LEARNING MATHEMATICS

CLASS IX

FIRST AND SECOND TERM
CBSE SYLLABUS

Prepared as per the Latest Term-wise Syllabus and Guidelines issued by the CBSE on Comprehensive and Continuous Evaluation (CCE)

M.L. Aggarwal
Former Head of P.G. Department of Mathematics
D.A.V. College, Jalandhar

AVICHAL PUBLISHING COMPANY
8, INDUSTRIAL AREA, TRILOKPUR ROAD, KALA AMB 173 030, DISTT. SIRMOUR (H.P.)
DELHI OFFICE : 1002 Faiz Road (opp. Hanuman Murti), Karol Bagh, New Delhi-100 005

Every effort has been made to avoid errors or omissions in this publication. In spite of this, some errors might have crept in. Any mistake, error or discrepancy noted may be brought to our notice which shall be taken care of in the next edition. It is notified that neither the publisher nor the author or seller will be responsible for any damage or loss of action to any one, of any kind, in any manner, therefrom.

For binding mistakes, misprints or for missing pages etc., the publisher's liability is limited to replacement within one month of purchase by similar edition. All expenses in this connection are to be borne by the purchaser.

No part of this book may be reproduced or copied in any form or by any means [graphic, electronic or mechanical, including photocopying, recording, taping or information retrieval system] or reproduced on any disc, tape, perforated media or other information storage device etc., without the written permission of the publishers. Breach of this condition is liable for legal action.

Published by :

AVICHAL PUBLISHING COMPANY
8, Industrial Area, Trilokpur Road
Kala Amb - 173 030, Distt. Sirmour (H.P.)

Delhi Office:
1002 Faiz Road (opp. Hanuman Murti),
Karol Bagh, New Delhi - 110 005 (India)
Phone : 011-28752604, 28752745
Fax : 011-28756921
Email : info@apcbooks.co.in
Website : www.apcbooks.co.in

ISBN-978-81-7739-448-1

© Author

First Edition : 2015

Price : ₹ 320.00

Laser Typeset at :
Laser Tech Prints

Printed at :
G. Offset Printers
Dayabasti, Delhi

PREFACE

This book has been written as per the latest guidelines and syllabus prescribed by the National Council of Educational Research and Training (NCERT), New Delhi. The guidelines prepared by CBSE on Continuous and Comprehensive Evaluation (CCE) have also been taken care of.

The subject matter contained in this book has been explained in a simple language and includes many examples from real life situations. Emphasis has been laid on basic facts, terms, principles, concepts and on their applications. Carefully selected examples consist of detailed step-by-step solutions so that students get prepared to tackle all the problems given in the exercises.

Salient features of the book:

1. To fulfill the requirements of the teachers and the students in the best manner possible, the matter has been divided into sections and sub-sections. Consequently, the students will be able to learn at their own pace.
2. Various new concepts have been developed with the help of class activity method.
3. Results, wherever possible, have been verified by lab activity method.
4. Very Short Answer Type Questions have also been included chapter wise.
5. Questions in the form of Fill in the Blanks, True/False Statements and Multiple Choice Questions have been given under the heading **'Mental Maths'**.
6. Multiple Choice Questions have been framed in a manner such that they holistically cover all the concepts covered in chapter and also, prepare students for the competitive exams.
7. Some **'Value Based Questions'** have also been included to impart value among students.
8. In addition to normal questions, some **'Higher Order Thinking Skills (HOTS)'** questions have been given to enhance the analytical thinking of the students.
9. A **'Chapter Test'** has been put in the end of each chapter which serves as the brief revision of the entire chapter.
10. Term-wise **Model Question Papers** for Formative and Summative Assessments have been given at proper places.

It has been my sincere endeavour to present the concepts, examples and questions in a coherent and an interesting manner so that the students develop interest in 'learning' and 'understanding' mathematics.

I am grateful to my publishers 'M/s Avichal Publishing Company' and 'Laser Tech Prints' and thank them for their friendly cooperation and untiring efforts in bringing out this book in an excellent form.

I would highly appreciate if you suggest any improvement you would like to see in the book in its next edition.

—M.L. Aggarwal

Syllabus

MATHEMATICS

(CODE NO. : 041)

- As per CCE guidelines, the syllabus of Mathematics for class IX has been divided term-wise.
- The units specified for each term shall be assessed through both Formative and Summative Assessments.
- In each term, there will be two Formative Assessments, each carrying 10% weightage.
- The Summative Assessment in the Term I will carry 30% weightage and the Summative Assessment in the Term II will carry 30% weightage.
- Listed laboratory activities and projects will necessarily be assessed through Formative Assessments.

COURSE STRUCTURE

CLASS IX

First Term	**Marks : 90**
Units	**Marks**
I. NUMBER SYSTEMS	17
II. ALGEBRA	25
III. GEOMETRY	37
IV. CO-ORDINATE GEOMETRY	06
V. MENSURATION	05
TOTAL	90

Second Term	**Marks : 90**
Units	**Marks**
II. ALGEBRA (contd.)	16
III. GEOMETRY (contd.)	38
V. MENSURATION (contd.)	18
VI. STATISTICS AND PROBABILITY	18
TOTAL	90

First Term

UNIT I : NUMBER SYSTEMS

1. Real Numbers (Periods 18)

1. Review of representation of natural numbers, integers, rational numbers on the number line. Representation of terminating/non-terminating recurring decimals, on the number line through successive magnification. Rational numbers as recurring/terminating decimals.

2. Examples of non-recurring/non-terminating decimals such as $\sqrt{2}, \sqrt{3}, \sqrt{5}$ etc. Existence of non-rational numbers (irrational numbers) such as $\sqrt{2}, \sqrt{3}$ and their representation on

the number line. Explaining that every real number is represented by a unique point on the number line and conversely, every point on the number line represents a unique real number.

3. Rational numbers as recurring/terminating decimals.
4. Existence of $\sqrt{x}$ for a given positive real number x (visual proof to be emphasized).
5. Definition of nth root of a real number.
6. Recall of laws of exponents with integral powers. Rational exponents with positive real bases (to be done by particular cases, allowing learner to arrive at the general laws).
7. Rationalization (with precise meaning) of real numbers of the type (and their combinations) $\frac{1}{a + b\sqrt{x}}$ and $\frac{1}{\sqrt{x} + \sqrt{y}}$, where x and y are natural numbers and a and b are integers.

UNIT II : ALGEBRA

1. Polynomials (Periods 23)

Definition of a polynomial in one variable, its coefficients with examples and counter examples, its terms, zero polynomial. Degree of a polynomial. Constant, linear, quadratic and cubic polynomials; monomials, binomials, trinomials. Factors and multiples. Zeroes/ roots of a polynomial/equation. State and motivate the Remainder Theorem with examples and analogy to integers. Statement and proof of the Factor Theorem. Factorization of $ax^2 + bx + c$, $a \neq 0$ where a, b and c are real numbers, and of cubic polynomials using the Factor Theorem.

Recall of algebraic expressions and identities. Further verification of identities of the type

$$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx,$$
$$(x \pm y)^3 = x^3 \pm y^3 \pm 3xy\,(x \pm y),$$
$$x^3 \pm y^3 = (x \pm y)\,(x^2 \pm xy + y^2),$$
$$x^3 + y^3 + z^3 - 3xyz = (x + y + z)\,(x^2 + y^2 + z^2 - xy - yz - zx)$$

and their use in factorization of polynomials. Simple expressions reducible to these polynomials.

UNIT III : GEOMETRY

1. Introduction to Euclid's Geometry (Periods 6)

History - Geometry in India and Euclid's geometry. Euclid's method of formalizing observed phenomenon into rigorous mathematics with definitions, common/obvious notions, axioms/postulates and theorems. The five postulates of Euclid. Equivalent versions of the fifth postulate. Showing the relationship between axiom and theorem. For example,

(Axiom) 1. Given two distinct points, there exists one and only one line through them.

(Theorem) 2. (Prove) Two distinct lines cannot have more than one point in common.

2. Lines and Angles (Periods 10)

1. (Motivate) If a ray stands on a line, then the sum of the two adjacent angles so formed is 180° and the converse.
2. (Prove) If two lines intersect, the vertically opposite angles are equal.
3. (Motivate) Results on corresponding angles, alternate angles, interior angles when a transversal intersects two parallel lines.
4. (Motivate) Lines, which are parallel to a given line, are parallel.
5. (Prove) The sum of the angles of a triangle is 180°.
6. (Motivate) If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles.

3. Triangles (Periods 20)

1. (Motivate) Two triangles are congruent if any two sides and the included angle of one triangle is equal to any two sides and the included angle of the other triangle (SAS Congruence).
2. (Prove) Two triangles are congruent if any two angles and the included side of one triangle is equal to any two angles and the included side of the other triangle (ASA Congruence).
3. (Motivate) Two triangles are congruent if the three sides of one triangle are equal to three sides of the other triangle (SSS Congruence).
4. (Motivate) Two right triangles are congruent if the hypotenuse and a side of one triangle are equal (respectively) to the hypotenuse and a side of the other triangle.
5. (Prove) The angles opposite to equal sides of a triangle are equal.
6. (Motivate) The sides opposite to equal angles of a triangle are equal.
7. (Motivate) Triangle inequalities and relation between 'angle and facing side' inequalities in triangles.

UNIT IV : COORDINATE GEOMETRY

1. Coordinate Geometry (Periods 9)

The Cartesian plane, coordinates of a point, names and terms associated with the coordinate plane, notations, plotting points in the plane, graph of linear equations as examples; focus on linear equations of the type $Ax + By + C = 0$ by writing it as $y = mx + c$.

UNIT V : MENSURATION

1. Areas (Periods 4)

Area of a triangle using Heron's Formula (without proof) and its application in finding the area of a quadrilateral.

Second Term

UNIT II : ALGEBRA (CONTD.)

2. Linear Equations in Two Variables (Periods 14)

Recall of linear equations in one variable. Introduction to the equation in two variables. Prove that a linear equation in two variables has infinitely many solutions and justify their being written as ordered pairs of real numbers, plotting them and showing that they seem to lie on a line. Examples, problems from real life, including problems on Ratio and Proportion and with algebraic and graphical solutions being done simultaneously.

UNIT III : GEOMETRY (CONTD.)

4. Quadrilaterals (Periods 10)

1. (Prove) The diagonal divides a parallelogram into two congruent triangles.
2. (Motivate) In a parallelogram opposite sides are equal and conversely.
3. (Motivate) In a parallelogram opposite angles are equal and conversely.
4. (Motivate) A quadrilateral is a parallelogram if a pair of its opposite sides is parallel and equal.
5. (Motivate) In a parallelogram, the diagonals bisect each other and conversely.
6. (Motivate) In a triangle, the line segment joining the mid-points of any two sides is parallel to the third side and (motivate) its converse.

5. Area (Periods 4)

Review concept of area, recall area of a rectangle.

1. (Prove) Parallelograms on the same base and between the same parallels have the same area.

2. (Motivate) Triangles on the same base and between the same parallels are equal in area and its converse.

6. Circles (Periods 15)

Through examples, arrive at definitions of circle related concepts, radius, circumference, diameter, chord, arc, subtended angle.

1. (Prove) Equal chords of a circle subtend equal angles at the centre and (motivate) its converse.
2. (Motivate) The perpendicular from the centre of a circle to a chord bisects the chord and conversely, the line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.
3. (Motivate) There is one and only one circle passing through three given non-collinear points.
4. (Motivate) Equal chords of a circle (or of congruent circles) are equidistant from the centre(s) and conversely.
5. (Prove) The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.
6. (Motivate) Angles in the same segment of a circle are equal.
7. (Motivate) If a line segment joining two points subtends equal angle at two other points lying on the same side of the line containing the segment, the four points lie on a circle.
8. (Motivate) The sum of the either pair of the opposite angles of a cyclic quadrilateral is 180° and its converse.

7. Constructions (Periods 10)

1. Construction of bisectors of line segments and angles of 60°, 90°, 45°, etc. equilateral triangles.
2. Construction of a triangle given its base, sum/difference of the other two sides and one base angle.
3. Construction of a triangle of given perimeter and base angles.

UNIT V : MENSURATION (CONTD.)

2. Surface Areas and Volumes (Periods 12)

Surface areas and volumes of cubes, cuboids, spheres (including hemispheres) and right circular cylinders/cones.

UNIT VI : STATISTICS AND PROBABILITY

1. Statistics (Periods 13)

Introduction to Statistics : Collection of data, presentation of data—tabular form, ungrouped/ grouped, bar graphs, histograms (with varying base lengths), frequency polygons, qualitative analysis of data to choose the correct form of presentation for the collected data. Mean, median, mode of ungrouped data.

2. Probability (Periods 12)

History, Repeated experiments and observed frequency approach to probability. Focus is on empirical probability. (A large amount of time to be devoted to group and to individual activities to motivate the concept; the experiments to be drawn from real - life situations, and from examples used in the chapter on Statistics).

Contents

Term – I

Chapters	Pages
1. **Number Systems**	1
2. **Polynomials**	36
• Model Question Paper 1 (Formative Assessment I – Term I)	70
3. **Introduction to Euclid's Geometry**	71
4. **Lines and Angles**	80
5. **Triangles**	111
• Model Question Paper 2 (Formative Assessment II – Term I)	147
6. **Coordinate Geometry**	149
7. **Heron's Formula**	163
• Model Question Paper 3 (Summative Assessment—First Term)	180

Term – II

8. **Linear Equations in Two Variables**	183
9. **Quadrilaterals**	202
10. **Areas of Parallelograms and Triangles**	235
• Model Question Paper 4 (Formative Assessment III – Term II)	262
11. **Circles**	263
12. **Constructions**	309
13. **Surface Areas and Volumes**	319
• Model Question Paper 5 (Formative Assessment IV – Term II)	359
14. **Statistics**	360
15. **Probability**	400
• Model Question Paper 6 (Summative Assessment—Second Term)	415
Answers	419

Term - I

Number Systems



INTRODUCTION

You are already familiar with the system of natural numbers, whole numbers, integers, rational numbers and their representation on the number line. You also know the four fundamental operations of arithmetic on them—addition, subtraction, multiplication and division. In this chapter, we shall extend our study to irrational numbers and real numbers. We shall learn the representation of terminating/non-terminating decimal numbers on the number line through successive magnifications and shall explain that every real number can be represented by a unique point on the number line and conversely corresponding to every point on the number line there is a unique real number. We shall also learn the rationalisation of irrational numbers of the type $\frac{1}{a + b\sqrt{x}}$ and $\frac{1}{\sqrt{x} + \sqrt{y}}$ where x, y are natural numbers and a, b are integers; and laws of exponents for real numbers.

1.1 RATIONAL NUMBERS

Recall the definition of a rational number.

Any number that can be expressed in the form $\frac{p}{q}$, *where* p *and* q *are both integers and* $q \neq 0$ *is called a* ***rational number****.*

The word 'rational' comes from the word 'ratio'. Thus, every rational number can be written as the ratio of two integers.

Note that every integer (positive, negative or zero) can be written in the form $\frac{p}{q}$ where $q = 1$. For example,

$5 = \frac{5}{1}, -7 = \frac{-7}{1}, 0 = \frac{0}{1}$.

Hence, every integer is a rational number.

We know that the rational numbers do not have a unique representation in the form $\frac{p}{q}$, where p and q are both integers and $q \neq 0$. For example, $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{7}{14} = \frac{53}{106}$, etc. In fact, these are equivalent rational numbers. However, if we write a rational number in the form $\frac{p}{q}$, where p and q are both integers and $q \neq 0$ and p and q have no common factors except 1, then among the infinitely many rational numbers equivalent to $\frac{1}{2}$, we choose $\frac{1}{2}$ to represent all of them on the number line.

Thus, a rational number can be uniquely expressed as $\frac{p}{q}$ *where* p *and* q *are both integers,* $q \neq 0$ *and* p *and* q *have no common factors except 1 i.e.* p *and* q *are co-prime.*

It is called in the **lowest terms** or **simplest form**.

1.1.1 Representation of rational numbers

Recall the representation of natural numbers, whole numbers, integers and rational numbers on the number line.

Let l be a straight line which extends endlessly on both sides. Mark the positive direction to the right by an arrowhead. Take a point O on l and label it 0 (zero). Next choose another point, say A, on l towards the right of O and label it 1 (one). Thus the points O and A represent the numbers 0 and 1 respectively. The length of the segment OA represents unit length. Now mark points on l to the right of A at unit length intervals and label these 2, 3, 4, ... Similarly, mark points on l to the left of O at unit length intervals and label them $-1, -2, -3, -4, \ldots$, shown in fig. 1.1. *Thus, every integer has been represented by one and only one point on the line l.*

Next, we consider the representation of rational numbers on the line l. Take one-half of the unit length and mark points on l on both sides of O; these points will represent the numbers $\frac{1}{2}, \frac{2}{2}, \frac{3}{2}$ and $-\frac{1}{2}, -\frac{2}{2}, -\frac{3}{2}$ ..., shown in fig. 1.1.

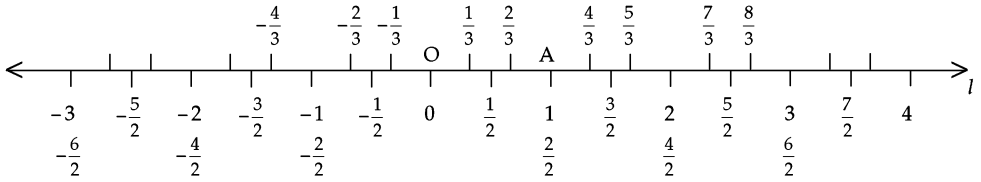


Fig. 1.1

Similarly, take one-third of the unit length and mark points on l on both sides of O; these points will represent the numbers $\frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}$, ... and $-\frac{1}{3}, -\frac{2}{3}, -\frac{3}{3}, -\frac{4}{3}$, ... and so on. *Thus, every rational number has been represented by one and only one point on the line l.*

Remark

If a and b are two different rational numbers, then $\frac{a+b}{2}$ is a rational number and it lies between them *i.e.* if $a < b$ then $a < \frac{a+b}{2} < b$. Continuing this process, we find that there are infinitely many rational numbers between two rational numbers.

Illustrative Examples

Example 1. *Find a rational number between* $\frac{4}{9}$ *and* $\frac{5}{7}$.

Solution. We know that if a and b are two rational numbers then $\frac{a+b}{2}$ is a rational number and it lies between them.

Therefore, a rational number between $\frac{4}{9}$ and $\frac{5}{7}$

$$= \frac{\frac{4}{9}+\frac{5}{7}}{2} = \frac{\frac{28+45}{63}}{2} = \frac{73}{126}.$$

Note

Since infinitely many rational numbers lie between two different rational numbers, $\frac{73}{126}$ is not the only rational number between $\frac{4}{9}$ and $\frac{5}{7}$.

Example 2. *Find five rational numbers between 2 and 3.*

Solution. We shall approach the problem in two ways.

Method I. A rational number between 2 and 3 $= \frac{2+3}{2} = \frac{5}{2}$.

A rational number between 2 and $\frac{5}{2} = \frac{2+\frac{5}{2}}{2} = \frac{9}{4}$.

A rational number between 2 and $\frac{9}{4} = \frac{2+\frac{9}{4}}{2} = \frac{17}{8}$.

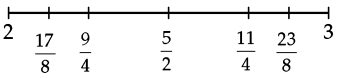


Fig. 1.2

A rational number between $\frac{5}{2}$ and 3 $= \frac{\frac{5}{2}+3}{2} = \frac{11}{4}$.

A rational number between $\frac{11}{4}$ and 3 $= \frac{\frac{11}{4}+3}{2} = \frac{23}{8}$.

Thus, five rational numbers between 2 and 3 are $\frac{17}{8}, \frac{9}{4}, \frac{5}{2}, \frac{11}{4}, \frac{23}{8}$.

Method II. The other way is to find all the five rational numbers in one step.

The given numbers 2 and 3 can be written as $2 = \frac{2}{1}$ and $3 = \frac{3}{1}$.

Since we want to find five rational numbers between the given numbers, multiplying the numerator and denominator of each of the above numbers by 5 + 1 *i.e.* by 6, we get $\frac{12}{6}$ and $\frac{18}{6}$, which are equivalent to the given numbers.

As $12 < 13 < 14 < 15 < 16 < 17 < 18$,

$$\frac{12}{6} < \frac{13}{6} < \frac{14}{6} < \frac{15}{6} < \frac{16}{6} < \frac{17}{6} < \frac{18}{6}$$

$$\Rightarrow \quad 2 < \frac{13}{6} < \frac{7}{3} < \frac{5}{2} < \frac{8}{3} < \frac{17}{6} < 3.$$

Therefore, five rational numbers between 2 and 3 are:

$$\frac{13}{6}, \frac{7}{3}, \frac{5}{2}, \frac{8}{3}, \frac{17}{6}.$$

Example 3. *Find six rational numbers between* $\frac{3}{5}$ *and* $\frac{4}{5}$.

Solution. Since we want to find six rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$, multiplying the numerator and denominator of each of the given numbers by 6 + 1 *i.e.* by 7, we get $\frac{21}{35}$ and $\frac{28}{35}$, which are equivalent to the given numbers.

As $21 < 22 < 23 < 24 < 25 < 26 < 27 < 28$,

$$\frac{21}{35} < \frac{22}{35} < \frac{23}{35} < \frac{24}{35} < \frac{25}{35} < \frac{26}{35} < \frac{27}{35} < \frac{28}{35}$$

$$\Rightarrow \quad \frac{3}{5} < \frac{22}{35} < \frac{23}{35} < \frac{24}{35} < \frac{5}{7} < \frac{26}{35} < \frac{27}{35} < \frac{4}{5}.$$

Therefore, six rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$ are:

$$\frac{22}{35}, \frac{23}{35}, \frac{24}{35}, \frac{5}{7}, \frac{26}{35}, \frac{27}{35}.$$

Example 4. *Insert eight rational numbers between* $-\frac{1}{3}$ *and* $\frac{2}{7}$.

Solution. Writing the given numbers with same denominator 21 (L.C.M. of 3 and 7), we get $-\frac{1}{3} = \frac{-7}{21}$ and $\frac{2}{7} = \frac{6}{21}$.

As $-7 < -6 < -5 < -4 < -3 < -2 < -1 < 0 < 1 < 6$,

$$-\frac{7}{21} < -\frac{6}{21} < -\frac{5}{21} < -\frac{4}{21} < -\frac{3}{21} < -\frac{2}{21} < -\frac{1}{21} < 0 < \frac{1}{21} < \frac{6}{21}$$

$$\Rightarrow \quad -\frac{1}{3} < -\frac{2}{7} < -\frac{5}{21} < -\frac{4}{21} < -\frac{1}{7} < -\frac{2}{21} < -\frac{1}{21} < 0 < \frac{1}{21} < \frac{2}{7}$$

Therefore, eight rational numbers between $-\frac{1}{3}$ and $\frac{2}{7}$ are:

$$-\frac{2}{7}, -\frac{5}{21}, -\frac{4}{21}, -\frac{1}{7}, -\frac{2}{21}, -\frac{1}{21}, 0, \frac{1}{21}$$

Example 5. *Find four rational numbers between* $\frac{3}{4}$ *and* $\frac{5}{6}$.

Solution. Writing the given numbers with same denominator 12 (L.C.M. of 4 and 6), we get $\frac{3}{4} = \frac{9}{12}$ and $\frac{5}{6} = \frac{10}{12}$.

Since we want to find four rational numbers between the given numbers, multiplying the numerator and denominator of the above numbers by 4 + 1 *i.e.* by 5, we get $\frac{45}{60}$ and $\frac{50}{60}$, which are equivalent to the given numbers.

As $45 < 46 < 47 < 48 < 49 < 50$,

$$\frac{45}{60} < \frac{46}{60} < \frac{47}{60} < \frac{48}{60} < \frac{49}{60} < \frac{50}{60}$$

$$\Rightarrow \quad \frac{3}{4} < \frac{23}{30} < \frac{47}{60} < \frac{4}{5} < \frac{49}{60} < \frac{5}{6}.$$

Therefore, four rational numbers between $\frac{3}{4}$ and $\frac{5}{6}$ are: $\frac{23}{30}, \frac{47}{60}, \frac{4}{5}, \frac{49}{60}$.

Exercise 1.1

1. Find a rational number between $\frac{1}{3}$ and $\frac{2}{7}$.
2. Find a rational number between $\frac{2}{9}$ and $\frac{3}{8}$, and arrange in descending order.
3. Find two rational numbers between $\frac{1}{3}$ and $\frac{1}{4}$.
4. Find three rational numbers between

   (*i*) 0.1 and 0.11 *(NCERT Exemplar)* (*ii*) $\frac{5}{7}$ and $\frac{6}{7}$ *(NCERT Exemplar)*
5. Find six rational numbers between 3 and 4. *(NCERT)*
6. Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$. *(NCERT)*
7. Find ten rational numbers between $-\frac{2}{5}$ and $\frac{1}{7}$.
8. Find six rational numbers between $\frac{1}{2}$ and $\frac{2}{3}$.

Answers

1.2 IRRATIONAL NUMBERS

Look at the number line *l* (shown in fig. 1.1) again and think of the situation in another way. As far as you can imagine, there are infinitely many numbers on the number line.

You may start collecting only natural numbers *i.e.* the numbers 1, 2, 3, 4, ... You know that this list of natural numbers is endless *i.e.* there are infinitely many natural numbers. The collection of **natural numbers** is denoted by **N**.

If you put the number zero (0) in the above list, then you have the collection of **whole numbers** which is denoted by **W**.

Further, if you put all the negative integers in the collection of whole numbers then you get the collection of **all integers** which is denoted by **I** (or **Z**).

Are there still more numbers left on the number line? Yes! There are numbers like $\frac{1}{2}, \frac{2}{3}, \frac{6}{7}, -\frac{5}{11}, \frac{231}{14}, \frac{3331}{112}$, etc. If you put all these numbers in the collection of all integers, you get the collection of **rational numbers** which is denoted by **Q**.

The question arises: Have you collected all the numbers on the number line? Not, as yet! In fact, there are infinitely many more numbers left on the number line. There are gaps in between the places of the numbers you have collected and not just a few gaps but infinitely many gaps. You will be surprised to realise that there are infinitely many more numbers (other than rational numbers) in these gaps.

Pythagoreans (followers of great mathematician Pythagoras), were the first to discover such numbers. These numbers are called **irrational numbers**, because these numbers cannot be written in the form of ratio of two integers.

Formally, we define these numbers as:

A number that cannot be expressed in the form $\frac{p}{q}$, *where p and q are both integers and* $q \neq 0$, *p and q have no common factors except 1, is called an* ***irrational number.***

There are infinitely many irrational numbers. Some examples of these numbers are:

$$\sqrt{2}, \sqrt{3}, \sqrt{7}, -\sqrt{6}, \sqrt{20}, \frac{1}{\sqrt{5}}, 2 + \sqrt{3}, \pi, \text{ etc.}$$

Next, to represent irrational numbers on the line l, we use a result from geometry, known as Pythagoras theorem.

The square of the hypotenuse of a right angled triangle is equal to the sum of squares of the other two sides.

Construct a right angled triangle OAC, right angle at A, such that OA = AC =1, then by Pythagoras theorem,

$OC^2 = OA^2 + AC^2 \Rightarrow OC^2 = 1^2 + 1^2 = 2 \Rightarrow OC = \sqrt{2}$.

Now mark a point, say P, on l on the right of O such that OP = OC = $\sqrt{2}$, then the point P represents the irrational number $\sqrt{2}$ (shown in fig. 1.3).

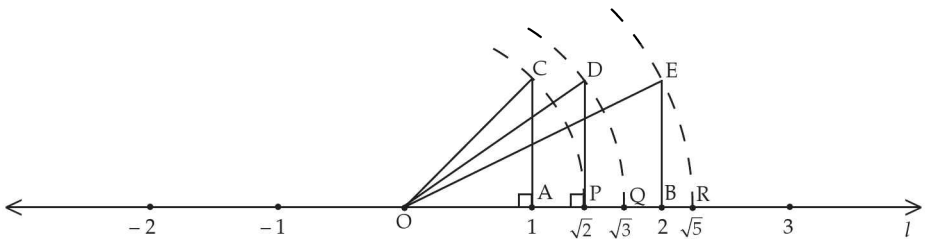


Fig. 1.3

To represent the irrational number $\sqrt{3}$ on the line l, construct a right angled triangle OPD, right angled at P such that DP = 1, then by Pythagoras theorem,

$$OD^2 = OP^2 + DP^2 = (\sqrt{2})^2 + 1^2 = 2 + 1 = 3 \Rightarrow OD = \sqrt{3}.$$

Now mark a point, say Q, on l to the right of O such that OQ = OD = $\sqrt{3}$, then the point Q represents the irrational number $\sqrt{3}$ (shown in fig. 1.3).

Next, to represent the irrational number $\sqrt{5}$ on the line l, construct a right angled triangle OBE, right angled at B such that BE = 1. Then by Pythagoras theorem,

$$OE^2 = OB^2 + BE^2 = 2^2 + 1^2 = 4 + 1 = 5 \Rightarrow OE = \sqrt{5}.$$

Now mark a point, say R, on l to the right of O such that OR = OE = $\sqrt{5}$, then the point R represents the irrational number $\sqrt{5}$ (shown in fig. 1.3).

Thus, geometrical constructions can be devised to identify the points on the number line l which correspond to the irrational numbers $\sqrt{2}, \sqrt{3}, \sqrt{5}$, etc. and so on.

Hence, corresponding to every real number (rational or irrational) there exists one and only one point on the line l and conversely corresponding to every point on the line l there exists one and only one real number.

The line l is called the ***real axis*** *or the* ***number line****.*

Remark

When we use the symbol $\sqrt{\ }$, we assume that it is the positive square root of a number. Thus $\sqrt{4} = 2$, $\sqrt{\frac{9}{16}} = \frac{3}{4}$, etc. You are already familiar with the number π which is the ratio of the circumference of a circle to its diameter. The number π is irrational (which we accept without proof at this stage).

1.2.1 Construction of square root spiral

Procedure

1. Mark a point O and draw OP = 1 unit.
2. At P, draw $P_1P \perp OP$ and cut off PP_1 = 1 unit.
3. Join OP_1, then ΔOPP_1 is right angled at P.
   By Pythagoras theorem, we get
   $OP_1^2 = OP^2 + PP_1^2 = 1^2 + 1^2 = 2 \Rightarrow OP_1 = \sqrt{2}$.

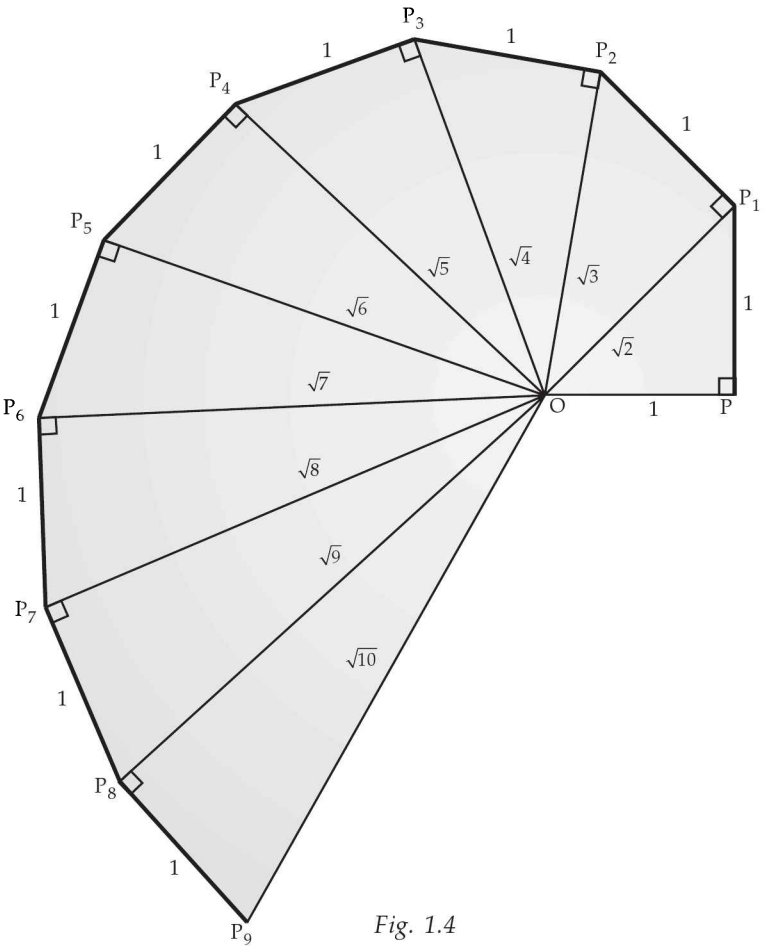


Fig. 1.4

4. At P_1, draw $P_1P_2 \perp OP_1$ and cut off $P_1P_2 = 1$ unit.
5. Join OP_2, then ΔOP_1P_2 is right angled at P_1.
   By Pythagoras theorem, we get
   $OP_2^2 = OP_1^2 + P_1P_2^2 = (\sqrt{2})^2 + 1 = 3 \Rightarrow OP_2 = \sqrt{3}$.
6. Repeat the above steps 4 and 5 till you get $\sqrt{10}$.
   We observe that the length of line segments OP_1, OP_2, OP_3, OP_4, OP_5, ... are $\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \ldots$.
   The shape formed by dark line segments is known as **square root spiral** (shown in fig. 1.4).

1.3 REAL NUMBERS

The collection of all rational numbers and irrational numbers together form the collection of real numbers. This collection is denoted by **R**.

Every real number is either a rational number or an irrational number. Thus, a real number which is not rational is an irrational number.

Look at the adjoining diagram to know the various number systems.

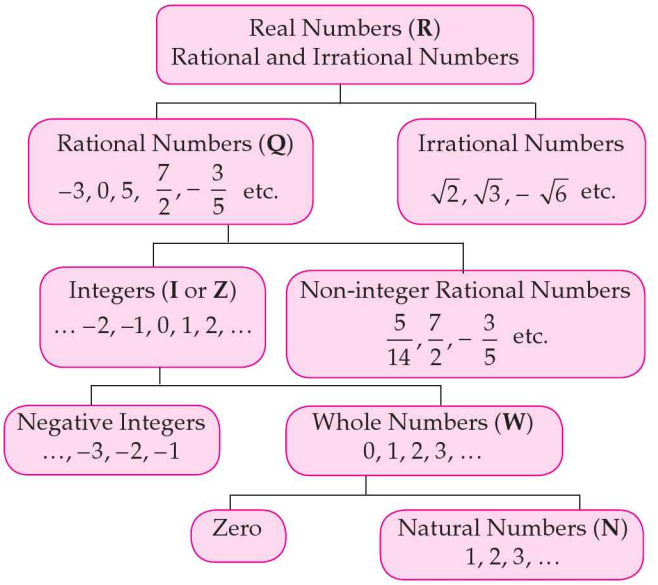


Fig. 1.5

1.4 DECIMAL EXPANSIONS OF REAL NUMBERS

In this section, we shall find the expansions of real numbers and see if we can use these expansions to distinguish between rational and irrational numbers.

As we are more familiar with rational numbers, we begin with the expansions of rational numbers and shall pay special attention to the remainders and see if they follow any pattern.

For example, let us find the decimal expansions of the following rational numbers:

(*i*) $\frac{13}{50}$ (*ii*) $\frac{25}{16}$ (*iii*) $\frac{10}{3}$ (*iv*) $\frac{15}{11}$ (*v*) $\frac{1}{7}$ (*vi*) $\frac{67}{13}$

By actual division, we get

(*i*)

$$\begin{array}{r|l} & 0.26 \\ \hline 50 & 13.0 \\ & 100 \\ \hline & 300 \\ & 300 \\ \hline & 0 \end{array}$$

Remainders: 3, 0

Divisor: 50

(*ii*)

$$\begin{array}{r|l} & 1.5625 \\ \hline 16 & 25 \\ & 16 \\ \hline & 90 \\ & 80 \\ \hline & 100 \\ & 96 \\ \hline & 40 \\ & 32 \\ \hline & 80 \\ & 80 \\ \hline & 0 \end{array}$$

Remainders: 9, 10, 4, 8, 0

Divisor: 16

(*iii*)

	3.333…
3	10
	9
	10
	9
	10
	9
	10
	9
	1 ⟵ Repeated

Remainders: 1, 1, 1, 1, …
Divisor: 3

(*iv*)

	1.3636…
11	15
	11
	40
	33
	70
	66
	40
	33
	70
	66
	4 ⟵ Repeated

Remainders: 4, 7, 4, 7, 4, 7, …
Divisor: 11

(*v*)

	0.142857…
7	1.0
	7
	30
	28
	20
	14
	60
	56
	40
	35
	50
	49
	1 ⟵ Repeated

Remainders: 3, 2, 6, 4, 5, 1, …
Divisor: 7

(*vi*)

	5.153846…
13	67
	65
	20
	13
	70
	65
	50
	39
	110
	104
	60
	52
	80
	78
	2 ⟵ Repeated

Remainders: 2, 7, 5, 11, 6, 8, …
Divisor: 13

What have you observed? Have you obtained some patterns? Yes! You must have observed the following things:

(*i*) In all cases, each remainder is smaller than the divisor, which must be true for all divisors.

(*ii*) The remainders either become zero after a certain stage or start repeating.

(*iii*) The number of items in the repeating chain of remainders is less than the divisor.

In case of $\frac{10}{3}$, there is just one entry, namely 1, which repeats itself and the divisor is 3.

In case of $\frac{15}{11}$, there are two entries, namely 4 and 7, which repeat themselves and the divisor is 11.

In case of $\frac{1}{7}$, there are six entries, namely 3, 2, 6, 4, 5 and 1, which repeat themselves and the divisor is 7.

In case of $\frac{67}{13}$, there are six entries, namely 2, 7, 5, 11, 6 and 8, which repeat themselves and the divisor is 13.

(*iv*) If the remainders repeat, we get a repeating block of digits in the quotient.

For $\frac{10}{3}$, 3 repeats in the quotient; for $\frac{15}{11}$, the repeating block of digits is 36; for $\frac{1}{7}$, the repeating block of digits is 142857 and for $\frac{67}{13}$, the repeating block of digits is 153846.

So far, we have observed these patterns only for the above examples, however, it is true for all rational numbers.

If we consider any positive rational number $\frac{p}{q}$, where p and q are both positive integers, on dividing p by q, two things happen — either the remainder becomes zero after a certain stage or it never becomes zero and we get a repeating chain of remainders. We shall discuss the two cases separately.

Case I. *When the remainder becomes zero.*

In case of $\frac{13}{50}$, we find that the remainder becomes zero after some steps and $\frac{13}{50} = 0.26$; in case of $\frac{25}{16}$, the remainder becomes zero after some steps and $\frac{25}{16} = 1.5625$. In these cases, the decimal expansions terminate after a finite number of steps.

Such decimal expansions are called **terminating**.

Thus, the rational numbers $\frac{13}{50}$ and $\frac{25}{16}$ have terminating decimal expansions.

Some other examples are:

$\frac{1}{2} = 0.5$, $\frac{7}{5} = 1.4$, $\frac{253}{4} = 63.25$, $\frac{7}{8} = 0.875$, $\frac{9}{80} = 0.1125$.

The decimal numbers 0.26, 1.5625, 0.5, 1.4, 63.25, 0.875 and 0.1125 are called **terminating decimals.**

Case II. *When the remainder never becomes zero.*

In case of $\frac{10}{3}$, $\frac{15}{11}$, $\frac{1}{7}$ and $\frac{67}{13}$, we find that the remainder never becomes zero and they repeat after a certain stage which force the decimal expansions to go forever. In these cases, we get a repeating block of digits in the quotient.

In the above example, we obtained

$\frac{10}{3} = 3.3333\ldots$, $\frac{15}{11} = 1.363636\ldots$

$\frac{1}{7} = 0.142857142857\ldots$ and $\frac{67}{13} = 5.153846153846\ldots$

Such decimal expansions are called **non-terminating recurring**.

The decimal numbers 3.3333..., 1.363636..., 0.142857142857... and 5.153846153846... are called **non-terminating recurring decimals** or simply **recurring (repeating) decimals.**

Notation for non-terminating recurring decimals:

(*i*) If the recurring decimal contains only one repeating digit, put a *dot* or *line* (called *vinculum*) above the repeating digit. For example,

$\frac{10}{3} = 3.3333\ldots = 3.\dot{3}$ or $3.\overline{3}$.

(*ii*) If the recurring decimal contains two repeating digits, put a dot above each repeating digit or a line above both the repeating digits. For example,

$\frac{15}{11} = 1.363636\ldots = 1.\dot{3}\,\dot{6}$ or $1.\overline{36}$.

(*iii*) If the recurring decimal contains more than two repeating digits, put a dot on the first repeating digit and another on the last digit or draw a line covering the entire block of the repeating digits. For example,

$\frac{1}{7} = 0.142857142857 \ldots = 0.\dot{1}42857\dot{}$ or $0.\overline{142857}$,

$\frac{67}{13} = 5.153846153846 \ldots = 5.\dot{1}5384\dot{6}$ or $5.\overline{153846}$.

In the above example, we have seen that a rational number has either a terminating or a non-terminating recurring (repeating) decimal expansion. The question arises, can a rational number have a non-terminating non-repeating decimal expansion? No! Since each successive remainder is less than the divisor, there comes a stage when a remainder repeats and the digits in the quotient start repeating. Therefore, the decimal expansion of a rational number has only two choices—either it is terminating or it is non-terminating recurring (repeating).

Remarks

- All integers (positive, zero or negative) are terminating decimals.
- The decimal expansion of a rational number $\frac{p}{q}$, where p, q are integers, $q > 0$, p, q have no common factors (except 1) is:
  - (*i*) terminating if q can be expressed as $q = 2^m 5^n$, where m, n are whole numbers.
  - (*ii*) non-terminating recurring if q has a prime factor other than 2 or 5.

Conversely, can every terminating decimal or a non-terminating recurring (repeating) decimal is a rational number? Yes!

Let us explain it through some examples.

Example 1. *Show that the following terminating decimals are rational numbers:*

(i) 0.075 (ii) 23.7812 (iii) 3.142678 (NCERT)

Solution. We know that any number which can be expressed in the form $\frac{p}{q}$ where p and q are both integers and $q \neq 0$ is a rational number. So, we are to show that the given numbers can be expressed in this form.

(*i*) $0.075 = \frac{75}{1000}$, which is a rational number.

(*ii*) $23.7812 = \frac{237812}{10000}$, which is a rational number.

(*iii*) $3.142678 = \frac{3142678}{1000000}$, which is a rational number.

Example 2. *Show that the following repeating decimals are rational numbers:*

(i) $0.\overline{6}$ *(NCERT)* *(ii)* $1.\overline{27}$ *(NCERT)* *(iii)* $0.2\overline{35}$ *(NCERT)*

(iv) $0.12\overline{3}$ *(NCERT Exemplar)* *(v)* $0.00\overline{32}$ *(NCERT Exemplar)*

Solution. (*i*) Let $x = 0.\overline{6} = 0.66666\ldots$ ...(1)

As there is one repeating digit after the decimal point, so multiplying both sides of (1) by 10, we get

$10x = 6.66666\ldots$ ...(2)

Subtracting (1) from (2), we get

$9x = 6 \Rightarrow x = \frac{2}{3}$, which is a rational number.

(*ii*) Let $x = 1.\overline{27} = 1.272727\ldots$ ...(1)

As there are two repeating digits after the decimal point, so multiplying both sides of (1) by 100, we get

$100x = 127.2727\ldots$ ...(2)

Subtracting (1) from (2), we get

$$99x = 126 \Rightarrow x = \frac{14}{11}, \text{ which is a rational number.}$$

(*iii*) Let $x = 0.2\overline{35} = 0.2353535...$ ...(1)

There is one non-repeating digit after the decimal, multiplying both sides of (1) by 10, we get

$$10x = 2.353535 \quad ...(2)$$

As there are two repeating digits after the decimal, multiplying both sides of (2) by 100, we get

$$1000x = 235.3535... \quad ...(3)$$

Subtracting (2) from (3), we get

$$990x = 233 \Rightarrow x = \frac{233}{990}, \text{ which is a rational number.}$$

(*iv*) Let $x = 0.12\overline{3} = 0.123333...$ ...(1)

There are two non-repeating digits after the decimal, multiplying both sides of (1) by 100, we get

$$100x = 12.3333... \quad ...(2)$$

As there is one repeating digit after the decimal, multiplying both sides of (2) by 10, we get

$$1000x = 123.333... \quad ...(3)$$

Subtracting (2) from (3), we get

$$900x = 111 \Rightarrow x = \frac{37}{300}, \text{ which is a rational number.}$$

(*v*) Let $x = 0.00\overline{32} = 0.00323232...$ ...(1)

There are two non-repeating digits after the decimal, multiplying both sides of (1) by 100, we get

$$100x = 0.323232... \quad ...(2)$$

As there are two repeating digits after the decimal, so multiplying both sides of (2) by 100, we get

$$10000x = 32.3232... \quad ...(3)$$

Subtracting (2) from (3), we get

$$9900x = 32 \Rightarrow x = \frac{8}{2475}, \text{ which is a rational number.}$$

From the above examples, we see that every terminating decimal or non-terminating recurring (repeating) decimal number is a rational number. This is true in general. We have already seen that every rational number has either a terminating or a non-terminating recurring (repeating) decimal expansion. We summarise these results as:

The decimal expansion of every rational number is either terminating or non-terminating recurring (repeating) and conversely, any number with terminating or non-terminating recurring (repeating) decimal expansion is a rational number.

So, you know what the decimal expansion of a rational number can be. What can you say about the decimal expansion of an irrational number? Because of the above result we can say that the decimal expansion of an irrational number is non-terminating non-recurring and conversely, every non-terminating non-recurring decimal number is an irrational number. We state these results as:

The decimal expansion of every irrational number is non-terminating non-recurring and conversely, any number with non-terminating non-recurring decimal expansion is an irrational number.

Consider the decimal number

29.101001000100001… …(*i*)

Observe that in (*i*) above, on the right of the decimal point, there are either 1's or 0's and 1's are separated by one zero, then by two zeros, then by three zeros and so on. Thus, the number of zeros separating two successive 1's goes on increasing successively by one. As we can go on writing this endlessly, this decimal number is non-terminating. Moreover, as no group of integers repeats, so it is non-recurring. Thus, the above decimal number is an irrational number.

