,\text{m/s}^2$)



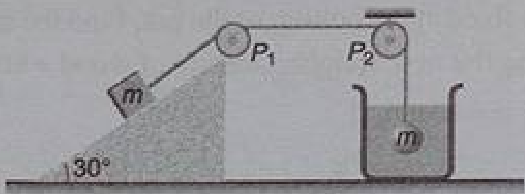
3. Figure shows a container having liquid of variable density. The density of liquid varies as $\rho = \rho_0\left(4 - \dfrac{3h}{h_0}\right)$. Here, h_0 and ρ_0 are constants and h is measured from bottom of the container. A solid block of small dimensions whose density is $\dfrac{5}{2}\rho_0$ and mass m is released from bottom of the tank. Prove that the block will execute simple harmonic motion. Find the frequency of oscillation.



4. A cylindrical tank 1 m in radius rests on a platform 5 m high. Initially the tank is filled with water to a height of 5 m. A plug whose area is $10^{-4}\,\text{m}^2$ is removed from an orifice on the side of the tank at the bottom. Calculate (a) initial speed with which the water flows from the orifice, (b) initial speed with which water strikes the ground, (c) time taken to empty the tank to half its original value. ($g = 10\,\text{m/s}^2$)

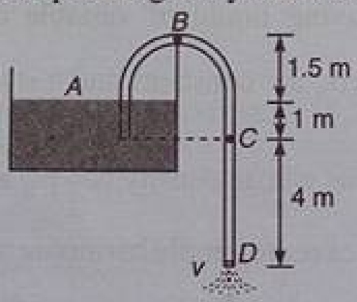


5. A block of mass m is kept over a fixed smooth wedge. Block is attached to a sphere of same mass through fixed massless pullies P_1 and P_2. Sphere is dipped inside the water as shown. If specific gravity of material of sphere is 2. Find the acceleration of sphere.



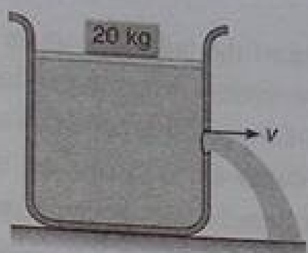
6. A cubic body floats on mercury with 0.25 fraction of its volume below the surface. What fraction of the volume of the body will be immersed in the mercury if a layer of water poured on top of the mercury covers the body completely ?

7. A siphon tube is discharging a liquid of specific gravity 0.9 from a reservoir as shown in the figure.

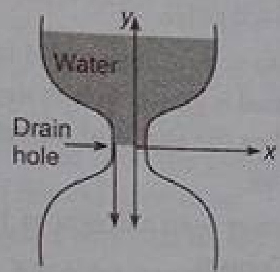


(a) Find the velocity of the liquid through the siphon.
(b) Find the pressure at the highest point B.
(c) Find the pressure at point C.

8. A long cylindrical tank of cross-sectional area 0.5 m^2 is filled with water. It has a small hole at a height 50 cm from the bottom. A movable piston of cross-sectional area almost equal to 0.5 m^2 is fitted on the top of the tank such that it can slide in the tank freely. A load of 20 kg is applied on the top of the water by piston, as shown in the figure. Calculate the speed of the water jet with which it hits the surface when piston is 1 m above the bottom. (Ignore the mass of the piston).

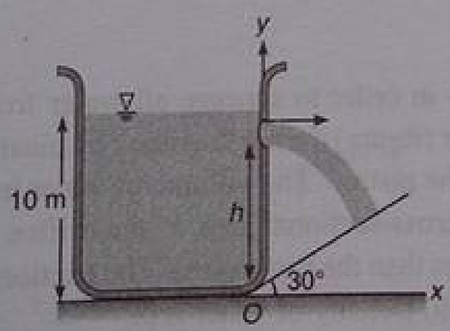


9. The shape of an ancient water clock jug is such that water level descends at a constant rate at all times. If the water level falls by 4 cm every hour, determine the shape of the jar, *i.e.*, specify x as a function of y. The radius of drain hole is 2 mm and can be assumed to be very small compared to x.



10. A spring is attached to the bottom of an empty swimming pool, with the axis of the spring oriented vertically. An 8.00 kg block of wood ($\rho = 840\ kg/m^3$) is fixed to the top of the spring and compresses it. Then the pool is filled with water, completely covering the block. The spring is now observed to be stretched twice as much as it had been compressed. Determine the percentage of the block's total volume that is hollow. Ignore any air in the hollow space.