In fact, you can obtain different such numbers by replacing the number 1 in (*i*) by any natural number of your choice. Since these are infinitely many, you have infinitely many non-terminating non-recurring decimals *i.e.* irrational numbers. Some other examples of irrational numbers are

0.0101101110…, 3.212112111211112…

Let us examine the decimal expansion of $\sqrt{2}$. We find the square root of 2 by division method.

	1.4142135…
1	$2.\overline{00}\,\overline{00}\,\overline{00}\,\overline{00}\,\overline{00}\,\overline{00}\,\overline{00}$ 1
24	100 96
281	400 281
2824	11900 11296
28282	60400 56564
282841	383600 282841
2828423	10075900 8485269
28284265	159063100 141421325
	17641775

So $\sqrt{2}$ = 1.414235…

You may find some more digits in the above process. You will observe that this process will neither terminate nor a block of digits repeat in the process. Thus, $\sqrt{2}$ has non-terminating non-recurring decimal expansion and hence, it is an irrational number.

All decimal numbers (terminating, recurring or non-terminating and non-recurring) are ***real numbers****.*

Look at the following diagram:

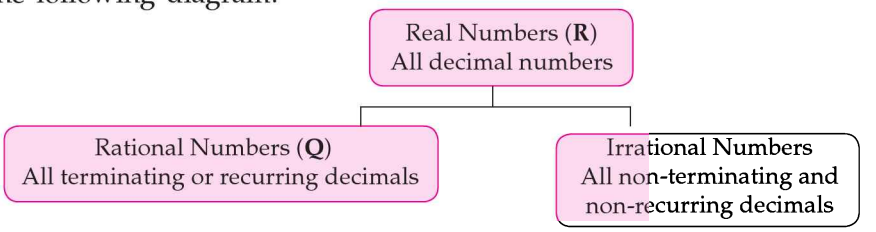


Fig. 1.6

Illustrative Examples

Example 1. *Locate* $\sqrt{13}$ *on the number line.* *(NCERT Exemplar)*

Solution. We write 13 as the sum of squares of two natural numbers:

$13 = 9 + 4 = 3^2 + 2^2$.

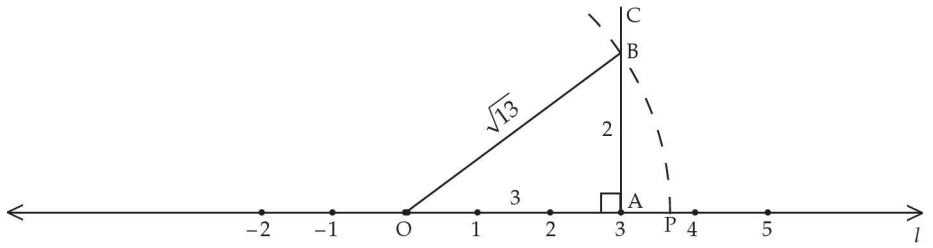


Fig. 1.7

Let l be the number line. If point O represents number 0 and point A represents number 3, then length of line segment OA = 3 units.

At A, draw AC ⊥ OA. From AC, cut off AB = 2 units.

We observe that OAB is a right angled triangle at A. By Pythagoras theorem, we get

$OB^2 = OA^2 + AB^2 = 3^2 + 2^2 = 13 \Rightarrow OB = \sqrt{13}$ units.

With O as centre and radius = OB, we draw an arc of a circle to meet the number line l at point P.

As OP = OB = $\sqrt{13}$ units, the point P will represent the number $\sqrt{13}$ on the number line (shown in fig. 1.7).

Example 2. *Without actual division, find which of the following rational numbers are terminating decimals:*

(i) $\frac{11}{50}$ *(ii)* $\frac{48}{300}$ *(iii)* $\frac{27}{56}$.

Solution. (*i*) First, we note that the given rational number $\frac{11}{50}$ is in its lowest terms.

Denominator of the given rational number = $50 = 2 \times 25 = 2^1 \times 5^2$, which is of the form $2^m 5^n$, where m and n are whole numbers.

$\therefore$ The given rational number *i.e.* $\frac{11}{50}$ is a terminating decimal.

(*ii*) The given rational number = $\frac{48}{300} = \frac{4}{25}$, in its lowest terms.

Its denominator = $25 = 5^2 = 2^0 \times 5^2$, which is of the form $2^m 5^n$, where m and n are whole numbers.

$\therefore$ The given rational number $\frac{4}{25}$ *i.e.* $\frac{48}{300}$ is a terminating decimal.

(*iii*) First, we note that the given rational number $\frac{27}{56}$ is in the lowest terms.

Its denominator = $56 = 8 \times 7 = 2^3 \times 7$, which contains a prime factor other than 2 and 5.

$\therefore$ The given rational number *i.e.* $\frac{27}{56}$ is a non-terminating recurring decimal.

Example 3. *Find an irrational number between* $\frac{1}{7}$ *and* $\frac{2}{7}$. *(NCERT)*

Solution. The decimal expansions of the given numbers are:

$\frac{1}{7} = 0.\overline{142857}$ (obtain it) and $\frac{2}{7} = 2 \times \frac{1}{7} = 2 \times 0.\overline{142857} = 0.\overline{285714}$

To find an irrational number between $\frac{1}{7}$ and $\frac{2}{7}$, we find a decimal number which is non-terminating non-recurring and lying between them *i.e.* between $0.\overline{142857}$ and $0.\overline{285714}$. There are infinitely many such numbers.

One such number is 0.202002000200002....

Example 4. *Find three irrational numbers between* $\frac{5}{7}$ *and* $\frac{9}{11}$. *(NCERT)*

Solution. The decimal expansions of the given numbers are:

$\frac{5}{7} = 0.\overline{714285}$ and $\frac{9}{11} = 0.\overline{81}$ (obtain these)

To find three irrational numbers between $\frac{5}{7}$ and $\frac{9}{11}$, we find three decimal numbers which are non-terminating non-recurring and lying between them *i.e.* between $0.\overline{714285}$ and $0.\overline{81}$. There are infinitely many such numbers. Three such numbers are

0.7201001000100001..., 0.7301001000100001.... and 0.7401001000100001....

Example 5. *Find a rational number between* $\sqrt{5}$ *and* $\sqrt{7}$.

Solution. Consider the squares of $\sqrt{5}$ and $\sqrt{7}$.

$(\sqrt{5})^2 = 5$ and $(\sqrt{7})^2 = 7$.

Take any rational number between 5 and 7 which is a perfect square of a rational number. One such number is 6.25 and $6.25 = (2.5)^2$ *i.e.* $\sqrt{6.25} = 2.5$

As $5 < 6.25 < 7 \Rightarrow \sqrt{5} < \sqrt{6.25} < \sqrt{7}$

$\Rightarrow \sqrt{5} < 2.5 < \sqrt{7}$.

Hence, 2.5 is a rational number between $\sqrt{5}$ and $\sqrt{7}$.

Note

As 5.76 and 6.76 are also perfect squares of rational numbers lying between 5 and 7,

$\sqrt{5} < \sqrt{5.76} < \sqrt{6.76} < \sqrt{7} \Rightarrow \sqrt{5} < 2.4 < 2.6 < \sqrt{7}$.

So, rational numbers 2.4, 2.6 also lie between $\sqrt{5}$ and $\sqrt{7}$.

In fact, there are many more rational numbers lying between $\sqrt{5}$ and $\sqrt{7}$.

Example 6. *Find two irrational numbers between* $\sqrt{2}$ *and* $\sqrt{7}$.

Solution. Consider the squares of $\sqrt{2}$ and $\sqrt{7}$.

$(\sqrt{2})^2 = 2$ and $(\sqrt{7})^2 = 7$.

As $2 < 3 < 5 < 7$, it follows that $\sqrt{2} < \sqrt{3} < \sqrt{5} < \sqrt{7}$, therefore $\sqrt{3}$ and $\sqrt{5}$ lie between $\sqrt{2}$ and $\sqrt{7}$.

Hence, two irrational numbers between $\sqrt{2}$ and $\sqrt{7}$ are $\sqrt{3}$ and $\sqrt{5}$.

Note

Since infinitely many irrational numbers lie between two distinct irrational numbers, $\sqrt{3}$ and $\sqrt{5}$ are not the only irrational numbers between $\sqrt{2}$ and $\sqrt{7}$.

Exercise 1.2

1. Locate $\sqrt{10}$ and $\sqrt{17}$ on the number line. *(NCERT Exemplar)*
2. Write the decimal expansion of each of the following numbers and say what kind of decimal expansion each has:

(*i*) $\frac{36}{100}$ *(NCERT)* (*ii*) $4\frac{1}{8}$ *(NCERT)* (*iii*) $\frac{2}{9}$

(*iv*) $\frac{2}{11}$ *(NCERT)* (*v*) $\frac{3}{13}$ *(NCERT)* (*vi*) $\frac{329}{400}$ *(NCERT)*

3. Without actual division, find which of the following rational numbers are terminating decimals:

(*i*) $\frac{7}{16}$ (*ii*) $\frac{21}{80}$ (*iii*) $\frac{136}{250}$ (*iv*) $\frac{5}{6}$

(*v*) $\frac{54}{60}$ (*vi*) $\frac{48}{55}$ (*vii*) $\frac{44}{63}$ (*viii*) $\frac{115}{640}$.

4. Without performing long division, determine which of the following rational numbers has a terminating decimal expansion:

$$\frac{7}{13}, \frac{15}{48}, 2\frac{1}{6}, 3\frac{19}{35}$$

Also write the decimal expansion of the number.

5. Write the decimal expansion of $\frac{1}{7}$. Hence, write the decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}$ and $\frac{6}{7}$. *(NCERT)*

6. What can be the maximum number of digits in the repeating block of digits in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer. *(NCERT)*

7. Express the following numbers in the form $\frac{p}{q}$, where p and q are both integers and $q \neq 0$:

(*i*) $0.\overline{3}$ *(NCERT)* (*ii*) $5.\overline{2}$ *(NCERT Exemplar)* (*iii*) 0.404040... *(NCERT Exemplar)*

(*iv*) $0.4\overline{7}$ *(NCERT)* (*ii*) $0.1\overline{34}$ *(NCERT Exemplar)* (*iii*) $0.\overline{001}$ *(NCERT)*

8. Express 0.9999... in the form $\frac{p}{q}$. Are you surprised by your answer? *(NCERT)*

9. Classify the following numbers as rational or irrational:

(*i*) $\sqrt{23}$ (*ii*) $\sqrt{225}$ (*iii*) 0.3796

(*iv*) 7.478478... (*v*) 1.101001000100001... *(NCERT)*

10. Write three numbers whose decimal expansions are non-terminating non-recurring.

11. Insert a rational number and an irrational number between the following:

(*i*) 2 and 3 (*ii*) 0 and 0.1 (*iii*) $\frac{1}{3}$ and $\frac{1}{2}$

(*iv*) $-\frac{2}{5}$ and $\frac{1}{2}$ (*v*) 0.15 and 0.16 *(NCERT Exemplar)*

12. Write two irrational numbers between $\frac{4}{9}$ and $\frac{7}{11}$.

13. Find a rational number between $\sqrt{2}$ and $\sqrt{3}$.

14. Find two rational numbers between $2\sqrt{3}$ and $\sqrt{15}$.

Hint

$2\sqrt{3} = \sqrt{12}$; $12 < 12.25 < 12.96 < 15$

$\Rightarrow \sqrt{12} < \sqrt{12.25} < \sqrt{12.96} < \sqrt{15}$.

15. Insert an irrational number between $\sqrt{5}$ and $\sqrt{7}$.

16. Insert two irrational numbers between $\sqrt{3}$ and $\sqrt{7}$.

Answers

1.5 REPRESENTING REAL NUMBERS ON THE NUMBER LINE

In the previous section, we learnt the decimal expansions of real numbers. This helps us in representing real numbers on the number line.

We shall explain the process of representation of a real number (decimal number) on the number line through examples.

Suppose we want to locate the point on the number line which represents the number 1.358. We know that this number lies between 1 and 2.

Look at the portion of the number line which lies between 1 and 2. Divide this portion into ten equal parts and mark each point of the division as shown in fig. 1.8(*i*). Then the first mark on the right of 1 will represent 1.1, the second 1.2 and so on. You might be having some difficulty in observing these points of division between 1 and 2 (shown in fig. 1.8(*i*)). To have a clear view of the same, take a **magnifying glass** and look at the portion of the number line between 1 and 2. It will look like what you see in fig. 1.8(*ii*).

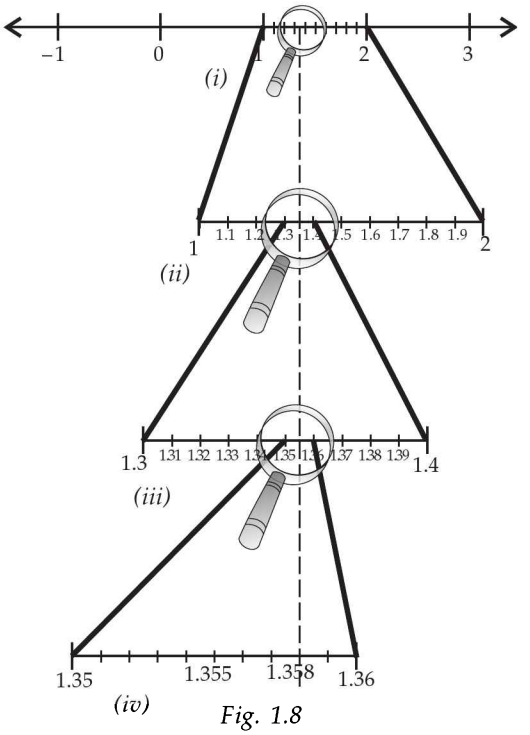


Fig. 1.8

As 1.358 lies between 1.3 and 1.4, look at the portion of the number line between 1.3 and 1.4. Divide this portion again into ten equal parts. The first mark will represent 1.31, the next 1.32 and so on. To have a clear view of the same, magnify this portion of the number line as shown in fig. 1.8(*iii*). Further, as 1.358 lies between 1.35 and 1.36. Divide this portion of the number line into ten equal parts. We magnify this portion *i.e.* the portion between 1.35 and 1.36 as shown in fig. 1.8(*iv*). The first mark represents 1.351, the next 1.352 and so on. Thus, 1.358 is the 8th mark in this subdivision.

The above process of visualisation of representation of numbers on the number line through a magnifying glass is called the process of ***successive magnification.***

Thus, we have seen that it is possible by successive magnifications to visualise (locate) the point on the number line which represents a real number having a terminal decimal expansion.

Remark

You must have noticed that each time we magnify a portion of the number line, we take up a smaller portion of the number line and then magnify this portion. Thus, on successive magnification, the portions of the number line successively go on decreasing.

Now, we shall try to visualise the point on the number line which represents a real number having a non-terminating recurring decimal expansion.

Suppose we want to visualise the point on the number line which represents the number $2.\overline{36}$ upto 5 decimal places *i.e.* upto 2.36666.

We shall proceed by the method of successive magnification and look at appropriate intervals through a magnifying glass and visualise the point on the number line which represents the number $2.\overline{36}$. First, look at the portion of the number line, which lies between 2 and 3. Divide it into ten equal parts and mark each point of the division as

shown in fig. 1.9(*i*), then $2.3\bar{6}$ is located between 2.3 and 2.4. To have a clear view of the same, look at the portion of the number line between 2 and 3 through a magnifying glass. It will look like what you see in fig. 1.9(*ii*). As $2.3\bar{6}$ lies between 2.3 and 2.4, look at the portion of the number line between 2.3 and 2.4. Divide it into ten equal parts, then $2.3\bar{6}$ is located between 2.36 and 2.37. To have a clear view, magnify this portion of the number line as shown in fig. 1.9(*iii*).

To visualise $2.3\bar{6}$ more accurately, we again divide the portion of the number line between 2.36 and 2.37 into ten equal parts and visualise the representation of $2.3\bar{6}$ by magnification as shown in fig. 1.9(*iv*), we see that $2.3\bar{6}$ is located between 2.366 and 2.367. Divide the portion of the numbers line between 2.366 and 2.367 into ten equal parts and magnify it as shown in fig. 1.9(*v*). Notice that $2.3\bar{6}$ is more closer to 2.3667 than to 2.3666. The point on the number line which represents the number $2.3\bar{6}$ upto 5 places is shown in fig. 1.9(*v*).

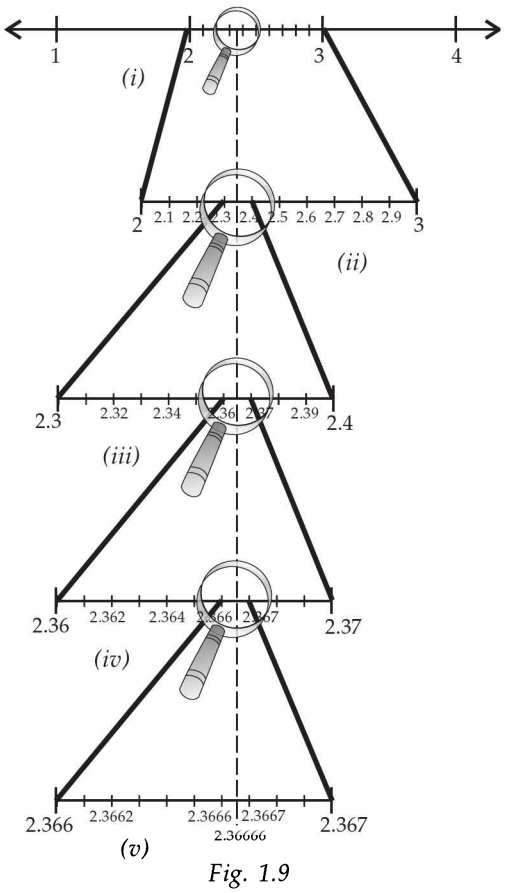


Fig. 1.9

Remark

We can proceed endlessly in the above manner, successively looking through a magnifying glass and simultaneously imagining the decrease in the length of the portion of the number line on which $2.3\bar{6}$ is located. The size of the portion of the number line depends upon the degree of accuracy we desire for the visualisation of the point on the number line representing the number $2.3\bar{6}$.

By now, you must have realised that the above process can be used to visualise the representation of a real number with non-terminating non-recurring decimal representation on the number line.

In the light of the above discussions and visualisations, we restate that:

Every real number can be represented by a unique point on the number line and conversely, corresponding to every point on the number line there is a unique real number.

Exercise 1.3

1. Visualise 3.47 on the number line, using successive magnification.
2. Visualise 3.765 on the number line, using successive magnification. *(NCERT)*
3. Visualise $5.3\bar{7}$ on the number line, upto 4 decimal places. *(NCERT)*

1.6 OPERATIONS ON REAL NUMBERS

In previous classes, you have learnt the four fundamental operations of arithmetic *i.e.* addition, subtraction, multiplication and division on rational numbers and found that the rational numbers satisfy the commutative, associative and distributive laws for addition and multiplication. Moreover, if we add, subtract, multiply or divide (except by zero) two rational numbers, we get a rational number *i.e.* the rational numbers are 'closed' with respect

to addition, subtraction, multiplication and division. It turns out that irrational numbers also satisfy the commutative, associative and distributive laws for addition and multiplication. However, the sum, difference, product and quotient of two irrational numbers are not *always* irrational numbers.

For example,

$(\sqrt{5}) + (-\sqrt{5}), (\sqrt{3}) - (\sqrt{3}), (\sqrt{7})(\sqrt{7})$ and $\frac{\sqrt{13}}{\sqrt{13}}$ are rational numbers.

Let us see what happens when we add and multiply an irrational number with a rational number. For example, $\sqrt{2}$ is irrational. What about $5 + \sqrt{2}$ and $5\sqrt{2}$? As $\sqrt{2}$ has a non-terminating and non-recurring decimal expansion, the same is true for $5 + \sqrt{2}$ and $5\sqrt{2}$. Therefore, $5 + \sqrt{2}$ and $5\sqrt{2}$ are also irrational numbers. In general, let us see what happens if we add, subtract, multiply or divide two irrational numbers.

Look at the following examples.

Example 1. *Add* $2 + 5\sqrt{3}$ *and* $-7 - 5\sqrt{3}$.

Solution. $(2 + 5\sqrt{3}) + (-7 - 5\sqrt{3}) = (2 - 7) + (5 - 5)\sqrt{3}$
$= -5 + 0 = -5.$

Example 2. *Subtract* $\sqrt{2} - 3\sqrt{3}$ *from* $3\sqrt{2} + 5\sqrt{3}$.

Solution. $(3\sqrt{2} + 5\sqrt{3}) - (\sqrt{2} - 3\sqrt{3}) = (3 - 1)\sqrt{2} + (5 + 3)\sqrt{3}$
$= 2\sqrt{2} + 8\sqrt{3}.$

Example 3. *Multiply* $7\sqrt{5}$ *by* $3\sqrt{5}$.

Solution. $7\sqrt{5} \times 3\sqrt{5} = 7 \times 3 \times \sqrt{5} \times \sqrt{5} = 21 \times 5 = 105.$

Example 4. *Divide* $10\sqrt{21}$ *by* $2\sqrt{7}$.

Solution. $10\sqrt{21} \div 2\sqrt{7} = \frac{10\sqrt{21}}{2\sqrt{7}} = \frac{5 \times 2 \times \sqrt{7} \times \sqrt{3}}{2 \times \sqrt{7}} = 5\sqrt{3}.$

The above examples may lead you to expect the following facts, **which are true:**

(i) The sum or difference of a rational number and an irrational number is an irrational number.

(ii) The product or quotient of a non-zero rational number with an irrational number is an irrational number.

(iii) If we add, subtract, multiply or divide two irrational numbers, then the result may be a rational number or an irrational number.

1.6.1 Square root of a positive real number

Recall that if a is a natural number then $\sqrt{a} = b$ if and only if $b^2 = a$, $b > 0$. The same definition can be extended to positive real numbers.

If a is any positive real number then $\sqrt{a} = b$ if and only if $b^2 = a$, $b > 0$.

Finding $\sqrt{x}$ for any given positive real number x geometrically:

Procedure

1. Draw a line and mark a point A on it. Take a point O on this line such that length of segment OA = x units.
2. From O, mark a point B on the line such that length of segment OB = 1 unit (as shown in fig. 1.10).
3. Find the mid-point, say M, of the segment AB. With M as centre and radius = AM, draw a semicircle (as shown in fig. 1.10).
4. At O, draw a perpendicular to AB meeting the semicircle at C (as shown in fig. 1.10).

5. Join CM (shown dotted in fig. 1.10).

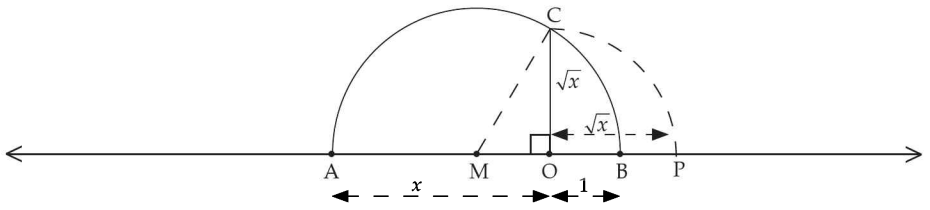


Fig. 1.10

From fig. 1.10, AO = x units, OB = 1 unit $\Rightarrow$ AB = $(x + 1)$ units.

As M is mid-point of AB, AM = $\frac{x+1}{2}$ units

Since AM is radius of circle, therefore,

$$AM = BM = CM = \frac{x+1}{2} \text{ units.}$$

Now, $MO = MB - OB = \left(\frac{x+1}{2} - 1\right)$ units $= \frac{x-1}{2}$ units.

We observe that MOC is a right angled triangle at the point O.

By Pythagoras theorem, we get

$$CM^2 = MO^2 + OC^2$$

$$\Rightarrow \quad OC^2 = CM^2 - MO^2 = \left(\frac{x+1}{2}\right)^2 - \left(\frac{x-1}{2}\right)^2 = \frac{4x}{4} = x$$

$$\Rightarrow \quad OC = \sqrt{x} \text{ units.}$$

If you want to represent $\sqrt{x}$ on the number line, then treat the line AB as the number line with O representing zero and B representing 1, and so on. Draw an arc with O as centre and radius = OC to meet the number line at P (as shown in fig. 1.10), then the point P will represent the number $\sqrt{x}$.

In fact, you can take any positive real number and proceed as above to find its square root.

nth root of a positive real number:

The concept of square root can be extended to cube root, fourth root, fifth root, ..., nth root, where n is a natural number.

Let $a > 0$ be a real number and be a natural number, then $\sqrt[n]{a} = b$ if and only if $b^n = a$, $b > 0$.

For example: $\sqrt[3]{8} = 2$ because $2^3 = 8$; $\sqrt[5]{243} = 3$ because $3^5 = 243$. Note that the symbol '$\sqrt{\ }$' used in $\sqrt{5}, \sqrt[3]{8}, \sqrt[5]{243}$ is called the **radical sign**.

1.6.2 Some facts about real numbers

1. If a and b are positive real numbers, then the following results hold:

(*i*) $\sqrt{ab} = \sqrt{a}\sqrt{b}$ (*ii*) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

(*iii*) $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$ (*iv*) $(a + \sqrt{b})(a - \sqrt{b}) = a^2 - b$

(*v*) $(\sqrt{a} + \sqrt{b})^2 = a + 2\sqrt{ab} + b$ (*vi*) $(\sqrt{a} - \sqrt{b})^2 = a - 2\sqrt{ab} + b$.

2. If a, b, c and d are positive real numbers, then

$(\sqrt{a} + \sqrt{b})(\sqrt{c} + \sqrt{d}) = \sqrt{ac} + \sqrt{ad} + \sqrt{bc} + \sqrt{bd}$.

3. If a, b, c and d are rational numbers, $\sqrt{p}$ is an irrational number and $a + b\sqrt{p} = c + d\sqrt{p}$, then $a = c$ and $b = d$.

1.6.3 Rationalisation

If the product of two irrational numbers is a rational number, then each number is called the **rationalising factor** of the other number.

The process of multiplying an irrational number by its rationalising factor is called **rationalisation**.

Rule to rationalise the denominator of a fraction:

Multiply and divide the numerator and denominator of the given fraction by the rationalising factor of its denominator and simplify.

Illustrative Examples

Example 1. *Check whether the following numbers are irrational numbers or not:*

(i) $\frac{1}{\sqrt{2}}$ *(NCERT)* *(ii)* $\frac{7}{\sqrt{5}}$ *(NCERT)* *(iii)* $\pi - 2$ *(NCERT)*

Solution. *(i)* $\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{1}{2} \times \sqrt{2}$.

We know that $\frac{1}{2}$ is a (non-zero) rational number and $\sqrt{2}$ is an irrational number, so their product *i.e.* $\frac{1}{2} \times \sqrt{2}$ is an irrational number. Hence, $\frac{1}{\sqrt{2}}$ is an irrational number.

(ii) $\frac{7}{\sqrt{5}} = \frac{7}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{7}{5} \times \sqrt{5}$.

We know that $\frac{7}{5}$ is a (non-zero) rational number and $\sqrt{5}$ is an irrational number, so their product *i.e.* $\frac{7}{5} \times \sqrt{5}$ is an irrational number. Hence, $\frac{7}{\sqrt{5}}$ is an irrational number.

(iii) We know that π is an irrational number and 2 is a rational number, therefore, their difference is an irrational number.

Hence, $\pi - 2$ is an irrational number.

Example 2. *Simplify the following:*

(i) $(3 + \sqrt{5})(2 + \sqrt{3})$ *(ii)* $(5 + \sqrt{7})(5 - \sqrt{7})$

(iii) $(\sqrt{5} - \sqrt{3})^2$ *(iv)* $(\sqrt{11} + \sqrt{7})(\sqrt{11} - \sqrt{7})$. *(NCERT)*

Solution. *(i)* $(3 + \sqrt{5})(2 + \sqrt{3}) = 6 + 3\sqrt{3} + 2\sqrt{5} + \sqrt{5}\sqrt{3}$

$= 6 + 3\sqrt{3} + 2\sqrt{5} + \sqrt{15}$.

(ii) $(5 + \sqrt{7})(5 - \sqrt{7}) = 5^2 - (\sqrt{7})^2 = 25 - 7 = 18$.

(iii) $(\sqrt{5} - \sqrt{3})^2 = (\sqrt{5})^2 - 2\sqrt{5}\sqrt{3} + (\sqrt{3})^2 = 5 - 2\sqrt{15} + 3$

$= 8 - 2\sqrt{15}$.

(iv) $(\sqrt{11} + \sqrt{7})(\sqrt{11} - \sqrt{7}) = (\sqrt{11})^2 - (\sqrt{7})^2 = 11 - 7 = 4$.

Example 3. *If* $\sqrt{3} = 1.732$, *then find the value of:*

$\sqrt{27} - 3\sqrt{75} + 5\sqrt{48} + 2\sqrt{108}$

Solution. $\sqrt{27} - 3\sqrt{75} + 5\sqrt{48} + 2\sqrt{108}$

$= \sqrt{9 \times 3} - 3\sqrt{25 \times 3} + 5\sqrt{16 \times 3} + 2\sqrt{36 \times 3}$

$= 3\sqrt{3} - 3 \times 5\sqrt{3} + 5 \times 4\sqrt{3} + 2 \times 6\sqrt{3}$

$= 3\sqrt{3} - 15\sqrt{3} + 20\sqrt{3} + 12\sqrt{3}$

$= (3 - 15 + 20 + 12)\sqrt{3} = 20 \times \sqrt{3}$

$= 20 \times 1.732 = 34.64$

Example 4. *Give examples to show that:*

(i) the sum of two irrational numbers may not be irrational.

(ii) the difference of two irrational numbers may not be irrational.

(iii) the product of two irrational numbers may not be irrational.

Solution. (*i*) Let $a = 5 + \sqrt{2}$ and $b = 3 - \sqrt{2}$, then a and b both being the sum of rational and irrational numbers are irrational.

Their sum $= a + b = (5 + \sqrt{2}) + (3 - \sqrt{2}) = 8$, which is rational.

Hence, the sum of two irrational numbers may not be irrational.

(*ii*) Let $a = 3 + \sqrt{5}$ and $b = 7 + \sqrt{5}$, then a and b are both irrational.

Their difference $= a - b = (3 + \sqrt{5}) - (7 + \sqrt{5}) = -4$, which is rational.

Hence, the difference of two irrational numbers may not be irrational.

(*iii*) Let $a = 5 - \sqrt{6}$ and $b = 5 + \sqrt{6}$, then a and b are both irrational.

Their product $= ab = (5 - \sqrt{6})(5 + \sqrt{6}) = 25 - 6 = 19$, which is rational.

Hence, the product of two irrational numbers may not be irrational.

Example 5. *Rationalise the denominator of the following:*

(*i*) $\frac{1}{7 + 3\sqrt{2}}$ *(NCERT)* (*ii*) $\frac{5}{\sqrt{3} - \sqrt{5}}$. *(NCERT)* (*iii*) $\frac{4\sqrt{3} + 5\sqrt{2}}{\sqrt{48} + \sqrt{18}}$ *(NCERT Exemplar)*

Solution. (*i*) $\frac{1}{7 + 3\sqrt{2}} = \frac{1}{7 + 3\sqrt{2}} \times \frac{7 - 3\sqrt{2}}{7 - 3\sqrt{2}} = \frac{7 - 3\sqrt{2}}{7^2 - (3\sqrt{2})^2}$

$= \frac{7 - 3\sqrt{2}}{49 - 18} = \frac{7 - 3\sqrt{2}}{31}$

(*ii*) $\frac{5}{\sqrt{3} - \sqrt{5}} = \frac{5}{\sqrt{3} - \sqrt{5}} \times \frac{\sqrt{3} + \sqrt{5}}{\sqrt{3} + \sqrt{5}} = \frac{5(\sqrt{3} + \sqrt{5})}{(\sqrt{3})^2 - (\sqrt{5})^2}$

$= \frac{5(\sqrt{3} + \sqrt{5})}{3 - 5} = -\frac{5(\sqrt{3} + \sqrt{5})}{2}$

(*iii*) $\frac{4\sqrt{3} + 5\sqrt{2}}{\sqrt{48} + \sqrt{18}} = \frac{4\sqrt{3} + 5\sqrt{2}}{\sqrt{48} + \sqrt{18}} \times \frac{\sqrt{48} - \sqrt{18}}{\sqrt{48} - \sqrt{18}} = \frac{(4\sqrt{3} + 5\sqrt{2})(4\sqrt{3} - 3\sqrt{2})}{(\sqrt{48})^2 - (\sqrt{18})^2}$

$= \frac{16 \times 3 - 12\sqrt{6} + 20\sqrt{6} - 15 \times 2}{48 - 18} = \frac{18 + 8\sqrt{6}}{30} = \frac{9 + 4\sqrt{6}}{15}$.

Example 6. *If a and b are rational numbers and $\frac{5 + 2\sqrt{3}}{7 + 4\sqrt{3}} = a - b\sqrt{3}$, find the values of a and b.* *(NCERT Exemplar)*

Solution. $\frac{5 + 2\sqrt{3}}{7 + 4\sqrt{3}} = \frac{5 + 2\sqrt{3}}{7 + 4\sqrt{3}} \times \frac{7 - 4\sqrt{3}}{7 - 4\sqrt{3}} = \frac{35 - 20\sqrt{3} + 14\sqrt{3} - 8 \times 3}{7^2 - (4\sqrt{3})^2}$

$= \frac{11 - 6\sqrt{3}}{49 - 48} = \frac{11 - 6\sqrt{3}}{1} = 11 - 6\sqrt{3}$

$\therefore \quad a - b\sqrt{3} = 11 - 6\sqrt{3}$

$\Rightarrow \quad a = 11$ and $b = 6$ (Using fact 3)

Example 7. *Simplify:* $\frac{7\sqrt{3}}{\sqrt{10} + \sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6} + \sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15} + 3\sqrt{2}}$. *(NCERT Exemplar)*

Solution. (*i*) First term $= \frac{7\sqrt{3}}{\sqrt{10} + \sqrt{3}} = \frac{7\sqrt{3}}{\sqrt{10} + \sqrt{3}} \times \frac{\sqrt{10} - \sqrt{3}}{\sqrt{10} - \sqrt{3}}$

$= \frac{7\sqrt{3}(\sqrt{10} - \sqrt{3})}{10 - 3} = \frac{7\sqrt{3}(\sqrt{10} - \sqrt{3})}{7} = \sqrt{30} - 3;$

second term $= \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} = \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} \times \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}}$

$= \frac{2\sqrt{5}(\sqrt{6}-\sqrt{5})}{6-5} = 2\sqrt{30} - 10$ and

third term $= \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}} = \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}} \times \frac{\sqrt{15}-3\sqrt{2}}{\sqrt{15}-3\sqrt{2}}$

$= \frac{3\sqrt{2}(\sqrt{15}-3\sqrt{2})}{15-18} = \frac{3\sqrt{2}(\sqrt{15}-3\sqrt{2})}{-3} = -\sqrt{30} + 6.$

$\therefore \quad \frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$

$= (\sqrt{30} - 3) - (2\sqrt{30} - 10) - (-\sqrt{30} + 6)$

$= \sqrt{30} - 3 - 2\sqrt{30} + 10 + \sqrt{30} - 6 = 1.$

Example 8. *If $x = 2 - \sqrt{3}$, find the value of $\left(x - \frac{1}{x}\right)^3$.*

Solution. Given $x = 2 - \sqrt{3}$,

$\therefore \quad \frac{1}{x} = \frac{1}{2-\sqrt{3}} = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{2+\sqrt{3}}{2^2-(\sqrt{3})^2}$

$= \frac{2+\sqrt{3}}{4-3} = \frac{2+\sqrt{3}}{1} = 2+\sqrt{3}.$

$\therefore \quad x - \frac{1}{x} = (2 - \sqrt{3}) - (2 + \sqrt{3}) = -2\sqrt{3}.$

$\therefore \quad \left(x - \frac{1}{x}\right)^3 = (-2\sqrt{3})^3 = (-2)^3 (\sqrt{3})^3$

$= -8 \times 3\sqrt{3} = -24\sqrt{3}.$

Example 9. *If $a = 5 + 2\sqrt{6}$, then find the value of $a^2 + \frac{1}{a^2}$.* *(NCERT Exemplar)*

Solution. Given $a = 5 + 2\sqrt{6}$.

$\therefore \quad \frac{1}{a} = \frac{1}{5+2\sqrt{6}} = \frac{1}{5+2\sqrt{6}} \times \frac{5-2\sqrt{6}}{5-2\sqrt{6}} = \frac{5-2\sqrt{6}}{5^2-(2\sqrt{6})^2}$

$= \frac{5-2\sqrt{6}}{25-24} = \frac{5-2\sqrt{6}}{1} = 5 - 2\sqrt{6}$

$\therefore \quad a + \frac{1}{a} = (5 + 2\sqrt{6}) + (5 - 2\sqrt{6}) = 10$ ...(*i*)

We know that $\left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2$

$\Rightarrow \quad a^2 + \frac{1}{a^2} = \left(a + \frac{1}{a}\right)^2 - 2$

$\Rightarrow \quad a^2 + \frac{1}{a^2} = (10)^2 - 2$ [using (*i*)]

$\Rightarrow \quad a^2 + \frac{1}{a^2} = 100 - 2 = 98.$

Example 10. *If $p = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ and $q = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$, find the value of $p^2 + q^2$.*

Solution. Given $p = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ and $q = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$,

$$\therefore \quad p + q = \frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}} + \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}} = \frac{(\sqrt{5} - \sqrt{3})^2 + (\sqrt{5} + \sqrt{3})^2}{(\sqrt{5})^2 - (\sqrt{3})^2}$$

$$= \frac{(5 + 3 - 2\sqrt{5}\sqrt{3}) + (5 + 3 + 2\sqrt{5}\sqrt{3})}{5 - 3} = \frac{16}{2} = 8 \qquad \ldots(i)$$

Also $pq = \frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}} \times \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}} = 1$ $\qquad \ldots(ii)$

We know that $(p + q)^2 = p^2 + q^2 + 2pq$

$\Rightarrow \quad p^2 + q^2 = (p + q)^2 - 2pq$

$\Rightarrow \quad p^2 + q^2 = 8^2 - 2 \times 1$ (Using (i) and (ii))

$\Rightarrow \quad p^2 + q^2 = 64 - 2 = 62.$

Exercise 1.4

1. Classify the following numbers as rational or irrational:

(i) $2 - \sqrt{5}$ *(NCERT)* $\quad$ (ii) $(3 + \sqrt{23}) - \sqrt{23}$ *(NCERT)* $\quad$ (iii) $7\sqrt{5}$ *(NCERT)*

(iv) $\frac{2\sqrt{7}}{7\sqrt{7}}$ *(NCERT)* $\quad$ (v) $\left(\frac{2}{5}\sqrt{7}\right)^2$ $\quad$ (vi) $2\pi - 3$

2. Add: $2\sqrt{2} + 5\sqrt{3}$ and $\sqrt{2} - 3\sqrt{3}$ *(NCERT)*

3. Simplify the following:

(i) $\sqrt{45} - 3\sqrt{20} + 4\sqrt{5}$ *(NCERT Exemplar)* $\quad$ (ii) $3\sqrt{3} + 2\sqrt{27} + \frac{7}{\sqrt{3}}$ *(NCERT Exemplar)*

(iii) $6\sqrt{5} \times 2\sqrt{5}$ *(NCERT)* $\quad$ (iv) $8\sqrt{15} \div 2\sqrt{3}$ *(NCERT)*

(v) $\frac{\sqrt{24}}{8} + \frac{\sqrt{54}}{9}$ *(NCERT Exemplar)* $\quad$ (vi) $\frac{3}{\sqrt{8}} + \frac{1}{\sqrt{2}}$ *(NCERT Exemplar)*

4. Simplify the following:

(i) $(5 + \sqrt{7})(2 + \sqrt{5})$ *(NCERT)* $\quad$ (ii) $(5 + \sqrt{5})(5 - \sqrt{5})$ *(NCERT)*

(iii) $(\sqrt{5} + \sqrt{2})^2$ *(NCERT)* $\quad$ (iv) $(\sqrt{3} - \sqrt{7})^2$

(v) $(\sqrt{2} + \sqrt{3})(\sqrt{5} + \sqrt{7})$ $\quad$ (vi) $(4 + \sqrt{5})(\sqrt{3} - \sqrt{7})$

5. Represent the following numbers on the number line:

(i) $\sqrt{3.5}$ *(NCERT)* $\quad$ (ii) $\sqrt{9.3}$ *(NCERT)* $\quad$ (iii) $\sqrt{5.6}$ *(NCERT Exemplar)*

Hint: (i) Take $x = 3.5$ and proceed as in article 1.6.1]

6. If $\sqrt{2} = 1.414$, then find the value of:

(i) $\sqrt{8} + \sqrt{50} + \sqrt{72} + \sqrt{98}$ $\quad$ (ii) $3\sqrt{32} - 2\sqrt{50} + 4\sqrt{128} - 20\sqrt{18}$.

7. If $\sqrt{3} = 1.732$, then find the value of:

(i) $\sqrt{27} + \sqrt{75} + \sqrt{108} - \sqrt{243}$ $\quad$ (ii) $5\sqrt{12} - 3\sqrt{48} + 6\sqrt{75} + 7\sqrt{108}$.

8. Rationalise the denominator of the following:

(i) $\frac{1}{2 + \sqrt{3}}$ *(NCERT)* $\quad$ (ii) $\frac{1}{\sqrt{7} - 2}$ *(NCERT)* $\quad$ (iii) $\frac{16}{\sqrt{41} - 5}$ *(NCERT Exemplar)*

(iv) $\frac{1}{\sqrt{7} - \sqrt{6}}$ *(NCERT)* $\quad$ (v) $\frac{1}{\sqrt{5} + \sqrt{2}}$ *(NCERT)* $\quad$ (vi) $\frac{3\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}}$ *(NCERT Exemplar)*

9. Given a and b are rational numbers. Find a and b if:

(*i*) $\frac{3-\sqrt{5}}{3+2\sqrt{5}} = -\frac{19}{11} + a\sqrt{5}$ *(NCERT Exemplar)*

(*ii*) $\frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = a - b\sqrt{6}$ *(NCERT Exemplar)*

(*iii*) $\frac{7+\sqrt{5}}{7-\sqrt{5}} - \frac{7-\sqrt{5}}{7+\sqrt{5}} = a + \frac{7}{11}b\sqrt{5}$ *(NCERT Exemplar)*

10. Simplify: $\frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}} + \frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}}$.

11. Rationalise the denominator of the following and hence evaluate by taking $\sqrt{2} = 1.414$ and $\sqrt{3} = 1.732$, upto three places of decimal:

(*i*) $\frac{\sqrt{2}}{2+\sqrt{2}}$ *(NCERT Exemplar)* (*ii*) $\frac{1}{\sqrt{3}+\sqrt{2}}$ *(NCERT Exemplar)*

12. If $a = 2 + \sqrt{3}$, then find the value of $a - \frac{1}{a}$. *(NCERT Exemplar)*

13. If $x = 1 - \sqrt{2}$, find the value of $\left(x - \frac{1}{x}\right)^4$.

14. If $x = 5 - 2\sqrt{6}$, find the value of $x^2 + \frac{1}{x^2}$.

15. If $p = \frac{2-\sqrt{5}}{2+\sqrt{5}}$ and $q = \frac{2+\sqrt{5}}{2-\sqrt{5}}$, find the values of:

(*i*) $p + q$ (*ii*) $p - q$ (*iii*) $p^2 + q^2$ (*iv*) $p^2 - q^2$

Answers

1.7 LAWS OF EXPONENTS FOR REAL NUMBERS

Recall the laws of exponents which you have studied in your earlier classes.

If a, b are rational numbers and m, n are integers, then the following results hold:

(*i*) $a^m \cdot a^n = a^{m+n}$ (*ii*) $(a^m)^n = a^{mn}$ (*iii*) $\frac{a^m}{a^n} = a^{m-n}$, $a \neq 0$

(*iv*) $a^m \cdot b^m = (ab)^m$ (*v*) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, $b \neq 0$ (*vi*) $a^0 = 1$, $a \neq 0$

(*vii*) $a^{-n} = \left(\frac{1}{a}\right)^n = \frac{1}{a^n}$, $a \neq 0$ (*viii*) $a^n = b^n$, $n \neq 0 \Rightarrow a = b$

(*ix*) $a^m = a^n$, $a \neq 1 \Rightarrow m = n$.

The question arises, can we extend these laws when the base is a positive real number and the exponents are rational numbers. Before we state these laws, let us try to understand what is $8^{\frac{2}{3}}$?

In section 1.6, we defined $\sqrt[n]{a}$ for real number $a > 0$ and n a natural number as follows:

If $a > 0$ be a real number and n a natural number, then $\sqrt[n]{a} = b$ if and only if $b^n = a$, $b > 0$.

In the language of exponents, we write $\sqrt[n]{a} = a^{\frac{1}{n}}$.

So, in particular, we write $\sqrt[3]{5} = 5^{\frac{1}{3}}$ and $\sqrt[5]{2} = 2^{\frac{1}{5}}$.

Now, look at $8^{\frac{2}{3}}$. There are two ways:

$$8^{\frac{2}{3}} = \left(8^{\frac{1}{3}}\right)^2 = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4;$$

$$8^{\frac{2}{3}} = (8^2)^{\frac{1}{3}} = (64)^{\frac{1}{3}} = \sqrt[3]{64} = 4$$

This leads us to the following definition:

If $a > 0$ is a real number and m, n are integers, $n > 0$, m, n have no common factors except 1, then

$$a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}\,.$$

Thus, we have the following laws of exponents for real numbers:

If a, b are positive real numbers and m, n are rational numbers, then the following results hold:

(*i*) $a^m \cdot a^n = a^{m+n}$ (*ii*) $(a^m)^n = a^{mn}$ (*iii*) $\frac{a^m}{a^n} = a^{m-n}$

(*iv*) $a^m \cdot b^m = (ab)^m$ (*v*) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ (*vi*) $a^{-n} = \left(\frac{1}{a}\right)^n = \frac{1}{a^n}$

(*vii*) $a^n = b^n, n \neq 0 \Rightarrow a = b$ (*viii*) $a^m = a^n \Rightarrow m = n$ provided $a \neq 1$.

Illustrative Examples

Example 1. *Simplify the following:*

(*i*) $\dfrac{7^{\frac{1}{5}}}{7^{\frac{1}{3}}}$ (*NCERT*) (*ii*) $\left(\frac{1}{27}\right)^{-\frac{2}{3}}$ (*NCERT Exemplar*) (*iii*) $\left[\left((625)^{-\frac{1}{2}}\right)^{-\frac{1}{4}}\right]^2$ (*NCERT Exemplar*)

Solution. (*i*) $\dfrac{7^{\frac{1}{5}}}{7^{\frac{1}{3}}} = 7^{\frac{1}{5}-\frac{1}{3}} = 7^{\frac{3-5}{15}} = 7^{-\frac{2}{15}}$.

(*ii*) $\left(\frac{1}{27}\right)^{-\frac{2}{3}} = (27)^{\frac{2}{3}}$ $\left(\because a^{-n} = \left(\frac{1}{a}\right)^n\right)$

$$= \left((27)^{\frac{1}{3}}\right)^2 = \left(\left(3^3\right)^{\frac{1}{3}}\right)^2 = \left(3^{3\times\frac{1}{3}}\right)^2 = \left(3^1\right)^2 = 3^{1\times 2} = 3^2 = 9\,.$$

(*iii*) $\left[\left((625)^{-\frac{1}{2}}\right)^{-\frac{1}{4}}\right]^2 = \left[(625)^{\left(-\frac{1}{2}\right)\times\left(-\frac{1}{4}\right)}\right]^2 = \left[(625)^{\frac{1}{8}}\right]^2$

$$= (625)^{\frac{1}{8}\times 2} = (625)^{\frac{1}{4}} = (5^4)^{\frac{1}{4}} = 5^{4\times\frac{1}{4}} = 5^1 = 5.$$

Example 2. *Simplify the following:*

(*i*) $\dfrac{9^{\frac{1}{3}} \times 27^{-\frac{1}{2}}}{3^{\frac{1}{6}} \times 3^{-\frac{2}{3}}}$ (*NCERT Exemplar*) (*ii*) $(256)^{-\left(4^{-\frac{3}{2}}\right)}$ (*NCERT Exemplar*)

(*iii*) $\sqrt[4]{81} - 8\sqrt[3]{216} + 15\sqrt[5]{32} + 2\sqrt{225}$

Solution. (*i*) $\dfrac{9^{\frac{1}{3}} \times 27^{-\frac{1}{2}}}{3^{\frac{1}{6}} \times 3^{-\frac{2}{3}}} = \dfrac{(3^2)^{\frac{1}{3}} \times (3^3)^{-\frac{1}{2}}}{3^{\frac{1}{6}} \times 3^{-\frac{2}{3}}} = \dfrac{3^{\frac{2}{3}} \times 3^{-\frac{3}{2}}}{3^{\frac{1}{6}} \times 3^{-\frac{2}{3}}}$

$$= 3^{\frac{2}{3}+\left(-\frac{3}{2}\right)-\frac{1}{6}-\left(-\frac{2}{3}\right)} = 3^{\frac{2}{3}-\frac{3}{2}-\frac{1}{6}+\frac{2}{3}}$$

$$= 3^{\frac{4-9-1+4}{6}} = 3^{-\frac{2}{6}} = 3^{-\frac{1}{3}}.$$

(*ii*) Note that $4^{-\frac{3}{2}} = (2^2)^{-\frac{3}{2}} = 2^{2\times\left(-\frac{3}{2}\right)} = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$.

$$\therefore \quad (256)^{-\left(4^{-\frac{3}{2}}\right)} = (256)^{-\frac{1}{8}} = (2^8)^{-\frac{1}{8}} = 2^{8\times\left(-\frac{1}{8}\right)} = 2^{-1} = \frac{1}{2^1} = \frac{1}{2}.$$

(*iii*) $\sqrt[4]{81} - 8\sqrt[3]{216} + 15\sqrt[5]{32} + 2\sqrt{225}$

$$= (3^4)^{\frac{1}{4}} - 8(6^3)^{\frac{1}{3}} + 15(2^5)^{\frac{1}{5}} + 2(15^2)^{\frac{1}{2}}$$

$$= 3^{4\times\frac{1}{4}} - 8\left(6^{3\times\frac{1}{3}}\right) + 15\left(2^{5\times\frac{1}{5}}\right) + 2\left(15^{2\times\frac{1}{2}}\right)$$

$$= 3^1 - 8 \times 6^1 + 15 \times 2^1 + 2 \times 15^1$$
$$= 3 - 8 \times 6 + 15 \times 2 + 2 \times 15$$
$$= 3 - 48 + 30 + 30 = 15.$$

Example 3. *Find the value of* $\frac{4}{(216)^{-\frac{2}{3}}} + \frac{1}{(256)^{-\frac{3}{4}}} + \frac{2}{(243)^{-\frac{1}{5}}}$. *(NCERT Exemplar)*

Solution. $\frac{4}{(216)^{-\frac{2}{3}}} + \frac{1}{(256)^{-\frac{3}{4}}} + \frac{2}{(243)^{-\frac{1}{5}}}$

$$= 4(216)^{\frac{2}{3}} + (256)^{\frac{3}{4}} + 2(243)^{\frac{1}{5}}$$

$$= 4(6^3)^{\frac{2}{3}} + (4^4)^{\frac{3}{4}} + 2(3^5)^{\frac{1}{5}} = 4\left(6^{3\times\frac{2}{3}}\right) + 4^{4\times\frac{3}{4}} + 2\left(3^{5\times\frac{1}{5}}\right)$$

$$= 4(6^2) + 4^3 + 2(3^1) = 4 \times 36 + 64 + 6$$
$$= 144 + 64 + 6 = 214.$$

Example 4. *Write the following in simplest form:*

(i) $\sqrt[3]{2} \times \sqrt[4]{3}$ *(ii)* $\sqrt[4]{28} \div \sqrt[3]{7}$

Solution. (*i*) $\sqrt[3]{2} \times \sqrt[4]{3} = 2^{\frac{1}{3}} \times 3^{\frac{1}{4}}$.

L.C.M. of 3 and 4 = 12, so write each number with exponent $\frac{1}{12}$.

$2^{\frac{1}{3}} = (2^4)^{\frac{1}{12}}$ and $3^{\frac{1}{4}} = (3^3)^{\frac{1}{12}}$

$$\therefore \quad \sqrt[3]{2} \times \sqrt[4]{3} = (2^4)^{\frac{1}{12}} \times (3^3)^{\frac{1}{12}} = (2^4 \times 3^3)^{\frac{1}{12}}$$

$$= (16 \times 27)^{\frac{1}{12}} = (432)^{\frac{1}{12}} = \sqrt[12]{432}$$

(*ii*) $\sqrt[4]{28} \div \sqrt[3]{7} = (28)^{\frac{1}{4}} \div (7)^{\frac{1}{3}}$

L.C.M. of 4 and 3 = 12, so write each number with exponent 12.

$(28)^{\frac{1}{4}} = ((28)^3)^{\frac{1}{12}}$ and $7^{\frac{1}{3}} = (7^4)^{\frac{1}{12}}$

$$\therefore \quad \sqrt[4]{28} \div \sqrt[3]{7} = \frac{((28)^3)^{\frac{1}{12}}}{(7^4)^{\frac{1}{12}}} = \left(\frac{(28)^3}{7^4}\right)^{\frac{1}{12}} = \left(\frac{28 \times 28 \times 28}{7 \times 7 \times 7 \times 7}\right)^{\frac{1}{12}}$$

$$= \left(\frac{4 \times 4 \times 4}{7}\right)^{\frac{1}{12}} = \left(\frac{64}{7}\right)^{\frac{1}{12}} = \sqrt[12]{\frac{64}{7}}$$

Example 5. *Write in ascending order:* $4\sqrt{5}, 5\sqrt{3}, 10, 3\sqrt{7}, 6\sqrt{2}$.

Solution. Write all the numbers as square roots under one radical.

$4\sqrt{5} = \sqrt{16} \times \sqrt{5} = \sqrt{80}, 5\sqrt{3} = \sqrt{25} \times \sqrt{3} = \sqrt{75}, 10 = \sqrt{100}$,

$3\sqrt{7} = \sqrt{9} \times \sqrt{7} = \sqrt{63}$ and $6\sqrt{2} = \sqrt{36} \times \sqrt{2} = \sqrt{72}$

Since $63 < 72 < 75 < 80 < 100 \Rightarrow \sqrt{63} < \sqrt{72} < \sqrt{75} < \sqrt{80} < \sqrt{100}$

$\Rightarrow \quad 3\sqrt{7} < 6\sqrt{2} < 5\sqrt{3} < 4\sqrt{5} < 10$.

Hence, the given numbers in ascending order are $3\sqrt{7}, 6\sqrt{2}, 5\sqrt{3}, 4\sqrt{5}, 10$.

Example 6. *Arrange the following numbers in ascending order:*

$\sqrt{3}, \sqrt[3]{5}, \sqrt[4]{8}$

Solution. The given numbers are $\sqrt{3}, \sqrt[3]{5}, \sqrt[4]{8}$ *i.e.* $3^{\frac{1}{2}}, 5^{\frac{1}{3}}, 8^{\frac{1}{4}}$.

L.C.M. of 2, 3 and 4 = 12, so write each number with exponent $\frac{1}{12}$.

$\sqrt{3} = 3^{\frac{1}{2}} = (3^6)^{\frac{1}{12}} = (729)^{\frac{1}{12}}$,

$\sqrt[3]{5} = 5^{\frac{1}{3}} = (5^4)^{\frac{1}{12}} = (625)^{\frac{1}{12}}$ and

$\sqrt[4]{8} = 8^{\frac{1}{4}} = (8^3)^{\frac{1}{12}} = (512)^{\frac{1}{12}}$.

As $512 < 625 < 729 \Rightarrow (512)^{\frac{1}{12}} < (625)^{\frac{1}{12}} < (729)^{\frac{1}{12}}$

$\sqrt[4]{8} < \sqrt[3]{5} < \sqrt{3}$.

Hence, the given numbers in ascending order are $\sqrt[4]{8}, \sqrt[3]{5}, \sqrt{3}$.

Exercise 1.5

1. Simplify the following:

(*i*) $2^{\frac{2}{3}}.2^{\frac{1}{3}}$ *(NCERT)* (*ii*) $32^{\frac{2}{5}}$ *(NCERT)* (*iii*) $13^{\frac{1}{5}}.17^{\frac{1}{5}}$ *(NCERT)*

(*iv*) $9^{\frac{3}{2}}$ *(NCERT)* (*v*) $(125)^{-\frac{1}{3}}$ *(NCERT)* (*vi*) $\left(\frac{81}{16}\right)^{-\frac{3}{4}}$

2. Simplify the following:

(*i*) $\left(1\frac{61}{64}\right)^{-\frac{2}{3}}$ (*ii*) $(64)^{-\frac{2}{3}} \div 9^{-\frac{3}{2}}$ (*iii*) $\sqrt[5]{(32)^{-3}}$

3. Simplify the following:

(*i*) $\left(\frac{3}{5}\right)^4\left(\frac{8}{5}\right)^{-12}\left(\frac{32}{5}\right)^6$ *(NCERT Exemplar)*

(*ii*) $\frac{8^{\frac{1}{3}} \times 16^{\frac{1}{3}}}{32^{-\frac{1}{3}}}$ *(NCERT Exemplar)*

(*iii*) $64^{-\frac{1}{3}}(64^{\frac{1}{3}} - 64^{\frac{2}{3}})$ *(NCERT Exemplar)*

4. Find the value of the following:

(*i*) $16^{\frac{3}{4}} + 2\left(\frac{1}{2}\right)^{-1}$ (*ii*) $(81)^{\frac{3}{4}} - \left(\frac{1}{32}\right)^{-\frac{2}{5}} + (8)^{\frac{1}{3}}\left(\frac{1}{2}\right)^{0}$

5. Write the following in simplest form:

(*i*) $\sqrt[3]{7} \times \sqrt[6]{6}$ (*ii*) $\sqrt[3]{2} \times \sqrt{5}$ (*iii*) $\sqrt[3]{2} \times \sqrt[4]{5}$

(*iv*) $\sqrt[3]{7} \div \sqrt[6]{2}$ (*v*) $\sqrt[4]{4} \div \sqrt[3]{3}$

6. Write the following numbers in ascending order:

(*i*) $3\sqrt{2}, 2\sqrt{3}, \sqrt{15}, 4$ (*ii*) $3\sqrt{2}, 2\sqrt{8}, 4, \sqrt{50}, 4\sqrt{3}$

7. Arrange the following numbers in ascending order:

(*i*) $\sqrt[3]{2}, \sqrt{3}, \sqrt[6]{5}$ (*ii*) $\sqrt{3}, \sqrt[3]{4}, \sqrt[4]{5}$

8. Arrange $\sqrt[3]{2}, \sqrt[4]{3}$ and $\sqrt[3]{4}$ in descending order.

Answers

Very Short Answer Type Questions

1. Find a rational number between $\frac{2}{3}$ and $\frac{3}{5}$.
2. Find an irrational number between $\frac{3}{5}$ and $\frac{2}{3}$.
3. Find a rational number between 1 and $\sqrt{2}$.
4. Find an irrational number between $\sqrt{2}$ and $\sqrt{3}$.
5. How many rational numbers lie between 0.01 and 0.1?
6. How many irrational numbers lie between 0.01 and 0.1?
7. Without actual division, determine whether the decimal expansion of the rational number $\frac{51}{84}$ is terminating or non-terminating recurring.
8. Find the value of: $4\sqrt{3} - 3\sqrt{12} + 2\sqrt{75}$.
9. Find the value of: $16\sqrt{15} \div 9\sqrt{60}$.
10. Simplify: $(4 + 3\sqrt{5})(4 - 3\sqrt{5})$.
11. Write the rationalising factor of $\frac{1}{3 - \sqrt{11}}$.
12. If $x = 7 + 4\sqrt{3}$, then find the value of $x + \frac{1}{x}$.
13. If $x = \sqrt{2} - 1$, then find the value of $x^2 + \frac{1}{x^2}$.
14. Simplify: $\left(\frac{32}{243}\right)^{-\frac{1}{5}}$
15. Simplify: $\left(2\frac{10}{27}\right)^{\frac{2}{3}}$.

Answers

Objective Type Questions

(FOR FORMATIVE/SUMMATIVE ASSESSMENT)

Mental Maths

1. Fill in the blanks:

(*i*) The reciprocal of every (non-zero) rational number is a ..... number.

(*ii*) The sum of a rational and an irrational number is always ..... number.

(*iii*) The difference of a rational and an irrational number is always ..... number.

(*iv*) The decimal expansion of every rational number is either ..... or non-terminating ..... .

(*v*) The decimal expansion of every irrational number is always ..... .

(*vi*) Every number whose decimal expansion is non-terminating non-recurring is ..... number.

(*vii*) Between two distinct rational numbers there lie ..... rational numbers.

(*viii*) Between two distinct rational numbers there lie ..... irrational numbers.

(*ix*) Between two distinct irrational numbers there lie ..... rational numbers.

(*x*) Corresponding to every real number, we can find ..... point on the number line.

(*xi*) The decimal expansion of a rational number $\frac{p}{q}$, where p, q are integers, $q > 0$, p, q have no common factors (except 1) is non-terminating recurring if q has a prime factor ..... .

2. State whether the following statements are true or false. Justify your answer.

(*i*) Every natural number is a whole number. *(NCERT)*

(*ii*) Every integer is a whole number. *(NCERT)*

(*iii*) Every integer can be written in the form $\frac{p}{q}$, where p, q are integers, $q \neq 0$.

(*iv*) Every rational number is an integer.

(*v*) Every real number is an irrational number. *(NCERT)*

(*vi*) Every irrational number is a real number. *(NCERT)*

(*vii*) Every point on the number line represents a number of the form $\sqrt{n}$, where n is a natural number. *(NCERT)*

(*viii*) The product of a rational and an irrational number is an irrational number.

(*ix*) Reciprocal of every rational number is a rational number.

(*x*) The square roots of all positive integers are irrational numbers. *(NCERT)*

(*xi*) There are infinitely many integers between any two integers.

(*xii*) The square of an irrational number is always a rational number.

Multiple Choice Questions

Choose the correct answer from the given four options (3 to 37):

3. The smallest natural number is

(*a*) -1 (*b*) 0 (*c*) 1 (*d*) 2

4. The smallest whole number is

(*a*) -1 (*b*) 0 (*c*) 1 (*d*) 2

5. Choose the wrong statement:

(*a*) There is no largest natural number.

(*b*) There is no largest integer. (*c*) There is no smallest integer.

(*d*) The collection of rational numbers has largest as well as smallest.

6. Choose the wrong statement:

(*a*) Every natural number is a whole number.

(*b*) Every integer is a rational number.

(*c*) Every rational number is an integer.

(*d*) Every rational number is a real number.

7. Every rational number is
   (*a*) a natural number (*b*) an integer
   (*c*) a real number (*d*) a whole number *(NCERT Exemplar)*
8. Between two rational numbers
   (*a*) there is no rational number
   (*b*) there is exactly one rational number
   (*c*) there are infinitely many rational numbers
   (*d*) there are only rational numbers and no irrational numbers. *(NCERT Exemplar)*
9. Decimal representation of a rational number cannot be
   (*a*) terminating (*b*) non-terminating
   (*c*) non-terminating repeating (*d*) non-terminating non-repeating
   (NCERT Exemplar)
10. The product of any two irrational numbers is
   (*a*) always an irrational number (*b*) always a rational number
   (*c*) always an integer
   (*d*) sometimes rational, sometimes irrational *(NCERT Exemplar)*
11. The decimal expansion of the number $\sqrt{2}$ is
   (*a*) a finite decimal (*b*) 1.41421
   (*c*) non-terminating recurring (*d*) non-terminating non-recurring
   (NCERT Exemplar)
12. Which of the following is an irrational number?
   (*a*) $\sqrt{\frac{4}{9}}$ (*b*) $\frac{\sqrt{12}}{\sqrt{3}}$ (*c*) $\sqrt{7}$ (*d*) $\sqrt{81}$
   (NCERT Exemplar)
13. Which of the following is not a rational number?
   (*a*) $\sqrt{2}$ (*b*) $\sqrt{4}$ (*c*) $\sqrt{9}$ (*d*) $\sqrt{25}$
14. Which of the following is an irrational number?
   (*a*) $\sqrt{\frac{9}{25}}$ (*b*) $\sqrt{\frac{2}{8}}$ (*c*) $\sqrt{\frac{4}{27}}$ (*d*) $\sqrt{\frac{9}{49}}$
15. Which of the following is different from others?
   (*a*) $\sqrt{7}$ (*b*) $\sqrt{8}$ (*c*) $\sqrt{9}$ (*d*) $\sqrt{10}$
16. The number of rational numbers between the rational numbers $\frac{1}{3}$ and $\frac{1}{2}$ is
   (*a*) 1 (*b*) 2 (*c*) 6 (*d*) infinitely many.
17. Choose the rational number which does not lie between the rational numbers $-\frac{2}{5}$ and $-\frac{1}{5}$.
   (*a*) $-\frac{3}{10}$ (*b*) $\frac{3}{10}$ (*c*) $-\frac{1}{4}$ (*d*) $-\frac{7}{20}$
18. Which of the following numbers has terminating decimal representation?
   (*a*) $\frac{3}{7}$ (*b*) $\frac{3}{5}$ (*c*) $\frac{1}{3}$ (*d*) $\frac{3}{11}$
19. Which of the following is an irrational number?
   (*a*) 0.14 (*b*) $0.14\overline{16}$ (*c*) $0.\overline{1416}$ (*d*) 0.4014001400014...
   (NCERT Exemplar)

20. Choose a rational number which does not lie between $\frac{2}{5}$ and $\frac{3}{4}$.

(*a*) $\frac{17}{20}$ (*b*) $\frac{13}{20}$ (*c*) $\frac{11}{20}$ (*d*) $\frac{9}{20}$

21. A rational number between $\sqrt{2}$ and $\sqrt{3}$ is

(*a*) $\frac{\sqrt{2}+\sqrt{3}}{2}$ (*b*) $\frac{\sqrt{2}\times\sqrt{3}}{2}$ (*c*) 1.5 (*d*) 1.8

(NCERT Exemplar)

22. The value of 1.999... in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is

(*a*) $\frac{19}{20}$ (*b*) $\frac{1999}{1000}$ (*c*) 2 (*d*) $\frac{1}{9}$

(NCERT Exemplar)

23. The number $0.\overline{25}$ is equal to

(*a*) $\frac{65}{99}$ (*b*) $\frac{37}{99}$ (*c*) $\frac{5}{9}$ (*d*) $\frac{25}{99}$

24. $\sqrt{10}\times\sqrt{15}$ is equal to

(*a*) $6\sqrt{5}$ (*b*) $5\sqrt{6}$ (*c*) $\sqrt{25}$ (*d*) $10\sqrt{5}$

(NCERT Exemplar)

25. $2\sqrt{3}+\sqrt{3}$ is equal to

(*a*) $2\sqrt{6}$ (*b*) 6 (*c*) $3\sqrt{3}$ (*d*) $4\sqrt{6}$

(NCERT Exemplar)

26. The value of $\sqrt{8}+\sqrt{18}$ is

(*a*) $\sqrt{26}$ (*b*) $2(\sqrt{2}+\sqrt{3})$ (*c*) $5\sqrt{2}$ (*d*) $6\sqrt{2}$

27. The number $(2-\sqrt{3})^2$ is

(*a*) a natural number (*b*) an integer

(*c*) a rational number (*d*) an irrational number

28. The value of $\frac{\sqrt{32}+\sqrt{48}}{\sqrt{8}+\sqrt{12}}$ is equal to

(*a*) $\sqrt{2}$ (*b*) 2 (*c*) 4 (*d*) 8

(NCERT Exemplar)

29. The number obtained on rationalising the denominator of $\frac{1}{\sqrt{7}-2}$ is

(*a*) $\frac{\sqrt{7}+2}{3}$ (*b*) $\frac{\sqrt{7}-2}{3}$ (*c*) $\frac{\sqrt{7}+2}{5}$ (*d*) $\frac{\sqrt{7}+2}{45}$

(NCERT Exemplar)

30. $\frac{1}{\sqrt{9}-\sqrt{8}}$ is equal to

(*a*) $\frac{1}{2}(3-2\sqrt{2})$ (*b*) $\frac{1}{3+2\sqrt{2}}$ (*c*) $3-2\sqrt{2}$ (*d*) $3+2\sqrt{2}$

(NCERT Exemplar)

31. After rationalising the denominator of $\frac{7}{3\sqrt{3}-2\sqrt{2}}$, we get the denominator as

(*a*) 13 (*b*) 19 (*c*) 5 (*d*) 35

(NCERT Exemplar)

32. If $\sqrt{2}=1.4142$, then $\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$ is equal to

(*a*) 2.4142 (*b*) 5.8284 (*c*) 0.4142 (*d*) 0.1718

(NCERT Exemplar)

33. $\sqrt[4]{\sqrt[3]{2^2}}$ is equal to

(*a*) $2^{-\frac{1}{6}}$ (*b*) 2^{-6} (*c*) $2^{\frac{1}{6}}$ (*d*) 2^6

(NCERT Exemplar)

34. The product $\sqrt[3]{2}.\sqrt[4]{2}.\sqrt[12]{32}$ equals

(*a*) $\sqrt{2}$ (*b*) 2 (*c*) $\sqrt[12]{2}$ (*d*) $\sqrt[12]{32}$

(NCERT Exemplar)

35. The value of $\sqrt[4]{(81)^{-2}}$ is

(*a*) $\frac{1}{9}$ (*b*) $\frac{1}{3}$ (*c*) 9 (*d*) $\frac{1}{81}$

(NCERT Exemplar)

36. Value of $(256)^{0.16} \times (256)^{0.09}$ is

(*a*) 4 (*b*) 16 (*c*) 64 (*d*) 256.25

(NCERT Exemplar)

37. Which of the following is equal to x?

(*a*) $x^{\frac{12}{7}} - x^{\frac{5}{7}}$ (*b*) $\sqrt[12]{(x^4)^{\frac{1}{3}}}$ (*c*) $(\sqrt{x^3})^{\frac{2}{3}}$ (*d*) $x^{\frac{12}{7}} \times x^{\frac{7}{12}}$

(NCERT Exemplar)

Answers

Higher Order Thinking Skills (HOTS)

1. Rationalise the denominator of the following:

(*i*) $\frac{1}{\sqrt{3} - \sqrt{2} + 1}$ (*ii*) $\frac{1}{\sqrt{5} + \sqrt{6} - \sqrt{11}}$.

2. Solve the following equations for x:

(*i*) $(\sqrt[3]{4})^{2x+\frac{1}{2}} = \frac{1}{32}$ (*ii*) $9 \times (81)^x = \frac{1}{27^{x-2}}$ (*iii*) $\sqrt{\left(\frac{3}{5}\right)^{1-2x}} = 4\frac{17}{27}$.

Answers

Summary

- Any number that can be expressed in the form $\frac{p}{q}$, where p, q are integers, $q \neq 0$, is called a **rational number.** Usually, we take $q > 0$, while p may be positive, negative or zero. Collection of rational numbers is denoted by **Q**.
  Rational numbers do not have unique representation.
  For example, $\frac{1}{2} = \frac{2}{4} = \frac{7}{14} = \frac{40}{80}$ etc.
  Every rational number can be written in the form $\frac{p}{q}$, *where p, q are integers,* $q \neq 0$, *p, q have no common factors (except 1).* It is called in the **lowest terms** or **simplest form.**
- A number that cannot be expressed in the form $\frac{p}{q}$, where p, q are integers, $q \neq 0$, p, q have no common factors (except 1) is called an **irrational number.**
  For example, $\sqrt{2}, \sqrt{3}, \sqrt{6}, -\sqrt{7}, 2 - \sqrt{5}, \pi$ are all irrational numbers.
- The decimal expansion of a rational number is either terminating or non-terminating recurring (repeating). Conversely, a number whose decimal expansion is terminating or non-terminating recurring is rational.

- The decimal expansion of a rational number $\frac{p}{q}$, where p, q are integers, $q > 0$, p and q have no common factors except 1 is:
  (*i*) terminating if prime factors of q are 2 or 5 or both.
  (*ii*) non-terminating recurring if q has a prime factor other than 2 or 5.
- The decimal expansion of an irrational number is non-terminating non-recurring. Conversely, a number whose decimal expansion is non-terminating non-recurring is irrational.
- All rational numbers and all irrational numbers are **real numbers.** The collection of real numbers is denoted by **R.**
- Every real number (rational or irrational) can be represented by a unique point on the number line. Conversely, every point on the number line represents a unique real number.
- If x is rational and y is irrational, then $x + y$, $x - y$ and $y - x$ are irrational numbers.
- If x is a non-zero rational number and y is an irrational number, then xy, $\frac{x}{y}$ and $\frac{y}{x}$ are irrational numbers.
- If a and b are positive real numbers, then the following results hold:

  (*i*) $\sqrt{ab} = \sqrt{a}\sqrt{b}$ (*ii*) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

  (*iii*) $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$ (*iv*) $(a + \sqrt{b})(a - \sqrt{b}) = a^2 - b$

  (*v*) $(\sqrt{a} + \sqrt{b})^2 = a + 2\sqrt{ab} + b$ (*vi*) $(\sqrt{a} - \sqrt{b})^2 = a - 2\sqrt{ab} + b$
- If the product of two irrational numbers is a rational number, then each number is called the **rationalising factor** of the other number.

  The process of multiplying an irrational number by its rationalising factor is called **rationalisation.**

  Rule to rationalise the denominator of a fraction:

  Multiply and divide the numerator and denominator of the given fraction by the rationalising factor of its denominator and simplify.
- If $a > 0$ is a real number and n is a positive integer, then $\sqrt[n]{a} = b$ if and only if $b^n = a$, $b > 0$. $\sqrt[n]{a}$ is also written as $a^{1/n}$.
- If $a > 0$ is a real number and m, n are integers, $n > 0$, m, n have no common factors except 1, then

$$a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = (\sqrt[n]{a})^m = \sqrt[n]{a^m}\ .$$

- If a, b are positive real numbers and m, n are rational numbers, then the following results hold:

  (*i*) $a^m \cdot a^n = a^{m+n}$ (*ii*) $(a^m)^n = a^{mn}$ (*iii*) $\frac{a^m}{a^n} = a^{m-n}$

  (*iv*) $a^m \cdot b^m = (ab)^m$ (*v*) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ (*vi*) $a^{-n} = \left(\frac{1}{a}\right)^n = \frac{1}{a^n}$

  (*vii*) $a^n = b^n$, $n \neq 0 \Rightarrow a = b$ (*viii*) $a^m = a^n \Rightarrow m = n$ provided $a \neq 1$.

Chapter Test

Choose the correct answer from the given four options (1 to 7):

1. A rational number between $\frac{1}{2}$ and $\frac{1}{3}$ is

(*a*) $\frac{1}{5}$ (*b*) $\frac{2}{5}$ (*c*) $\frac{3}{5}$ (*d*) $\frac{4}{5}$

2. An irrational number between 2 and 3 is

(*a*) $\sqrt{2}$ (*b*) $\sqrt{3}$ (*c*) $\sqrt{6}$ (*d*) $\sqrt{12}$

3. Which of the following is an irrational number?

(*a*) 3.758 (*b*) 3.1010010001...

(*c*) $3.\overline{23789}$ (*d*) 37.56489125648912...

4. The number $0.\overline{27}$ is equal to

(*a*) $\frac{2}{11}$ (*b*) $\frac{3}{11}$ (*c*) $\frac{4}{11}$ (*d*) $\frac{5}{11}$.

5. The rationalising factor of $\frac{1}{5+2\sqrt{6}}$ is

(*a*) $\sqrt{5}+2\sqrt{6}$ (*b*) $-\sqrt{5}+2\sqrt{6}$ (*c*) $5-2\sqrt{6}$ (*d*) $-5-2\sqrt{6}$

6. If $x = \frac{1}{3+2\sqrt{2}}$, then the value of $x - \frac{1}{x}$ is

(*a*) 6 (*b*) −6 (*c*) $4\sqrt{2}$ (*d*) $-4\sqrt{2}$

7. The value of $\left(5\frac{1}{16}\right)^{-\frac{3}{4}}$ is

(*a*) $\frac{4}{9}$ (*b*) $\frac{9}{4}$ (*c*) $\frac{27}{8}$ (*d*) $\frac{8}{27}$

8. Without performing long division, determine which of the following rational numbers has a terminating decimal expansion:

$$\frac{7}{15}, -\frac{12}{840}, \frac{3}{80}, \frac{15}{220}$$

Also write its decimal expansion.

9. What can be the maximum number of entries in the repeating string of remainders in the decimal expansion of $\frac{3}{7}$? Perform the division and write the repeating string of remainders.

10. Insert four rational numbers between $\frac{4}{5}$ and $\frac{5}{6}$.

11. Express the number $0.2\overline{45}$ in the form $\frac{p}{q}$, where p and q are integers, $q \neq 0$.

12. Insert four irrational numbers between $3\sqrt{2}$ and $2\sqrt{3}$.

13. Represent the numbers −1.5 and $2+\sqrt{2}$ on the number line in the same diagram.

14. Represent geometrically the number $\sqrt{8.1}$ on the number line. *(NCERT Exemplar)*

15. Express $0.6 + 0.\overline{7} + 0.4\overline{7}$ in the form $\frac{p}{q}$, where p and q are integers, $q \neq 0$. *(NCERT Exemplar)*

16. If a and b are rational numbers and $\frac{6}{3\sqrt{2}-2\sqrt{3}} = a\sqrt{2} - b\sqrt{3}$, then find the values of a and b. *(NCERT Exemplar)*

17. If a and b are rational numbers and $\frac{\sqrt{2}+2\sqrt{3}}{2\sqrt{2}+\sqrt{3}} = a + b\sqrt{6}$, find the values of a and b.

18. If $\sqrt{2} = 1.414$ and $\sqrt{3} = 1.732$, then find the value of $\frac{4}{3\sqrt{3}-2\sqrt{2}} + \frac{3}{3\sqrt{3}+2\sqrt{2}}$. *(NCERT Exemplar)*

19. If $a = \frac{3+\sqrt{5}}{2}$, then find the value of $a^2 + \frac{1}{a^2}$. *(NCERT Exemplar)*

20. If $x = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ and $y = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, then find the value of $x^2 + y^2$. *(NCERT Exemplar)*

21. Arrange the following numbers in ascending order:

$\sqrt{2}, \sqrt[3]{4}, \sqrt[4]{5}, \sqrt[4]{3}$.

Answers

Polynomials



INTRODUCTION

In earlier classes, you have studied algebraic expressions and the basic operations on them *i.e.* their addition, subtraction, multiplication and division. In this chapter, we shall study particular types of algebraic expressions called **polynomials** and the terms related to them. We shall also study **Remainder Theorem** and **Factor Theorem** and their use in finding the zeroes of a polynomial/roots of a polynomial equation and in the factorisation of polynomials. In previous classes, you also have studied some algebraic identities. In this chapter, we shall review those algebraic identities and shall study some more algebraic identities and their use in the factorisation and in evaluating some given algebraic expressions.

2.1 POLYNOMIAL IN ONE VARIABLE AND RELATED TERMS

Definition. An algebraic expression of the form

$a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_1 x + a_0$, where a_n, a_{n-1}, a_{n-2}, …, a_0 are real numbers and n is a whole number, is called a **polynomial** in the variable x.

Polynomials in the variable x are usually denoted by $p(x)$, $f(x)$, $g(x)$ etc. Thus,

$p(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_1 x + a_0$.

(*i*) If $a_n \neq 0$, then n is called the **degree** of the polynomial $p(x)$. It is written as *deg* $p(x) = n$.

(*ii*) $a_n x^n$, $a_{n-1} x^{n-1}$, $a_{n-2} x^{n-2}$, …, $a_1 x$, a_0 are called the **terms** of the polynomial $p(x)$; a_0 is called the **constant term.**

(*iii*) A polynomial of degree n in one variable has atmost $(n + 1)$ terms.

(*iv*) a_n, a_{n-1}, a_{n-2}, …, a_1, a_0 are called the **coefficients** of the polynomial $p(x)$.

(*v*) If $a_n \neq 0$, then $a_n x^n$ is called the **leading term** and a_n is called the **leading coefficient** of the polynomial $p(x)$.

(*vi*) If all the coefficients a_n, a_{n-1}, a_{n-2}, …, a_1, a_0 are zero, then $p(x)$ is called a **zero polynomial.** The zero polynomial is usually denoted by the symbol 0. *The degree of the zero polynomial is not defined.*

(*vii*) The degree of a polynomial is zero if and only if it is a non-zero constant polynomial *i.e.* if $p(x) = c$, $c \neq 0$.

Remember that $x^0 = 1$.

If $p(x) = c$, $c \neq 0$, then $p(x)$ can be written as $p(x) = c = c.1 = cx^0$, so *deg* $p(x) = 0$.

Converse is also true. If *deg* $p(x) = 0$, then $p(x) = cx^0$, $c \neq 0$, so $p(x) = c$.

Remark

In a polynomial, the exponent of the variable of each term must be a whole number.

2.1.1 Some particular polynomials in one variable

A polynomial of degree one is called a **linear polynomial.** It is of the form $ax + b$, where a and b are real numbers and $a \neq 0$.

A polynomial of degree two is called a **quadratic polynomial.** It is of the form $ax^2 + bx + c$, where a, b and c are real numbers and $a \neq 0$.

A polynomial of degree three is called a **cubic polynomial.** It is of the form $ax^3 + bx^2 + cx + d$, where a, b, c and d are real numbers and $a \neq 0$.

A polynomial of degree four is called a biquadratic polynomial, etc.

For example:

(a) $3x + 5$ is a linear polynomial in the variable x.

(b) $5x^2 - 7x + \sqrt{2}$ is a quadratic polynomial in the variable x.

(c) $2y^3 + 3y^2 - \sqrt{3}y + \frac{5}{7}$ is a cubic polynomial in the variable y.

(d) $3u^4 - 7u^2 + 5u - \frac{2}{3}$ is a biquadratic polynomial in the variable u.

2.1.2 Monomial, Binomial, Trinomial, Multinomial

(i) A polynomial having one term is called a **monomial.**

(ii) A polynomial having two terms is called a **binomial.**

(iii) A polynomial having three terms is called a **trinomial.**

(iv) A polynomial having two or more terms is called a **multinomial.**

For example:

Polynomial	*No. of terms*	*Name*	*Degree of polynomial*
(i) $7x^5$	1	monomial	5
(ii) $3x^2 - 5$	2	binomial	2
(iii) $5u^7 - 3u^2 + 1$	3	trinomial	7
(iv) $9y^4 - \sqrt{2}\,y^2 + 3y + 1$	4	multinomial	4
(v) $3x^2 - 7x^9 + \pi$	3	trinomial	9

2.1.3 Value of a polynomial $p(x)$ at $x = \alpha$

Let $p(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_1 x + a_0$ be a polynomial in x and α be a real number, then the real number

$$a_n \alpha^n + a_{n-1}\alpha^{n-1} + a_{n-2}\alpha^{n-2} + \ldots + a_1 \alpha + a_0$$

is called the ***value of p(x) at*** α.

Thus, if $p(x)$ is a polynomial in x and α is a real number then value obtained by replacing x by α in $p(x)$ is called the value of $p(x)$ at $x = \alpha$. It is denoted by $p(\alpha)$.

For example:

1. Let $p(x) = 3x + 5$ be a linear polynomial in x, then

$p(2) = 3 \times 2 + 5 = 11$,

$p(0) = 3 \times 0 + 5 = 5$,

$p(-5) = 3 \times (-5) + 5 = -10$,

$p\left(-\frac{5}{3}\right) = 3 \times \left(-\frac{5}{3}\right) + 5 = -5 + 5 = 0.$

2. Let $p(x) = 2x^2 + 3x - 2$ be a quadratic polynomial in x, then

$p(3) = 2 \times 3^2 + 3 \times 3 - 2 = 18 + 9 - 2 = 25$,

$p(-2) = 2 \times (-2)^2 + 3(-2) - 2 = 8 - 6 - 2 = 0,$

$p\left(\frac{1}{2}\right) = 2 \times \left(\frac{1}{2}\right)^2 + 3 \times \frac{1}{2} - 2 = \frac{1}{2} + \frac{3}{2} - 2 = 0,$

$p(\sqrt{2}) = 2 \times (\sqrt{2})^2 + 3 \times \sqrt{2} - 2 = 4 + 3\sqrt{2} - 2 = 2 + 3\sqrt{2}.$

3. Let $p(x) = x^3 - 5x^2 + 7x - 3$ be a cubic polynomial in x, then

$p(0) = 0^3 - 5 \times 0^2 + 7 \times 0 - 3 = -3,$

$p(1) = 1^3 - 5 \times 1^2 + 7 \times 1 - 3 = 1 - 5 + 7 - 3 = 0,$

$p(-2) = (-2)^3 - 5(-2)^2 + 7(-2) - 3 = -8 - 20 - 14 - 3 = -45.$

4. Let $p(y) = 2y^5 - 3y^2 + 7$ be a polynomial in y, then

$p(2) = 2 \times 2^5 - 3 \times 2^2 + 7 = 64 - 12 + 7 = 59,$

$p(-1) = 2 \times (-1)^5 - 3 \times (-1)^2 + 7 = -2 - 3 + 7 = 2.$

Illustrative Examples

Example 1. *Which of the following algebraic expressions are polynomials in one variable and which are not? State reason for your answer.*

(i) $7x^2 - 5x + 11$ *(ii)* $3y^3 - 2y + \sqrt{7}$ *(iii)* $\frac{1}{\sqrt{5}}x^{\frac{1}{2}} + 1$ *(NCERT Exemplar)*

(iv) $y + \frac{2}{y}$ *(NCERT)* *(v)* $7x^4 - \sqrt{3}\,x^3 + \frac{2}{3}x + \pi$ *(vi)* $x^2 + 3xy - 9.$

Solution. *(i)* $7x^2 - 5x + 11$ is a polynomial in one variable x.

(ii) $3y^3 - 2y + \sqrt{7}$ is a polynomial in one variable y.

(iii) $\frac{1}{\sqrt{5}}x^{\frac{1}{2}} + 1$ is not a polynomial because the exponent of the variable x in the term $\frac{1}{\sqrt{5}}x^{\frac{1}{2}}$ is not a whole number.

(iv) $y + \frac{2}{y}$ is not a polynomial because the exponent of the variable y in the term $\frac{2}{y}$ *i.e.* $2y^{-1}$ is not a whole number.

(v) $7x^4 - \sqrt{3}\,x^3 + \frac{2}{3}x + \pi$ is a polynomial in one variable x.

(vi) $x^2 + 3xy - 9$ is not a polynomial in one variable. Of course, it is a polynomial in two variables x and y.

Example 2. *Write the coefficient of x^2 in each of the following polynomials. Also write the number of terms and the degree of each.*

(i) $3x^3 - 5x^2 + \sqrt{3}$ *(ii)* $5x^4 - 3x^3 + \frac{1}{2}x^2 - 7\sqrt{3}$

(iii) $7x^5 - \sqrt{5}\,x + 2$ *(iv)* $3 - 5x - 2\sqrt{5}\,x^2 + 5x^7$

(v) $(2x - 5)(2x^2 - 3x + 1).$ *(NCERT Exemplar)*

Solution. *(i)* Coefficient of $x^2 = -5$, number of terms = 3, degree = 3.

(ii) Coefficient of $x^2 = \frac{1}{2}$, number of terms = 4, degree = 4.

(iii) As the given polynomial contains no term with x^2, so coefficient of $x^2 = 0$, number of terms = 3, degree = 5.

(iv) Coefficient of $x^2 = -2\sqrt{5}$, number of terms = 4, degree = 7.

(*v*) $(2x - 5)(2x^2 - 3x + 1) = 4x^3 - 6x^2 + 2x - 10x^2 + 15x - 5$

$= 4x^3 - 16x^2 + 17x - 5$

It is a polynomial in one variable x.

Coefficient of $x^2 = -16$, number of terms = 4, degree = 3.

Example 3. *(i) Is* $\frac{6\sqrt{x} + x^{\frac{3}{2}}}{\sqrt{x}}$ *a polynomial,* $x \neq 0$*? Justify your answer.* *(NCERT Exemplar)*

(ii) Find the coefficient of x^2 *in the polynomial* $\sqrt{2}\,x - 1$*.* *(NCERT)*

(iii) Is -5 *a polynomial? If so, then find its degree.*

Solution. (*i*) $\frac{6\sqrt{x} + x^{\frac{3}{2}}}{\sqrt{x}} = \frac{6\sqrt{x}}{\sqrt{x}} + \frac{x^{\frac{3}{2}}}{\sqrt{x}} = 6 + x$, which is a polynomial in one variable x.

(*ii*) $\sqrt{2}\,x - 1$ is a polynomial in x. It can be written as $0x^2 + \sqrt{2}\,x - 1$, so coefficient of $x^2 = 0$.

Hence, the coefficient of x^2 in the given polynomial = 0.

(*iii*) Here, the only term is -5. It can be written as $-5x^0$, which is a polynomial in one variable x. As the exponent of x is 0, so its degree is 0.

Example 4. *Classify the following polynomials as linear, quadratic and cubic. Also identify monomial, binomial, trinomial and multinomials:*

(i) $5x^2 - 3$ *(ii)* $\sqrt{7}\,x^3 - 2x^2 + 3x - 1$ *(iii)* $5t - \sqrt{7}$ *(NCERT)*

(iv) $7x^3$ *(NCERT)* *(v)* $\frac{2}{3} - \sqrt{7}\,x + 9x^2$.

Solution. (*i*) Quadratic; binomial (*ii*) Cubic; multinomial

(*iii*) Linear; binomial (*iv*) Cubic; monomial

(*v*) Quadratic; trinomial.

Example 5. *Find the value of* $f(x) = 4x^3 - 3x^2 + 5x + 7$ *at*

(i) $x = 0$ *(ii)* $x = -2$ *(iii)* $x = \frac{1}{2}$.

Solution. Given $f(x) = 4x^3 - 3x^2 + 5x + 7$

(*i*) $f(0) = 4 \times 0^3 - 3 \times 0^2 + 5 \times 0 + 7 = 0 - 0 + 0 + 7 = 7$.

(*ii*) $f(-2) = 4 \times (-2)^3 - 3 \times (-2)^2 + 5(-2) + 7$

$= -32 - 12 - 10 + 7 = -47$.

$f\left(\frac{1}{2}\right) = 4 \times \left(\frac{1}{2}\right)^3 - 3 \times \left(\frac{1}{2}\right)^2 + 5 \times \frac{1}{2} + 7$

$= 4 \times \frac{1}{8} - 3 \times \frac{1}{4} + \frac{5}{2} + 7 = \frac{1}{2} - \frac{3}{4} + \frac{5}{2} + 7$

$= \frac{2 - 3 + 10 + 28}{4} = \frac{37}{4}$.

Example 6. *If* $p(x) = x^2 - 4x + 3$*, evaluate:* $p(2) - p(-1) + p\left(\frac{1}{2}\right)$ *(NCERT Exemplar)*

Solution. Given $p(x) = x^2 - 4x + 3$

$\therefore$ $p(2) = 2^2 - 4 \times 2 + 3 = 4 - 8 + 3 = -1$,

$p(-1) = (-1)^2 - 4 \times (-1) + 3 = 1 + 4 + 3 = 8$ and

$p\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - 4 \times \frac{1}{2} + 3 = \frac{1}{4} - 2 + 3 = \frac{1}{4} + 1 = \frac{5}{4}$

$\therefore$ $p(2) - p(-1) + p\left(\frac{1}{2}\right) = -1 - 8 + \frac{5}{4} = -9 + \frac{5}{4} = -\frac{31}{4}$.

Exercise 2.1

1. Which of the following algebraic expressions are polynomials in one variable? If so, write their degree.

(*i*) $\frac{2}{5}x^4 - \sqrt{3}\,x^2 + x - 7$ (*ii*) $7x^3 - \frac{3}{x^2} + \sqrt{5}$ (*iii*) $5x^3 - 5x^2 + 7\sqrt[3]{x} - 1$

(*iv*) $9t^5 - \frac{2}{3}t^3 + \sqrt{7}\,t + 9$ (*v*) $y^2 + \sqrt{2}$ (*vi*) $2 - y^2 - y^3 + 7y^8$

(*vii*) $x^3 + 3xy^2 - 7y^3 + 5$ (*viii*) $x^{10} + y^3 + t^{50}$ (*ix*) $\frac{1}{5x^{-2}} + 5x + 7$

(NCERT) *(NCERT Exemplar)*

2. Write the coefficient of x^2 in each of the following polynomials:

(*i*) $\frac{\pi}{2}x^2 + x$ (*ii*) $\frac{\pi}{6}x + x^2 - 1$ (*iii*) $(x - 1)(3x - 4)$

(NCERT) *(NCERT Exemplar)* *(NCERT Exemplar)*

3. (*i*) Write the coefficient of x^2 and the leading term in the polynomial

$5 - 7x^2 + 7x^3 + \sqrt{11}\,x^5$.