11. A rectangular tank of height 10 m filled with water, is placed near the bottom of a plane inclined at an angle 30° with horizontal. At height h from bottom a small hole is made (as shown in figure) such that the stream coming out from hole, strikes the inclined plane normally. Calculate h.



12. A ball of density d is dropped onto a horizontal solid surface. It bounces elastically from the surface and returns to its original position in a time t_1. Next, the ball is released and it falls through the same height before striking the surface of a liquid of density d_L.
(a) If $d < d_L$, obtain an expression (in terms of d, t_1 and d_L) for the time t_2 the ball takes to come back to the position from which it was released.
(b) Is the motion of the ball simple harmonic ?

(c) If $d = d_L$, how does the speed of the ball depend on its depth inside the liquid ? Neglect all frictional and other dissipative forces. Assume the depth of the liquid to be large.

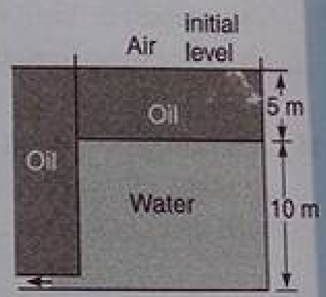
13. There is an air bubble of radius 1.0 mm in a liquid of surface tension 0.075 N/m and density 1000 kg/m^3. The bubble is at a depth of 10 cm below the free surface. By what amount is the pressure inside the bubble greater than the atmospheric pressure ? (Take $g = 9.8$ m/s^2)

14. A metal sphere of radius 1 mm and mass 50 mg falls vertically in glycerine. Find (a) the viscous force exerted by the glycerine on the sphere when the speed of the sphere is 1 cm/s, (b) the hydrostatic force exerted by the glycerine on the sphere and (c) the terminal velocity with which the sphere will move down without acceleration. Density of glycerine = 1260 kg/m^3 and its coefficient of viscosity at room temperature = 8.0 poise.

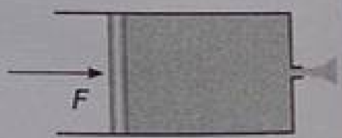
15. A wire forming a loop is dipped into soap solution and taken out, so that a film of soap solution is formed. A loop of 6.28 cm long thread is gently put on the film and the film is pricked with a needle inside the loop. The thread loop takes the shape of a circle. Find the tension in the thread. Surface tension of soap solution = 0.030 N/m.

16. A cylindrical vessel is filled with water upto a height of 1 m. The cross-sectional area of the orifice at the bottom is (1/400) that of the vessel.
(a) What is the time required to empty the tank through the orifice at the bottom?
(b) What is the time required for the same amount of water to flow out if the water level in tank is maintained always at a height of 1 m from orifice?

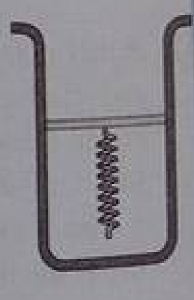
17. A tank having a small circular hole contains oil on top of water. It is immersed in a large tank of the same oil. Water flows through the hole. What is the velocity of this flow initially? When the flow stops, what would be the position of the oil-water interface in the tank from the bottom. The specific gravity of oil is 0.5.



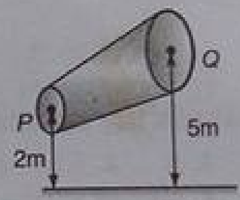
18. What work should be done in order to squeeze all water from a horizontally located cylinder (figure) during the time t by means of a constant force acting on the piston? The volume of water in the cylinder is equal to V, the cross-sectional area of the orifice is s, with s being considerably less than the piston area. The friction and viscosity are negligibly small. Density of water is ρ.



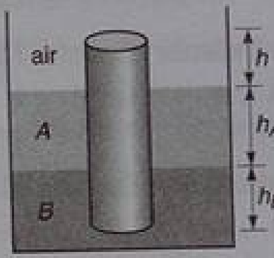
19. A cylinder is fitted with a piston, beneath which is a spring, as in the figure. The cylinder is open at the top. Friction is absent. The spring constant of the spring is 3600 N/m. The piston has a negligible mass and a radius of 0.025 m. (a) When air beneath the piston is completely pumped out, how much does the atmospheric pressure cause the spring to compress? (b) How much work does the atmospheric pressure do in compressing the spring?