(*ii*) Write the coefficient of y^3 and the leading coefficient in the polynomial

$-3y^7 + 5y^6 + 3y^2 - \frac{2}{3}$.

4. For the polynomial $\frac{x^3 + 2x + 1}{5} - \frac{7}{2}x^2 - x^6$, write:

(*i*) the degree of the polynomial (*ii*) the coefficient of x^3

(*iii*) the coefficient of x^6 (*iv*) the constant term. *(NCERT Exemplar)*

5. Identify the following polynomials as monomials, binomials, trinomials and multinomials:

(*i*) $3y^3 - 5$ (*ii*) $5y^2 - 7y + 18$ (*iii*) $5t^3$

(*iv*) $5x^4 - 3x^2 + 5x + \sqrt{3}$ (*v*) $5x^7 - 5x^2 + \pi$ (*vi*) $\sqrt{7}\,t^5 + \frac{2}{3}$.

6. Give one example each of a binomial of degree 35, and a monomial of degree 100. *(NCERT)*

7. Classify the following polynomials as constant, linear, quadratic, cubic and biquadratic. Also find the number of terms in each.

(*i*) $3x^3 - 5x^2 + \frac{2}{3}$ (*ii*) $\sqrt{7}\,y^2 - 5y + 4\sqrt{3}$

(*iii*) $5t - \sqrt{7}$ (*iv*) $7x^4 - 3x^2 + \sqrt{13}\,x + \pi$

(*v*) 3 *(NCERT Exemplar)* (*vi*) r^2 *(NCERT)*

8. Find $p(0)$, $p(1)$ and $p(-2)$ for each of the following polynomials:

(*i*) $p(x) = 10x - 4x^2 - 3$ (*ii*) $p(y) = (y + 2)(y - 2)$

(NCERT Exemplar) *(NCERT Exemplar)*

(*iii*) $p(t) = 2 + t + 2t^2 - t^3$ *(NCERT)*

9. Find the value of the following polynomials at the indicated value of the variables:

(*i*) $p(x) = 2 + x + 2x^2 - x^3$ at $x = 1$ (*ii*) $f(y) = 3y^3 - 4y + \sqrt{11}$ at $y = 2$

(NCERT)

(*iii*) $f(x) = 2x^4 + 9x^3 + 11x^2 + 4x - 6$ at $x = -3$.

Answers

2.2 ZEROES OF A POLYNOMIAL

Definition. *A real number α is a* **zero** *of the polynomial $p(x)$ if and only if $p(\alpha) = 0$.*

For example:

(*i*) If $p(x) = x + 2$, then $p(-2) = -2 + 2 = 0$, therefore, -2 is a zero of $p(x)$.

Note that a zero of a polynomial may not be 0.

(*ii*) If $p(x) = 3x - 5$, then $p\left(\frac{5}{3}\right) = 3 \times \frac{5}{3} - 5 = 5 - 5 = 0$, therefore, $\frac{5}{3}$ is a zero of $p(x)$.

(*iii*) If $p(x) = 2x^2 + x - 3$, then $p(1) = 2 \times 1^2 + 1 - 3 = 2 + 1 - 3 = 0$.

Also $p\left(-\frac{3}{2}\right) = 2 \times \left(-\frac{3}{2}\right)^2 + \left(-\frac{3}{2}\right) - 3 = \frac{9}{2} - \frac{3}{2} - 3 = 0$.

Therefore, 1 and $-\frac{3}{2}$ are zeroes of the polynomial $p(x)$.

Note that a polynomial may have more than one zero.

❑ **Rule to find zeroes of a polynomial $p(x)$**

Put $p(x) = 0$ and solve it for x. The values of x will be the zeroes of the polynomial $p(x)$.

For example:

To find a zero of the polynomial $p(x) = 7x + 3$, on putting $p(x) = 0$, we get

$$7x + 3 = 0$$

$$\Rightarrow \quad 7x = -3 \Rightarrow x = -\frac{7}{3}.$$

Therefore, $-\frac{7}{3}$ is a zero of the given polynomial $7x + 3$.

❑ **A polynomial may not have real zeroes**

For example, consider the polynomial

$p(x) = x^2 + 2x + 7$,

then $p(x) = (x + 1)^2 + 6$ but $(x + 1)^2 \geq 0$ for all real values of x

$\Rightarrow$ $p(x) \geq 6$ for all real values of x.

Therefore, $p(x) \neq 0$ for any real value of x.

Hence, $p(x)$ has no real zeroes.

In fact, the number of real zeroes of a polynomial may be less than the degree of the polynomial.

❑ **Number of zeroes of a polynomial**

(*i*) By convention, every real number is taken as a zero of the zero polynomial. Thus, zero polynomial has infinitely many zeroes.

(*ii*) A non-zero constant polynomial has no zeroes.

For example, consider the non-zero constant polynomial $p(x) = 5$.

Replacing x by any real number in 5 *i.e.* in $5x^\circ$, we still get 5. Therefore, $p(x) \neq 0$ for any real value of x.

Hence, $p(x)$ has no real zeroes.

(*iii*) Every linear polynomial has one and only one zero.

Let $p(x) = ax + b$, where a and b are real numbers and $a \neq 0$,

then $p\left(-\frac{b}{a}\right) = a \times \left(-\frac{b}{a}\right) + b = -b + b = 0$.

Therefore, $-\frac{b}{a}$ is a zero of $p(x)$.

(*iv*) A quadratic polynomial can have atmost 2 zeroes.

(*v*) A cubic polynomial can have atmost 3 zeroes.

(*vi*) A polynomial of degree $n (> 1)$ can have atmost n zeroes.

Remarks

- **Integral zeroes of a polynomial**

  If $f(x)$ is a polynomial with integral coefficients and leading coefficient is 1, then every integral zero of $f(x)$ must be a factor of the constant term of $f(x)$.

  For example, if $f(x) = x^2 + x - 6$ then the possible integral zeroes of $f(x)$ are the factors of -6 *i.e.* $\pm 1, \pm 2, \pm 3, \pm 6$.

- **Rational zeroes of a polynomial**

  If $f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_0$ is a polynomial with integral coefficients and a rational number $\frac{p}{q}$ (in lowest terms) is a zero of $f(x)$, then p must be a factor of a_0 and q must be a factor of a_n.

  For example, if $f(x) = 2x^2 + 5x - 3$, then the possible rational zeroes of $f(x)$ are ± 1, ± 3, $\pm \frac{1}{2}$, $\pm \frac{3}{2}$.

2.2.1 Root of a polynomial equation

Definition. *A real number α is a* ***root*** *of the polynomial equation $p(x) = 0$ if and only if $p(\alpha) = 0$.*

For example:

(*i*) If $p(x) = 2x - 5$, then $p\left(\frac{5}{2}\right) = 2 \times \frac{5}{2} - 5 = 5 - 5 = 0$,

therefore, $\frac{5}{2}$ is a root of the polynomial equation $p(x) = 0$

i.e. of the polynomial equation $2x - 5 = 0$.

(*ii*) If $p(x) = 3x^2 - x - 2$, then $p(1) = 3 \times 1^2 - 1 - 2 = 3 - 1 - 2 = 0$.

Also $p\left(-\frac{2}{3}\right) = 3 \times \left(-\frac{2}{3}\right)^2 - \left(-\frac{2}{3}\right) - 2 = 3 \times \frac{4}{9} + \frac{2}{3} - 2 = \frac{4}{3} + \frac{2}{3} - 2 = 0$.

Therefore, 1 and $-\frac{2}{3}$ are roots of the polynomial equation $p(x) = 0$ *i.e.* of the polynomial equation $3x^2 - x - 2 = 0$.

Zero–Product Rule

If a and b are two real numbers or expressions and if $ab = 0$, then either $a = 0$ or $b = 0$ or both $a = 0$ and $b = 0$.

Using the above rule, the roots of the equation $(x - 1)(x - 2) = 0$ can be found by putting each factor equal to zero and then solving for x. Thus, we get

$x - 1 = 0$ or $x - 2 = 0$

$\Rightarrow x = 1$ or $x = 2$.

Hence, the roots of the equation $(x - 1)(x - 2) = 0$ are 1 and 2.

Illustrative Examples

Example 1. *Check whether 0 and 2 are zeroes of the polynomial $x^2 - 2x$.* *(NCERT)*

Solution. Let $p(x) = x^2 - 2x$.

Then $p(0) = 0^2 - 2 \times 0 = 0 - 0 = 0$

$\Rightarrow$ 0 is a zero of the given polynomial.

Also $p(2) = 2^2 - 2 \times 2 = 4 - 4 = 0$

$\Rightarrow$ 2 is a zero of the given polynomial.

Hence, 0 and 2 are both zeroes of the given polynomial $x^2 - 2x$.

Note that 0 may be a zero of a polynomial.

Example 2. *Which of the numbers −1, 2 and −3 are zeroes of the polynomial* $2x^4 + 9x^3 + 11x^2 + 4x - 6$?

Solution. Let $p(x) = 2x^4 + 9x^3 + 11x^2 + 4x - 6$.

Then $p(-1) = 2 \times (-1)^4 + 9 \times (-1)^3 + 11 \times (-1)^2 + 4 \times (-1) - 6$

$= 2 - 9 + 11 - 4 - 6 = -6$

$\Rightarrow$ -1 is not a zero of the given polynomial.

$p(2) = 2 \times 2^4 + 9 \times 2^3 + 11 \times 2^2 + 4 \times 2 - 6$

$= 32 + 72 + 44 + 8 - 6 = 150$

$\Rightarrow$ 2 is not a zero of the given polynomial.

$p(-3) = 2 \times (-3)^4 + 9 \times (-3)^3 + 11 \times (-3)^2 + 4 \times (-3) - 6$

$= 162 - 243 + 99 - 12 - 6 = 0$

$\Rightarrow$ -3 is a zero of the given polynomial.

Hence, -1 and 2 are not the zeroes of the given polynomial whereas -3 is a zero of the given polynomial.

Example 3. *Find the zeroes of the following polynomials:*

(i) $p(x) = 2x + 5$ *(ii)* $g(x) = 3 - 6x$ *(iii)* $p(x) = ax$, $a \neq 0$.

(NCERT) (NCERT Exemplar) (NCERT)

Solution. (*i*) Finding zero of $p(x)$ is the same as solving $p(x) = 0$.

Now $p(x) = 0 \Rightarrow 2x + 1 = 0$

$\Rightarrow$ $2x = -1 \Rightarrow x = -\frac{1}{2}$.

Hence, $-\frac{1}{2}$ is a zero of the given polynomial $2x + 1$.

(*ii*) $g(x) = 0 \Rightarrow 3 - 6x = 0 \Rightarrow 3 = 6x \Rightarrow x = \frac{1}{2}$

$\Rightarrow$ $\frac{1}{2}$ is a zero of the given polynomial $g(x)$.

(*iii*) $p(x) = 0 \Rightarrow ax = 0 \Rightarrow x = 0$ $(\because\ a \neq 0)$

$\Rightarrow$ 0 is a zero of the given polynomial.

Example 4. *Find the roots of the following polynomial equations:*

(i) $3x - 7 = 0$ *(ii)* $\sqrt{7}\,x + 11 = 0$ *(iii)* $(x + 3)(x + 2) = 0$.

Solution. (*i*) $3x - 7 = 0 \Rightarrow 3x = 7 \Rightarrow x = \frac{7}{3}$

$\Rightarrow$ $\frac{7}{3}$ is a root of the given polynomial equation.

(*ii*) $\sqrt{7}\,x + 11 = 0 \Rightarrow \sqrt{7}\,x = -11 \Rightarrow x = -\frac{11}{\sqrt{7}}$

$\Rightarrow$ $-\frac{11}{\sqrt{7}}$ is a root of the given polynomial equation.

(*iii*) $(x + 3)(x - 2) = 0 \Rightarrow x + 3 = 0$ or $x - 2 = 0$ (using zero-product rule)

$\Rightarrow$ $x = -3$ or $x = 2$.

Hence, the roots of the given equation are -3 and 2.

Example 5. *Find the zeroes of the polynomial* $p(x) = (x - 2)^2 - (x + 2)^2$. *(NCERT Exemplar)*

Solution. Given $p(x) = (x - 2)^2 - (x + 2)^2$

$= (x^2 - 4x + 4) - (x^2 + 4x + 4) = -8x$.

Now $p(x) = 0 \Rightarrow -8x = 0 \Rightarrow x = 0$

$\Rightarrow$ 0 is the only zero of the given polynomial $p(x)$.

Example 6. *Find the integral zeroes of the polynomial* $x^3 + x^2 + x - 3$.

Solution. Let $p(x) = x^3 + x^2 + x - 3$.

The possible integral zeroes of $p(x)$ are the factors of -3 *i.e.* 1, -1, 3 and -3.

Now $p(1) = 1^3 + 1^2 + 1 - 3 = 1 + 1 + 1 - 3 = 0$

$\Rightarrow$ 1 is a zero of the given polynomial.

$p(-1) = (-1)^3 + (-1)^2 + (-1) - 3 = -1 + 1 - 1 - 3 \neq -4$

$\Rightarrow$ -1 is not a zero of the given polynomial.

$p(3) = 3^3 + 3^2 + 3 - 3 = 27 + 9 + 3 - 3 = 36$

$\Rightarrow$ 3 is not a zero of the given polynomial.

$p(-3) = (-3)^3 + (-3)^2 + (-3) - 3 = -27 + 9 - 3 - 3 = -24$

$\Rightarrow$ -3 is not a zero of the given polynomial.

Hence, 1 is the only integral zero of the given polynomial.

Exercise 2.2

1. Check whether 2 and -2 are zeroes of the polynomial $x + 2$. *(NCERT)*
2. Check whether 0 and -3 are zeroes of the polynomial $x^2 + 3x$.
3. Show that 3 is a zero of the polynomial $x^3 - 8x^2 + 8x + 21$.
4. Check whether the number(s) are zeroes of the polynomials, indicated against them:

   (*i*) $p(x) = 3x + 1$, $x = -\frac{1}{3}$ *(NCERT)* (*ii*) $p(x) = 5x - \pi$, $x = \frac{4}{5}$ *(NCERT)*

   (*iii*) $p(x) = x^2 - 1$, $x = 1, -1$ *(NCERT)* (*iv*) $p(x) = (x + 1)(x - 2)$, $x = -1, 2$ *(NCERT)*

   (*v*) $p(x) = 7x$, $x = 0, 7$ (*vi*) $p(x) = 3x^2 - 1$, $x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$. *(NCERT)*

5. (*i*) Show that 5 is a zero of the polynomial $2x^3 - 7x^2 - 16x + 5$.

   (*ii*) Show that -1, 1 and 3 are all zeroes of the polynomial $x^3 - 3x^2 - x + 3$.
6. Find the zeroes of the following polynomials:

   (*i*) $2x + 5$ *(NCERT)* (*ii*) $ax + b$, $a \neq 0$ (*iii*) $(x + 1)(2x + 3)$
7. Find the roots of the following polynomial equations:

   (*i*) $5x - 3 = 0$ (*ii*) $9x^2 - 4 = 0$ (*iii*) $5x^2 - 9 = 0$

   (*iv*) $(x - 1)(3x + 2) = 0$ (*v*) $(x + 2)(x - 3)(2x - 7) = 0$.
8. Find the integral zeroes of $4x^3 + 20x^2 - x - 5$.
9. Show that the polynomial $3x^3 + 8x^2 - 1 = 0$ has no integral zeroes.
10. Show that the polynomial $x^2 + 2x + 7$ has no zeroes.

Answers

2.3 DIVISION ALGORITHM FOR POLYNOMIALS

Let us divide $3x^3 + 5x^2 + x$ by the monomial x.

We have, $(3x^3 + 5x^2 + x) \div x = \frac{3x^3}{x} + \frac{5x^2}{x} + \frac{x}{x}$

$= 3x^2 + 5x + 1.$

In fact, we notice that x is common to each term of $3x^3 + 5x^2 + x$. So, we can write $2x^3 + 5x^2 + x$ as $x(2x^2 + 5x + 1)$.

We say that x and $2x^2 + 5x + 1$ are **factors** of $2x^3 + 5x^2 + x$ and $2x^3 + 5x^2 + x$ is a **multiple** of x as well as a **multiple** of $2x^2 + 5x + 1$.

Now, let us divide $x^3 - 5x^2 + 7x + 9$ by $x - 2$.

We have:

$$
\begin{array}{r|l}
 & x^2 - 3x + 1 \\
\hline
x - 2 & x^3 - 5x^2 + 7x + 9 \\
 & x^3 - 2x^2 \\
 & - \quad + \\
\hline
 & -3x^2 + 7x + 9 \\
 & -3x^2 + 6x \\
 & + \quad - \\
\hline
 & x + 9 \\
 & x - 2 \\
 & - \quad + \\
\hline
 & 11
\end{array}
$$

$\therefore \quad x^3 - 5x^2 + 7x + 9 = (x - 2)(x^2 - 3x + 1) + 11.$

Here, dividend $= x^3 - 5x^2 + 7x + 9$, divisor $= x - 2$,

quotient $= x^2 - 3x + 1$ and remainder $= 11$.

In general, we have:

If $p(x)$ and $g(x)$ are any two polynomials with $g(x) \neq 0$, then there exists (unique) polynomials $q(x)$ and $r(x)$ such that

$$p(x) = g(x)\, q(x) + r(x)$$

where either $r(x) = 0$ or deg $r(x) <$ deg $g(x)$.

Dividend = Divisor × Quotient + Remainder.

Here, dividend $= p(x)$, divisor $= g(x)$,

quotient $= q(x)$ and remainder $= r(x)$.

In the above example, $p(x) = x^3 - 5x^2 + 7x + 9$, $g(x) = x - 2$,

$q(x) = x^2 - 3x + 1$ and remainder $= r(x) = 11$.

2.4 REMAINDER THEOREM

Let $p(x)$ be a polynomial of degree greater than or equal to 1 and α be a real number. If $p(x)$ is divided by $x - \alpha$, then the remainder is $p(\alpha)$.

Proof. By division algorithm, there exists unique polynomials $q(x)$ and $r(x)$ such that

$$p(x) = (x - \alpha)\, q(x) + r(x)$$

where either $r(x) = 0$ or deg $r(x) <$ deg $(x - \alpha)$.

But deg $(x - \alpha) = 1$, therefore, $r(x)$ is constant, say r.

Hence, for all value of α,

$$p(x) = (x - \alpha)\, q(x) + r \qquad \ldots(i)$$

Putting $x = \alpha$ in (i), we get

$$p(\alpha) = (\alpha - \alpha)\, q(\alpha) + r \Rightarrow p(\alpha) = 0 + r$$

$\therefore \quad p(\alpha) = r.$

Hence, remainder $= p(\alpha)$.

Corollary 1. If $p(x)$ is divided by $x + \alpha$, then remainder $= p(-\alpha)$.

Corollary 2. If $p(x)$ is divided by $ax - b$ where a and b are real numbers and $a \neq 0$, then remainder $= p\left(\frac{b}{a}\right)$.

Corollary 3. If $p(x)$ is divided by $ax + b$ where a and b are real numbers and $a \neq 0$, then remainder $= p\left(-\frac{b}{a}\right)$.

Illustrative Examples

Example 1. *Divide $p(x)$ by $g(x)$ where $p(x) = x + 3x^2 - 1$ and $g(x) = 1 + x$. Also verify the division algorithm.* *(NCERT)*

Solution. Writing $p(x)$ and $g(x)$ in descending order of their degrees *i.e.* in standard form, we get

$p(x) = 3x^2 + x - 1$ and $g(x) = x + 1$.

Let us divide $3x^2 + x - 1$ by $x + 1$, we have

$$\begin{array}{r|l} & 3x - 2 \\ \hline x + 1 & 3x^2 + x - 1 \\ & 3x^2 + 3x \\ & - \quad - \\ \hline & \quad -2x - 1 \\ & \quad -2x - 2 \\ & \quad + \quad + \\ \hline & \qquad\quad 1 \end{array}$$

$\therefore \quad 3x^2 + x - 1 = (x + 1)(3x - 2) + 1.$

Here, dividend $= p(x) = 3x^2 + x - 1$, divisor $= g(x) = x + 1$,

quotient $= q(x) = 3x - 2$ and remainder $= r(x) = 1$.

Now $g(x)\, q(x) + r(x) = (x + 1)(3x - 2) + 1$

$= 3x^2 + 3x - 2x - 2 + 1 = 3x^2 + x - 1 = p(x)$.

Hence, division algorithm is verified.

Example 2. *Find the remainder when the polynomial $x^4 + x^3 - 2x^2 + x + 1$ is divided by $x - 1$.* *(NCERT)*

Solution. Let $p(x) = x^4 + x^3 - 2x^2 + x + 1$.

When $p(x)$ is divided by $x - 1$, by Remainder theorem,

remainder $= p(1) = 1^4 + 1^3 - 2 \times 1^2 + 1 + 1$

$= 1 + 1 - 2 + 1 + 1 = 2$.

Example 3. *Find the remainder when the polynomial $4y^3 - 3y^2 - 5y + 1$ is divided by $2y + 3$.*

Solution. Let $p(y) = 4y^3 - 3y^2 - 5y + 1$.

When $p(y)$ is divided by $2y + 3$, by Remainder theorem,

$$\text{remainder} = p\left(-\frac{3}{2}\right) = 4\left(-\frac{3}{2}\right)^3 - 3\left(-\frac{3}{2}\right)^2 - 5\left(-\frac{3}{2}\right) + 1$$

$$= -\frac{27}{2} - \frac{27}{4} + \frac{15}{2} + 1 = \frac{-54 - 27 + 30 + 4}{4} = -\frac{47}{4}.$$

Example 4. *Check whether the polynomial $f(t) = 4t^3 + 4t^2 - t - 1$ is a multiple of $2t + 1$.* *(NCERT)*

Solution. We know that the polynomial $f(t)$ will be a multiple of $2t + 1$ if on dividing $f(t)$ by $2t + 1$, the remainder is zero.

On dividing $f(t)$ by $2t + 1$, by Remainder theorem,

$$\text{remainder} = f\left(-\frac{1}{2}\right) = 4\left(-\frac{1}{2}\right)^3 + 4\left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right) - 1$$

$$= -\frac{1}{2} + 1 + \frac{1}{2} - 1 = 0.$$

Therefore, the given polynomial is a multiple of $2t + 1$.

Example 5. *If the polynomials* $az^3 + 4z^2 + 3z - 4$ *and* $z^3 - 4z + a$ *leave the same remainder when divided by* $z - 3$, *find the value of* a. *(NCERT Exemplar)*

Solution. Let $p(z) = az^3 + 4z^2 + 3z - 4$ and $f(z) = z^3 - 4z + a$.

Given that when $p(z)$ and $f(z)$ are divided by $z - 3$ they leave the same remainder, by Remainder theorem,

$$p(3) = f(3) \Rightarrow a \times 3^3 + 4 \times 3^2 + 3 \times 3 - 4 = 3^3 - 4 \times 3 + a$$

$$\Rightarrow \quad 27a + 36 + 9 - 4 = 27 - 12 + a$$

$$\Rightarrow \quad 27a + 41 = 15 + a \Rightarrow 26a = -26$$

$$\Rightarrow \quad a = -1.$$

Hence, the value of a is -1.

Exercise 2.3

1. If $p(x) = x^3 + x^2 - 2x + 3$ and $g(x) = x + 3$, find the quotient and remainder on dividing $p(x)$ by $g(x)$. Also verify the division algorithm.
2. By actual division, find the quotient and remainder when $x^4 + 1$ is divided by $x - 1$. *(NCERT Exemplar)*
3. If $p(x) = x^4 + x^3 + x^2 - 5x + 1$ and $g(x) = x + 1$, find the remainder on dividing $p(x)$ by $g(x)$ (use actual division).
4. Use Remainder theorem to find the remainder when $f(x)$ is divided by $g(x)$ in the following:
   (*i*) $f(x) = x^2 - 5x + 7$, $g(x) = x + 3$
   (*ii*) $f(x) = x^3 + 1$, $g(x) = x + 1$ *(NCERT)*
   (*iii*) $f(x) = x^3 + 3x^2 + 3x + 1$, $g(x) = 5 + 2x$ *(NCERT)*
   (*iv*) $f(x) = x^3 - 6x^2 + 2x - 4$, $g(x) = 1 - \frac{3}{2}x$ *(NCERT Exemplar)*
   (*v*) $f(x) = x^4 - 3x^2 + 2x + 6$, $g(x) = x + 1$
5. Using Remainder theorem, check whether $7 + 3x$ is a factor of $3x^3 + 7x$. *(NCERT)*
6. Using Remainder theorem, check whether $3x^3 + 11x^2 + x - 15$ is a multiple of $x - 1$.
7. Using Remainder theorem, check whether $p(x)$ is a multiple of $g(x)$ or not:
   (*i*) $p(x) = x^3 - 5x^2 + 4x - 3$, $g(x) = x - 2$ *(NCERT Exemplar)*
   (*ii*) $p(x) = 2x^3 - 11x^2 - 4x + 5$, $g(x) = 2x + 1$ *(NCERT Exemplar)*
8. Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by $x - a$. *(NCERT)*
9. If the remainder on dividing the polynomial $2x^4 - kx^2 + 5x - 3k + 3$ by $(x + 2)$ is 4, then find the value of k.
10. If the polynomials $ax^3 + 3x^2 - 13$ and $5x^3 - 8x + a$ leave the same remainder when divided by $x + 1$, then find the value of a.
11. The polynomials $ax^3 - 3x^2 + 4$ and $2x^3 - 5x + a$ when divided by $(x - 2)$ leave the remainder p and q respectively. If $p - 2q = 4$, find the value of a.

Answers

2.5 FACTOR THEOREM

If $p(x)$ is a polynomial of degree ≥ 1 and α is a real number, then

(*i*) $x - \alpha$ is a factor of $p(x)$ if $p(\alpha) = 0$ and

(*ii*) conversely, $p(\alpha) = 0$ if $x - \alpha$ is a factor of $p(x)$.

Proof. When $p(x)$ is divided by $x - \alpha$, by Remainder theorem,

remainder $= p(\alpha)$.

(*i*) If $p(\alpha) = 0$, then remainder $= 0$.

Therefore, $x - \alpha$ is a factor of $p(x)$.

(*ii*) Conversely, if $x - \alpha$ is a factor of $p(x)$, then remainder = 0

$\Rightarrow$ $p(\alpha) = 0$.

Corollary. If $p(x)$ is a polynomial of degree ≥ 1 and α is a real number, then

(*i*) $x - \alpha$ is a factor of $p(x)$ if α is a zero of $p(x)$ and

(*ii*) conversely, α is a zero of $p(x)$ if $x - \alpha$ is a factor of $p(x)$.

Illustrative Examples

Example 1. *Show that $x + 2$ is a factor of $x^3 + 3x^2 + 5x + 6$.* *(NCERT)*

Solution. Let $p(x) = x^3 + 3x^2 + 5x + 6$.

By Factor theorem, $x + 2$ is a factor of $p(x)$ if $p(-2) = 0$.

Now, $p(-2) = (-2)^3 + 3(-2)^2 + 5(-2) + 6$

$= -8 + 12 - 10 + 6 = 0$.

Therefore, $x + 2$ is a factor of the given polynomial.

Example 2. *Check whether $g(x)$ is a factor of $p(x)$ or not, where $p(x) = 8x^3 - 6x^2 - 4x + 3$ and $g(x) = \frac{x}{3} - \frac{1}{4}$.* *(NCERT Exemplar)*

Solution. $g(x) = \frac{x}{3} - \frac{1}{4} = 0 \Rightarrow x = \frac{3}{4}$.

By Factor theorem, $g(x)$ will be a factor of $p(x)$ if $p\left(\frac{3}{4}\right) = 0$.

Now, $p\left(\frac{3}{4}\right) = 8\left(\frac{3}{4}\right)^3 - 6\left(\frac{3}{4}\right)^2 - 4\left(\frac{3}{4}\right) + 3$

$= 8 \times \frac{27}{64} - 6 \times \frac{9}{16} - 3 + 3 = \frac{27}{4} - \frac{27}{4} - 3 + 3 = 0$.

Therefore, $g(x)$ is a factor of $p(x)$.

Example 3. *If both $x - 2$ and $2x - 1$ are factors of $px^2 + 5x + r$, show that $p = r$.* *(NCERT Exemplar)*

Solution. Let $f(x) = px^2 + 5x + r$.

As both $x - 2$ and $2x - 1$ are factors of $f(x)$, by Factor theorem,

$f(2) = 0$ and $f\left(\frac{1}{2}\right) = 0$.

$\therefore$ $p \times 2^2 + 5 \times 2 + r = 0$

$\Rightarrow$ $4p + r + 10 = 0$ ...(*i*)

and $p \times \left(\frac{1}{2}\right)^2 + 5 \times \frac{1}{2} + r = 0 \Rightarrow \frac{p}{4} + \frac{5}{2} + r = 0$

$\Rightarrow$ $p + 4r + 10 = 0$ ...(*ii*)

Subtracting (*ii*) from (*i*), we get

$3p - 3r = 0 \Rightarrow 3p = 3r \Rightarrow p = r$.

Example 4. *Without actual division, prove that $2x^4 - 5x^3 + 2x^2 - x + 2$ is divisible by $x^2 - 3x + 2$.* *(NCERT Exemplar)*

Solution. Let $p(x) = 2x^4 - 5x^3 + 2x^2 - x + 2$ and $g(x) = x^2 - 3x + 2$, then

$g(x) = x^2 - 2x - x + 2 = x(x - 2) - 1(x - 2) = (x - 2)(x - 1)$.

Now, $p(2) = 2 \times 2^4 - 5 \times 2^3 + 2 \times 2^2 - 2 + 2$

$= 32 - 40 + 8 - 2 + 2 = 0$ and

$p(1) = 2 \times 1^4 - 5 \times 1^3 + 2 \times 1^2 - 1 + 2 = 2 - 5 + 2 - 1 + 2 = 0$

$\Rightarrow$ $(x - 2)$ and $(x - 1)$ are both factors of $p(x)$

$\Rightarrow$ $p(x)$ is divisible by $(x - 2)(x - 1)$

$\Rightarrow$ $p(x)$ is divisible by $g(x)$

$\Rightarrow$ $2x^4 - 5x^3 + 2x^2 - x + 2$ is divisible by $x^2 - 3x + 2$.

Example 5. *Find the value of k, if $x - 1$ is a factor of $4x^3 + 3x^2 - 4x + k$.* *(NCERT)*

Solution. Let $p(x) = 4x^3 + 3x^2 - 4x + k$.

Given $x - 1$ is a factor of $p(x)$, therefore, by Factor theorem,

$p(1) = 0 \Rightarrow 4 \times 1^3 + 3 \times 1^2 - 4 \times 1 + k = 0$

$\Rightarrow$ $4 + 3 - 4 + k = 0 \Rightarrow 3 + k = 0 \Rightarrow k = -3$.

Hence, the value of k is -3.

Example 6. *Find the value of m so that $2x - 1$ be a factor of $8x^4 + 4x^3 - 16x^2 + 10x + m$.*

Solution. Let $p(x) = 8x^4 + 4x^3 - 16x^2 + 10x + m$.

Now, $2x - 1$ will be a factor of $p(x)$, if $p\left(\frac{1}{2}\right) = 0$

$$\Rightarrow \quad 8 \times \left(\frac{1}{2}\right)^4 + 4 \times \left(\frac{1}{2}\right)^3 - 16 \times \left(\frac{1}{2}\right)^2 + 10 \times \frac{1}{2} + m = 0$$

$$\Rightarrow \quad 8 \times \frac{1}{16} + 4 \times \frac{1}{8} - 16 \times \frac{1}{4} + 5 + m = 0$$

$$\Rightarrow \quad \frac{1}{2} + \frac{1}{2} - 4 + 5 + m = 0 \Rightarrow 2 + m = 0 \Rightarrow m = -2.$$

Hence, the value of m is -2.

Example 7. *If $x - 1$ and $x + 2$ are factors of $x^3 + 10x^2 + ax + b$, then find the values of a and b.*

Solution. Let $f(x) = x^3 + 10x^2 + ax + b$.

Given $x - 1$ and $x + 2$ are factors of $f(x)$

$\Rightarrow$ $f(1) = 0$ and $f(-2) = 0$.

Now $f(1) = 0 \Rightarrow 1^3 + 10 \times 1^2 + a \times 1 + b = 0$

$\Rightarrow$ $1 + 10 + a + b = 0 \Rightarrow a + b + 11 = 0$ ...(*i*)

and $f(-2) = 0 \Rightarrow (-2)^3 + 10 \times (-2)^2 + a \times (-2) + b = 0$

$\Rightarrow$ $-8 + 40 - 2a + b = 0 \Rightarrow -2a + b + 32 = 0$

$\Rightarrow$ $2a - b - 32 = 0$ ...(*ii*)

On adding (*i*) and (*ii*), we get

$3a - 21 = 0 \Rightarrow 3a = 21 \Rightarrow a = 7$.

Substituting this value of a in (*i*), we get

$7 + b + 11 = 0 \Rightarrow b = -18$.

Hence, $a = 7$ and $b = -18$.

Remark

Examples and questions involving solution of linear equations in two variables may be taken up after learning Chapter 8 (on Linear Equations in Two Variables).

Example 8. *If the polynomial $2x^3 - ax^2 + bx + 4$ has $(x + 1)$ as a factor and leaves remainder 4 when divided by $(2x + 1)$, find the values of a and b.*

Solution. Let $f(x) = 2x^3 - ax^2 + bx + 4$.

Given that $f(x)$ has $(x + 1)$ as a factor and leaves remainder 4 when divided by $(2x + 1)$

$\Rightarrow$ $f(-1) = 0$ and $f\left(-\frac{1}{2}\right) = 4$.

Now $f(-1) = 0 \Rightarrow 2(-1)^3 - a(-1)^2 + b(-1) + 4 = 0$

$\Rightarrow \quad -2 - a - b + 4 = 0 \Rightarrow a + b - 2 = 0$ ...(*i*)

and $f\left(-\frac{1}{2}\right) = 4 \Rightarrow 2\left(-\frac{1}{2}\right)^3 - a\left(-\frac{1}{2}\right)^2 + b\left(-\frac{1}{2}\right) + 4 = 4$

$\Rightarrow \quad -\frac{1}{4} - \frac{a}{4} - \frac{b}{2} = 0 \Rightarrow a + 2b + 1 = 0$ ...(*ii*)

Subtracting (*i*) from (*ii*), we get $b + 3 = 0 \Rightarrow b = -3$.

Substituting this value of b in (*i*), we get $a - 3 - 2 = 0 \Rightarrow a = 5$.

Hence, $a = 5$ and $b = -3$.

Exercise 2.4

1. Show that $x - 3$ is a factor of the polynomial $x^3 + x^2 - 17x + 15$.
2. Without actual division, show that $2x^3 + 13x^2 + x - 70$ is exactly divisible by $x - 2$.
3. By using Factor theorem, check whether $x + 2$ is a factor of $x^3 + 3x^2 + 3x + 1$. *(NCERT)*
4. Use Factor theorem to determine whether $g(x)$ is a factor of $p(x)$ in each of the following:
   (*i*) $p(x) = 2x^3 + x^2 - 2x - 1$, $g(x) = x + 1$ *(NCERT)*
   (*ii*) $p(x) = x^3 - 4x^2 + x + 6$, $g(x) = x - 3$ *(NCERT)*
   (*iii*) $p(x) = x^3 - x + 1$, $g(x) = 2 - 3x$ *(NCERT Exemplar)*
   (*iv*) $p(x) = x + 2x^3 - 4x^2 + 12$, $g(x) = 2x - 3$ *(NCERT Exemplar)*
5. Find the value of k, if $x - 1$ is a factor of $p(x)$ in each of the following:
   (*i*) $p(x) = kx^2 - 3x + k$ *(NCERT)* (*ii*) $p(x) = kx^2 - \sqrt{2}x + 1$ *(NCERT)*
   (*iii*) $p(x) = 2x^2 + kx + \sqrt{2}$ *(NCERT)* (*iv*) $p(x) = kx^3 - 3x^2 + 5kx + 10$.
6. (*i*) If $x + 1$ is a factor of $ax^3 + x^2 - 2x + 4a - 9$, find the value of a. *(NCERT Exemplar)*
   (*ii*) If $x - a$ is a factor of $x^3 - ax^2 + 2x + a - 1$, find the value of a. *(NCERT Exemplar)*
   (*iii*) If $x + 2a$ is a factor of $x^5 - 4a^2x^3 + 2x + 2a + 3$, find the value of a. *(NCERT Exemplar)*
7. (*i*) For what value of m is $x^3 - 2mx^2 + 16$ divisible by $x + 2$?
   (*ii*) Find the value of a for which the polynomial $2y^4 + 3y^3 + 2ay^2 + 3y + 6$ is divisible by $y + 2$.
   (*iii*) For what value of m is the polynomial $2x^4 - mx^3 + 4x^2 + 2x + 1$ has $1 - 2x$ as one of its factors?
8. If $x - 1$ and $x + 1$ are factors of $ax^3 + x^2 - 2x + b$, then find the values of a and b.
9. If $p(x) = x^3 - ax^2 + bx + 3$ leaves a remainder -19 when divided by $(x + 2)$ and remainder 17 when divided by $(x - 2)$, prove that $a + b = 6$.
10. If $f(x) = x^4 - 2x^3 + 3x^2 - ax + b$ is divided by $(x - 1)$ and $(x + 1)$, it leaves the remainders 5 and 19 respectively. Find the values of a and b.

Answers

2.6 FACTORISATION

We know that the product of $3x + 7$ and $3x - 7 = (3x + 7)(3x - 7) = 9x^2 - 49$; we say that $3x + 7$ and $3x - 7$ are factors of $9x^2 - 49$. We write it as

$$9x^2 - 49 = (3x + 7)(3x - 7).$$

Similarly, the product of $2x + 1$ and $x - 3 = (2x + 1)(x - 3) = 2x^2 - 5x - 3$; we say that $2x + 1$ and $x - 3$ are factors of $2x^2 - 5x - 3$. We write it as

$$2x^2 - 5x - 3 = (2x + 1)(x - 3).$$

Thus, when an algebraic expression can be written as the product of two or more algebraic expressions, then each of these expression is called a **factor** of the given expression.

To find the factors of a given algebraic expression means to obtain two or more expressions whose product is the given expression. The process of finding two or more expressions whose product is the given expression is called **factorisation.**

Thus, factorisation is the reverse process of multiplication.

For example:

Product	Factors
(*i*) $(2x + 5)(2x - 5) = 4x^2 - 25$	$4x^2 - 25 = (2x + 5)(2x - 5)$
(*ii*) $(p + 3)(p - 7) = p^2 - 4p - 21$	$p^2 - 4p - 21 = (p + 3)(p - 7)$
(*iii*) $(2y + 3)(3y - 5) = 6y^2 - y - 15$	$6y^2 - y - 15 = (2y + 3)(3y - 5)$.

2.6.1 Factorisation of trinomials

In this section, we shall learn the factorisation of trinomials of the form $ax^2 + bx + c$, where a, b and c are real numbers.

Rule to factorise trinomial $ax^2 + bx + c$, where a, b and c are real numbers:

Split b (the coefficient of x) into two real numbers such that the algebraic sum of these two numbers is b and their product is ac.

We illustrate the method with the help of following examples:

Illustrative Examples

Example 1. *Factorise the following trinomials:*

(i) $x^2 + 9x + 18$ *(NCERT Exemplar)* *(ii)* $y^2 - 3y - 54$

Solution. (*i*) To factorise $x^2 + 9x + 18$, we want to find two real numbers whose sum is 9 and product is 18. By trial, we see that $3 + 6 = 9$ and $3 \times 6 = 18$.

$$\therefore \quad x^2 + 9x + 18 = x^2 + 3x + 6x + 18$$
$$= x(x + 3) + 6(x + 3)$$
$$= (x + 3)(x + 6).$$

(*ii*) To factorise $y^2 - 3y - 54$, we want to find two real numbers whose sum is -3 and product is -54. By trial, we see that $(-9) + 6 = -3$ and $(-9) \times 6 = -54$.

$$\therefore \quad y^2 - 3y - 54 = y^2 - 9y + 6y - 54$$
$$= y(y - 9) + 6(y - 9)$$
$$= (y - 9)(y + 6).$$

Example 2. *Factorise the following trinomials:*

(i) $6x^2 + 17x + 5$ *(NCERT)* *(ii)* $12x^2 - 7x + 1$ *(NCERT)*

(iii) $2x^2 - 7x - 15$ *(NCERT Exemplar)* *(iv)* $84 - 2r - 2r^2$ *(NCERT Exemplar)*

Solution. (*i*) To factorise $6x^2 + 17x + 5$, we want to find two real numbers whose sum is 17 and product is 6×5 *i.e.* 30. By trial, we see that $2 + 15 = 17$ and $2 \times 15 = 30$.

$$\therefore \quad 6x^2 + 17x + 5 = 6x^2 + 2x + 15x + 5$$
$$= 2x(3x + 1) + 5(3x + 1)$$
$$= (3x + 1)(2x + 5).$$

(*ii*) To factorise $12x^2 - 7x + 1$, we want to find two real numbers whose sum is -7 and product is 12×1 *i.e.* 12. By trial, we see that $(-3) + (-4) = -7$ and $(-3) \times (-4) = 12$.

$$\therefore \quad 12x^2 - 7x + 1 = 12x^2 - 3x - 4x + 1$$
$$= 3x(4x - 1) - 1(4x - 1)$$
$$= (4x - 1)(3x - 1).$$

(*iii*) To factorise $2x^2 - 7x - 15$, we want to find two real numbers whose sum is -7 and product is $2 \times (-15)$ *i.e.* -30. By trial, we see that $(-10) + 3 = -7$ and $(-10) \times 3 = -30$.

$$\therefore \quad 2x^2 - 7x - 15 = 2x^2 - 10x + 3x - 15$$
$$= 2x(x - 5) + 3(x - 5)$$
$$= (x - 5)\,(2x + 3).$$

(*iv*) We note that $84 - 2r - 2r^2 = 2(42 - r - r^2)$.

To factorise $42 - r - r^2$, we want to find two real numbers whose sum $= -1$ and product is $42 \times (-1)$ *i.e.* -42. By trial, we see that $(-7) + 6 = -1$ and $(-7) \times 6 = -42$.

$$\therefore \quad 84 - 2r - 2r^2 = 2(42 - r - r^2)$$
$$= 2(42 - 7r + 6r - r^2)$$
$$= 2[7(6 - r) + r(6 - r)]$$
$$= 2(6 - r)\,(7 + r).$$

Example 3. *Factorise:* $7\sqrt{2}\,x^2 - 10x - 4\sqrt{2}$

Solution. To factorise $7\sqrt{2}\,x^2 - 10x - 4\sqrt{2}$, we want to find two real numbers whose sum is -10 and product is $(7\sqrt{2}) \times (-4\sqrt{2})$ *i.e.* -56. By trial, we see that $(-14) + 4 = -10$ and $(-14) \times 4 = -56$.

$$\therefore \quad 7\sqrt{2}\,x^2 - 10x - 4\sqrt{2} = 7\sqrt{2}\,x^2 - 14x + 4x - 4\sqrt{2}$$
$$= 7\sqrt{2}\,x(x - \sqrt{2}) + 4(x - \sqrt{2})$$
$$= (x - \sqrt{2})\,(7\sqrt{2}\,x + 4).$$

Example 4. *Factorise* $y^2 - 5y + 6$ *by using factor theorem.* *(NCERT)*

Solution. Let $p(y) = y^2 - 5y + 6$.

For the factors of $p(y)$, we look for the factors of 6.

The factors of 6 are $\pm 1, \pm 2, \pm 3, \pm 6$.

Here, $p(2) = 2^2 - 5 \times 2 + 6 = 4 - 10 + 6 = 0$.

$\Rightarrow$ $y - 2$ is a factor of $p(y)$.

Also, $p(3) = 3^2 - 5 \times 3 + 6 = 9 - 15 + 6 = 0$

$\Rightarrow$ $y - 3$ is a factor of $p(y)$.

$\therefore$ $y^2 - 5y + 6 = (y - 2)\,(y - 3)$.

Example 5. *If* $x^2 - 3x + 2$ *is a factor of* $x^4 - ax^2 + b$, *then find the values of* a *and* b.

Solution. Let $f(x) = x^4 - ax^2 + b$.

$$x^2 - 3x + 2 = x^2 - 2x - x + 2$$
$$= x(x - 2) - 1(x - 2)$$
$$= (x - 2)\,(x - 1).$$

Given $x^2 - 3x + 2$ is a factor of $f(x)$

$\Rightarrow$ $(x - 2)\,(x - 1)$ is a factor of $f(x)$

$\Rightarrow$ $(x - 2)$ and $(x - 1)$ are both factors of $f(x)$

$\Rightarrow$ $f(2) = 0$ and $f(1) = 0$.

Now $f(2) = 0 \Rightarrow 2^4 - a \times 2^2 + b = 0$

$\Rightarrow$ $16 - 4a + b = 0 \Rightarrow 4a - b - 16 = 0$ ...(*i*)

and $f(1) = 0 \Rightarrow 1^4 - a \times 1^2 + b = 0$

$\Rightarrow$ $1 - a + b = 0 \Rightarrow a - b - 1 = 0$ ...(*ii*)

On subtracting (*ii*) from (*i*), we get

$3a - 15 = 0 \Rightarrow 3a = 15 \Rightarrow a = 5$.

Substituting this value of a in (*ii*), we get

$5 - b - 1 = 0 \Rightarrow 4 - b = 0 \Rightarrow b = 4$.

Hence, $a = 5$ and $b = 4$.

Exercise 2.5

Factorise the following trinomials (1 to 6):

1. (*i*) $x^2 + 22x + 85$ (*ii*) $y^2 - 23y + 42$.

2. (*i*) $m^2 - 7m - 30$ (*ii*) $t^2 + 23t - 108$.