20. A non-viscous liquid of constant density 1000 kg/m^3 flows in a streamline motion along a tube of variable cross-section. The tube is kept inclined in the vertical plane as shown in the figure. The area of cross-section of the tube at two points P and Q at heights of 2 m and 5 m are respectively $4 \times 10^{-3} \text{ m}^2$ and $8 \times 10^{-3} \text{ m}^2$. The velocity of the liquid at point P is 1 m/s. Find the work done per unit volume by the pressure and the gravity forces as the fluid flows from point P to Q. Take $g = 9.8 \text{ m/s}^2$.



21. A uniform solid cylinder of density 0.8 g/cm^3 floats in equilibrium in a combination of two non-mixing liquids A and B with its axis vertical. The densities of the liquids A and B are 0.7 g/cm^3 and 1.2 g/cm^3, respectively. The height of liquid A is $h_A = 1.2$ cm. The length of the part of the cylinder immersed in liquid B is $h_B = 0.8$ cm.



(a) Find the total force exerted by liquid A on the cylinder.

(b) Find h, the length of the part of the cylinder in air.

(c) The cylinder is depressed in such a way that its top surface is just below the upper surface of liquid A and is then released. Find the acceleration of the cylinder immediately after it is released.

ANSWERS

Introductory Exercise 13.1

1. B 2. 6.92×10^5 Pa 3. 0.8
4. Mercury will rise in the arm containing sprit. The difference in level is 0.221 cm
5. (i) Absolute pressure = 96 cm of Hg, Gauge pressure = 20 cm of Hg for (a),
absolute pressure = 58 cm of Hg, gauge pressure = − 18 cm of Hg for (b)
(ii) Mercury would rise in the left limb such that the difference in the levels in the two limbs becomes 19 cm.

Introductory Exercise 13.2

1. 3.5×10^3 kg/m^3 2. 7 g/cm^3, 3 g/cm^3 3. It will float at the same level 4. Zero 5. False
6. 80 kg 7. It will remain same 8. 17 cm 9. 600 kg/m^3, 705 kg/m^3 10. 6.03 g

Introductory Exercise 13.3

1. 1.44×10^{-5} J 2. $8\pi R^2 T$ 3. It will increase by $8\pi r^2 T$ 4. The capillaries formed in threads disappear
5. (a) 2.25 m/s (b) zero 6. (a) 7.0×10^{-4} m^3/s (b) 2.24×10^4 Pa (c) 0.5×10^{-3} m^3/s

AIEEE Corner

Subjective Questions (Level 1)

2. 3840 kg/m^3 3. 28 cm^3 4. 23.7 N 5. 0.33 m^3 6. (a) 2×10^{-6} m^3, 5000 kg/m^3 (b) 750 kg/m^3
7. 0.206 N 8. 32 mm in mercury and 28 mm in water 9. (a) 14.7 m/s^2 (b) 0.63 s
10. $T = 19.6$ N, $V = 32 \times 10^{-3}$ m^3 11. (a) 24 N (b) 12 m/s^2 12. (a) 10.9 cm (b) 2.5 cm
13. $\tan\theta = \dfrac{\rho - \sigma}{\rho + \sigma}$ 14. 31 N
15. 12.6 cm 16. (a) $\dfrac{\rho g l h^2}{2}$ (b) $\dfrac{\rho g l h^3}{6}$ (c) $\dfrac{h}{3}$ 17. (a) 1470 Pa (b) 13.9 cm 18. 18 cm
19. 0.58 cm 20. 6.7 kPa, 8.2 kPa 21. $R^2/4h$ 22. $v = \sqrt{\dfrac{2F}{A\rho}}$ 23. 12.3 m/s
24. $v_B \approx 3.3$ m/s 25. 146 cm^3/s 26. 2.13 cm 27. 0.87 mm/s 28. 19.5 mm of Hg
29. $v' = (2)^{2/3} v$ 30. $v = \dfrac{u_0 \ln(R_2/r)}{\ln(R_2/R)}$ 31. $F = 0.02$ N 32. 10^{-3} N/m^2 33. $T_m/T_w = 7.23$
35. 1.4 mm 36. $h = 1$ cm

Objective Questions (Level 1)

1. (a)	2. (d)	3. (d)	4. (d)	5. (d)	6. (d)	7. (a)	8. (a)	9. (b)	10. (c)
11. (a)	12. (c)	13. (c)	14. (c)	15. (b)	16. (a)	17. (b)	18. (b)	19. (a)	20. (c)
21. (a)	22. (a)	23. (d)	24. (c)	25. (b)	26. (a)	27. (b)	28. (c)	29. (b)	30. (c)
31. (a)	32. (c)	33. (d)	34. (d)	35. (d)	36. (d)	37. (c)	38. (c)		

JEE Corner

Assertion and Reason

1. (d)	2. (a)	3. (d)	4. (a)	5. (d)	6. (d)	7. (b)	8. (a)	9. (d)	10. (d)
11. (c)									

Objective Questions (Level 2)

1. (b) 2. (a) 3. (d) 4. (c) 5. (c) 6. (c) 7. (a) 8. (b) 9. (d) 10. (a)
11. (d) 12. (d) 13. (b) 14. (a) 15. (c) 16. (d) 17. (b) 18. (c) 19. (d) 20. (d)
21. (d) 22. (b) 23. (d) 24. (a) 25. (b) 26. (b) 27. (d) 28. (a) 29. (c) 30. (c)
31. (b) 32. (b) 33. (d) 34. (d) 35. (a) 36. (b) 37. (c) 38. (c)

More than One Correct Options

1. (a,c) 2. (b,c) 3. (a,b,c) 4. (a,c,d) 5. (b,c,d) 6. (a,b,c) 7. (a,c)
8. (a,c,d) 9. (a,d) 10. (a,c,d)

Match the Columns

1. (a) → r (b) → p (c) → r (d) → p
2. (a) → s (b) → s (c) → p (d) → s
3. (a) → r (b) → r (c) → s (d) → s
4. (a) → r (b) → s (c) → p (d) → q
5. (a) → p (b) → q,r (c) → s (d) → s

Subjective Questions (Level 2)

1. 45° 2. 0.354 N 3. $\frac{1}{2\pi}\sqrt{\frac{6g}{5h_0}}$ 4. (a) 10 m/s (b) 14.1 m/s (c) 9200 s

5. zero 6. 0.19 7. (a) 9.9 m/s (b) 4.36×10^4 Pa (c) 6.6×10^4 Pa 8. 4.51 m/s

9. $y = 0.4x^4$ 10. 60.41% 11. 8.33 m

12. (a) $\frac{t_1 d_L}{d_L - d}$ (b) No (c) The ball will continue to move with constant velocity $v = \frac{gt_1}{2}$ inside the liquid.

13. 1130 Pa 14. (a) 1.5×10^{-4} N (b) 5.2×10^{-5} N (c) 32.5 m/s 15. 3.0×10^{-4} N

16. (a) 3 min (b) 1.5 min 17. 9.8 m/s, 5.0 m

18. $\frac{1}{2}\frac{\rho V^3}{s^2 t^2}$ 19. (a) 5.5 cm (b) 5.445 J 20. 29025 J/m^3, 29400 J/m^3

21. (a) zero (b) 0.25 cm (c) $\frac{g}{6}$ (upwards)

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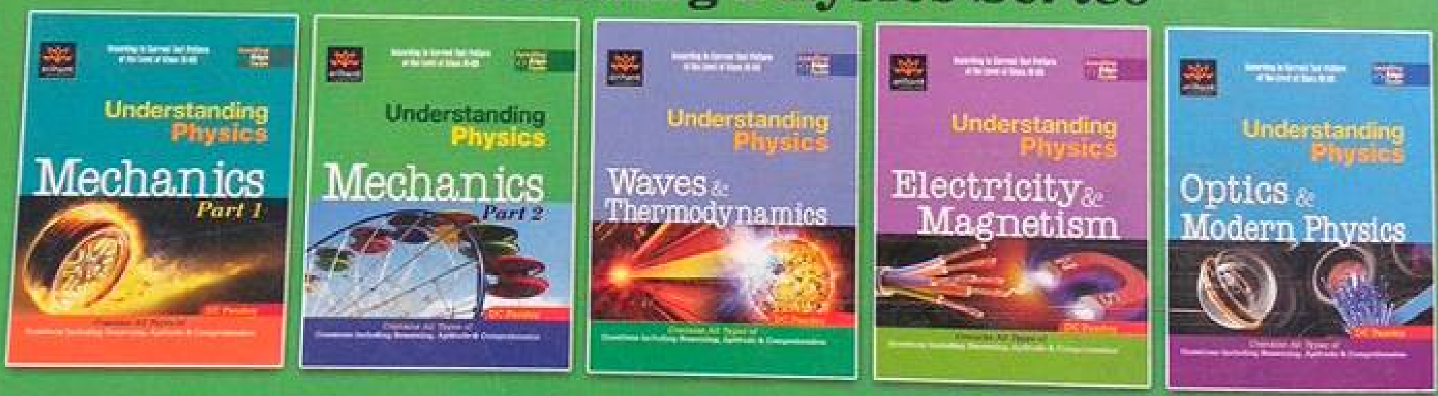

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