3. (*i*) $2x^2 + 7x + 3$ *(NCERT)* (*ii*) $6x^2 + 5x - 6$. *(NCERT)*

4. (*i*) $3x^2 - x - 4$ *(NCERT)* (*ii*) $6x^2 + 7x - 3$. *(NCERT Exemplar)*

5. (*i*) $1 - 18y - 63y^2$ (*ii*) $x(12x + 7) - 10$.

Hint:

(*ii*) $x(12x + 7) - 10 = 12x^2 + 7x - 10$.

6. (*i*) $x^2 - (1 + \sqrt{2})\, x + \sqrt{2}$ (*ii*) $\sqrt{2}\, x^2 + 7x + 5\sqrt{2}$.

7. Factorise the following trinomials by using factor theorem:

(*i*) $x^2 + 4x - 5$ (*ii*) $y^2 - 2y - 15$.

Answers

2.6.2 Factorisation of cubic polynomials

Rules to factorise a cubic polynomial:

(*i*) Find one (linear) factor of the given cubic polynomial by using Factor theorem.

(*ii*) Divide the given polynomial by the factor obtained in step (*i*) and find the quotient, which will be a trinomial.

(*iii*) Factorise the trinomial.

We illustrate the method with the help of following examples:

Illustrative Examples

Example 1. *Factorise the cubic* $x^3 - 2x^2 - x + 2$. *(NCERT)*

Solution. Let $p(x) = x^3 - 2x^2 - x + 2$.

For the factors of $p(x)$, we look for the factors of 2.

The factors of 2 are 1, –1, 2 and –2.

Here, $p(1) = 1^3 - 2.1^2 - 1 + 2 = 1 - 2 - 1 + 2 = 0$

$\Rightarrow$ $x - 1$ is a factor of $p(x)$.

Dividing $p(x)$ by $x - 1$ by long division method, we have

$$
\begin{array}{r|l}
 & x^2 - x - 2 \\
\hline
x - 1 & x^3 - 2x^2 - x + 2 \\
 & x^3 - x^2 \\
 & \quad - \quad + \\
\hline
 & \quad -x^2 - x + 2 \\
 & \quad -x^2 + x \\
 & \quad + \quad - \\
\hline
 & \quad\quad -2x + 2 \\
 & \quad\quad -2x + 2 \\
 & \quad\quad + \quad - \\
\hline
 & \quad\quad\quad 0
\end{array}
$$

$\therefore$ $x^3 - 2x^2 - x + 2 = (x - 1)(x^2 - x - 2)$.

Now $x^2 - x - 2 = x^2 + x - 2x - 2$

$= x(x + 1) - 2(x + 1) = (x + 1)(x - 2)$.

Hence, $x^3 - 2x^2 - x + 2 = (x - 1)(x + 1)(x - 2)$.

Example 2. *Factorise the cubic* $2x^3 - x^2 - 13x - 6$.

Solution. Let $p(x) = 2x^3 - x^2 - 13x - 6$.

For the factors of $p(x)$, we look for the factors of -6.

The factors of -6 are $\pm 1, \pm 2, \pm 3$ and ± 6.

Here, $p(-2) = 2 \times (-2)^3 - (-2)^2 - 13 \times (-2) - 6$

$= -16 - 4 + 26 - 6 = 0$

$\Rightarrow$ $x + 2$ is a factor of $p(x)$.

Dividing $p(x)$ by $x + 2$ by long division method, we have

$$
\begin{array}{r|l}
 & 2x^2 - 5x - 3 \\
\hline
x + 2 & 2x^3 - x^2 - 13x - 6 \\
 & 2x^3 + 4x^2 \\
 & \quad - \quad - \\
\hline
 & \quad - 5x^2 - 13x - 6 \\
 & \quad - 5x^2 - 10x \\
 & \quad + \quad + \\
\hline
 & \quad\quad\quad -3x - 6 \\
 & \quad\quad\quad -3x - 6 \\
 & \quad\quad\quad + \quad + \\
\hline
 & \quad\quad\quad\quad 0
\end{array}
$$

$\therefore$ $2x^3 - x^2 - 13x - 6 = (x + 2)(2x^2 - 5x - 3)$.

Now, $2x^2 - 5x - 3 = 2x^2 - 6x + x - 3$

$= 2x(x - 3) + 1(x - 3)$

$= (x - 3)(2x + 1)$

Hence, $2x^3 - x^2 - 13x - 6 = (x + 2)(x - 3)(2x + 1)$.

Exercise 2.6

1. By Factor theorem, show that $x - 1$ is a factor of $x^3 - 23x^2 + 142x - 120$. Hence, factorise the given polynomial completely. *(NCERT)*

Factorise the following cubic polynomials:

2. (*i*) $x^3 - 3x^2 - 9x - 5$ *(NCERT)* (*ii*) $x^3 + x^2 - 4x - 4$. *(NCERT Exemplar)*

3. (*i*) $x^3 - 6x^2 + 11x - 6$ *(NCERT Exemplar)* (*ii*) $x^3 + 13x^2 + 32x + 20$. *(NCERT)*

4. (*i*) $2y^3 + y^2 - 2y - 1$ *(NCERT)* (*ii*) $3x^3 - x^2 - 3x + 1$. *(NCERT Exemplar)*

Answers

2.7 ALGEBRAIC IDENTITIES

An algebraic equation which is true for all real values of the variables occurring in the equation is called an **algebraic identity.**

You have already studied the following algebraic identities in the earlier classes:

1. $(x + y)^2 = x^2 + 2xy + y^2$

2. $(x - y)^2 = x^2 - 2xy + y^2$

3. $x^2 - y^2 = (x + y)(x - y)$

4. $(x + a)(x + b) = x^2 + (a + b)x + ab$

Some more algebraic identities are:

5. $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

6. $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

$= x^3 + y^3 + 3x^2y + 3xy^2$

7. $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$= x^3 - y^3 - 3x^2y + 3xy^2$

8. $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

9. $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

10. $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$$= \frac{1}{2}(x + y + z)((x - y)^2 + (y - z)^2 + (z - x)^2)$$

11. If $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$.

The students are advised to memorize the above results (without resorting to actual multiplication) until they can recognise each, both the product from the factors and the factors from the product.

Illustrative Examples

Example 1. *Using appropriate identities, find the following products:*

(i) $(2x + 3)(2x + 3)$ *(ii)* $(x + 8)(x - 10)$ *(NCERT)*

(iii) $(3x + 4)(3x - 5)$ *(NCERT)* *(iv)* $(3 - 2x)(3 + 2x)$. *(NCERT)*

Solution. *(i)* Using identity 1 *i.e.* $(x + y)^2 = x^2 + 2xy + y^2$, we have

$$(2x + 3)(2x + 3) = (2x + 3)^2 = (2x)^2 + 2(2x)(3) + (3)^2$$
$$= 4x^2 + 12x + 9.$$

(ii) Using identity 4 *i.e.* $(x + a)(x + b) = x^2 + (a + b)x + ab$, we have

$$(x + 8)(x - 10) = x^2 + (8 + (-10))x + 8(-10)$$
$$= x^2 - 2x - 80.$$

(iii) Using identity 4 *i.e.* $(x + a)(x + b) = x^2 + (a + b)x + ab$, we have

$$(3x + 4)(3x - 5) = (3x)^2 + (4 + (-5))(3x) + 4(-5)$$
$$= 9x^2 - 3x - 20.$$

(iv) Using identity 3 *i.e.* $(x + y)(x - y) = x^2 - y^2$, we have

$$(3 - 2x)(3 + 2x) = (3 + 2x)(3 - 2x) = (3)^2 - (2x)^2$$
$$= 9 - 4x^2.$$

Example 2. *Using proper identities, factorise the following:*

(i) $49a^2 + 70ab + 25b^2$ *(NCERT)* *(ii)* $9y^2 - 66yz + 121z^2$ *(NCERT Exemplar)*

(iii) $\frac{25}{4}x^2 - \frac{y^2}{9}$ *(NCERT)* *(iv)* $9a^2 + 6a + 1 - 25b^2$

Solution. *(i)* $49a^2 + 70ab + 25b^2 = (7a)^2 + 2 \times 7a \times 5b + (5b)^2$

$$= (7a + 5b)^2 \quad \text{(using identity 1)}$$
$$= (7a + 5b)(7a + 5b).$$

(ii) $9y^2 - 66yz + 121z^2 = (3y)^2 - 2 \times 3y \times 11z + (11z)^2$

$$= (3y - 11z)^2 \quad \text{(using identity 2)}$$
$$= (3y - 11z)(3y - 11z).$$

(iii) $\frac{25}{4}x^2 - \frac{y^2}{9} = \left(\frac{5}{2}x\right)^2 - \left(\frac{y}{3}\right)^2$

$$= \left(\frac{5}{2}x + \frac{y}{3}\right)\left(\frac{5}{2}x - \frac{y}{3}\right) \quad \text{(using identity 3)}$$

(iv) $9a^2 + 6a + 1 - 25b^2 = ((3a)^2 + 2 \times 3a \times 1 + (1)^2) - (5b)^2$

$$= (3a + 1)^2 - (5b)^2 \quad \text{(using identity 1)}$$
$$= (3a + 1 + 5b)(3a + 1 - 5b) \quad \text{(using identity 3)}$$
$$= (3a + 5b + 1)(3a - 5b + 1).$$

Example 3. *Expand the following:*

(i) $\left(2p - \frac{q}{5} - 3\right)\left(2p + \frac{q}{5} + 3\right)$ *(ii)* $(3x - 2y)\,(3x + 2y)\,(9x^2 + 4y^2)$.

Solution. *(i)* Given expression $= \left(2p - \frac{q}{5} - 3\right)\left(2p + \frac{q}{5} + 3\right)$

$= \left\{2p - \left(\frac{q}{5} + 3\right)\right\}\left\{2p + \left(\frac{q}{5} + 3\right)\right\}$

$= (2p)^2 - \left(\frac{q}{5} + 3\right)^2$ (using identity 3)

$= 4p^2 - \left\{\left(\frac{q}{5}\right)^2 + 2\left(\frac{q}{5}\right)(3) + (3)^2\right\}$ (using identity 1)

$= 4p^2 - \frac{q^2}{25} - \frac{6q}{5} - 9.$

(ii) Given expression $= (3x - 2y)\,(3x + 2y)(9x^2 + 4y^2)$

$= ((3x - 2y)\,(3x + 2y))\,(9x^2 + 4y^2)$

$= ((3x)^2 - (2y)^2)\,(9x^2 + 4y^2)$

$= (9x^2 - 4y^2)\,(9x^2 + 4y^2)$

$= (9x^2)^2 - (4y^2)^2$

$= 81x^4 - 16y^4.$

Example 4. *Expand the following:*

(i) $(4a - 2b - 3c)^2$ *(NCERT)* *(ii)* $\left(\frac{1}{4}a - \frac{1}{2}b + 1\right)^2$ *(NCERT)*

Solution. Using identity 5, we have

(i) $(4a - 2b - 3c)^2 = (4a + (-2b) + (-3c))^2$

$= (4a)^2 - (-2b)^2 + (-3c)^2 + 2(4a)\,(-2b) + 2(-2b)\,(-3c) + 2(-3c)\,(4a)$

$= 16a^2 + 4b^2 + 9c^2 - 16ab + 12bc - 24ca.$

(ii) $\left(\frac{1}{4}a - \frac{1}{2}b + 1\right)^2 = \left(\frac{1}{4}a + \left(-\frac{1}{2}b\right) + 1\right)^2$

$= \left(\frac{1}{4}a\right)^2 + \left(-\frac{1}{2}b\right)^2 + 1^2 + 2\left(\frac{1}{4}a\right)\left(-\frac{1}{2}b\right) + 2\left(-\frac{1}{2}b\right)(1) + 2(1)\left(\frac{1}{4}a\right)$

$= \frac{1}{16}a^2 + \frac{1}{4}b^2 + 1 - \frac{1}{4}ab - b + \frac{1}{2}a$

Example 5. *By using proper identity, factorise the following:*

(i) $4x^2 + y^2 + z^2 - 4xy - 2yz + 4xz$ *(NCERT)*

(ii) $2x^2 + y^2 + 8z^2 + 2\sqrt{2}\,xy - 4\sqrt{2}\,yz - 8xz.$ *(NCERT)*

Solution. *(i)* $4x^2 + y^2 + z^2 - 4xy - 2yz + 4xz$

$= (2x)^2 + (-y)^2 + z^2 + 2(2x)\,(-y) + 2(-y)z + 2(2x)z$

$= (2x + (-y) + z)^2$ (using identity 5)

$= (2x - y + z)\,(2x - y + z).$

(ii) $2x^2 + y^2 + 8z^2 + 2\sqrt{2}\,xy - 4\sqrt{2}\,yz - 8xz$

$= (\sqrt{2}\,x)^2 + (y)^2 + (-2\sqrt{2}\,z)^2 + 2(\sqrt{2}\,x)y + 2y(-2\sqrt{2}\,z) + 2(\sqrt{2}\,x)\,(-2\sqrt{2}\,z)$

$= (\sqrt{2}\,x + y + (-2\sqrt{2}\,z))^2$ (using identity 5)

$= (\sqrt{2}\,x + y - 2\sqrt{2}\,z)\,(\sqrt{2}\,x + y - 2\sqrt{2}\,z).$

Example 6. *Write the following cubes in the expanded form:*

(i) $(3a + 4b)^3$ *(NCERT)* *(ii)* $\left(x - \frac{2}{3}y\right)^3$ *(NCERT)* *(iii)* $(x + y - 1)^3$.

Solution. *(i)* $(3a + 4b)^3 = (3a)^4 + (4b)^3 + 3(3a)\,(4b)\,(3a + 4b)$ (using identity 6)

$= 27a^3 + 64b^3 + 36ab\,(3a + 4b)$

$= 27a^3 + 64b^3 + 108a^2b + 144ab^2$.

(ii) $\left(x - \frac{2}{3}y\right)^3 = x^3 - \left(\frac{2}{3}y\right)^3 - 3(x)\left(\frac{2}{3}y\right)\left(x - \frac{2}{3}y\right)$ (using identity 7)

$= x^3 - \frac{8}{27}y^3 - 2xy\left(x - \frac{2}{3}y\right)$

$= x^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4}{3}xy^2$

(iii) $(x + y - 1)^3 = (\overline{x + y} - 1)^3$ [consider $x + y$ as one term]

$= (x + y)^3 - (1)^3 - 3(x + y)\,(1)\,(\overline{x + y} - 1)$ (using identity 7)

$= (x + y)^3 - 1 - 3(x + y)\,(\overline{x + y} - 1)$

$= [x^3 + y^3 + 3xy\,(x + y)] - 1 - 3\,(x + y)^2 + 3(x + y)$

$= x^3 + y^3 + 3x^2y + 3xy^2 - 1 - 3\,(x^2 + 2xy + y^2) + 3x + 3y$

$= x^3 + y^3 + 3x^2y + 3xy^2 - 3x^2 - 6xy - 3y^2 + 3x + 3y - 1$.

Example 7. *By using proper identity, factorise the following:*

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$ *(NCERT)* *(ii)* $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$ *(NCERT)*

Solution. *(i)* $8a^3 + b^3 + 12a^2b + 6ab^2 = (2a)^3 + b^3 + 3(2a)^2b + 3(2a)\,(b)^2$

$= (2a + b)^3$ (using identity 6)

$= (2a + b)\,(2a + b)\,(2a + b)$.

(ii) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p = (3p)^3 - \left(\frac{1}{6}\right)^3 - 3 \times (3p)^2 \times \frac{1}{6} + 3 \times (3p) \times \left(\frac{1}{6}\right)^2$

$= \left(3p - \frac{1}{6}\right)^3$ (using identity 7)

$= \left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)$.

Example 8. *Using appropriate identity, find the following products:*

(i) $(3x + 5y)\,(9x^2 - 15xy + 25y^2)$ *(ii)* $(x^2 - 1)\,(x^4 + x^2 + 1)$ *(NCERT Exemplar)*

Solution. *(i)* We know that $(a + b)\,(a^2 - ab + b^2) = a^3 + b^3$. (Identity 8)

$\therefore$ Given expression $= (3x + 5y)\,[(3x)^2 - (3x)\,(5y) + (5y)^2]$

$= (3x)^3 + (5y)^3$

$= 27x^3 + 125y^3$.

(ii) We know that $(a - b)\,(a^2 + ab + b^2) = a^3 - b^3$. (Identity 9)

$\therefore$ Given expression $= (x^2 - 1)\,((x^2)^2 + (x^2)\,(1) + (1)^2)$

$= (x^2)^3 - (1)^3 = x^6 - 1$.

Example 9. *Using proper identity, factorise the following:*

(i) $27y^3 + 125z^3$ *(NCERT)* *(ii)* $a^3 - 2\sqrt{2}\,b^3$ *(NCERT Exemplar)*

Solution. *(i)* $27y^3 + 125z^3 = (3y)^3 + (5z)^3$

$= (3y + 5z)\,((3y)^2 - (3y)\,(5z) + (5z)^2)$ (using identity 8)

$= (3y + 5z)\,(9y^2 - 15yz + 25z^2)$.

(*ii*) $a^3 - 2\sqrt{2}\,b^3 = (a)^3 - (\sqrt{2}\,b)^3$

$= (a - \sqrt{2}\,b)\,(a^2 + a(\sqrt{2}\,b) + (\sqrt{2}\,b)^2)$ (using identity 9)

$= (a - \sqrt{2}\,b)\,(a^2 + \sqrt{2}\,ab + 2b^2)$

Example 10. *Using proper identity, find the following products:*

(i) $(x + 3y + 5z)\,(x^2 + 9y^2 + 25z^2 - 3xy - 15yz - 5zx)$

(ii) $(-z + x - 2y)\,(x^2 + 4y^2 + z^2 + 2xy - 2yz + zx)$ *(NCERT Exemplar)*

Solution. We know that

$(a + b + c)\,(a^2 + b^2 + c^2 - ab - bc - ca) = a^3 + b^3 + c^3 - 3abc.$ (Identity 10)

(*i*) Given expression $= (x + 3y + 5z)[(x)^2 + (3y)^2 + (5z)^2 - (x)\,(3y) - (3y)(5z) - (5z)\,(x)]$

$= (x)^3 + (3y)^3 + (5z)^3 - 3(x)\,(3y)\,(5z)$

$= x^3 + 27y^3 + 125z^3 - 45xyz.$

(*ii*) Given expression $= (x - 2y - z)\,(x^2 + 4y^2 + z^2 + 2xy - 2yz + zx)$

$= (x + (-2y) + (-z))\,(x^2 + (-2y)^2 + (-z)^2 - x(-2y) - (-2y)\,(-z) - (-z)x)$

$= (x)^3 + (-2y)^3 + (-z)^3 - 3(x)\,(-2y)\,(-z)$

$= x^3 - 8y^3 - z^3 - 6xyz.$

Example 11. *Using proper identity, factorise the following:*

(i) $8x^3 + y^3 + 27z^3 - 18xyz$ *(NCERT)* *(ii)* $a^3 - b^3 + 1 + 3ab$

(iii) $2\sqrt{2}\,a^3 + 8b^3 - 27c^3 + 18\sqrt{2}\,abc.$ *(NCERT Exemplar)*

Solution. (*i*) $8x^3 + y^3 + 27z^3 - 18xyz = (2x)^3 + y^3 + (3z)^3 - 3(2x)y\,(3z)$

$= (2x + y + 3z)\,((2x)^2 + y^2 + (3z)^2 - (2x)y - y(3z) - (3z)\,(2x))$

(using identity 10)

$= (2x + y + 3z)\,(4x^2 + y^2 + 9z^2 - 2xy - 3yz - 6zx).$

(*ii*) $a^3 - b^3 + 1 + 3ab = (a)^3 + (-b)^3 + (1)^3 - 3(a)\,(-b)\,(1)$

$= (a + (-b) + 1)\,(a^2 + (-b)^2 + 1^2 - a(-b) - (-b)\,(1) - (a)\,(1))$

(using identity 10)

$= (a - b + 1)\,(a^2 + b^2 + 1 + ab + b - a).$

(*iii*) $2\sqrt{2}\,a^3 + 8b^3 - 27c^3 + 18\sqrt{2}\,abc$

$= (\sqrt{2}\,a)^3 + (2b)^3 + (-3c)^3 - 3(\sqrt{2}\,a)\,(2b)\,(-3c)$

$= (\sqrt{2}\,a + 2b + (-3c))\,((\sqrt{2}\,a)^2 + (2b)^2 + (-3c)^2 - (\sqrt{2}\,a)\,(2b) - (2b)\,(-3c)$

$- (-3c)\,(\sqrt{2}\,a))$ (using identity 10)

$= (\sqrt{2}\,a + 2b - 3c)\,(2a^2 + 4b^2 + 9c^2 - 2\sqrt{2}\,ab + 6bc + 3\sqrt{2}\,ca).$

Example 12. *If* $x + y + z = 0$, *then prove that* $x^3 + y^3 + z^3 = 3xyz.$ *(NCERT)*

Solution. Given $x + y + z = 0 \Rightarrow x + y = -z$ ...(*i*)

$\Rightarrow$ $(x + y)^3 = (-z)^3$ (cubing both sides of (*i*))

$\Rightarrow$ $x^3 + y^3 + 3xy\,(x + y) = -z^3$ (using identity 6)

$\Rightarrow$ $x^3 + y^3 + 3xy\,(-z) = -z^3$ (using (*i*))

$\Rightarrow$ $x^3 + y^3 + z^3 = 3xyz$, as required.

Example 13. *Without finding cubes, factorise* $(x - y)^3 + (y - z)^3 + (z - x)^3$.

(NCERT Exemplar)

Solution. We know that if $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$ (Identity 11)

Here, $(x - y) + (y - z) + (z - x) = 0.$

Therefore, by using the above identity, we have

$(x - y)^3 + (y - z)^3 + (z - x)^3 = 3(x - y)\,(y - z)\,(z - x).$

Exercise 2.7

1. Using appropriate identities, find the following products:

(*i*) $(x + 3)(x + 3)$ *(NCERT)* (*ii*) $(x - 3)(x + 5)$ *(NCERT)*

(*iii*) $\left(\frac{2}{3}y - 1\right)\left(\frac{2}{3}y - 1\right)$ (*iv*) $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$ *(NCERT)*

2. Using proper identities, factorise the following:

(*i*) $4x^2 + 20x + 25$ *(NCERT Exemplar)* (*ii*) $4y^2 - 4y + 1$ *(NCERT)*

(*iii*) $x^2 - \frac{y^2}{100}$ *(NCERT)* (*iv*) $\left(2x + \frac{1}{3}\right)^2 - \left(x - \frac{1}{2}\right)^2$ *(NCERT Exemplar)*

3. Using proper identities, find the following products:

(*i*) $(2x - y + 3)(2x - y - 3)$ (*ii*) $(3x + y - 5)(3x - y - 5)$

Using proper identities, expand the following (4 to 7):

4. (*i*) $(3a + 4b + 5c)^2$ *(NCERT)* (*ii*) $(3a - 5b - c)^2$ *(NCERT Exemplar)*

(*iii*) $(-2x + 3y + 2z)^2$ *(NCERT)* (*iv*) $(-2x + 5y - 3z)^2$ *(NCERT)*

5. (*i*) $(2x + 1)^3$ *(NCERT)* (*ii*) $(5p - 3q)^3$ *(NCERT)*

(*iii*) $\left(\frac{3}{2}x + 1\right)^3$ *(NCERT)* (*iv*) $\left(4 - \frac{1}{3x}\right)^3$, $x \neq 0$ *(NCERT Exemplar)*

6. (*i*) $(2p + 3q)(4p^2 - 6pq + 9q^2)$ (*ii*) $(3p - 4q)(9p^2 + 12pq + 16q^2)$

(*iii*) $\left(\frac{x}{2} + 2y\right)\left(\frac{x^2}{4} - xy + 4y^2\right)$ *(NCERT Exemplar)*

7. (*i*) $(2x + 3y + 4z)(4x^2 + 9y^2 + 16z^2 - 6xy - 12yz - 8zx)$

(*ii*) $(2x - y + 3z)(4x^2 + y^2 + 9z^2 + 2xy + 3yz - 6zx)$ *(NCERT Exemplar)*

Using proper identities, factorise the following (8 to 12):

8. (*i*) $a^2 - 1 - 2b - b^2$ (*ii*) $x^2 - y^2 + 6y - 9$

9. (*i*) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16zx$ *(NCERT)*

(*ii*) $25x^2 + 16y^2 + 4z^2 - 40xy + 16yz - 20zx$ *(NCERT Exemplar)*

(*iii*) $x^2 + y^2 + 2xy + 8x + 8y + 16$

10. (*i*) $8x^3 + 27y^3 + 36x^2y + 54xy^2$ *(NCERT)* (*ii*) $64a^3 - 27b^3 - 144a^2b + 108ab^2$ *(NCERT)*

(*iii*) $27 - 125a^3 - 135a + 225a^2$ *(NCERT)* (*iv*) $1 - 64a^3 - 12a + 48a^2$ *(NCERT Exemplar)*

11. (*i*) $64m^3 - 343n^3$ *(NCERT)* (*ii*) $1 + 64x^3$ *(NCERT Exemplar)*

12. (*i*) $27x^3 + y^3 + z^3 - 9xyz$ *(NCERT)* (*ii*) $a^3 - 8b^3 - 64c^3 - 24abc$ *(NCERT Exemplar)*

13. Without finding cubes, factorise

$(x - 2y)^3 + (2y - 3z)^3 + (3z - x)^3$. *(NCERT Exemplar)*

14. If $a + b + c = 0$, prove that $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = 3$. *(NCERT Exemplar)*

15. Give possible expressions for length and breadth of the rectangles whose areas are Given.

(*i*) Area $= 25x^2 - 35x + 12$ (*ii*) Area $= 35x^2 + 13x - 12$ *(NCERT)*

16. What are the possible expressions for the dimensions of the cuboids whose volumes are Given.

(*i*) Volume $= 3x^2 - 12x$ (*ii*) Volume $= 12ky^2 + 8ky - 20k$. *(NCERT)*

Answers

2.8 SOME APPLICATIONS OF ALGEBRAIC IDENTITIES

We shall illustrate the use of algebraic identities with the help of following examples:

Example 1. *Using suitable identities, evaluate the following:*

(i) 103 × 107 (NCERT) *(ii) 104 × 96 (NCERT)* *(iii) $(999)^2$ (NCERT Exemplar)*

Solution. (*i*) $103 \times 107 = (100 + 3) \times (100 + 7)$

$= (100)^2 + (3 + 7)(100) + 3 \times 7$ (using identity 4)

$= 10000 + 1000 + 21 = 11021.$

(*ii*) $104 \times 96 = (100 + 4)(100 - 4)$

$= (100)^2 - (4)^2$ (using identity 3)

$= 10000 - 16 = 9984.$

(*iii*) $(999)^2 = (1000 - 1)^2 = (1000)^2 - 2(1000)(1) + (1)^2$ (using identity 2)

$= 1000000 - 2000 + 1 = 998001.$

Example 2. *Using suitable identities, evaluate the following:*

(i) $(103)^3$ (NCERT Exemplar) *(ii) $(99)^3$ (NCERT)*

Solution. (*i*) $(103)^3 = (100 + 3)^3$

$= (100)^3 + (3)^3 + 3(100)(3)(100 + 3)$ (using identity 6)

$= 1000000 + 27 + 900 \times 103$

$= 1000000 + 27 + 92700 = 1092727.$

(*ii*) $(99)^3 = (100 - 1)^3$

$= (100)^3 - (1)^3 - 3(100)(1)(100 - 1)$

$= 1000000 - 1 - 300 \times 99$

$= 1000000 - 1 - 29700 = 970299.$

Example 3. *Without actually calculating the cubes, find the values of:*

(i) $(28)^3 + (-15)^3 + (-13)^3$ (NCERT) *(ii) $\left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3$ (NCERT Exemplar)*

Solution. We know that $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$ (Identity 11)

(*i*) Here, $28 + (-15) + (-13) = 0.$

Therefore, by using the above identity, we get

$(28)^3 + (-15)^3 + (-13)^3 = 3(28)(-15)(-13)$

$= 84 \times 195 = 16380$

(*ii*) Note that $\left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3 = \left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 + \left(-\frac{5}{6}\right)^3$

Here, $\frac{1}{2} + \frac{1}{3} + \left(-\frac{5}{6}\right) = 0.$

Therefore, by using the above identity, we get

$$\left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 + \left(-\frac{5}{6}\right)^3 = 3\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)\left(-\frac{5}{6}\right) = -\frac{5}{12}$$

$$\Rightarrow \quad \left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3 = -\frac{5}{12}.$$

Example 4. *If $a + b + c = 9$ and $ab + bc + ca = 26$, then find $a^2 + b^2 + c^2$.* *(NCERT Exemplar)*

Solution. Given $a + b + c = 9$ and $ab + bc + ca = 26.$

We know that

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$\Rightarrow \quad (9)^2 = a^2 + b^2 + c^2 + 2 \times 26$

$\Rightarrow \quad a^2 + b^2 + c^2 = 81 - 52 = 29.$

Example 5. *If $a + b = 3$ and $ab = 2$, find the values of:*

(i) $a^2 + b^2$ *(ii)* $a - b$ *(iii)* $a^2 - b^2$.

Solution. (*i*) We know that $(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow \quad a^2 + b^2 = (a + b)^2 - 2ab$ but $a + b = 3$ and $ab = 2$

$\Rightarrow \quad a^2 + b^2 = (3)^2 - 2 \times 2 = 9 - 4 = 5.$

(*ii*) $(a - b)^2 = a^2 + b^2 - 2ab = 5 - 2 \times 2 = 5 - 4 = 1$

$\Rightarrow \quad a - b = \pm 1.$

(*iii*) $a^2 - b^2 = (a + b)\,(a - b) = 3 \times (\pm 1) = \pm 3.$

Example 6. *If $x + y = 12$ and $xy = 27$, find the value of $x^3 + y^3$.* *(NCERT Exemplar)*

Solution. Given $x + y = 12$ and $xy = 27$.

We know that

$(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

$\Rightarrow \quad (12)^3 = x^3 + y^3 + 3 \times 27 \times 12$

$\Rightarrow \quad x^3 + y^3 = (12)^3 - 3 \times 27 \times 12 = 12(12^2 - 3 \times 27)$

$\Rightarrow \quad x^3 + y^3 = 12(144 - 81) = 12 \times 63 = 756.$

Example 7. *If $a - b = 7$ and $a^2 + b^2 = 85$, then find the value of $a^3 - b^3$.*

Solution. Given $a - b = 7$ ...(*i*) and $a^2 + b^2 = 85$ ...(*ii*)

We know that $(a - b)^2 = a^2 + b^2 - 2ab$

$\Rightarrow \quad 7^2 = 85 - 2ab$ (using (*i*) and (*ii*))

$\Rightarrow \quad 2ab = 85 - 49 \Rightarrow 2ab = 36$

$\Rightarrow \quad ab = 18$ ...(*iii*)

We know that $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

$\Rightarrow \quad 7^3 = a^3 - b^3 - 3 \times 18 \times 7$ (using (*i*), (*ii*) and (*iii*))

$\Rightarrow \quad 343 = a^3 - b^3 - 378$

$\Rightarrow \quad a^3 - b^3 = 343 + 378 \Rightarrow a^3 - b^3 = 721.$

Example 8. *If $a + b + c = 5$ and $ab + bc + ca = 10$, then prove that*

$a^3 + b^3 + c^3 - 3abc = -25.$ *(NCERT Exemplar)*

Solution. Given $a + b + c = 5$ and $ab + bc + ca = 10$.

We know that

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$\Rightarrow \quad (5)^2 = a^2 + b^2 + c^2 + 2 \times 10$

$\Rightarrow \quad a^2 + b^2 + c^2 = 25 - 20 = 5$

Also, we know that

$a^3 + b^3 + c^3 - 3abc = (a + b + c)\,[a^2 + b^2 + c^2 - (ab + bc + ca)]$

$= 5(5 - 10) = 5 \times (-5) = -25.$

Example 9. *Find the value of $x^3 - 8y^3 - 36xy - 216$ when $x = 2y + 6$.* *(NCERT Exemplar)*

Solution. Given $x = 2y + 6 \Rightarrow x - 2y - 6 = 0$

$\Rightarrow \quad x + (-2y) + (-6) = 0$

$\Rightarrow \quad x^3 + (-2y)^3 + (-6)^3 = 3 \times x \times (-2y) \times (-6)$

($\because$ if $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$)

$\Rightarrow \quad x^3 - 8y^3 - 216 = 36xy$

$\Rightarrow \quad x^3 - 8y^3 - 36xy - 216 = 0.$

Exercise 2.8

1. Using suitable identities, evaluate the following:
   (*i*) 105×106 *(NCERT)* (*ii*) 95×96 *(NCERT)* (*iii*) $(98)^2$
2. Using suitable identities, evaluate the following:
   (*i*) $(102)^3$ *(NCERT)* (*ii*) $(97)^3$ (*iii*) $(998)^3$ *(NCERT)*
3. Without actually calculating the cubes, find the values of:
   (*i*) $(-12)^3 + 7^3 + 5^3$ *(NCERT)* (*ii*) $(0.2)^3 - (0.3)^3 + (0.1)^3$ *(NCERT Exemplar)*
4. If $x + y = 8$ and $x - y = 2$, find the value of $2x^2 + 2y^2$.

   Hint: $2(x^2 + y^2) = (x + y)^2 + (x - y)^2$.
5. If $x + y = 6$ and $x - y = 4$, find (*i*) $x^2 + y^2$ (*ii*) xy.

   Hint: (*ii*) $4xy = (x + y)^2 - (x - y)^2$.
6. If $a + b + c = 12$ and $ab + bc + ca = 22$, find the value of $a^2 + b^2 + c^2$.
7. If $a + b + c = 12$ and $a^2 + b^2 + c^2 = 100$, find the value of $ab + bc + ca$.
8. If $a - b = 3$ and $ab = 4$, find the value of $a^3 - b^3$.
9. If $2a - 3b = 3$ and $ab = 2$, find the value of $8a^3 - 27b^3$.
10. If $a + b = 10$ and $a^2 + b^2 = 58$, find the value of $a^3 + b^3$.
11. If $a + b + c = 5$ and $ab + bc + ca = 10$, then find the value of $a^3 + b^3 + c^3 - 3abc$.
12. If $x + y = 4$, then find the value of $x^3 + y^3 + 12xy - 64$.

Answers

Very Short Answer Type Questions

1. Find the coefficient of x^2 in the polynomial $p(x) = (5 - 2x)(7x + 5x^2 - x^4)$.
2. Find the leading coefficient in the polynomial $p(x) = (7x - 3)(5x^3 - 9)$.
3. Find the degree of a the polynomial $p(y) = 3 - 7y^5 - 6y^2 + \sqrt{2}\,y$.
4. Give an example of a trinomial of degree 24 in the variable x.
5. If $p(x) = x^2 - 3\sqrt{3}\,x + 5$, then find the value of $p(2\sqrt{3})$.
6. If $p(x) = 2x^2 - 3x + 1$, then find the value of $\frac{p(1) - p(-1)}{2}$.
7. Find the zeroes of the polynomial $3x^2 - 1$.
8. If k is a zero of the polynomial $x^6 - kx^5 + 2x^4 - 2kx^3 + 3x - k + 2$, then find the value of k.
9. If the polynomial $3x^4 - 4x^3 - 3x - 1$ is divided by $x - 1$, then find the remainder.
10. If the remainder on dividing the polynomial $2x^3 + kx^2 + 3x + 9$ by $x + 3$ is -18, then find the value of k.
11. If $x + k$ is a factor of the polynomial $x^3 + kx^2 - 2x + k + 5$, then find the value of k.
12. Using suitable identity, find the value of $(97)^2$.
13. If $a + b + c = 11$ and $ab + bc + ca = 38$, find the value of $a^2 + b^2 + c^2$.
14. If $x + y = 5$ and $xy = 6$, then find the value of $x^3 + y^3$.
15. Write the factors of $x^3 - 8y^3$.

Answers

Objective Type Questions

(FOR FORMATIVE/SUMMATIVE ASSESSMENT)

Mental Maths

1. Fill in the blanks:

(*i*) An algebraic expression of the form $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n$, where $a_0, a_1, a_2, \ldots, a_n$ are real numbers and n is a ..... is called a polynomial in the ..... .

(*ii*) The degree of the zero polynomial is ..... .

(*iii*) The degree of a non-zero constant polynomial is ..... .

(*iv*) A cubic polynomial in one variable has atmost ..... terms.

(*v*) If $p(y) = y^3 + 3y^2 - 5y - 6$, then $p(-3)$ = ...

(*vi*) A cubic polynomial in one variable has atmost ..... zeroes.

(*vii*) A linear polynomial in one variable has ..... and only ..... zero.

(*viii*) A non-zero constant polynomial has ..... zeroes.

(*ix*) Every real number is a zero of the ..... polynomial.

(*x*) If $f(x)$ is a polynomial of degree ≥ 1 and α is a real number, then $x - \alpha$ is a factor of $f(x)$ if and only if α is a ..... of $f(x)$.

(*xi*) If $a + b + c = 0$, then $a^3 + b^3 + c^3 - 3abc$ = ...

2. State whether the following statements are true or false. Justify your answer.

(*i*) A quadratic polynomial in two variables has atmost three terms.

(*ii*) A binomial can have atmost two terms. *(NCERT Exemplar)*

(*iii*) Every polynomial is a binomial. *(NCERT Exemplar)*

(*iv*) The degree of a constant polynomial is zero.

(*v*) A binomial may have degree 5. *(NCERT Exemplar)*

(*vi*) Zero of a polynomial is always 0. *(NCERT Exemplar)*

(*vii*) 0 may be a zero of a polynomial.

(*viii*) A polynomial cannot have more than one zero. *(NCERT Exemplar)*

(*ix*) Every quadratic polynomial in one variable has two real zeroes.

(*x*) If 3 is a zero of a polynomial $f(x)$, then 3 is also a zero of the polynomial $3f(x)$.

(*xi*) A polynomial of degree 5 in one variable can have three terms.

(*xii*) The degree of the sum of two polynomials each of degree 5 is always 5. *(NCERT Exemplar)*

(*xiii*) If 2 is one of the zeroes of a polynomial $f(x)$ and 3 is one of the zeroes of a polynomial $g(x)$, then 5 is one of the zeroes of the polynomial $f(x) + g(x)$.

Multiple Choice Questions

Choose the correct answer from the given four options (3 to 31):

3. Which one of the following is a polynomial?

(*a*) $\frac{x^2}{2} - \frac{2}{x^2}$ (*b*) $\sqrt{2x} - 1$ (*c*) $x^2 + \frac{3x^{\frac{3}{2}}}{\sqrt{x}}$ (*d*) $\frac{x-1}{x+1}$

(NCERT Exemplar)

4. Which of the following algebraic expression is a polynomial in one variable?

(*a*) $7x^2 - \frac{2}{x}$ (*b*) $7x^2 - 3x + \sqrt{2}$ (*c*) $\frac{(x-2)(x-4)}{x}$ (*d*) $7x^2 - xy + 5y + 2$

5. Which of the following algebraic expression is a cubic polynomial in one variable?

(*a*) $2x - 5x^2$ (*b*) $2x^2y - 3xy + 5$

(*c*) $2 + 7y - 3y^2 + \sqrt{3}\,y^3$ (*d*) $3x^4 + 7x^2 - 5$

6. $\sqrt{2}$ is a polynomial of degree

(*a*) 2 (*b*) 0 (*c*) 1 (*d*) $\frac{1}{2}$

(NCERT Exemplar)

7. Degree of the polynomial $4x^4 + 0x^3 + 0x^5 + 5x + 7$ is

(*a*) 4 (*b*) 5 (*c*) 3 (*d*) 7 *(NCERT Exemplar)*

8. If $p(x) = 7x^3(x^2 - 5)$, then the degree of the polynomial $p(x)$ is

(*a*) 3 (*b*) 2 (*c*) 5 (*d*) 7

9. Which of the following algebraic expressions is a linear polynomial?

(*a*) $3x$ (*b*) $3x^2$ (*c*) $3x^2 + 2$ (*d*) $3x - \frac{1}{x}$

10. If $p(x) = (3x^2 - 1)(2x^3 + 1)$, then the leading coefficient of the polynomial $p(x)$ is

(*a*) 3 (*b*) 2 (*c*) 5 (*d*) 6

11. Which of the following algebraic expressions is a trinomial?

(*a*) $7x^3 + 2x$ (*b*) $2x^2 + 3x - x^5$ (*c*) $2x^3 + 4x^2 - 5x + 1$ (*d*) xyz

12. A polynomial in one variable of degree 4 has atmost

(*a*) 3 terms (*b*) 4 terms (*c*) 5 terms (*d*) 6 terms

13. If $p(x) = x^2 - 2\sqrt{2}\ x + 1$, then $p(2\sqrt{2})$ is equal to

(*a*) 0 (*b*) 1 (*c*) $4\sqrt{2}$ (*d*) $8\sqrt{2} + 1$

(NCERT Exemplar)

14. If $p(x) = x + 3$, then $p(x) + p(-x)$ is equal to

(*a*) 3 (*b*) $2x$ (*c*) 0 (*d*) 6

(NCERT Exemplar)

15. One of the zeroes of the polynomial $2x^2 + 7x - 4$ is

(*a*) 2 (*b*) $\frac{1}{2}$ (*c*) $-\frac{1}{2}$ (*d*) -2

(NCERT Exemplar)

16. If $p(x) = kx$, $k \neq 0$, then zero of $p(x)$ is

(*a*) 0 (*b*) 1 (*c*) k (*d*) $-k$

17. The zeroes of the polynomial $5x(x + 2)(x - 3)$ are

(*a*) $-2, 0, 3, 5$ (*b*) $-2, 3$ (*c*) $5, -2, 3$ (*d*) $0, -2, 3$

18. The roots of the polynomial equation $3x^3 - 12x = 0$ are

(*a*) $3, 2, -2$ (*b*) $3, 0, 4$ (*c*) $0, 2, -2$ (*d*) $3, 0, 2, -2$

19. If $p(x) = (x - 2)^2 - (x + 2)^2$, then

(*a*) the zeroes of $p(x)$ are $2, -2$ (*b*) the zeroes of $p(x)$ are $0, 2, -2$

(*c*) the zero of $p(x)$ is 0 (*d*) $p(x)$ has no real zeroes

20. If the polynomial $x^3 + 3x^2 + 3x + 1$ is divided by $2x - 1$, then the remainder is

(*a*) 1 (*b*) $\frac{27}{8}$ (*c*) $-\frac{27}{8}$ (*d*) 27

21. If $x^{51} + 51$ is divided by $x + 1$, then the remainder is

(*a*) 0 (*b*) 1 (*c*) 49 (*d*) 50

(NCERT Exemplar)

22. If the remainder on dividing the polynomial $2x^4 - kx^2 + 5x - 3k + 3$ by $x+2$ is 4, then the value of k is

(*a*) -3 (*b*) 3 (*c*) $\frac{25}{7}$ (*d*) $-\frac{25}{7}$

23. If $x + 1$ is a factor of $2x^2 + kx$, then the value of k is

(*a*) -3 (*b*) 4 (*c*) 2 (*d*) -2

(NCERT Exemplar)

24. $x + 1$ is a factor of the polynomial

(*a*) $x^3 + x^2 - x + 1$ (*b*) $x^3 + x^2 + x + 1$

(*c*) $x^4 + x^3 + x^2 + 1$ (*d*) $x^4 + 3x^3 + 3x^2 + x + 1$

(NCERT Exemplar)

25. If $(3x - 4)(5x + 7) = 15x^2 - kx - 28$, then the value of k is

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) -1

26. One of the factors of $(25x^2 - 1) + (1 + 5x)^2$ is

(*a*) $5 + x$ (*b*) $5 - x$ (*c*) $5x - 1$ (*d*) $10x$

(NCERT Exemplar)

27. The factorisation of $4x^2 + 8x + 3$ is

(*a*) $(x + 1)(x + 3)$ (*b*) $(2x + 1)(2x + 3)$

(*c*) $(2x + 2)(2x + 5)$ (*d*) $(2x - 1)(2x - 3)$ *(NCERT Exemplar)*

28. The value of $249^2 - 248^2$ is

(*a*) 1^2 (*b*) 477 (*c*) 487 (*d*) 497

(NCERT Exemplar)

29. Which of the following is a factor of $(x + y)^3 - (x^3 + y^3)$?

(*a*) $x^2 + xy + 2xy$ (*b*) $x^2 + y^2 - xy$ (*c*) xy^2 (*d*) $3xy$

(NCERT Exemplar)

30. If $\frac{x}{y} + \frac{y}{x} = -1$ $(x, y \neq 0)$, then the value of $x^3 - y^3$ is

(*a*) 1 (*b*) -1 (*c*) 0 (*d*) $\frac{1}{2}$

31. If $a + b + c = 0$, then the value of $a^3 + b^3 + c^3$ is

(*a*) 0 (*b*) abc (*c*) $2abc$ (*d*) $3abc$

(NCERT Exemplar)

Answers

Higher Order Thinking Skills (HOTS)

1. If on dividing the polynomial $f(x) = 2x^4 - 3x^2 - ax + b$ by $x-1$ and $x + 1$ the remainders are -3 and 7 respectively, then find the remainder when $f(x)$ is divided by $x - 2$.

2. If $f(x) = ax^5 + bx^3 + cx + 7$, where a, b, c are real numbers, and $f(-3) = -3$, then find the value of $f(3)$.

3. When a polynomial $p(x)$ is divided by $(x - 1)$, the remainder is 5, and when it is divided by $(x - 2)$, the remainder is 7. Find the remainder when $f(x)$ is divided by $(x - 1)(x - 2)$.

4. If the sum of two numbers is 7 and the sum of their cubes is 133, then find the sum of their squares.

5. If $a + b + c = 0$, find the value of $\frac{(b + c)^2}{bc} + \frac{(c + a)^2}{ca} + \frac{(a + b)^2}{ab}$.

6. Simplify: $\frac{(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3}{(a - b)^3 + (b - c)^3 + (c - a)^3}$.

Answers

Summary

- **Polynomial in one variable and related terms**

  A polynomial $p(x)$ in one variable x is an algebraic expression in x of the form $p(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_1 x + a_0$, where a_n, a_{n-1}, a_{n-2}, ..., a_0 are real numbers and n is a whole number.

  Polynomials in the variable x are usually denoted by $p(x)$, $f(x)$, $g(x)$ etc. Thus,

  $$p(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_1 x + a_0.$$

  (*i*) If $a_n \neq 0$, then n is called the **degree** of the polynomial $p(x)$. It is written as *deg* $p(x) = n$.

  (*ii*) $a_n x^n$, $a_{n-1}x^{n-1}$, $a_{n-2}x^{n-2}$, ..., $a_1 x$, a_0 are called the **terms** of the polynomial $p(x)$; a_0 is called the **constant term.**

  (*iii*) A polynomial of degree n in one variable has atmost $(n + 1)$ terms.

  (*iv*) a_n, a_{n-1}, a_{n-2}, ..., a_1, a_0 are called the **coefficients** of the polynomial $p(x)$.

  (*v*) If $a_n \neq 0$, then $a_n x^n$ is called the **leading term** and a_n is called the **leading coefficient** of the polynomial $p(x)$.

  (*vi*) If all the coefficients a_n, a_{n-1}, a_{n-2}, ..., a_1, a_0 are zero, then $p(x)$ is called a **zero polynomial.** The zero polynomial is usually denoted by the symbol 0. *The degree of the zero polynomial is not defined.*

  (*vii*) The degree of a polynomial is zero if and only if it is a non-zero constant polynomial.

- **Some particular polynomials in one variable**

  A polynomial of degree one is called a **linear polynomial.** It is of the form $ax + b$, where a, b are real numbers and $a \neq 0$.

  A polynomial of degree two is called a **quadratic polynomial.** It is of the form $ax^2 + bx + c$, where a, b, c are real numbers and $a \neq 0$.

  A polynomial of degree three is called a **cubic polynomial.** It is of the form $ax^3 + bx^2 + cx + d$, where a, b, c, d are real numbers and $a \neq 0$.

  A polynomial of degree four is called a biquadratic polynomial etc.

- **Monomial, Binomial, Trinomial, Multinomial**

  (*i*) A polynomial having one term is called a **monomial.**

  (*ii*) A polynomial having two terms is called a **binomial.**

  (*iii*) A polynomial having three terms is called a **trinomial.**

  (*iv*) A polynomial having two or more terms is called a **multinomial.**

- **Value of a polynomial $p(x)$ at $x = \alpha$**

  If $p(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_1 x + a_0$ is a polynomial in x and α be a real number, then the real number $a_n\alpha^n + a_{n-1}\alpha^{n-1} + a_{n-2}\alpha^{n-2} + \ldots + a_1\alpha + a_0$ is called the **value of $p(x)$ at α.** It is denoted by $p(\alpha)$.

- *A real number α is a* **zero** *of the polynomial $p(x)$ if and only if $p(\alpha) = 0$.*

- *A real number α is a root of the polynomial equation $p(x) = 0$ if and only if $p(\alpha) = 0$.*

- *A polynomial may not have real zeroes. In fact, the number of real zeroes of a polynomial may be less than the degree of the polynomial.*

- **Number of zeroes of a polynomial**

  (*i*) By convention, every real number is taken as a zero of the zero polynomial. Thus, zero polynomial has infinitely many zeroes.

  (*ii*) A non-zero constant polynomial has no zeroes.

(*iii*) Every linear polynomial has one and only one zero.

(*iv*) A polynomial of degree $n(> 1)$ can have atmost n zeroes.

❍ **Division algorithm for polynomials**

If $p(x)$ and $g(x)$ are any two polynomials with $g(x) \neq 0$, then there exists (unique) polynomials $q(x)$ and $r(x)$ such that

$$p(x) = g(x)\, q(x) + r(x)$$

where either $r(x) = 0$ or $deg\ r(x) < deg\ g(x)$.

Dividend = Divisor × Quotient + Remainder.

❍ **Remainder theorem**

Let $p(x)$ be a polynomial of degree ≥ 1.

(*i*) If $p(x)$ is divided by $x - \alpha$ where α is a real number, then remainder $= p(\alpha)$.

(*ii*) If $p(x)$ is divided by $ax - b$ where a, b are real numbers and $a \neq 0$, then remainder $= p\left(\frac{b}{a}\right)$.

(*iii*) If $p(x)$ is divided by $ax + b$ where a, b are real numbers and $a \neq 0$, then remainder $= p\left(-\frac{b}{a}\right)$.

❍ **Factor theorem**

If $p(x)$ is a polynomial of degree ≥ 1 and α is a real number, then

(*i*) $x - \alpha$ is a factor of $p(x)$ if and only if $p(\alpha) = 0$.

(*ii*) $x - \alpha$ is a factor of $p(x)$ if and only if α is a zero of $p(x)$.

❍ **Algebraic Identities**

An algebraic equation which is true for all real values of the variables occurring in the equation is called an **algebraic identity.**

Some algebraic identities are:

1. $(x + y)^2 = x^2 + 2xy + y^2$
2. $(x - y)^2 = x^2 - 2xy + y^2$
3. $x^2 - y^2 = (x + y)\,(x - y)$
4. $(x + a)\,(x + b) = x^2 + (a + b)x + ab$
5. $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$
6. $(x + y)^3 = x^3 + y^3 + 3xy(x + y) = x^3 + y^3 + 3x^2y + 3xy^2$
7. $(x - y)^3 = x^3 - y^3 - 3xy(x - y) = x^3 - y^3 - 3x^2y + 3xy^2$
8. $x^3 + y^3 = (x + y)\,(x^2 - xy + y^2)$
9. $x^3 - y^3 = (x - y)\,(x^2 + xy + y^2)$
10. $x^3 + y^3 + z^3 - 3xyz = (x + y + z)\,(x^2 + y^2 + z^2 - xy - yz - zx)$
$$= \frac{1}{2}(x + y + z)\,((x - y)^2 + (y - z)^2 + (z - x)^2)$$
11. If $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$.

Chapter Test

1. Match each of the following statements on the left with the same on the right:
   (*i*) Degree of $2x - 3x^4$ is — (*a*) $(2x + 3)(2x - 3)$
   (*ii*) Coefficient of y^2 in $3y^3 - 2y^2 + 5$ is — (*b*) $-\frac{5}{3}$
   (*iii*) Zero of the polynomial $3t + 5$ is — (*c*) 4
   (*iv*) Factors of $4x^2 - 9$ are — (*d*) -2
2. Fill in the blanks:
   (*i*) The coefficient of x^2 in the polynomial $f(x) = (2x^2 - 3x + 4)(2x^2 + 5x + 3)$ is ..... .
   (*ii*) When the polynomial $8t^3 - 6t^2 + 5t + 3$ is divided by $2t + 1$, then the remainder is ..... .
3. State whether the following statement is true or false. Justify your answer.
   If n is an odd natural number, then $x^n + 1$ is exactly divisible by $x + 1$.

Choose the correct answer from the given four options (4 to 8):

4. Zero of the zero polynomial
   (*a*) is 0 only (*b*) is 1 only (*c*) is any real number (*d*) does not exist
5. If $x + k$ is a factor of the polynomial $x^3 + kx^2 - 2x + k + 5$, then the value of k is
   (*a*) $-\frac{5}{3}$ (*b*) 5 (*c*) -5 (*d*) $\frac{5}{2}$
6. If r_1 and r_2 are the remainders when the polynomials $x^3 + 2x^2 - 5kx - 7$ and $x^3 + kx^2 - 12x + 6$ are divided by $x + 1$ and $x - 2$ respectively, and if $2r_1 + r_2 = 6$, then the value of k is
   (*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4
7. The factors of $2\sqrt{2}\,x^3 + y^3$ are
   (*a*) $(\sqrt{2}\,x + y)(2x^2 - \sqrt{2}\,xy + y^2)$ (*b*) $(\sqrt{2}\,x + y)(2x^2 + \sqrt{2}\,xy + y^2)$
   (*c*) $(\sqrt{2}\,x - y)(2x^2 + \sqrt{2}\,xy + y^2)$ (*d*) $(\sqrt{2}\,x - y)(2x^2 - \sqrt{2}\,xy + y^2)$
8. If $x^{1/3} + y^{1/3} + z^{1/3} = 0$, then
   (*a*) $x^3 + y^3 + z^3 = 0$ (*b*) $x^3 + y^3 + z^3 = 27xyz$
   (*c*) $(x + y + z)^3 = 27xyz$ (*d*) $x + y + z = 3xyz$
9. Show that the polynomial $3x^3 + 8x^2 - 1$ has no integral zeroes.
10. By using Remainder theorem, show that $2x + 1$ is a factor of the polynomial $f(x) = 4x^3 + 8x^2 - x - 2$.
11. If the polynomials $2x^3 + ax^2 + 3x - 5$ and $x^3 + 2x^2 - 5x - a$ leave the same remainder, when divided by $x - 1$, then find the value of a.
12. If $2x + k$ is a factor of the polynomial $8x^3 + 4kx^2 + 3x + k - 1$, then find the value of k.
13. If the polynomial $x^4 - 2x^3 + 3x^2 - px + 8$ is divided by $(x - 2)$, it leaves a remainder of 10. Find the value of p.
14. If $x - 1$ and $x + 3$ are factors of the polynomials $x^3 - ax^2 - 13x - b$, then find the values of a and b.
15. Factorise the following trinomials:
   (*i*) $15x^2 + x - 6$ (*ii*) $3\sqrt{2}\,x^2 - 5x + \sqrt{2}$.
16. Find the zeroes of the following polynomials:
   (*i*) $2x^2 + 7x + 3$ (*ii*) $3x^2 - x - 4$.
17. By using Factor theorem, show that $x + 2$ is a factor of the polynomial $3x^3 - x^2 - 20x - 12$. Hence, factorise the given polynomial.
18. Factorise the cubic polynomial: $2x^3 + 9x^2 + 10x + 3$.

19. Using proper identities, expand the following:

(*i*) $(-2x + 3y + 2z)^2$ *(NCERT)* (*ii*) $(2a - 3b)^3$ *(NCERT)*

(*iii*) $\left(\frac{1}{x} + \frac{y}{3}\right)^3$, $x \neq 0$. *(NCERT Exemplar)*

20. By using proper identities, find the following products:

(*i*) $(2x - 3y)(4x^2 + 6xy + 9y^2)$

(*ii*) $(3x - 2y + 5z)(9x^2 + 4y^2 + 25z^2 + 6xy + 10yz - 15zx)$

21. Using proper identities, factorise the following:

(*i*) $9x^2 - 12x + 4$ (*ii*) $8a^3 - b^3 - 12a^2b + 6ab^2$ *(NCERT Exemplar)*

(*iii*) $8p^3 + \frac{12}{5}p^2 + \frac{6}{25}p + \frac{1}{125}$ (*iv*) $27x^3 + 8y^3$ *(NCERT Exemplar)*

(*v*) $(2x - 5y)^3 - (2x + 5y)^3$ (*vi*) $8x^3 - 27y^3 + 64z^3 + 72xyz$ *(NCERT Exemplar)*

22. Without actually calculating the cubes, find the values of:

(*i*) $48^3 - 30^3 - 18^3$ (*ii*) $\left(\frac{1}{4}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{7}{12}\right)^3$ *(NCERT Exemplar)*

23. Factorise: $(2x - y - z)^3 + (2y - z - x)^3 + (2z - x - y)^3$.

24. If $a + 2b + 3c = 0$, prove that $a^3 + 8b^3 + 27c^3 = 18abc$.

25. (*i*) If $x - y = 2$ and $xy = 3$, find the values of $x^3 - y^3$.

(*ii*) If $a^2 + b^2 + c^2 = 250$ and $ab + bc + ca = 3$, then find the value of $a + b + c$.

26. If $a - b = 7$ and $a^3 - b^3 = 133$, then find the values of:

(*a*) ab (*b*) $a^2 + b^2$.

Answers

Model Question Paper 1

(FOR FORMATIVE ASSESSMENT I - TERM I)

Time: 1 Hour *Maximum Marks: 30*

Note. *Questions 1-2 carry 1 mark each, 3-4 carry 2 marks each, 5-8 carry 3 marks each and 9-11 carry 4 marks each.*

1. Simplify: $\sqrt[5]{(243)^{-2}}$
2. If $2x + k$ is a factor of the polynomial $8x^3 + 4kx^2 + 3x + k - 1$, then find the value of k.
3. If p and q are rational numbers and $\frac{2-\sqrt{3}}{2+\sqrt{3}} = p + q\sqrt{3}$, then find the values of p and q.
4. Check whether the polynomial $p(x) = 3x^3 + x^2 - 20x + 12$ is a multiple of $3x - 2$.
5. How many irrational numbers lie between $\sqrt{3}$ and $\sqrt{5}$? Write any three irrational numbers between $\sqrt{3}$ and $\sqrt{5}$.
6. Express the number $0.2\overline{37}$ in the form $\frac{p}{q}$, where p, q are integers, $q \neq 0$.
7. Show that the polynomial $5x^3 - 7x^2 + 3x - 2$ has no integral zeroes.
8. If $x + 2y = 5$, then find the value of $x^3 + 8y^3 + 30xy - 125$.
9. If $x = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ and $y = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, then find the value of $x^2 + y^2 + 3xy$.
10. If $p(x) = x^3 - ax^2 + bx + 3$ leaves a remainder -19 when divided by $(x + 2)$ and a remainder 17 when divided by $(x - 2)$, then prove that $a + b = 6$.
11. Factorise: $2x^3 - x^2 - 5x - 2$.

Answers

Introduction to Euclid's Geometry



INTRODUCTION

The word 'Geometry' is derived from the Greek words Geo (meaning Earth) and Metron (meaning measure). Ancient Egyptians, Greeks, Babylonians and Chinese used geometry for surveying, navigation, astronomy and other practical purposes. Geometric facts and principles discovered and derived by logical reasoning (by Thales, Pythagoras, Plato and Aristotle) were collected in the geometric text *'Elements'* written by *Euclid* (Greek mathematician) about 325 B.C.

In ancient India, the geometry of Vedic period originated with the construction of altars (or vedis) and fire places for performing Vedic rites. Squares and circular altars were used for household rituals, while altars, whose shapes were combinations of rectangles, triangles and trapeziums were required for public worship.

In this chapter, we shall discuss Euclid's approach to geometry and shall try to link it with the present day geometry.

3.1 EUCLID'S DEFINITIONS, AXIOMS AND POSTULATES

❑ **Point, Line and Plane**

Euclid gave some definitions of point, line and plane but these definitions required to define many other terms and we may get a long chain of definitions without an end. For such reasons, Euclid's definitions are not accepted these days. Mathematicians agree to leave such terms undefined.

Point, line and plane are some geometric terms which are not defined. However, we have an intuitive feeling for these geometric concepts. We have :

Point

A small dot marked by a sharp pencil on a sheet of paper or a prick made by a fine needle on a paper are examples of a point.



Point

A point determines a location in space. It has no length, breadth or thickness.

Line

A line has length only. It has no breadth or thickness. The basic concept of a line is its straightness and it extends indefinitely in both directions. The two arrowheads in the opposite directions indicate that the length of a line is unlimited *i.e.* it has no definite length. A line has no end points and it consists of an infinite (uncountable) number of points.



Line

Plane

A plane has length and breadth. It has no thickness. The basic concept of a plane is its flatness and it extends indefinitely in all directions. The length and breadth of a plane are unlimited *i.e.* a plane has no definite length and no definite breadth.

- **Space**

Space is the collection of all points in the universe. So, a point or a line or a plane is a part of space. In fact, everything we look at is a part of space.

- **Axioms and Postulates**

The assumptions which are actually 'obvious universal truths' are called **axioms** (or **postulates**).

Thus, an axiom is a statement which is accepted to be true without proof.

The assumptions that are specific to geometry are called **postulates** and the assumptions that are used throughout mathematics and not specifically linked to geometry are called **axioms**.

Now-a-days, postulates and axioms are terms that are used interchangeably and in the same sense.

- **Theorem, corollary**

A (mathematical) statement which can be proved by using axiom(s) or the other statements already proved is called a **theorem**.

A statement whose truth can easily be derived from a theorem is called its **corollary**.

- **Euclid's axioms**

Some of Euclid's axioms, not in his order, are given below :

1. Things which are equal to the same thing are equal to one another.
2. If equals are added to equals, the wholes are equals.
3. If equals are subtracted from equals, the remainders are equal.
4. Things which coincide with one another are equal to one another.
5. The whole is greater than the part.
6. Things which are double of the same thing are equal to one another.
7. Things which are halves of the same thing are equal to one another.

The above axioms refer to magnitudes of some kind. The magnitudes of same kind can be compared but the magnitudes of different kinds cannot be compared. For example, if the area of a triangle equals the area of a rectangle and the area of the rectangle equals the area of a square, then the area of the triangle is equal to the area of the square [using axiom (*i*)]. Note that the length of a line segment cannot be compared with measure of an angle.

The magnitudes of same kind can be added or subtracted but the magnitudes of different kinds cannot be added or subtracted.

For example, the area of a triangle can be added to the area of a square, but the area of a triangle cannot be added to the perimeter of a square.

- **Euclid's Five Postulates**

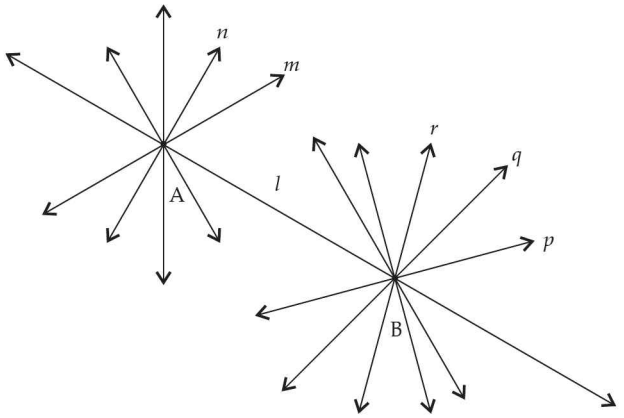
Euclid's five postulates are :

Postulate 1. *A straight line may be drawn from any one point to any other point.*

This postulate tells us that atleast one straight line passes through two distinct points, but it does not say that there cannot be more than one such line. However, he has assumed, without mentioning, that there is unique line joining two distinct points. So, we state this result in the form of an axiom as :

Axiom 3.1: *Given two distinct points, there is a unique line passing through them.*

Let A and B be two given distinct points. Draw lines l, m, n, ... through A. Also draw lines p, q, r, ... through B. Find the line (or lines) passing through both the points A and B.



We observe that exactly one line (here line l) passes through both the given points A and B in a plane. Thus, two distinct points determine a unique line.

Postulate 2. *A terminated line can be produced indefinitely.*

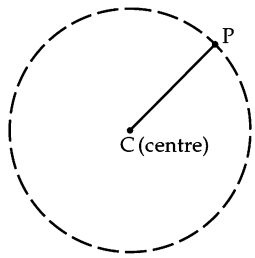
These days, Euclid's terminated line is called **line segment**. So, we restate the above postulate as :

A line segment can be extended on either side to form a line (shown in the adjoining figure).



Postulate 3. *A circle can be drawn with any centre and any radius.*

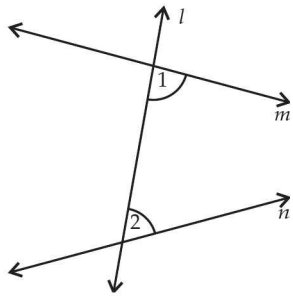
In other words, given any line segment, a circle can be drawn having the line segment as radius and one end point of the segment as centre.



Postulate 4. *All right angles are equal to one another.*

Postulate 5. *If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two lines, if produced indefinitely, meet on that side on which the sum of angles is less than two right angles.*

Look at the adjoining figure, the line l falls on the lines m and n, such that the sum of interior angles $\angle 1$ and $\angle 2$ is less than two right angles on the right side of the line l. Therefore, the lines m and n intersect on the right side of l.

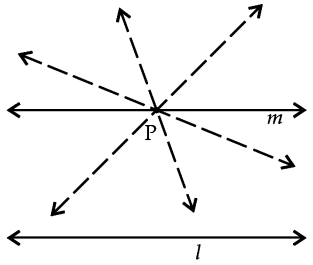


❑ **Equivalent versions of Euclid's fifth postulate**

Euclid's fifth postulate is very significant in the history of mathematics. It implies that the two lines will not meet if the sum of measures of the interior angles on the same side of the falling line is exactly 180°. There are several equivalent versions of Euclid's fifth postulate. One of them is 'Playfair's Axiom' (given by Scottish mathematician John Playfair in 1729). It states that :

For every line l and for every point P not lying on l, there exists a unique line m passing through P and parallel to l.

Look at the adjoining figure, of all the lines passing through the point P, only line m is parallel to line l.

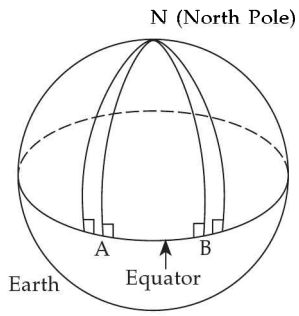


The above result can also be stated in the following form:

Two distinct intersecting lines cannot be parallel to the same line.

❑ **Non-Euclidean Geometries**

Many mathematicians, including Euclid, thought that the fifth postulate is actually a theorem that can be proved just using the first four postulates and other axioms. However, all attempts to prove the fifth postulate as a theorem failed. But these efforts led to a great achievement —the creation of other geometries. These are called **non-Euclidean geometries**. The geometry of the universe in which we live in is called **spherical geometry**. In spherical geometry, lines are not straight. These are parts of great circles (circles obtained by the intersection of a sphere and planes passing through centre of sphere).



In the adjoining figure, the lines AN and BN (which are parts of great circles) are perpendicular to the same line AB. Note that these lines are meeting at the same point N, though the sum of the angles on the same side of AB is not less than 180° (in fact, it is 90° + 90° = 180°). Also, note that in ΔABN, sum of angles is greater than 180° ($\because \angle A + \angle B = 180°$).

Thus, Euclidean geometry is valid only for figures in a plane.

Theorem 3.1: ***Two distinct lines cannot have more than one point in common.***

Proof. Let l and m be two given distinct lines, then we have to prove that these lines can have atmost one point in common.

Let us suppose that the lines l and m have two distinct points P and Q in common. Thus, we have two distinct lines l and m passing through two distinct points P and Q. But by axiom 3.1, there is a unique line passing through two distinct points. Therefore, our assumption is wrong.

Hence, we conclude that two distinct lines cannot have more than one point in common.

Note that two distinct lines may not have a point in common.

Illustrative Examples

Example 1. *Which of the following statements are true and which are false? Give reasons for your answer.*

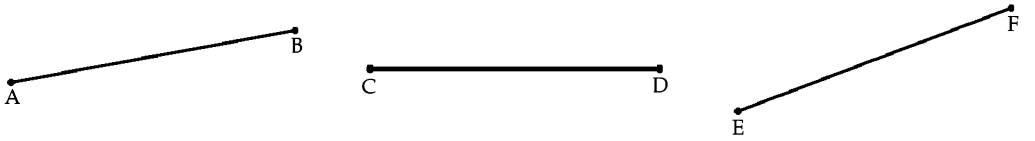
(i) Only one line can pass through a single point. *(NCERT)*

(ii) There are infinite number of lines which pass through two distinct points. *(NCERT)*

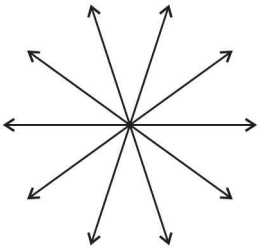
(iii) A terminated line can be produced indefinitely in both directions. *(NCERT)*

(iv) If two circles are equal, then their radii are equal. *(NCERT)*

(v) In the figure given below, if AB = CD and CD = EF, then AB = EF. *(NCERT)*



Solution. (*i*) False; in fact, many lines pass through a given point as shown in the adjoining figure.



(*ii*) False; because there is one and only one line which passes through two distinct points (Euclid's axiom 3.1).

(*iii*) True; it is Euclid's second postulate.

(*iv*) True; because if we superimpose the region bounded by one circle on the other, then they coincide. So, their centres and boundaries coincide. Therefore, their radii will coincide and hence they are equal.

(*v*) True; it is Euclid's Ist axiom.

Example 2. *Solve the equation $a - 15 = 25$ and state which axiom do you use here.* *(NCERT Exemplar)*

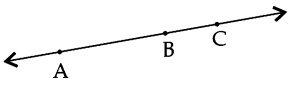
Solution. Given $a - 15 = 25$.

Adding 15 to both sides, we get

(using Euclid's second axiom *i.e.* if equals are added to equals, the wholes are equal)

$a - 15 + 15 = 25 + 15 \Rightarrow a = 40$.

Example 3. *If A, B and C are three points on a line, and B lies between A and C (shown in adjoining figure), then prove that AB + BC = AC.* *(NCERT)*



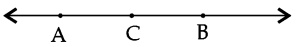
Solution. From the given figure, we note that AC coincides with AB + BC. Therefore, by Euclid's fourth axiom *i.e.* things which coincide with one another are equal to one another, it follows that AB + BC = AC.

Example 4. *(i) If a point C lies between points A and B such that AC = CB, then prove that* $AC = \frac{1}{2} AB$. *Explain by drawing the figure.* *(NCERT)*

(ii) Prove that every line segment has one and only one mid-point. *(NCERT)*

Solution. (*i*) Given the point C lies between A and B such that AC = CB.

Draw the figure as shown.



Adding AC to both sides, we get

(using Euclid's second axiom *i.e.* if equals are added to equals, wholes are equal)

$AC + AC = AC + CB$

$\Rightarrow$ $2AC = AB$ (AC + CB coincides with AB, axiom 1)

$\Rightarrow$ $AC = \frac{1}{2} AB$.

(*ii*) Point C is called mid-point of line segment AB. Thus, every line segment has atleast one mid-point.

Let C, D be two mid-points of the line segment AB.

Then by using part (*i*), we get

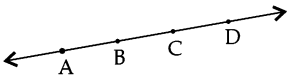
$AC = \frac{1}{2} AB$ and $AD = \frac{1}{2} AB$

$\Rightarrow$ $AC = AD$ (using axiom 1)

$\Rightarrow$ Points C and D coincide.

Hence, every line segment has one and only one mid-point.

Example 5. *In the adjoining figure, if AC = BD then prove that AB = CD.* *(NCERT)*



Solution. From given figure, point B lies between points A and C, point C lies between points B and D,

$\therefore$ AC = AB + BC and BD = BC + CD.

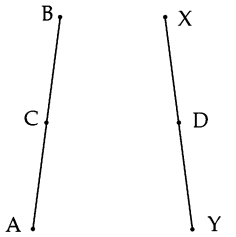
Given that AC = BD

$\Rightarrow$ AB + BC = BC + CD

By using Euclid's third axiom *i.e.* if equals are subtracted from equals then the remainders are equal, we get

AB = CD (subtracting BC)

Example 6. *In the adjoining figure, AC = XD, C is mid-point of AB and D is mid-point of XY. Using an Euclid's axiom, prove that AB = XY.* *(NCERT Exemplar)*



Solution. As C is mid-point of line segment AB, AB = 2AC.

Also, D is mid-point of line segment XY,

XY = 2XD

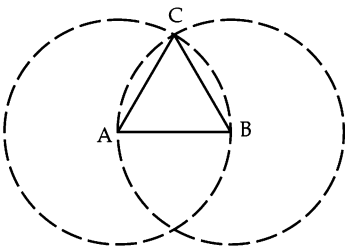
$\Rightarrow$ XY = 2AC ($\because$ XD = AC, given)

By using Euclid's sixth axiom *i.e.* things which are doubles of same thing are equal to one another, we have

AB = XY.

Example 7. *Prove that an equilateral triangle can be constructed on any given line segment.* *(NCERT)*

Solution. Let AB be a given line segment. Draw a circle with A as centre and AB as radius. Draw another circle with B as centre and AB as radius. Let these circles meet at C. Draw line segments AC and BC to form a triangle ABC.



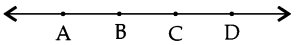
Now, AC = AB (radii of same circle)

BC = AB (radii of same circle)

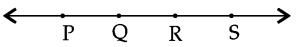
$\therefore$ AC = BC (Euclid's 1st axiom)

Exercise 3

1. In the adjoining figure, if AB = CD, then prove that AC = BD.

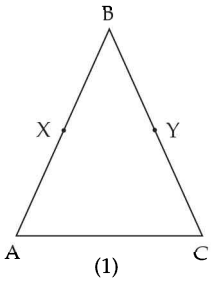


2. If AB = PQ and PQ = XY, then prove that AB = XY. Explain by drawing the figure.
3. In the adjoining figure, if PQ = QR = RS then prove that PQ = $\frac{1}{3}$ PS.

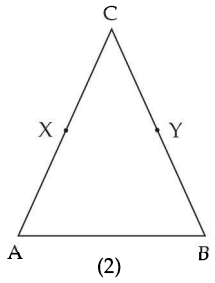


4. Why is axiom 5, in the list of Euclid's axioms, considered a universal truth? *(NCERT)*
5. Does Euclid's fifth postulate imply the existence of parallel lines? Explain. *(NCERT)*
6. It is known that $x + y = 10$ and $x = z$. Show that $y + z = 10$. *(NCERT Exemplar)*
7. (*i*) In figure (1) given below, if AB = BC and BX = BY then show that AX = CY. *(NCERT Exemplar)*

(*ii*) In figure (2) given below, if X and Y are mid-points of AB and BC and AX = CY then show that AC = BC. *(NCERT Exemplar)*



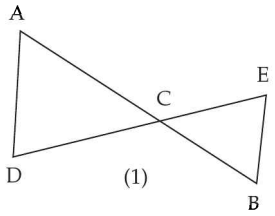
(1)



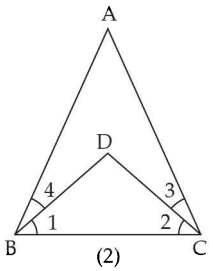
(2)

8. (*i*) In figure (1) given below, if AC = DC and CB = CE then show that AB = DE. *(NCERT Exemplar)*

(*ii*) In figure (2) given below, if $\angle ABC = \angle ACB$ and $\angle 3 = \angle 4$ then show that $\angle 1 = \angle 2$. *(NCERT Exemplar)*



(1)



(2)

Answers

Objective Type Questions

(FOR FORMATIVE/SUMMATIVE ASSESSMENT)

Mental Maths

1. Fill in the blanks:

(*i*) Euclid's 4th axiom "Things which coincide with one another are equal to one another" seems to say that if two things are identical, then they are ..... in all respects.

(*ii*) Euclid's 5th axiom "The whole is greater than the part" gives us the definition of "..... ."

(*iii*) The statements which are 'obvious universal truths' are called ..... or ..... .

(*iv*) The assumptions which are specifically linked to geometry are called ..... .

(*v*) The statements which can be proved by using axiom(s) or other statements already proved are called ..... .

(*vi*) Things which are halves of the same thing are ..... to one another.

(*vii*) All attempts to prove Euclid's 5th postulate failed but they led to the discovery of many other geometries, called ..... geometries.

2. Which of the following statements are true and which are false? Justify your answer.

(*i*) In geometry, we take a point, a line and a plane as undefined terms. *(NCERT Exemplar)*

(*ii*) The Euclidean geometry is valid only for figures in a plane. *(NCERT Exemplar)*

(*iii*) The whole is smaller than the part.

(*iv*) The boundaries of the solids are curves. *(NCERT Exemplar)*

(*v*) The edges of a surface are curves. *(NCERT Exemplar)*

(*vi*) If a quantity B is a part of another quantity A, then A can be written as the sum of B and some third quantity C. *(NCERT Exemplar)*

(*vii*) Two distinct intersecting lines cannot be parallel to the same line. *(NCERT Exemplar)*

Multiple Choice Questions

Choose the correct answer from the given four options (3 to 17):

3. Through two distinct points, there

(*a*) is no line that passes through them

(*b*) is a unique line that passes through them

(*c*) are two lines that pass through them

(*d*) are more than two lines that pass through them.

4. For every line l and for every point P not lying on l, there

(*a*) is no line passing through P and parallel to l

(*b*) is a unique line passing through P and parallel to l

(*c*) are two lines passing through P and parallel to l

(*d*) are infinitely many lines passing through P and parallel to l.

5. The things which are double of the same thing are

(*a*) equal (*b*) unequal

(*c*) halves of the same thing (*d*) double of the same thing

(NCERT Exemplar)

6. Euclid's fifth postulate is :

(*a*) The whole is greater than the part.

(*b*) A circle may be described with any centre and any radius.

(*c*) All right angles are equal to one another.

(*d*) If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines if produced indefinitely, meet on that side on which the sum of angles is less than two right angles. *(NCERT Exemplar)*

7. Axioms are assumed

(*a*) universal truths in all branches of mathematics

(*b*) universal truths specific to geometry

(*c*) theorems

(*d*) definitions *(NCERT Exemplar)*

8. Euclid belonged to the country:

(*a*) Babylonia (*b*) Egypt (*c*) Greece (*d*) India

(NCERT Exemplar)

9. In Ancient India, Altars with combinations of shapes like rectangles, triangles and trapeziums were used for:

(*a*) Public worship (*b*) Household rituals

(*c*) Both (*a*) and (*b*) (*d*) None of (*a*), (*b*) and (*c*)

(NCERT Exemplar)

10. The number of dimensions, a solid has:

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 0

(NCERT Exemplar)

11. The number of dimensions, a plane surface has:

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 0

(NCERT Exemplar)

12. The number of dimensions, a point has:

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

(NCERT Exemplar)

13. Boundaries of solid are:

(*a*) surfaces (*b*) curves (*c*) lines (*d*) points

(NCERT Exemplar)

14. Boundaries of surfaces are:

(*a*) surfaces (*b*) curves (*c*) lines (*d*) points

(NCERT Exemplar)

15. It is known if $x + y = 10$ then $x + y + z = 10 + z$. The Euclid's axiom that illustrates this statement is

(*a*) first axiom (*b*) second axiom (*c*) third axiom (*d*) fourth axiom

(NCERT Exemplar)

16. Which of the following needs a proof?

(*a*) Theorem (*b*) Axiom (*c*) Definition (*d*) Postulate

(NCERT Exemplar)

17. Euclid stated that all right angles are equal to each other in the form of

(*a*) an axiom (*b*) a definition (*c*) a postulate (*d*) a proof

(NCERT Exemplar)

Answers

Lines and Angles



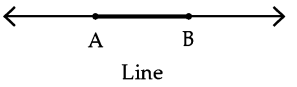
INTRODUCTION

In chapter 3, you have learnt that a unique line can be drawn passing through two distinct points and some basic terms of geometry – point, line and plane. You also studied some axioms and postulates and proved some other statements. In this chapter, we shall study some basic concepts and terms of geometry – line segment, ray, angle and types of angles. We shall learn about some pairs of related angles – complementary angles, supplementary angles, adjacent angles, linear pair, sum of angles at a point, sum of angles at a point on one side of a straight line and vertically opposite angles. We shall also learn about pairs of intersecting and parallel lines, transversal, angles made by a transversal and properties of angles associated with a pair of parallel lines. Further, we shall also learn angle sum property of a triangle.

4.1 BASIC TERMS AND DEFINITIONS

Recall, the basic concept of a line is its straightness and it extends indefinitely in both directions *i.e.* it has no definite length and it has no end points.

There is exactly one line passing through two distinct given points A and B in a plane. Also, if two distinct points of a line lie in a plane, then the line lies wholly in that plane.

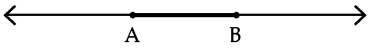


Line

The line is denoted by $\overleftrightarrow{AB}$. Sometimes small letters l, m, n etc. are used to denote lines.

- **Line segment**

  The portion of a line l between the points A and B (including the points A and B) is called the **line segment** AB, and A, B are called its end points.



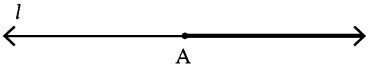
Line segment

  The line segment AB is denoted by $\overline{AB}$.

  There is exactly one line segment joining two given points, so a line segment is known completely if its end points are known.

- **Ray**

  By a ray we mean the part of line l that extends indefinitely in one direction from a point, say A, including that point. The point A is called the *initial* (or *end*) point of the ray. Thus, a ray has initial point as well as direction. A ray has only one end point, and it has unlimited length.



Ray

There is exactly one ray which has A as its initial point and passes through a given point B. It is denoted by $\overrightarrow{AB}$.

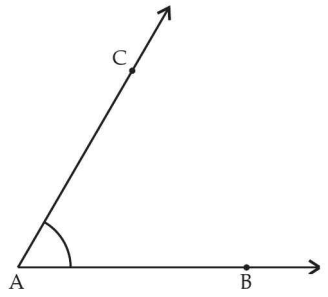
For convenience, we shall use the same symbol AB for the four concepts — line AB, line segment AB, length of line segment AB and ray AB.

❑ **Angle**

The adjoining diagram shows an angle.

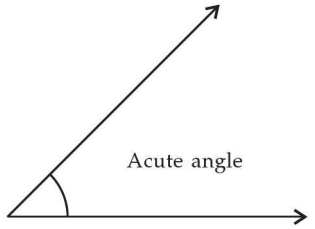
An ***angle*** is formed by two rays having same initial point.

The initial point is called its ***vertex*** and the two rays are called its ***arms.*** Note that each arm has a particular direction. The angle is named as ∠A or ∠CAB or ∠BAC.

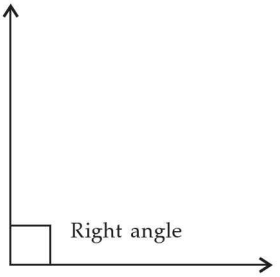


❑ **Types of Angles**

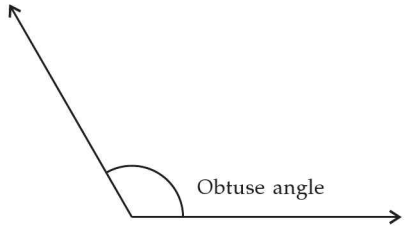
Acute angle — an angle whose measure lies between 0° and 90°.



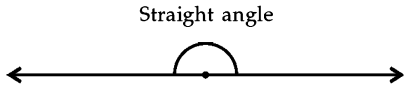
Right angle — an angle whose measure is 90°.



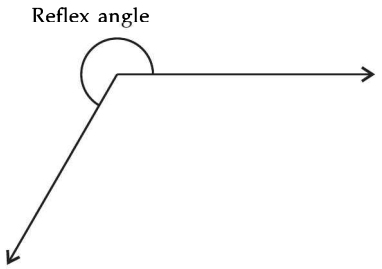
Obtuse angle — an angle whose measure lies between 90° and 180°.



Straight angle — an angle whose measure is 180°.

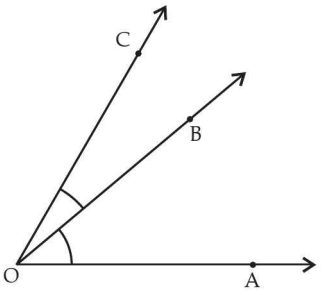


Reflex angle — an angle whose measure lies between 180°and 360°.



❑ **Adjacent angles**

Two angles are called ***adjacent angles*** if they have a common vertex, a common arm and their non-common arms lie on different sides of the common arm.



In the adjoining figure, ∠AOB and ∠BOC are adjacent angles. Ray OB is their common arm and point O is their common vertex. Ray OA and ray OC are their non-common arms.

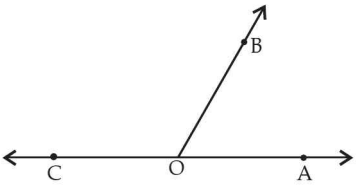
Note that ∠AOB and ∠AOC are not adjacent though these have a common arm OA.

Notice that ∠AOB + ∠BOC = ∠AOC.

This is known as ***addition axiom of adjacent angles***.

❑ **Linear pair of angles**

Two adjacent angles ∠AOB and ∠BOC are said to form a ***linear pair of angles*** if their non-common arms OA and OC form a straight line (as shown in the adjoining figure).



Note that ∠AOB + ∠BOC = ∠AOC.

As AOC is a straight line, ∠AOC is a straight angle, so ∠AOC = 180°.

Therefore, we can say that ∠AOB + ∠BOC = 180°.

We can state this result in the form of axiom as follows:

❑ **Axiom 4.1—*If a ray stands on a line, then the sum of two adjacent angles so formed is 180°.***

Converse is also true *i.e.* if the sum of two adjacent angles is 180°, then a ray stands on a line *i.e.* the non-common arms form a line. This result is stated in the form of an axiom as follows:

❑ **Axiom 4.2—*If the sum of two adjacent angles is 180°, then the non-common arms of the angles form a line.***

Axioms 4.1 and 4.2 taken together is called the ***linear pair axiom***.

❑ **Complementary angles**

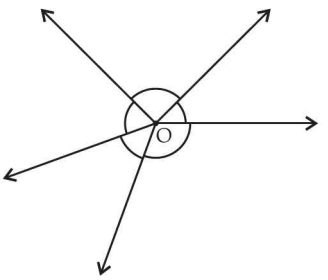
Two angles are called ***complementary angles*** if the sum of their measures is 90°. Each angle is called the complement of the other. Complementary angles need not be adjacent.

❑ **Supplementary angles**

Two angles are called ***supplementary angles*** if the sum of their measures is 180°. Each angle is called the supplement of the other. Supplementary angles need not be adjacent.

❑ **Angles at a point**

In the adjoining figure, the five angles together make one complete rotation, so they add upto 360°. This is true no matter how many angles are formed at a point. Thus:



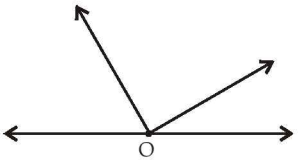
Sum of angles at a point = 360°.

Note that all the angles are drawn in the same plane.

❑ **Angles on one side of a line**

In the adjoining figure, the three angles together make a straight line, so they add upto 180°. This is true no matter how many angles make the straight line. Thus :

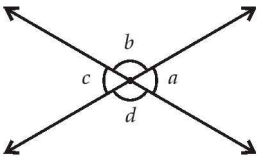
Sum of angles at a point on one side of a line = 180°.



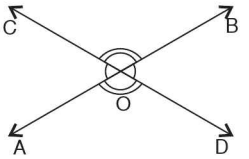
❑ **Vertically opposite angles**

When two lines intersect each other, they form four angles, say a, b, c and d.

Angles a and c are called vertically opposite angles (abbreviated vert. opp. ∠s), and so are angles b and d.



Theorem 4.1. *If two lines intersect each other, then the vertically opposite angles are equal.*



Given. Two lines AB and CD intersect each other at O.

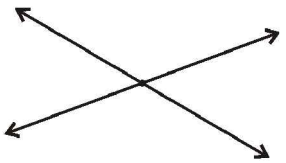
To prove. ∠AOC = ∠BOD and ∠COB = ∠AOD.

Proof.

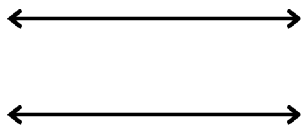
Statements	*Reasons*
1. ∠AOC + ∠COB = 180°	1. Ray OA stands on line AB, ∠AOC and ∠COB form a linear pair.
2. ∠COB + ∠BOD = 180°	2. Ray OB stands on line CD, ∠COB and ∠BOD form a linear pair.
3. ∠AOC + ∠COB = ∠COB + ∠BOD	3. From 1 and 2.
4. ∠AOC = ∠BOD Similarly, ∠COB = ∠AOD	4. ∠COB is common to both.

❑ **Intersecting and Parallel lines**

If two distinct lines are drawn in a plane, then they will intersect each other in a unique point or will be parallel as shown in the figures given below:



Intersecting lines



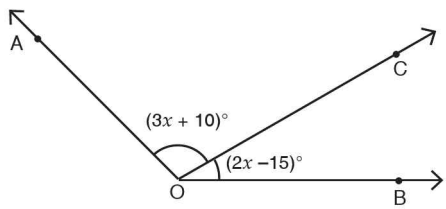
Parallel lines

Thus, two distinct lines drawn in a plane are parallel if and only if they are not intersecting.

Note that the lengths of the common perpendiculars at different points on parallel lines are the same *i.e.* the distance between parallel lines remains the same.

Illustrative Examples

Example 1. *In the adjoining figure, what value of x will make AOB a straight line?*



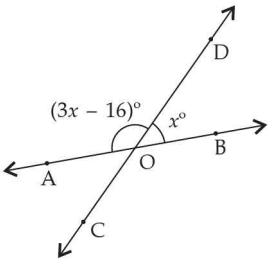
Solution. We observe that ∠BOC and ∠COA are adjacent angles. AOB will become straight line if ∠BOC + ∠COA = 180°

$\Rightarrow$ $(3x + 10) + (2x - 15) = 180$

$\Rightarrow$ $5x - 5 = 180 \Rightarrow 5x = 185$

$\Rightarrow$ $x = 37$.

Example 2. *In the adjoining figure, two lines AB and CD intersect at O. Find the value of x, and ∠AOC, ∠COB.*



Solution. As ray OD stands on line AB,

$\angle AOD + \angle DOB = 180°$ (linear pair of angles)

$\Rightarrow$ $(3x - 16) + x = 180$

$\Rightarrow$ $4x = 180 + 16 \Rightarrow 4x = 196$

$\Rightarrow$ $x = 49$, $3x - 16 = 3 \times 49 - 16 = 131$.

$\angle AOC = \angle BOD$ (vertically opposite angles)

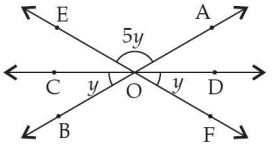
$\therefore$ $\angle AOC = 49°$.

Also $\angle COB = \angle AOD$ (vertically opposite angles)

$\therefore$ $\angle COB = 131°$.

Example 3. *In the adjoining figure, AB, CD and EF are three lines concurrent at O. Find the value of y.*

(NCERT Exemplar)



Solution. $\angle BOF = \angle AOE$ (vertically opposite angles)

But $\angle AOE = 5y$ (given)

$\therefore$ $\angle BOF = 5y$.

As the sum of angles at a point on one side of a line is 180°,

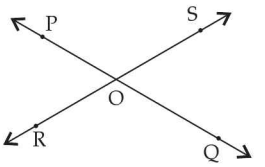
$\therefore$ $\angle COB + \angle BOF + \angle FOD = 180°$

$\Rightarrow$ $2y + 5y + 2y = 180°$

$\Rightarrow$ $9y = 180° \Rightarrow y = 20°$.

Example 4. *In the adjoining figure, lines PQ and RS intersect each other at the point O. If ∠POR : ∠ROQ = 5 : 7, find all the angles.*

(NCERT)



Solution. As ray OR stands on line PQ,

$\angle POR + \angle ROQ = 180°$ (linear pair of angles)

But $\angle POR : \angle ROQ = 5 : 7$ (given)

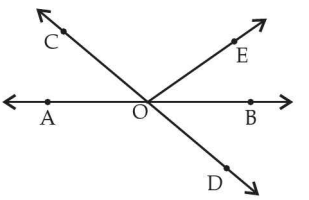
$\therefore$ $\angle POR = \frac{5}{12}$ of $180° = 75°$ and $\angle ROQ = \frac{7}{12}$ of $180° = 105°$.

$\angle QOS = \angle POR$ and $\angle SOP = \angle ROQ$ (vert. opp. ∠s)

$\therefore$ $\angle QOS = 75°$ and $\angle SOP = 105°$

Example 5. *In the adjoining figure, lines AB and CD intersect at O. If ∠AOC + ∠BOE = 70° and ∠BOD = 40°, then find ∠BOE and reflex ∠COE.*

(NCERT)



Solution. $\angle AOC = \angle BOD$ (vert. opp. ∠s)

$\Rightarrow$ $\angle AOC = 40°$ ($\because$ $\angle BOD = 40°$ given)

Now $\angle AOC + \angle BOE = 70°$ (given)

$\Rightarrow$ $40° + \angle BOE = 70° \Rightarrow \angle BOE = 30°$

As the sum of angles at a point on one side of a line is 180°,

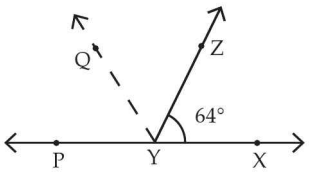
$\therefore$ $\angle AOC + \angle COE + \angle EOB = 180°$

$\Rightarrow$ $40° + \angle COE + 30° = 180° \Rightarrow \angle COE = 110°$

$\therefore$ Reflex $\angle COE = 360° - 110° = 250°$.

Example 6. *It is given that ∠XYZ = 64° and XY is produced to a point P. Draw a figure from the given information. If ray YQ bisects ∠ZYP, then find ∠XYQ and reflex ∠QYP.* *(NCERT)*

Solution. Since XY is produced to a point P, XYP is a straight line and the sum of angles at a point on one side of a line is 180°,



$\therefore$ $\angle ZYP + 64° = 180° \Rightarrow \angle ZYP = 116°$

As the ray YQ bisects ∠ZYP, $\angle ZYQ = \frac{1}{2}$ of $116° = 58°$

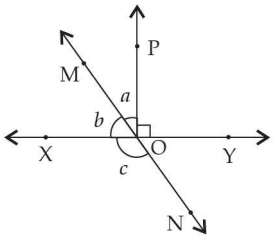
$\therefore$ $\angle XYQ = \angle XYZ + \angle ZYQ = 64° + 58° = 122°$.

As the ray YQ bisects ∠ZYP, $\angle QYP = \angle ZYQ$

$\Rightarrow$ $\angle QYP = 58°$ $(\because \angle ZYQ = 58°)$

$\therefore$ Reflex $\angle QYP = 360° - 58° = 302°$.

Example 7. *In the adjoining figure, lines XY and MN intersect at O. If ∠POY = 90° and a : b = 2 : 3, find c.* *(NCERT)*



Solution. As XOY is a straight line and the sum of angles at a point on one side of a line is 180°,

$\therefore$ $90° + a + b = 180° \Rightarrow a + b = 90°$.

Given $a : b = 2 : 3$,

$\therefore$ $a = \frac{2}{5}$ of $90° = 36°$ and $y = \frac{3}{5}$ of $90° = 54°$.

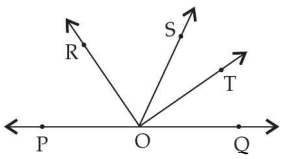
As the ray OX stands on the line MN,

$b + c = 180°$ (linear pair of angles)

$\Rightarrow$ $54° + c = 180°$

$\Rightarrow$ $c = 180° - 54° \Rightarrow c = 126°$.

Example 8. *In the adjoining figure, ray OS stands on the line PQ. If ray OR and ray OT are angle bisectors of ∠POS and ∠SOQ respectively, then find ∠ROT.* *(NCERT)*



Solution. Let $\angle QOS = x°$.

As ray OS stands on line PQ,

$\angle POS + \angle QOS = 180°$ (linear pair of angles)

$\Rightarrow$ $\angle POS + x° = 180° \Rightarrow \angle POS = 180° - x°$ ...(*i*)

As ray OT is bisector of ∠QOS, $\angle SOT = \frac{1}{2} \angle QOS$

$\Rightarrow$ $\angle SOT = \frac{1}{2}x°$.

Also, as ray OR is bisector of ∠POS, $\angle ROS = \frac{1}{2} \angle POS$

$\Rightarrow$ $\angle ROS = \frac{1}{2}(180° - x°)$ (using (*i*))

$\angle ROT = \angle ROS + \angle SOT$ (from figure)

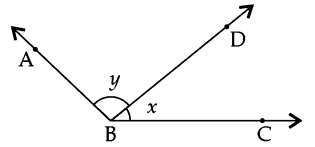
$\Rightarrow$ $\angle ROT = \frac{1}{2}x° + \frac{1}{2}(180° - x°) = \frac{1}{2}x° + 90° - \frac{1}{2}x°$

$\Rightarrow$ $\angle ROT = 90°$.

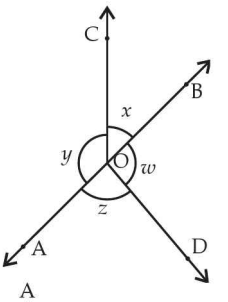
Exercise 4.1

1. In the adjoining figure, for what value of $x + y$ will ABC a straight line? Justify your answer.

(NCERT Exemplar)

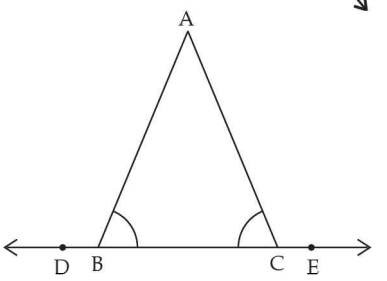


2. In the adjoining figure, if $x + y = z + w$, then prove that AOB is a straight line. *(NCERT)*



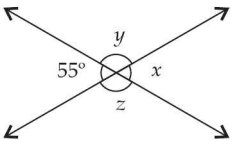
Hint: Sum of angles at a point = 360°.

3. In the adjoining figure, if ∠ABC = ∠ACB, then prove that ∠ABD = ∠ACE. *(NCERT)*

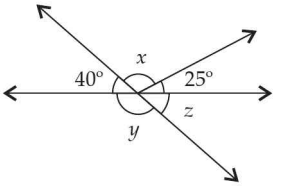


Hint: ∠ABC + ∠ABD (linear pair of angles).

4. Find the values of x, y and z in each of the following figures:

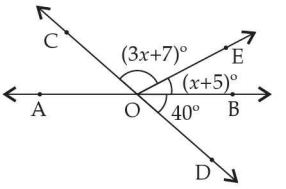


(i)

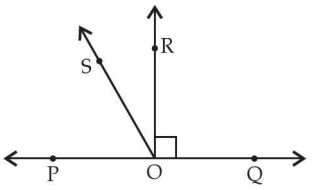


(ii)

5. In the adjoining figure, lines AB and CD intersect at O. Find x, ∠AOC, ∠AOD and ∠COE.



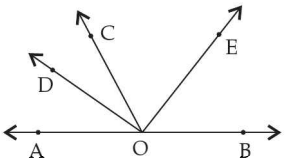
6. In the adjoining figure, ray OR is perpendicular to the line PQ. OS is another ray standing on line PQ. Prove that $\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$. *(NCERT)*



Hint: ∠QOS = 90° + ∠ROS and

∠POS = 90° − ∠ROS. Subtract.

7. In the adjoining figure, OD is bisector of ∠AOC, OE is bisector of ∠BOC and OD ⊥ OE. Show that the points A, O and B are collinear. *(NCERT Exemplar)*



Hint: ∠AOC = 2 ∠COD and ∠BOC = 2 ∠EOC.

∠COD + ∠EOC = ∠DOE = 90°

⇒ ∠AOC + ∠BOC = 2 × 90° = 180°

⇒ AOB is a straight line (axiom 4.2)

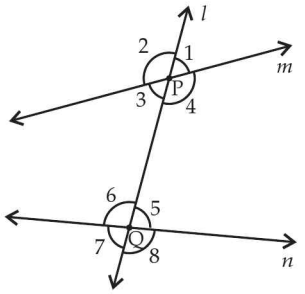
Answers

4.2 TRANSVERSAL AND PARALLEL LINES

❑ **Transversal**

A line that intersects two (or more) lines in a plane at different points is called a ***transversal.***

In the adjoining figure, line l intersects two lines m and n at distinct points P and Q, so line l is a transversal.



Note that four angles are formed at P and four angles are formed at Q. For convenience, we label them by the numerals 1 to 8 (as shown in the adjoining figure).

Classification of angles formed by a transversal

(*i*) The angles $\angle 3$, $\angle 4$, $\angle 5$ and $\angle 6$ are called ***interior angles***.

(*ii*) The angles $\angle 1$, $\angle 2$, $\angle 7$ and $\angle 8$ are called ***exterior angles***.

(*iii*) The four pairs $\angle 1$, $\angle 5$; $\angle 2$, $\angle 6$; $\angle 3$, $\angle 7$ and $\angle 4$, $\angle 8$ are called pairs of ***corresponding angles*** (abbreviated corres. $\angle$s).

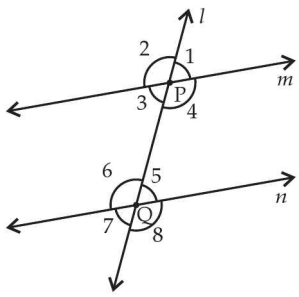
(*iv*) The two pairs $\angle 3$, $\angle 5$ and $\angle 4$, $\angle 6$ are called ***alternate interior angles*** or simply the ***alternate angles*** (abbreviated alt. $\angle$s).

The two pairs $\angle 1$, $\angle 7$ and $\angle 2$, $\angle 8$ are called ***alternate exterior angles*** (abbreviated alt. ext. $\angle$s).

(*v*) The two pairs $\angle 3$, $\angle 6$ and $\angle 4$, $\angle 5$ are pairs of angles of ***interior angles on the same side of the transversal***. These two pairs of angles are also called ***co-interior*** or ***consecutive interior*** or ***conjoined*** or ***allied angles*** (abbreviated co-int $\angle$s).

❑ **Properties of angles associated with parallel lines**

Let a transversal l intersect two parallel lines m and n at distinct points P and Q. Eight angles are formed—four at P and four at Q. For convenience, we label these angles by the numerals 1 to 8 (as shown in the adjoining figure).



Let us find out relations between pairs of angles when the lines m and n are parallel.

Now, measure any pair of corresponding angles and find out the relation between them. It will be found that $\angle 1 = \angle 5$, $\angle 2 = \angle 6$, $\angle 3 = \angle 7$ and $\angle 4 = \angle 8$. These results can be verified by an activity also. We state this result in the form of an axiom as follows:

Axiom 4.3—***If a transversal intersects two parallel lines, then each pair of corresponding angles is equal.***

This axiom is known as ***corresponding angles axiom***.

Converse is also true *i.e.* if a pair of corresponding angles is equal, then the two lines m and n are parallel. We state this result in the form of an axiom as follows :

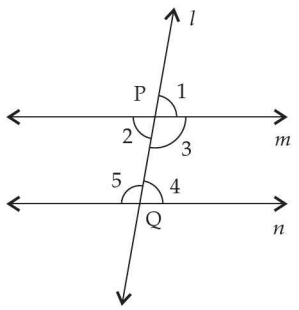
Axiom 4.4—*If a transversal intersects two lines such that a pair of corresponding angles is equal, then the two lines are parallel.*

The above two axioms taken together can be stated as :

If a transversal intersects two lines, then the lines are parallel if and only if each pair of corresponding angles is equal.

By using corresponding angles axiom and its converse, we can find out relations between pairs of alternate interior angles when a transversal intersects two parallel lines. In fact, we have :

Theorem 4.2. ***If a transversal intersects two parallel lines, then each pair of alternate interior angles is equal.***



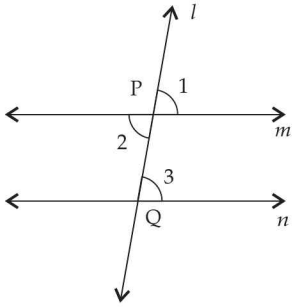
Given. Two parallel lines m and n. A transversal l intersects lines m and n at points P and Q respectively.

To prove. $\angle 2 = \angle 4$ and $\angle 3 = \angle 5$.

Proof.

Statements	*Reasons*
1. $\angle 1 = \angle 4$	1. Axiom 4.3; transversal l intersects two parallel lines m and n, each pair of corresponding angle is equal.
2. $\angle 1 = \angle 2$	2. Vertically opposite angles are equal.
3. $\angle 2 = \angle 4$ Similarly, $\angle 3 = \angle 5$	3. Using 1 and 2.

Converse is also true. In fact, we have:



Theorem 4.3. ***If a transversal intersects two lines such that a pair of alternate interior angles is equal, then the two lines are parallel.***

Given. A transversal l intersects two lines m and n at points P and Q respectively such that a pair of alternate interior angles is equal *i.e.* $\angle 2 = \angle 3$.

To prove. $m \parallel n$.

Proof.

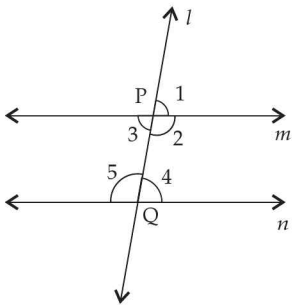
Statements	*Reasons*
1. $\angle 1 = \angle 2$	1. Vertically opposite angles are equal.
2. $\angle 2 = \angle 3$	2. Given.
3. $\angle 1 = \angle 3$	3. Using 1 and 2.
4. $m \parallel n$	4. Axiom 4.4; transversal l intersects lines m and n such that a pair of corresponding angles is equal, so lines m and n are parallel.

The above two theorems taken together can be stated as :

If a transversal intersects two lines, then the lines are parallel if and only if each pair of alternate interior angles is equal.

By using corresponding angles axiom and its converse, we can find out relations between pairs of interior angles on the same side of transversal when it intersects two parallel lines.

In fact, we have:



Theorem 4.4. ***If a transversal intersects two parallel lines, then each pair of interior angles on the same side of the transversal is supplementary.***

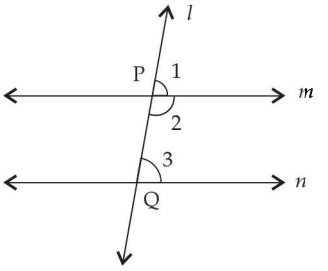
Given. Two parallel lines m and n. A transversal l intersects lines m and n at points P and Q.

To prove. $\angle 2 + \angle 4 = 180°$ and $\angle 3 + \angle 5 = 180°$

Proof.

Statements	*Reasons*
1. $\angle 1 = \angle 4$	1. Axiom 4.3; transversal l intersects two parallel lines m and n, each pair of corresponding angles is equal.
2. $\angle 2 + \angle 1 = 180°$	2. Linear pair axiom.
3. $\angle 2 + \angle 4 = 180°$ Similarly, $\angle 3 + \angle 5 = 180°$	3. Using 1 and 2.

Converse is also true. In fact, we have:



Theorem 4.5. ***If a transversal intersects two lines such that a pair of interior angles on the same side of the transversal is supplementary, then the two lines are parallel.***

Given. A transversal l intersects two lines m and n at points P and Q respectively such that a pair of interior angles on the same side of l is supplementary *i.e.* $\angle 2 + \angle 3 = 180°$.

To prove. $m \parallel n$

Proof.

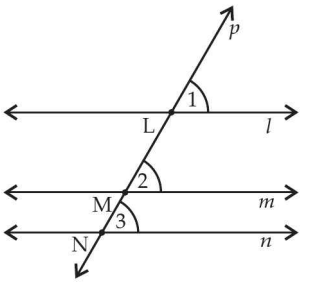
Statements	*Reasons*
1. $\angle 1 + \angle 2 = 180°$	1. Linear pair axiom.
2. $\angle 2 + \angle 3 = 180°$	2. Given.
3. $\angle 1 + \angle 2 = \angle 2 + \angle 3$	3. Using 1 and 2
4. $\angle 1 = \angle 3$	4. $\angle 2$ is common
5. $m \parallel n$	5. Axiom 4.4; transversal l intersects two lines m and n such that a pair of corresponding angles is equal, so lines m and n are parallel.

The above two theorems taken together can be stated as:

If a transversal intersects two lines, then the lines are parallel if and only if each pair of interior angles on the same side of the transversal is supplementary.

❑ **Lines parallel to the same line**

If two lines are parallel to the same line, then will the lines be themselves parallel? In fact, we have:



Theorem 4.6. ***If two lines are parallel to the same line, then the lines are themselves parallel.***

Given. Two lines m and n are parallel to the same line l *i.e.* $m \parallel l$ and $n \parallel l$.

To prove. $m \parallel n$.

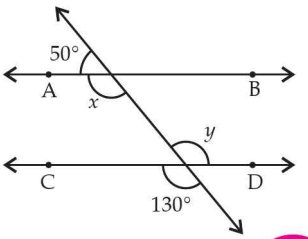
Construction. Draw a transversal p intersecting the lines l, m and n at the points L, M and N respectively.

Proof.

Statements	*Reasons*
1. $\angle 1 = \angle 2$	1. $m \parallel l$, corresponding angles are equal.
2. $\angle 1 = \angle 3$	2. $n \parallel l$, corresponding angles are equal.
3. $\angle 2 = \angle 3$	3. From 1 and 2.
4. $m \parallel n$	4. Transversal p intersects lines m and n such that a pair of corresponding angles is equal, so lines m and n are parallel.

Illustrative Examples

Example 1. *In the adjoining figure, find the values of x and y and then show that AB ∥ CD.* *(NCERT)*



Solution. $x + 50° = 180°$ (linear pair)

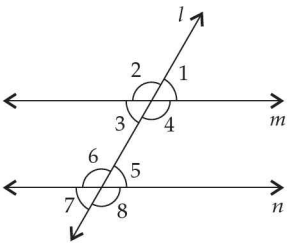
$\Rightarrow \quad x = 180° - 50° = 130°.$

$y = 130°$ (vert. opp. $\angle$s)

$\Rightarrow$ $x = y$ (each = 130°)

Thus a transversal intersects two lines AB and CD such that a pair of alternate interior angles is equal, therefore, AB || CD (Theorem 4.3).

Example 2. *In the adjoining figure, ∠1 = 60° and ∠6 = 120°. Show that lines m and n are parallel.* *(NCERT Exemplar)*



Solution. $\angle 3 = \angle 1$ (vert. opp. ∠s)

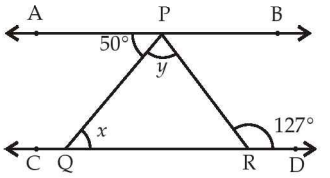
$\Rightarrow$ $\angle 3 = 60°$ ($\because$ $\angle 1 = 60°$, given)

$\therefore$ $\angle 3 + \angle 6 = 60° + 120°$ ($\because$ $\angle 6 = 120°$, given)

$\Rightarrow$ $\angle 3 + \angle 6 = 180°$

Thus a transversal l intersects two lines m and n such that a pair of interior angles on the same side is supplementary, therefore, the lines m and n are parallel (Theorem 4.5).

Example 3. *In the adjoining figure, AB || CD. If ∠APQ = 50° and ∠PRD = 127°, find x and y.* *(NCERT)*



Solution. Since AB || CD and transversal PQ meets them,

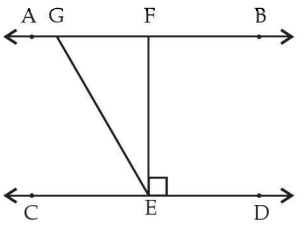
$\angle x = 50°$ (alt. int. ∠s)

As AB || CD and transversal PR meets them,

$y + 50° = 127°$ (alt. int. ∠s)

$\Rightarrow$ $y = 127° - 50° = 77°$.

Example 4. *In the adjoining figure, AB || CD and EF ⊥ CD. If ∠GED = 126°, find ∠AGE, ∠GEF and ∠FGE.* *(NCERT)*



Solution. Since AB || CD and transversal EG meets them,

$\angle AGE = \angle GED$ (alt. int. ∠s)

$\Rightarrow$ $\angle AGE = 126°$ ($\because$ $\angle GED = 126°$, given)

As EF ⊥ CD, $\angle FED = 90°$.

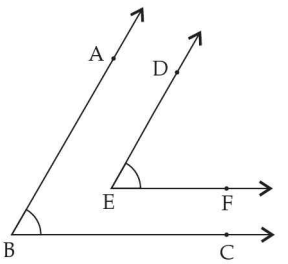
From fig., $\angle GEF = \angle GED - \angle FED = 126° - 90° = 36°$.

Now $\angle FGE + \angle AGE = 180°$ (linear pair)

$\Rightarrow$ $\angle FGE + 126° = 180°$ ($\because$ $\angle AGE = 126°$, proved above)

$\Rightarrow$ $\angle FGE = 180° - 126° = 54°$.

Example 5. *In the adjoining figure, BA || ED and BC || EF. Show that ∠ABC = ∠DEF.* *(NCERT Exemplar)*



Solution. Produce DE to intersect BC at P.

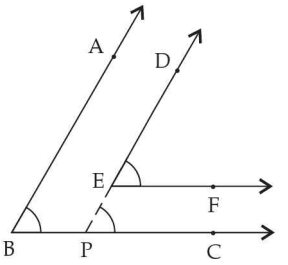
Since BA || ED *i.e.* BA || PD and BC is a transversal.

$\angle ABC = \angle DPC$ (corres. ∠s)

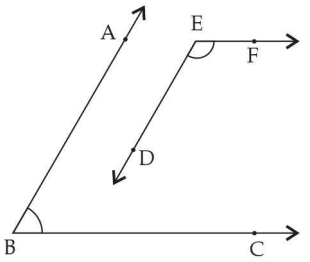
As BC || EF and PD is a transversal,

$\angle DEF = \angle DPC$.

$\therefore$ $\angle ABC = \angle DEF$ ($\because$ each = $\angle DPC$)



Example 6. *In the adjoining figure, BA || ED and BC || AF. Show that ∠ABC + ∠DEF = 180°.* *(NCERT Exemplar)*



Solution. Produce ED to intersect BC at P.

Since BA || ED and BC is a transversal,

$\angle ABC = \angle EPC$ (corres. ∠s)

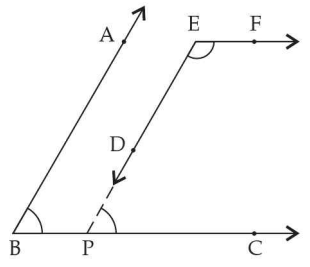
As BC || EF and EP is a transversal,

$\angle EPC + \angle PEF = 180°$

(pair of interior angles on same side of transversal)

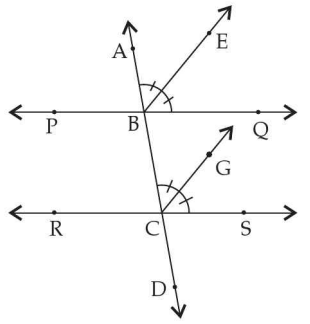
$\Rightarrow \angle ABC + \angle DEF = 180°$

($\because \angle EPC = \angle ABC$ and $\angle DEF = \angle PEF$)



Example 7. *If a transversal intersects two lines such that the bisectors of a pair of corresponding angles are parallel, then prove that the two lines are parallel.* *(NCERT)*

Solution. Let a transversal AD intersect two lines PQ and RS at the points B and C respectively. Ray BE is the bisector of ∠ABQ and Ray CG is the bisector of ∠BCS; and BE || CG.



We want to prove that PQ || RS.

Since BE || CG and AD is a transversal,

$\angle ABE = \angle BCG$ ...(*i*) (corresponding angles are equal)

As BE is bisector of ∠ABQ, $\angle ABE = \frac{1}{2}\angle ABQ$ ...(*ii*)

Also, as CG is bisector of ∠BCS, $\angle BCG = \frac{1}{2}\angle BCS$ ...(*iii*)

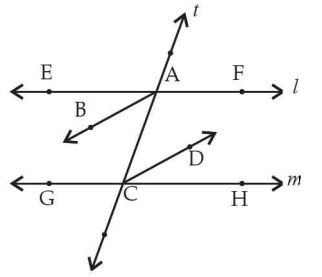
From (*i*), (*ii*) and (*iii*), we get

$\Rightarrow \frac{1}{2}\angle ABQ = \frac{1}{2}\angle BCS \Rightarrow \angle ABQ = \angle BCS.$

But ∠ABQ and ∠BCS are the corresponding angles formed by transversal AD with lines PQ and RS, therefore, PQ || RS.

Example 8. *In the adjoining figure, AB and CD are the bisectors of two alternate interior angles formed by the intersection of a transversal 't' with parallel lines l and m. Show that AB || CD.*

(NCERT Exemplar)



Solution. As $l \parallel m$ and t is a transversal,

$\angle EAC = \angle ACH$ (alt. int. ∠s are equal)

$\Rightarrow \frac{1}{2}\angle EAC = \frac{1}{2}\angle ACH$ ...(*i*)

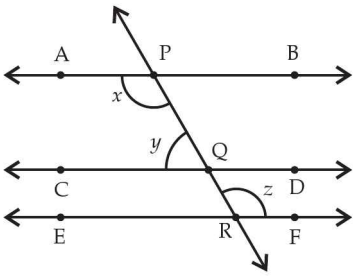
As AB and CD are bisectors of alternate interior angles *i.e.* of ∠EAC and ∠ACH respectively,

$\angle BAC = \frac{1}{2}\angle EAC$ and $\angle ACD = \frac{1}{2}\angle ACH$

$\Rightarrow \angle BAC = \angle ACD$ (using (*i*))

But ∠BAC and ∠ACD form a pair of alternate interior angles for AB and CD where t is a transversal, therefore, AB || CD.

Example 9. *In the adjoining figure, AB ‖ CD and AB ‖ EF. If y : z = 3 : 7, then find x.* *(NCERT)*



Solution. Since AB ‖ CD and AB ‖ EF,

$\therefore$ CD ‖ EF

(lines parallel to same line are themselves parallel)

$\angle PQD = \angle QRF$ (corres. $\angle$s are equal)

$\Rightarrow$ $\angle PQD = z$ ($\because$ $\angle QRF = z$, given)

Also $\angle CQP + \angle PQD = 180°$ (linear pair)

$\Rightarrow$ $y + z = 180°$.

But $y : z = 3 : 7$, so $z = \frac{7}{10}$ of $180° = 126°$.

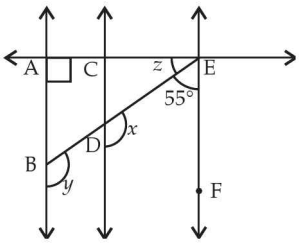
As AB ‖ EF and PR is a transversal,

$\therefore$ $\angle APQ = \angle QRF$ (alt. int. $\angle$s)

$\Rightarrow$ $x = z$

$\Rightarrow$ $x = 126°$ ($\because$ $z = 126°$)

Example 10. *In the adjoining figure, AB ‖ CD and AB ‖ EF. If EA ⊥ BA and ∠BEF = 55°, find the values of x, y and z.* *(NCERT)*



Solution. Since AB ‖ CD and AB ‖ EF,

$\therefore$ CD ‖ EF (lines parallel to same line)

$x + 55° = 180°$

(interior angles on the same side of transversal ED)

$\Rightarrow$ $x = 125°$.

As AB ‖ CD and BE is a transversal,

$y = x$ (corres. $\angle s$ are equal)

$\Rightarrow$ $y = 125°$ ($\because$ $x = 125°$)

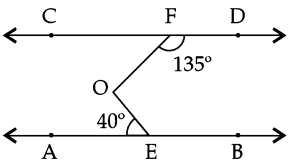
As AB ‖ EF and AE is a transversal,

$\angle BAE + \angle AEF = 180°$ (interior angles on same side of transversal)

$\Rightarrow$ $90° + z + 55° = 180°$

$\Rightarrow$ $z = 180° - 90° - 55° \Rightarrow z = 35°$.

Example 11. *In the adjoining figure, AB ‖ CD, ∠OFD = 135° and ∠OEA = 40°, find ∠EOF.* *(NCERT)*



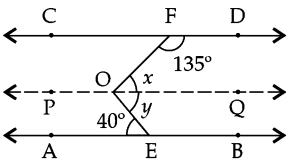
Solution. Through O, draw a line PQ parallel to AB. Mark the angles as shown in the adjoining figure.

As AB ‖ CD (given) and AB ‖ PQ (by construction)

PQ ‖ CD (lines parallel to same line)

Now AB ‖ PQ and OE is a transversal,

$y = 40°$ (alt. int. $\angle$s)



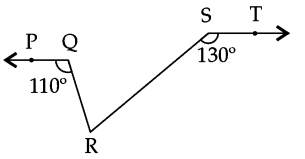
Also PQ ‖ CD and OF is a transversal,

$x + 135° = 180°$ (interior angles on the same side of transversal)

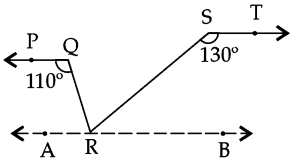
$\Rightarrow$ $x = 180° - 135° \Rightarrow x = 45°$.

$\therefore$ $\angle EOF = y + x = 40° + 45° = 85°$.

Example 12. *In the adjoining figure, PQ ‖ ST, ∠PQR = 110° and ∠RST = 130°. Find ∠QRS.* *(NCERT)*



Solution. Through R, draw a line AB parallel to PQ.



As PQ || ST (given) and PQ || AB (by construction)

ST || AB (lines parallel to same line)

Now PQ || AB and QR is a transversal,

$\angle ARQ + 110° = 180°$

(interior angles on same side of transversal)

$\Rightarrow \angle ARQ = 70°$ ...(*i*)

Also ST || AB and RS is a transversal,

$\angle SRB + 130° = 180°$ (interior angles on same side of transversal)

$\Rightarrow \angle SRB = 50°$ ...(*ii*)

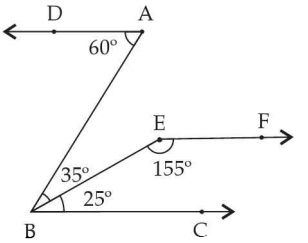
As ARB is a straight line,

$\angle ARQ + \angle QRS + \angle SRB = 180°$ (sum of angles at a point on same side of a line)

$\Rightarrow 70° + \angle QRS + 50° = 180°$ (using (*i*) and (*ii*))

$\Rightarrow \angle QRS = 180° - 70° - 50° \Rightarrow \angle QRS = 60°$.

Example 13. *In the adjoining figure, show that DA || EF.*



Solution. From given figure, we find that

$\angle ABC = 35° + 25° = 60°$

$\therefore \angle ABC = \angle DAB$ ($\because \angle DAB = 60°$, given)

So, a transversal AB intersects two lines DA and BC such that a pair of alternate interior angles is equal, therefore, DA || BC.

Also $\angle EBC + \angle BEF = 25° + 155° = 180°$.

$\Rightarrow$ $\angle EBC$ and $\angle BEF$ are supplementary.

Now, transversal BE intersect two lines BC and EF such that a pair of interior angles on the same side of transversal are supplementary, therefore, EF || BC.

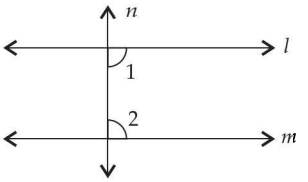
As DA || BC and EF || BC, DA || EF (lines parallel to same line)

Example 14. *If l, m and n are three lines such that l || m and n ⊥ l, then prove that n ⊥ m.*

Solution. Given *l*, *m* and *n* are three lines such that *l* || *m* and *n* ⊥ *l*.

Mark the angles as shown in figure.

As $n \perp l$, $\angle 1 = 90°$.



Now, *l* || *m* and *n* is a transversal,

$\therefore \angle 1 + \angle 2 = 180°$

(interior angles on the same side of transversal)

$\Rightarrow 90° + \angle 2 = 180°$ ($\because \angle 1 = 90°$)

$\Rightarrow \angle 2 = 90° \Rightarrow n \perp m$.

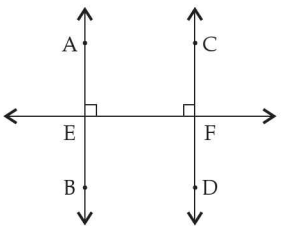
Example 15. *Prove that the lines which are perpendicular to the same line, are parallel to each other.*

Solution. Let the lines AB and CD be perpendicular to the same line EF.

As AB ⊥ EF, $\angle AEF = 90°$.

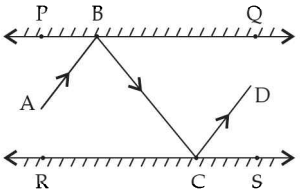
Also CD ⊥ EF, $\angle CFE = 90°$.

$\therefore \angle AEF + \angle CFE = 90° + 90° = 180°$.



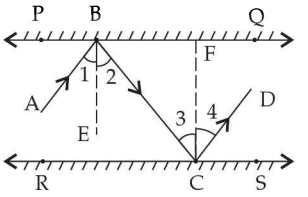
Thus the sum of interior angles on the same side of line EF (transversal) = 180°, therefore, AB || CD (Theorem 4.5)

Example 16. *In the adjoining figure, PQ and RS are two plane mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflected back along CD. Prove that AB ‖ CD.* *(NCERT)*



Solution. At B, draw BE ⊥ PQ and at C, draw CF ⊥ RS.

Mark the angles as shown in the figure.



As PQ ‖ RS and CF ⊥ RS,

so CF ⊥ PQ (see example 14)

since BE ⊥ PQ (by construction)

and CF ⊥ PQ (proved above)

so EB ‖ CF (see example 15)

Now EB ‖ CF and BC is a transversal,

$\angle 2 = \angle 3$ (alt. int. ∠s)

As AB is an incident ray and BC is reflected ray,

$\angle 1 = \angle 2$ (angle of incidence = angle of reflection)

Similarly, $\angle 3 = \angle 4$

But $\angle 2 = \angle 4$ (proved above)

$\Rightarrow \quad \angle 1 = \angle 4.$

$\therefore \quad \angle 1 + \angle 2 = \angle 4 + \angle 3 \Rightarrow \angle ABC = \angle BCD.$

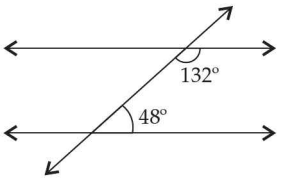
A transversal BC intersects two lines AD and BC such that ∠ABC = ∠BCD

i.e. a pair of alternate interior angle is equal, therefore,

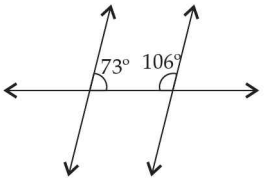
AB ‖ CD, as required.

Exercise 4.2

1. In the figure given below, which of the two lines are parallel? *(NCERT Exemplar)*

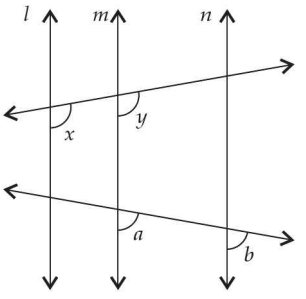


(i)

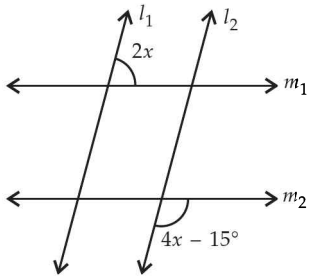


(ii)

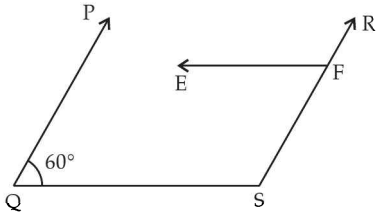
2. In the adjoining figure, $x = y$ and $a = b$. Prove that $l \parallel n$. *(NCERT Exemplar)*



3. (*a*) In the figure (*i*) given below, $l_1 \parallel l_2$ and $m_1 \parallel m_2$. Find the value of x.
(*b*) In the figure (*ii*) given below, QP $\parallel$ SR and FE $\parallel$ SQ. If $\angle PQS = 60°$, then find the measure of $\angle RFE$.

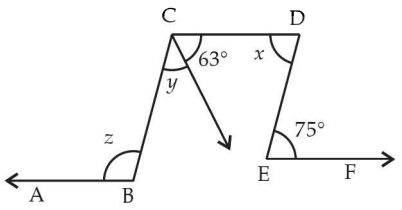


(*i*)

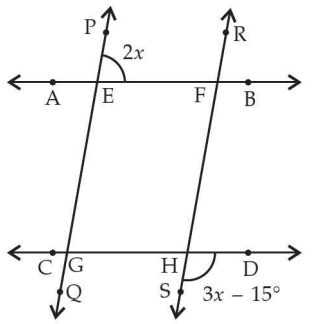


(*ii*)

4. (*a*) In the figure (*i*) given below, AB $\parallel$ CD, AB $\parallel$ EF and BC $\parallel$ ED. Find the values of x, y and z.
(*b*) In the figure (*ii*) given below, lines PQ and RS are parallel. For what value of x will the lines AB and CD be parallel?

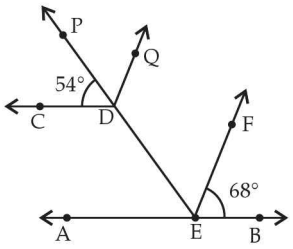


(*i*)

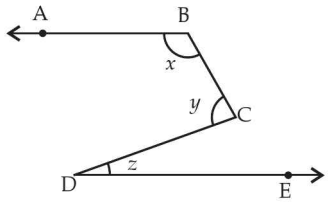


(*ii*)

5. (*a*) In the figure (*i*) given below, AB $\parallel$ CD and EF $\parallel$ DQ. Determine $\angle AED$, $\angle DEF$ and $\angle PDQ$.
(*b*) In the figure (*ii*) given below, AB $\parallel$ DE. Show that $\angle x + \angle y - \angle z = 180°$.



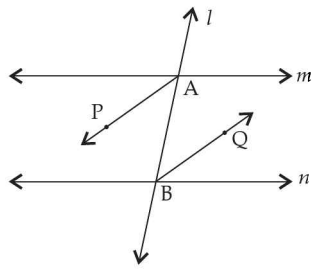
(*i*)



(*ii*)

6. Two lines are respectively perpendicular to two parallel lines. Show that they are parallel to each other. *(NCERT Exemplar)*

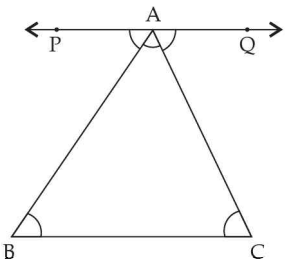
7. In the adjoining figure, bisectors AP and BQ of the alternate interior angles are parallel. Show that lines m and n are parallel. *(NCERT Exemplar)*



Answers

4.3 ANGLE SUM PROPERTY OF A TRIANGLE

In earlier classes, we have verified through activities that the sum of all the three angles of a triangle is 180°. We can prove this statement by using axioms and theorems related to parallel lines.



Theorem 4.7. ***The sum of angles of a triangle is 180°.***

Given. A triangle ABC.

To prove. $\angle A + \angle B + \angle C = 180°$.

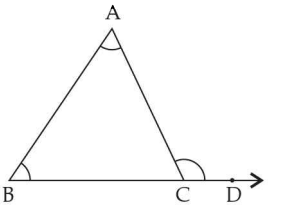
Construction. Through A, draw a line PQ parallel to BC.

Proof.

Statements	*Reasons*
1. $\angle PAB = \angle B$	1. Alt. int. $\angle$s, because PQ $\parallel$ BC and AB is a transversal.
2. $\angle CAQ = \angle C$	2. Alt. int. $\angle$s, because PQ $\parallel$ BC and AC is a transversal.
3. $\angle PAB + \angle A + \angle CAQ = 180°$	3. PQ is a line, sum of angles at a point on one side of a line is 180°.
4. $\angle B + \angle A + \angle C = 180°$ *i.e.* $\angle A + \angle B + \angle C = 180°$	4. Using 1 and 2.

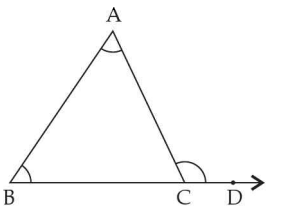
Thus, the sum of angles of a triangle is 180°.

Let ABC be a triangle. Produce the side BC to a point D as shown in the adjoining figure, then $\angle ACD$ is called an **exterior angle** of ΔABC and $\angle A$ and $\angle B$ are called **interior opposite angles** of $\angle ACD$.



In earlier classes, we have verified through activities at exterior angle of a triangle is equal to sum of its two opposite interior angles. In fact, this statement can be proved by using the above theorem and linear pair axiom.

Theorem 4.8. ***If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of its two interior opposite angles.***



Given. A triangle ABC, BC is produced to D and $\angle ACD$ is an exterior angle.

To prove. $\angle ACD = \angle A + \angle B$.

Proof.

Statements	*Reasons*
1. $\angle C + \angle ACD = 180°$	1. Linear pair axiom because BCD is a straight line.
2. $\angle A + \angle B + \angle C = 180°$	2. The sum of angles of a triangle is 180°.
3. $\angle C + \angle ACD = \angle A + \angle B + \angle C$	3. Using 1 and 2.
4. $\angle ACD = \angle A + \angle B$	4. $\angle C$ is common.

Thus, an exterior angle of a triangle is equal to sum of its two interior opposite angles.

Corollary 1. ***An exterior angle of a triangle is greater than either of its two interior opposite angles.***

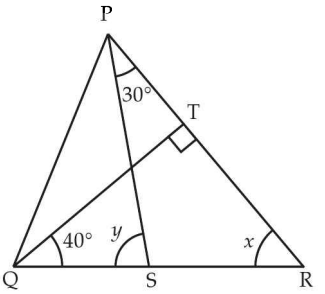
In the above figure, $\angle ACD = \angle A + \angle B$, therefore, $\angle ACD$ being the sum of $\angle A$ and $\angle B$ is greater than $\angle A$ as well as $\angle B$.

Corollary 2. ***If two angles of a triangle are equal to two angles of another triangle, then the third angles of both the triangles are equal.***

(∵ The sum of all the three angles of a triangle is 180°.)

Illustrative Examples

Example 1. *In the adjoining figure, QT ⊥ PR. If ∠SPR = 30° and ∠TQR = 40°, find x and y.* *(NCERT)*



Solution. As QT ⊥ PR, ∠QTR = 90°

In ΔQTR, ∠TQR + ∠QRT + ∠QTR = 180°

(sum of angles of a triangle)

⇒ 40° + x + 90° = 180°

⇒ x = 180° − 40° − 90° ⇒ x = 50°

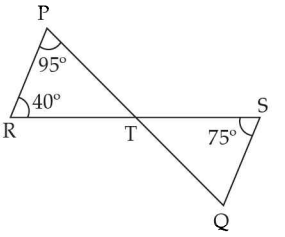
In ΔPRS, ∠QSP = ∠SRP + ∠SPR

(exterior angle = sum of two interior opposite angles)

⇒ y = 30° + x ⇒ y = 30° + 50° (∵ x = 50°)

⇒ y = 80°.

Example 2. *In the adjoining figure, lines PQ and RT intersect at point T such that ∠PRT = 40°, ∠RPT = 95° and ∠TSQ = 75°. Find ∠SQT.* *(NCERT)*



Solution. In ΔPRT, ∠PRT + ∠RPT + ∠PTR = 180°

(sum of angles of a triangle)

⇒ 40° + 95° + ∠PTR = 180°

⇒ ∠PTR = 180° − 40° − 95° = 45°

∠STQ = ∠PTR (vert. opp. ∠s)

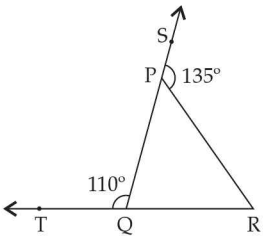
⇒ ∠STQ = 45° (∵ ∠PTR = 45°)

In ΔSTQ, ∠STQ + ∠TSQ + ∠SQT = 180° (sum of angles of a triangle)

⇒ 45° + 75° + ∠SQT = 180° (∵ ∠STQ = 45°)

⇒ ∠SQT = 180° − 45° − 75° = 60°.

Example 3. *In the adjoining figure, sides QP and RQ of ΔPQR are produced to points S and T respectively. If ∠SPR = 135° and ∠PQT = 110°, find ∠PRQ.* *(NCERT)*



Solution. ∠PQR + ∠PQT = 180° (linear pair)

⇒ ∠PQR + 110° = 180°

⇒ ∠PQR = 180° − 110° = 70°.

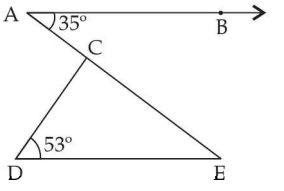
In ΔPQR, ∠SPR = ∠PRQ + ∠PQR

(exterior angle = sum of two interior opposite angles)

⇒ 135° = ∠PRQ + 70° (∵ ∠PQR = 70°)

⇒ ∠PRQ = 135° − 70° = 65°.

Example 4. *In the adjoining figure, AB ‖ DE. If ∠BAC = 35° and ∠CDE = 53°, find ∠DCE.* *(NCERT)*



Solution. As AB ‖ DE and AE is a transversal,

∠CED = ∠BAC (alt. int. ∠s)

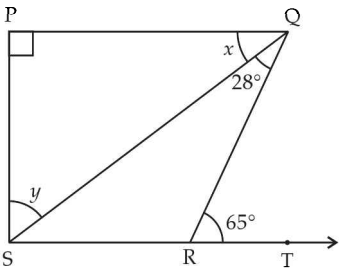
⇒ ∠CED = 35° (∵ ∠BAC = 35°, given)

In ΔCDE, $\angle CDE + \angle CED + \angle DCE = 180°$ (sum of angles of a triangle)

$\Rightarrow$ $53° + 35° + \angle DCE = 180°$

$\Rightarrow$ $\angle DCE = 180° - 53° - 35° = 92°$.

Example 5. *In the adjoining figure PQ || SR. If QP ⊥ SP, then find the values of x and y.* *(NCERT)*



Solution. As PQ || SR and QR is a transversal,

$\angle PQR = \angle QRT$ (alt. int. $\angle$s)

$\Rightarrow$ $\angle PQS + \angle SQR = 65°$ (from fig.)

$\Rightarrow$ $x + 28° = 65°$

$\Rightarrow$ $x = 65° - 28° = 37°$

As QP ⊥ SP, $\angle QPS = 90°$

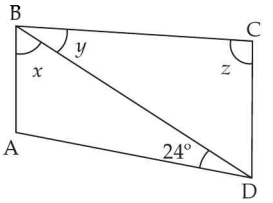
In ΔPQS, $\angle QPS + \angle PSQ + \angle PQS = 180°$ (sum of angles of a triangle)

$\Rightarrow$ $90° + y + x = 180°$

$\Rightarrow$ $90° + y + 37° = 180° \Rightarrow y = 180° - 90° - 37°$

$\Rightarrow$ $y = 53°$.

Example 6. *In the adjoining figure, AB || DC and $\angle ADB = 24°$. If $x = \frac{4y}{3}$ and $y = \frac{3z}{8}$, find $\angle BCD$, $\angle ABC$ and $\angle BAD$.*



Solution. Given $y = \frac{3z}{8} \Rightarrow z = \frac{8y}{3}$.

As AB || DC and BD is a transversal,

$\angle BDC = \angle ABD$ (alt. int. $\angle$s)

$\Rightarrow$ $\angle BDC = x$. ($\because$ $\angle ABD = x$)

In ΔBCD, $\angle CBD + \angle BCD + \angle BDC = 180°$ (sum of angles of a triangle)

$\Rightarrow$ $y + z + x = 180°$

$\Rightarrow$ $y + \frac{8y}{3} + \frac{4y}{3} = 180° \Rightarrow \frac{15y}{3} = 180°$

$\Rightarrow$ $5y = 180° \Rightarrow y = 36°$

$\Rightarrow$ $x = \frac{4}{3} \times 36° = 48°$ and $z = \frac{8}{3} \times 36° = 96°$.

$\therefore$ $\angle BCD = z = 96°$,

$\angle ABC = x + y = 48° + 36° = 84°$.

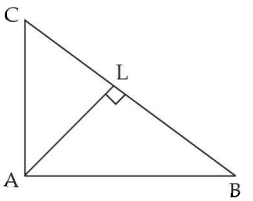
In ΔABD, $\angle ABD + \angle ADB + \angle BAD = 180°$ (sum of angles of a triangle)

$\Rightarrow$ $x + 24° + \angle BAD = 180° \Rightarrow 48° + 24° + \angle BAD = 180°$ ($\because$ $x = 48°$)

$\Rightarrow$ $\angle BAD = 180° - 48° - 24° \Rightarrow \angle BAD = 108°$.

Example 7. *A triangle is right angled at A. L is a point on BC such that AL ⊥ BC, prove that $\angle BAL = \angle ACB$.* *(NCERT Exemplar)*

Solution. Given ΔABC is right angled at A,



so $\angle A = 90°$.

As AL ⊥ BC, $\angle ALB = 90°$ and $\angle ALC = 90°$.

In ΔABL, $\angle BAL + \angle B + \angle ALB = 180°$

(sum of angles of a triangle)

$\Rightarrow$ $\angle BAL + \angle B + 90° = 180°$

$\Rightarrow$ $\angle BAL + \angle B = 90°$ ...(*i*)

In ΔABC, $\angle ACB + \angle B + \angle A = 180°$ (sum of angles of a triangle)

$\Rightarrow$ $\angle ACB + \angle B + 90° = 180°$

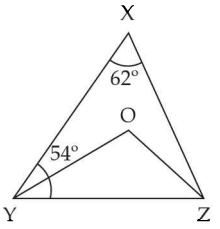
$\Rightarrow$ $\angle ACB + \angle B = 90°$ ...(*ii*)

From (*i*) and (*ii*), we get

$\angle BAL + \angle B = \angle ACB + \angle B$

$\Rightarrow \quad \angle BAL = \angle ACB.$

Example 8. *In the adjoining figure, $\angle X = 62°$, $\angle Y = 54°$. If YO and ZO are bisectors of $\angle XYZ$ and $\angle XZY$ respectively of ΔXYZ, find $\angle OZY$ and $\angle YOZ$.* *(NCERT)*



Solution. In ΔXYZ, $\angle X + \angle Y + \angle Z = 180°$ (sum of angles of a triangle)

$\Rightarrow \quad 62° + 54° + \angle Z = 180°$

$\Rightarrow \quad \angle Z = 180° - 62° - 54° = 64°.$

As OZ is bisector of $\angle XZY$,

$$\angle OZY = \frac{1}{2} \text{ of } \angle XZY = \frac{1}{2} \text{ of } 64° = 32°.$$

Also, OY is bisector of $\angle XYZ$,

$$\angle OYZ = \frac{1}{2} \text{ of } \angle XYZ = \frac{1}{2} \text{ of } 54° = 27°.$$

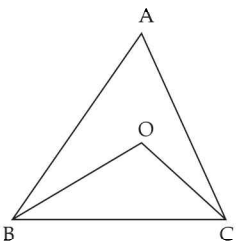
In ΔOYZ, $\angle YOZ + \angle OYZ + \angle OZY = 180°$ (sum of angles of a triangle)

$\Rightarrow \quad \angle YOZ + 27° + 32° = 180°$

$\Rightarrow \quad \angle YOZ = 180° - 27° - 32° \Rightarrow \angle YOZ = 121°.$

Example 9. *Bisectors of angles B and C of a triangle ABC intersect each other at the point O. Prove that $\angle BOC = 90° + \frac{1}{2} \angle A$.* *(NCERT Exemplar)*

Solution. Let ABC be any triangle as shown in the adjoining figure. Bisector of $\angle B$ and $\angle C$ intersects each other at O.



$$\angle OBC = \frac{1}{2} \text{ of } \angle B = \frac{1}{2} \angle B \qquad \ldots(i)$$

$$\text{and } \angle OCB = \frac{1}{2} \text{ of } \angle C = \frac{1}{2} \angle C \qquad \ldots(ii)$$

In ΔABC, $\angle A + \angle B + \angle C = 180°$ (sum of angles of a triangle)

$\Rightarrow \quad \angle B + \angle C = 180° - \angle A \qquad \ldots(iii)$

In ΔOBC, $\angle BOC + \angle OBC + \angle OCB = 180°$ (sum of angles of a triangle)

$$\Rightarrow \quad \angle BOC + \frac{1}{2} \angle B + \frac{1}{2} \angle C = 180° \quad \text{(using } (i) \text{ and } (ii))$$

$$\Rightarrow \quad \angle BOC = 180° - \frac{1}{2}(\angle B + \angle C)$$

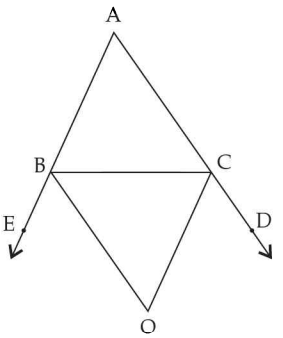
$$\Rightarrow \quad \angle BOC = 180° - \frac{1}{2}(180° - \angle A) \quad \text{(using } (i))$$

$$\Rightarrow \quad \angle BOC = 180° - 90° + \frac{1}{2} \angle A$$

$$\Rightarrow \quad \angle BOC = 90° + \frac{1}{2} \angle A.$$

Example 10. *In the adjoining figure, the sides AB and AC of ΔABC are produced to points E and D respectively. If the bisectors BO and CO of $\angle CBE$ and $\angle BCD$ respectively meet at the point O, then prove that*

$$\angle BOC = 90° - \frac{1}{2} \angle A. \qquad \textit{(NCERT)}$$



Solution. $\angle CBE + \angle B = 180°$ (linear pair)

and $\angle BCD + \angle C = 180°$ (linear pair)

$\Rightarrow \quad \angle CBE = 180° - \angle B$ and $\angle BCD = 180° - \angle C.$

As BO is bisector of $\angle CBE$,

$$\angle CBO = \frac{1}{2} \text{ of } \angle CBE = \frac{1}{2}(180° - \angle B) = 90° - \frac{1}{2}\angle B \quad \ldots(i)$$

Also CO is bisector of $\angle BCD$,

$$\angle BCO = \frac{1}{2} \text{ of } \angle BCD = \frac{1}{2}(180° - \angle C) = 90° - \frac{1}{2}\angle C \quad \ldots(ii)$$

In ΔABC, $\angle A + \angle B + \angle C = 180°$ (sum of angles of a triangle)

$\Rightarrow \angle B + \angle C = 180° - \angle A$ ...(*iii*)

In ΔBOC, $\angle BOC + \angle CBO + \angle BCO = 180°$ (sum of angles of a triangle)

$\Rightarrow \angle BOC + \left(90° - \frac{1}{2}\angle B\right) + \left(90° - \frac{1}{2}\angle C\right) = 180°$ (using (*i*) and (*ii*))

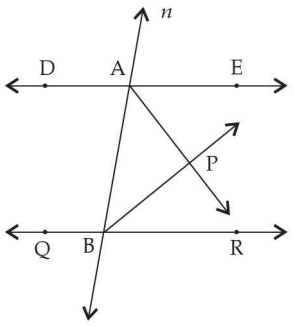
$\Rightarrow \angle BOC = \frac{1}{2}(\angle B + \angle C)$

$\Rightarrow \angle BOC = \frac{1}{2}(180° - \angle A)$ (using (*iii*))

$\Rightarrow \angle BOC = 90° - \frac{1}{2}\angle A.$

Example 11. *In the adjoining figure, DE ∥ QR, AP and BP are bisectors of ∠EAB and ∠RBA respectively. Find ∠APB.*

(NCERT Exemplar)



Solution. As DE ∥ QR and AB is a transversal,

$\angle EAB + \angle RBA = 180°$ ...(*i*)

(sum of angles on same side of a transversal)

As AP is bisector of $\angle EAB$,

$\angle PAB = \frac{1}{2}\angle EAB$ ...(*ii*)

Also BP is bisector of $\angle RBA$,

$\angle PBA = \frac{1}{2}\angle RBA$ ...(*iii*)

In ΔPAB, $\angle APB + \angle PAB + \angle PBA = 180°$ (sum of angles of a triangle)

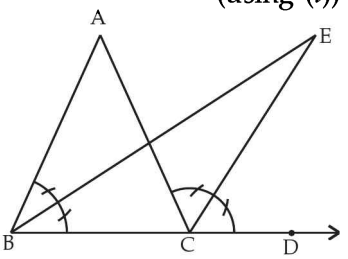
$\Rightarrow \angle APB + \frac{1}{2}\angle EAB + \frac{1}{2}\angle RBA = 180°$ (using (*ii*) and (*iii*))

$\Rightarrow \angle APB = 180° - \frac{1}{2}(\angle EAB + \angle RBA)$

$\Rightarrow \angle APB = 180° - \frac{1}{2}(180°)$ (using (*i*))

$\Rightarrow \angle APB = 180° - 90° \Rightarrow \angle APB = 90°.$

Example 12. *In the adjoining figure, side BC of ΔABC is produced to a point D. If the bisectors of ∠ABC and ∠ACD meet at E, prove that $\angle BEC = \frac{1}{2}\angle BAC$.* *(NCERT)*



Solution. Let $\angle ABC = x$ and $\angle BAC = y$, then

$\angle ACD = x + y$ (ext. ∠ = sum of two int. opp. ∠s)

As BE is bisector of $\angle ABC$, $\angle EBC = \frac{1}{2}x$ ...(*i*)

Also, as CE is bisector of $\angle ACD$, $\angle ECD = \frac{1}{2}(x + y)$ ...(*ii*)

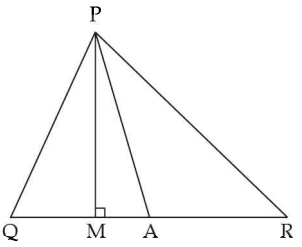
For ΔEBC, $\angle ECD = \angle EBC + \angle BEC$ (ext. ∠ = sum of two int. opp. ∠s)

$\Rightarrow \frac{1}{2}(x + y) = \frac{1}{2}x + \angle BEC$ (using (*i*) and (*ii*))

$\Rightarrow \angle BEC = \frac{1}{2}y \Rightarrow \angle BEC = \frac{1}{2}\angle BAC.$

Example 13. *In the adjoining figure, $\angle Q > \angle R$, PA is bisector of $\angle QPR$ and $MP \perp QR$. Prove that*

$$\angle APM = \frac{1}{2}(\angle Q - \angle R).$$ *(NCERT Exemplar)*



Solution. As PA is bisector of $\angle QPR$,

$\angle QPA = \angle APR$ ...(*i*)

As $MP \perp QR$, $\angle PMQ = 90°$ and $\angle PMR = 90°$.

In ΔPQM, $\angle Q + \angle QPM + \angle PMQ = 180°$ (sum of angles of a triangle)

$\Rightarrow$ $\angle Q + \angle QPM + 90° = 180°$

$\Rightarrow$ $\angle QPM = 90° - \angle Q$ ...(*ii*)

In ΔPRM, $\angle R + \angle RPM + \angle PMR = 180°$ (sum of angles of a triangle)

$\Rightarrow$ $\angle R + \angle RPM + 90° = 180°$

$\Rightarrow$ $\angle RPM = 90° - \angle R$ ...(*iii*)

Now, from figure, we have

$\angle APM = \angle QPA - \angle QPM$ ...(*iv*)

Also $\angle APM = \angle RPM - \angle APR$ ...(*v*)

Adding (*iv*) and (*v*), we get

$2\angle APM = \angle RPM - \angle QPM + \angle QPA - \angle APR$

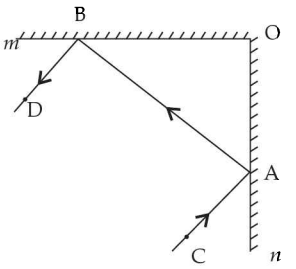
$\Rightarrow$ $2\angle APM = \angle RPM - \angle QPM$ (using (*i*))

$\Rightarrow$ $2\angle APM = (90° - \angle R) - (90° - \angle Q)$ (using (*iii*) and (*ii*))

$\Rightarrow$ $2\angle APM = \angle Q - \angle R$

$\Rightarrow$ $\angle APM = \frac{1}{2}(\angle Q - \angle R)$.

Example 14. *In the adjoining figure, m and n are two plane mirrors perpendicular to each other. Show that the incident ray CA is parallel to the reflected ray BD.* *(NCERT Exemplar)*



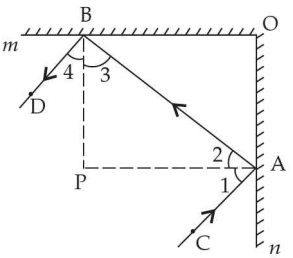
Solution. At A, draw perpendicular to mirror *n* and at B, draw perpendicular to mirror *m*. Let these perpendiculars meet at point P.

Mark the angles as shown in the figure.

As $PA \perp n$ and $m \perp n$, so PA || OB.

Also, as $PB \perp n$ and $m \perp n$, so BP || OA.

$\therefore$ $AP \perp BP \Rightarrow \angle APB = 90°$



In ΔAPB, $\angle 2 + \angle 3 + \angle APB = 180°$

(sum of angles of a triangle)

$\Rightarrow$ $\angle 2 + \angle 3 + 90° = 180°$

$\Rightarrow$ $\angle 2 + \angle 3 = 90°$ ...(*i*)

As angle of incidence = angle of reflection,

$\angle 1 = \angle 2$ and $\angle 3 = \angle 4$.

$\therefore$ $\angle 1 + \angle 4 = \angle 2 + \angle 3$

$\Rightarrow$ $\angle 1 + \angle 4 = 90°$ ...(*ii*) (using (*i*))

Adding (*i*) and (*ii*), we get

$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180°$

$\Rightarrow$ $(\angle 1 + \angle 2) + (\angle 3 + \angle 4) = 180°$

$\Rightarrow$ $\angle CAB + \angle DBA = 180°$.

Thus, a transversal AB intersects two lines CA and DB such that a pair of interior angles on the same side of transversal is supplementary, therefore, CA || DB.

Hence, incident ray CA is parallel to the reflected ray BD.

Exercise 4.3

1. (*i*) Can a triangle have all angles less than 60°? Give reason for your answer. *(NCERT Exemplar)*

(*ii*) Can a triangle have two right angles? Give reason for your answer.

(*iii*) Can a triangle have two obtuse angles? Give reason for your answer. *(NCERT Exemplar)*

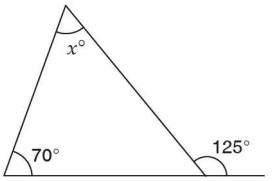
2. Prove that a triangle must have atleast two acute angles. *(NCERT Exemplar)*

3. How many triangles can be drawn having its angles as 45°, 64° and 72°? Give reason for your answer. *(NCERT Exemplar)*

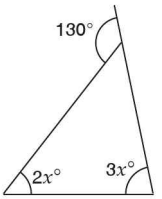
4. How many triangles can be drawn having its angles as 53°, 64° and 63°? Give reason for your answer. *(NCERT Exemplar)*

5. If the angles of a triangle are in the ratio 2 : 3 : 4, then find the measures of angles of the triangle. *(NCERT Exemplar)*

6. In each of the figures given below, find x :

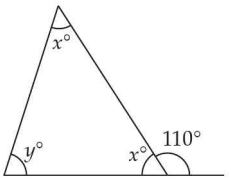


(*i*)

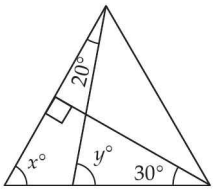


(*ii*)

7. In each of the figures given below, find the values of x and y :



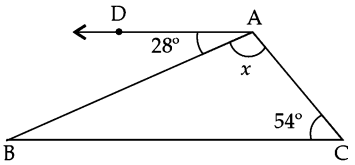
(*i*)



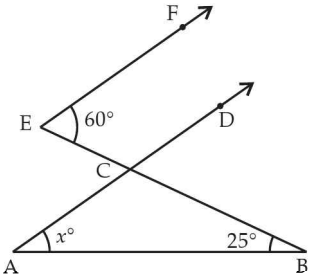
(*ii*)

8. (*a*) In the figure (*i*) given below, BC || DA. Find the value of x.

(*b*) In the figure (*ii*) given below, CD || EF. Find the value of x.



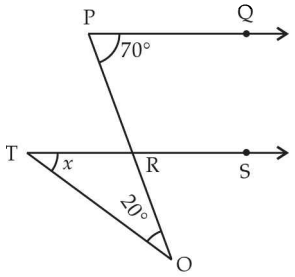
(*i*)



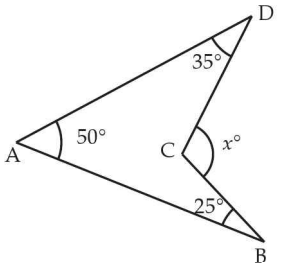
(*ii*)

9. (*a*) In the figure (*i*) given below, PQ || TS. If ∠QPR = 70° and ∠TOR = 20°, then find the value of x.

(*b*) In the figure (*ii*) given below, find the value of x.



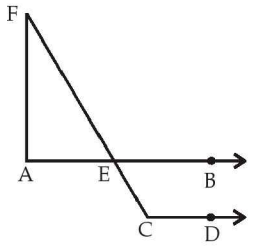
(*i*)



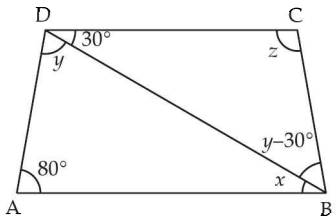
(*ii*)

10. (*a*) In the figure (*i*) given below, AB || CD. If FA ⊥ AB and ∠AFE = 40°, find ∠ECD.

(*b*) In the figure (*ii*) given below, AB || DC. Find the values of x, y and z.



(*i*)



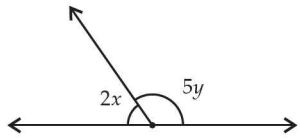
(*ii*)

Answers

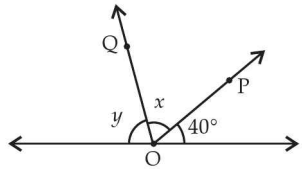
Very Short Answer Type Questions

1. If two angles form a linear pair and one of them measures 65°, then find the measure of the other angle.

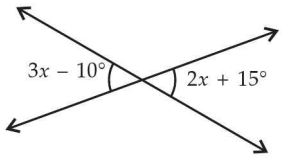
2. In the adjoining figure, a ray stands on a line. If $x = 30°$, then find the value of y.



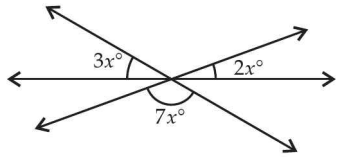
3. In the adjoining figure, rays OP and OQ stand on a line. If $x : y = 2 : 3$, then find the values of x and y.



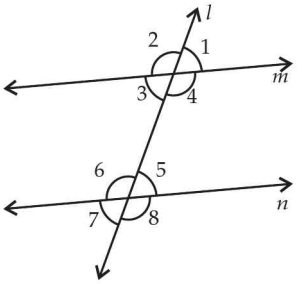
4. In the adjoining figure, then find the value of x.



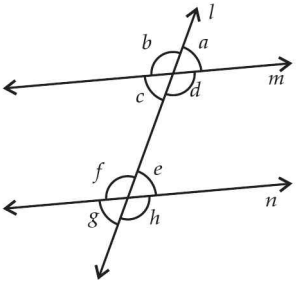
5. In the adjoining figure, then find the value of x.



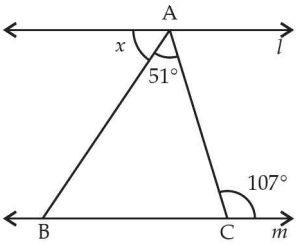
6. In the adjoining figure, if $m \parallel n$, then find $\angle 4 + \angle 7$.



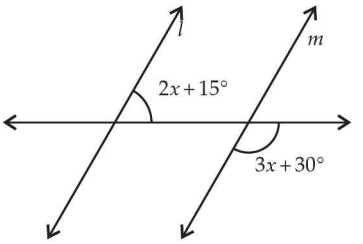
7. In the adjoining figure, if $m \parallel n$. If $\angle a : \angle b = 2 : 3$, then find the measure of $\angle h$.



8. In the adjoining figure, if $l \parallel m$, then find the value of x.

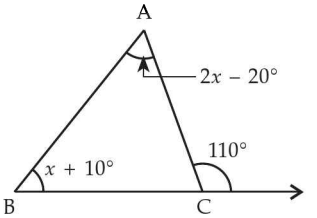


9. In the adjoining figure, find the value of x for which the lines l and m may be parallel.

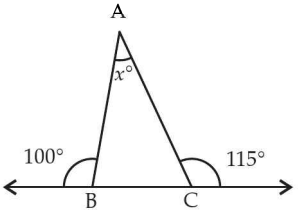


10. If the sum of two angles of a triangle is equal to the third angle, then the triangle is right angled.

11. In the adjoining figure, then find the value of x.



12. In the adjoining figure, then find the value of x.



Answers

Objective Type Questions

(FOR FORMATIVE/SUMMATIVE ASSESSMENT)

Mental Maths

1. Fill in the blanks:
   (*i*) Two angles are called adjacent angles if they have a common vertex, a common arm and their non-common arms lie on ..... of the common arm.
   (*ii*) If a ray stands on a line, then the sum of two adjacent angles so formed is ..... .
   (*iii*) If the sum of two adjacent angles is 180°, then the non-common arms of the angles form ..... .
   (*iv*) The sum of all the angles (drawn in a plane) at a point is ..... .
   (*v*) The sum of angles at a point on one side of a line is equal to ..... right angles.
   (*vi*) If two lines intersect each other, then the vertically opposite angles are ..... .
   (*vii*) Two lines drawn in a plane are called parallel if they ..... .
   (*viii*) The distance between the parallel lines ..... .
   (*ix*) Corresponding angles lie on the ..... side of the transversal.
   (*x*) Alternate angles lie on the ..... of the transversal.
   (*xi*) If a transversal intersects two parallel lines, then :
      (1) each pair of corresponding angles is ..... .
      (2) each pair of alternate interior angles is ..... .
      (3) sum of each pair of co-interior angles is ..... .
   (*xii*) If a transversal intersects two lines such that a pair of corresponding is equal, then the two lines ..... .
   (*xiii*) If a transversal intersects two lines such that a pair of alternate interior angles is ....., then the two lines are parallel.
   (*xiv*) If a transversal intersects two lines such that a pair of co-interior angles is ....., then the two lines are parallel.
   (*xv*) If two lines are parallel to a third line, then the two lines are themselves ..... .
   (*xvi*) The sum of all the angles of a triangle is ..... right angles.
   (*xvii*) An exterior angle of a triangle is equal to the sum of its two ..... angles.
   (*xviii*) An exterior angle of a triangle is ..... than either of its ..... interior opposite angles.
   (*xix*) If the non-common arms of adjacent angles form a line, then the adjacent angles are said to form a ..... .
2. State whether the following statements are true or false. If a statement is false, write the corresponding correct statement.
   (*i*) Two angles with a common vertex and a common arm are called adjacent angles.
   (*ii*) If a transversal intersects two lines, then each pair of corresponding angles is equal.
   (*iii*) If a transversal intersects two lines, then each pair of alternate interior angles is equal.

(*iv*) If a transversal intersects two lines, then the sum of each pair of consecutive interior angles is two right angles.

Multiple Choice Questions

Choose the correct answer from the given four options (3 to 17) :

3. If one angle of a triangle is equal to sum of other two angles, then the triangle is

(*a*) an obtuse angled triangle (*b*) a right angled triangle

(*c*) an isosceles triangle (*d*) an equilateral triangle

(NCERT Exemplar)

4. An exterior angle of a triangle is 105° and its two interior opposite angles are equal. Each of these equal angle is

(*a*) $37\frac{1^\circ}{2}$ (*b*) $52\frac{1^\circ}{2}$ (*c*) $72\frac{1^\circ}{2}$ (*d*) 75°

(NCERT Exemplar)

5. The angles of a triangle are in the ratio 2 : 4 : 3. The smallest angle of the triangle is

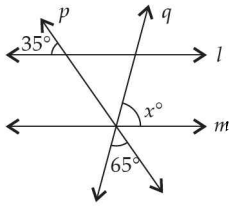
(*a*) 20° (*b*) 40° (*c*) 60° (*d*) 80°

(NCERT Exemplar)

6. If the sum of two adjacent angles is 100° and one of them is 35°, then the other is

(*a*) 70° (*b*) 65° (*c*) 135° (*d*) 145°

7. In the adjoining figure, if $l \parallel m$ then the value of x is



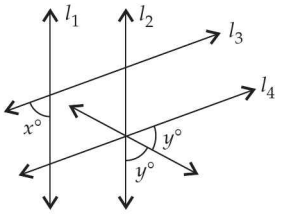
(*a*) 70

(*b*) 80

(*c*) 80°

(*d*) 85

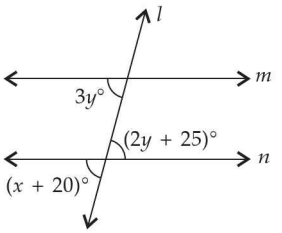
8. In the adjoining figure, if $l_1 \parallel l_2$ and $l_3 \parallel l_4$ then the value of y in terms of x is



(*a*) $90 + x$ (*b*) $90 - x$

(*c*) $90 + 2x$ (*d*) $90 - \frac{x}{2}$

9. In the adjoining figure, if $m \parallel n$ then the value of x is



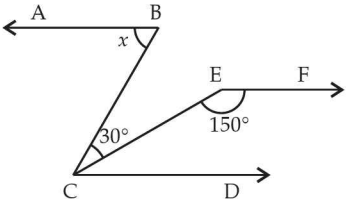
(*a*) 60

(*b*) 55

(*c*) 50

(*d*) 45

10. In the adjoining figure, AB || CD and AB || EF. The value of x is



(*a*) 40°

(*b*) 50°

(*c*) 60°

(*d*) 70°

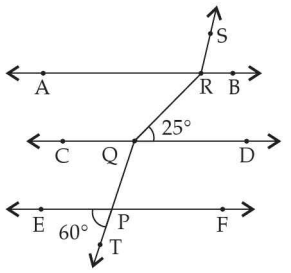
11. In the adjoining figure, AB || CD || EF and PQ || RS. If ∠CQP = 60° and ∠RQD = 25°, then ∠QRS is equal to

(*a*) 85°

(*b*) 135°

(*c*) 145°

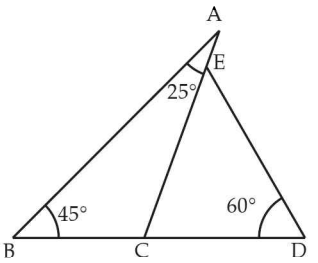
(*d*) 110° *(NCERT Exemplar)*



12. In the adjoining figure, the measure of ∠AED is

(*a*) 110° (*b*) 120°

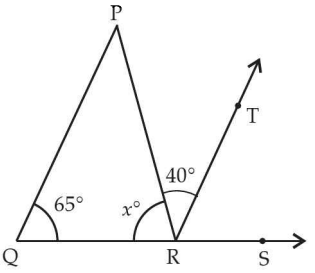
(*c*) 130° (*d*) 140°



13. In the adjoining figure, if QP || RT, then x is equal to

(*a*) 75 (*b*) 70

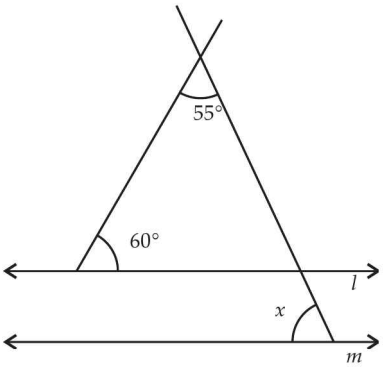
(*c*) 65 (*d*) 55



14. In the adjoining figure, if l || m then ∠x is equal to

(*a*) 55° (*b*) 65°

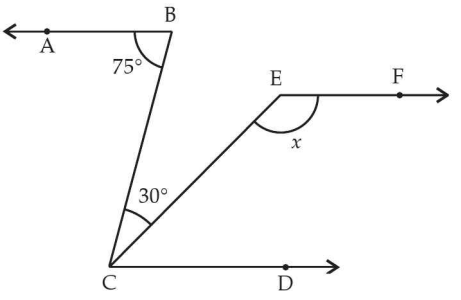
(*c*) 75° (*d*) 115°



15. In the adjoining figure, AB || CD. The value of x which will make EF || CD is

(*a*) 150° (*b*) 145°

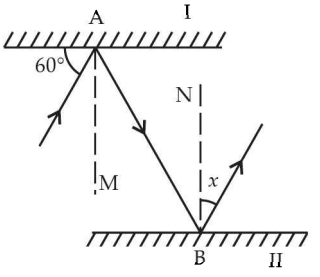
(*c*) 135° (*d*) 115°



16. In the adjoining figure, two plane mirrors I and II are placed parallel to each other. AM and BN are perpendiculars to the mirrors I and II respectively. If an incident ray strikes the mirror I at A and the reflected ray strikes the mirror II at B, then the value of x is

(*a*) 30° (*b*) 45°

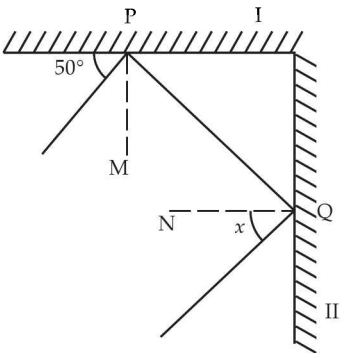
(*c*) 50° (*d*) 60°



17. In the adjoining figure, two plane mirrors I and II are placed perpendicular to each other. PM and QN are perpendiculars to the mirrors I and II respectively. If an incident ray strikes the mirror I at P and the reflected ray strikes the mirror II at Q, then the value of x is

(*a*) 60° (*b*) 50°

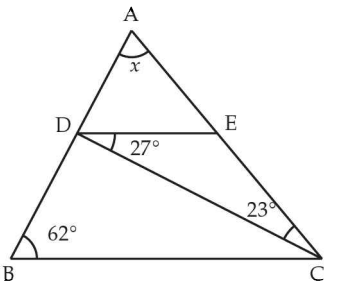
(*c*) 45° (*d*) 40°



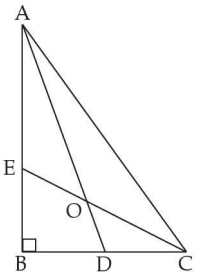
Answers

Higher Order Thinking Skills (HOTS)

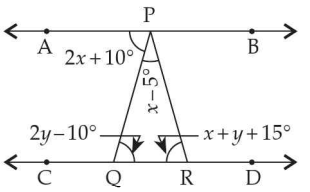
1. In the adjoining diagram, DE || BC. Find the value of x.



2. In the adjoining figure, AD and CE are bisectors of ∠A and ∠C respectively. If ∠ABC = 90°, then find ∠EOD.



3. In the adjoining figure, lines AB and CD are parallel. Find the values of x and y.



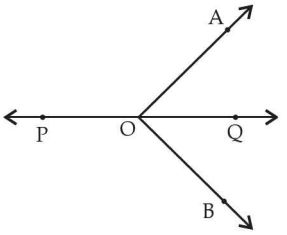
Answers

Chapter Test

1. (*a*) State whether the following statement is true or false. Justify your answer.
"All linear pairs are supplementary angles whereas all supplementary angles are not linear pairs."

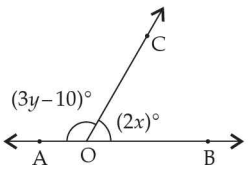
(*b*) Two adjacent angles are equal. Is it necessary that each of these angles will be a right angle? Justify your answer. *(NCERT Exemplar)*

2. In the adjoining figure, OQ bisects ∠AOB. OP is a ray opposite to ray OQ. Prove that ∠POA = ∠POB.

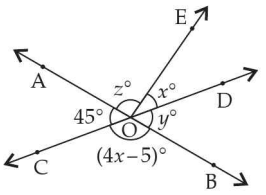


3. (*a*) In figure (*i*) given below, ray OC stands on line AB and $4x - 3y = 20$. Find the measures of ∠BOC and ∠AOC.

(*b*) In figure (*ii*) given below, lines AB and CD intersect each other at the point O. If ∠AOC = 45°, find the values of x, y and z.



(i)

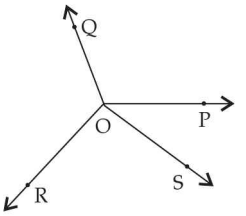


(ii)

4. In the adjoining figure, OP, OQ, OR and OS are four rays. Prove that

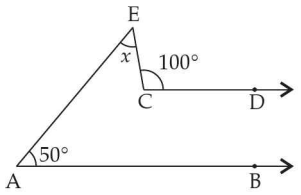
∠POQ + ∠QOR + ∠ROS + ∠SOP = 360°. *(NCERT)*

Hint: Produce any ray, say OP, backward to a point T so that TOP is a line.

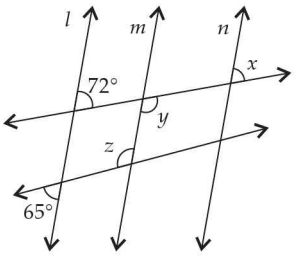


5. (*a*) In the figure (*i*) given below, AB || CD. If ∠EAB = 50° and ∠ECD = 100°, find ∠AEC.

(*b*) In the figure (*ii*) given below, $l \parallel m$ and $l \parallel n$. Find the values of x, y and z.

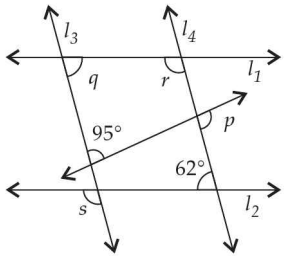


(i)

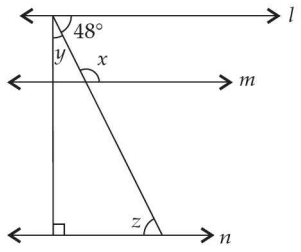


(ii)

6. (*a*) In the figure (*i*) given below, $l_1 \parallel l_2$ and $l_3 \parallel l_4$. Find the values of p, q, r and s.
(*b*) In the figure (*ii*) given below, $l \parallel m$ and $m \parallel n$. Find the values of x, y and z.



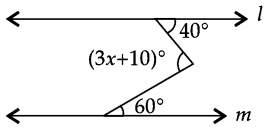
(*i*)



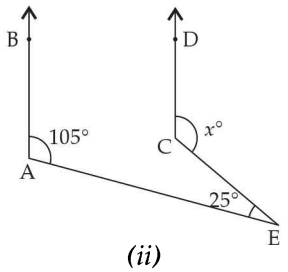
(*ii*)

7. (*a*) In the figure (*i*) given below, $l \parallel m$. Find the value of x.
(*b*) In the figure (*ii*) given below, AB $\parallel$ CD. Find the value of x.

Hint: (*b*) Produce DC to meet AE at F.



(*i*)

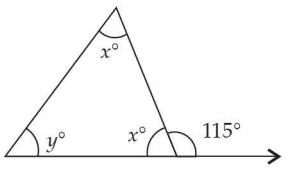


(*ii*)

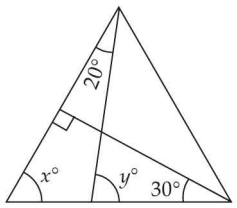
8. Prove that through a given point, we can draw only one perpendicular to a given line. *(NCERT Exemplar)*

Hint: Use proof by contradiction.

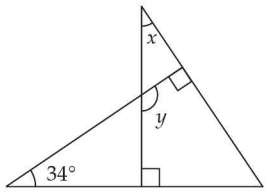
9. In each of the following figures, find the values of x and y.



(*i*)

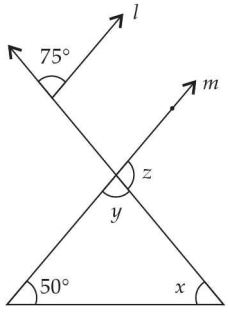


(*ii*)

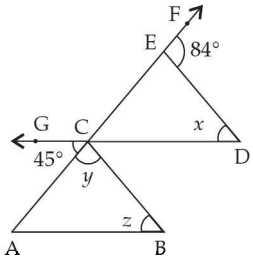


(*iii*)

10. (*a*) In the figure (*i*) given below, rays l and m are parallel. Find the values of x, y and z.
(*b*) In the figure (*ii*) given below, AB $\parallel$ CD and BC $\parallel$ DE. Find the values of x, y and z.



(*i*)



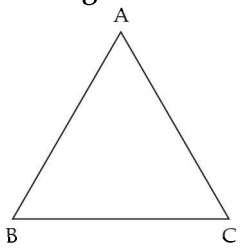
(*ii*)

Triangles



INTRODUCTION

In earlier classes, you have studied about triangles and their various properties. A triangle is a closed curve formed by three line segments. 'Tri' means 'three'. A triangle has three sides, three angles and three vertices.



The adjoining figure shows a triangle ABC. The line segments AB, BC and CA are called its **sides**. The angles $\angle A$, $\angle B$ and $\angle C$ are called its **interior angles** or simply **angles**. The points A, B and C are called its **vertices**. Three sides and the three angles are called its six **elements**.

In the previous chapter, you have studied angle sum property of triangles. In this chapter, you will study about congruence of triangles in detail, rules of congruence, some more properties of triangles and inequalities in a triangle.

5.1 CONGRUENCE OF TRIANGLES

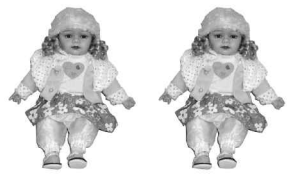
In our daily life, we see many objects around us which have same shape and size. Objects that have exactly same shape and same size are congruent to each other.

For example:

(*i*) Two coins of ₹ 5 minted at the same time are congruent.



(*ii*) Two toys made of same mould are congruent.



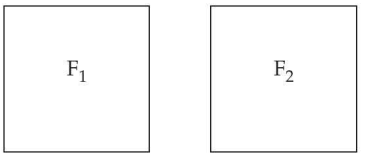
Two plane figures are called ***congruent*** *if and only if they have exactly the same shape and the same size. The relation of two figures being congruent is called* ***congruence.***

To check for congruency, take a trace-copy of one of the two figures and place it over the other figure. The two figures will be congruent if this trace-copy exactly covers the other figure.

Alternatively, we may cut out one figure and place it over the other.

This method of comparing two figures is called **method of superposition.**

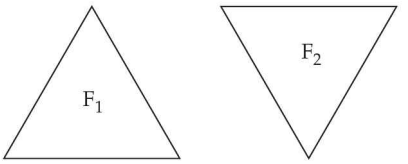
Look at the adjoining squares with sides of same measure.



Take a trace-copy of figure F_1 and place it over figure F_2. We note that this trace-copy of F_1 exactly covers the figure F_2. Therefore, F_1 and F_2 are congruent. We write as $F_1 \cong F_2$. The symbol '$\cong$' (read as 'congruent to') is used to indicate the congruency of two figures.

Sometimes, it may be necessary to rotate or flip the figure to check whether they are congruent.

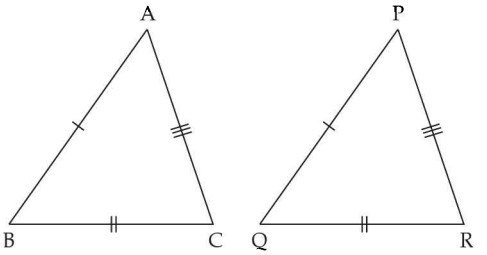
The adjoining equilateral triangles F_1 and F_2 with sides of same measure are congruent. Note that we have to rotate figure F_1 through 180°.



❑ **Congruent triangles**

Two triangles are called ***congruent*** *if and only if they have exactly the same shape and the same size.*

In the adjoining figure, two triangles ABC and PQR are congruent. It means that the sketch of one triangle can be slided onto the sketch of the other so that they fit each other exactly *i.e.* when one triangle is superimposed on the other, they cover each other exactly.



Notice that these triangles are such that

$AB = PQ$, $BC = QR$, $AC = PR$, $\angle A = \angle P$, $\angle B = \angle Q$ and $\angle C = \angle R$.

Thus these triangles have the same shape and same size, so they are congruent.

We express this as $\Delta ABC \cong \Delta PQR$.

It means that when we place a trace-copy of ΔABC on ΔPQR, vertex A falls on vertex P, vertex B on vertex Q and vertex C on vertex R. Then side AB falls on PQ, BC on QR and CA on RP. Also $\angle A$ falls on $\angle P$, $\angle B$ on $\angle Q$ and $\angle C$. Thus, the order in which the vertices match, automatically determines a correspondence between the sides and the angles of the two triangles. It follows that if the vertices of ΔABC match the vertices of ΔPQR in the order:

$A \leftrightarrow P$, $B \leftrightarrow Q$, $C \leftrightarrow R$

then all the six corresponding parts (3 sides and 3 angles) of two triangles are equal *i.e.*

$AB = PQ$, $BC = QR$, $CA = RP$, $\angle A = \angle P$, $\angle B = \angle Q$ and $\angle C = \angle R$.

However, if we place ΔABC on ΔPQR such that A falls on Q, then other vertices may not correspond suitably. Take a trace-copy of ΔABC and place vertex A on vertex Q and try to find out!

This shows that while talking about congruence of triangles, not only the measures of angles and the lengths of sides matter, but also the matching of vertices matter. In the above triangles ABC and PQR, the correspondence is

$A \leftrightarrow P$, $B \leftrightarrow Q$, $C \leftrightarrow R$.

Remarks

1. Congruent triangles are 'equal in all respects' *i.e.* they are the exact *duplicate* of each other.
2. If two triangles are congruent, then any one can be *superposed* on the other to cover it exactly.
3. In congruent triangles, the sides and the angles which coincide by superposition are called *corresponding sides* and *corresponding angles*.

4. The corresponding sides lie opposite to the equal angles and the corresponding angles lie opposite to the equal sides.

   In the above diagram, $\angle A = \angle P$, therefore, the corresponding sides BC and QR are equal. Also BC = QR, therefore, the corresponding angles A and P are equal.

   The abbreviation 'CPCT' or 'c.p.c.t.' will be used for corresponding parts of congruent triangles.

5. The order of the letters in the names of congruent triangles displays the corresponding relationship between the two triangles.

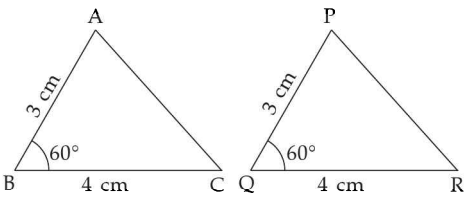
   Thus, when we write $\Delta ABC \cong \Delta PQR$, it means that A lies on P, B lies on Q and C lies on R *i.e.* $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$ and BC = QR, CA = RP, AB = PQ.

   Writing any other correspondence *i.e.* $\Delta ABC \cong \Delta PRQ$, $\Delta ABC \cong \Delta RPQ$ etc. will be incorrect.

5.2 CRITERIA FOR CONGRUENCE OF TRIANGLES

For any two triangles to be congruent, the *six elements* of one triangle *need not be proved* equal to the corresponding six elements of the other triangle.

Draw two triangles ABC and PQR such that BC = QR = 4 cm, $\angle B = \angle Q = 60°$ and AB = PQ = 3 cm.

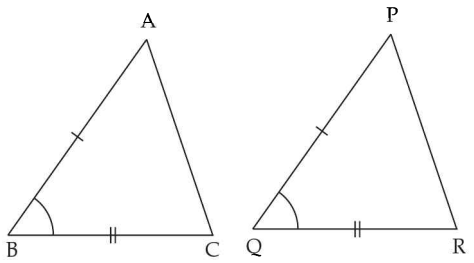


Make a trace-copy of ΔABC and place it over ΔPQR. We observe that the two triangles cover each other exactly and so they are congruent.

Repeat this activity with more pairs of triangles satisfying these conditions. We observe that equality of two sides and the included angle is enough for the congruence of two triangles. However, this result cannot be proved with the help of previously known results and hence it is accepted to be true as an axiom.

❑ **SAS axiom of congruency**

Two triangles are congruent if two sides and the included angle of one triangle are equal to two sides and the included angle of the other triangle.



In the adjoining figure,

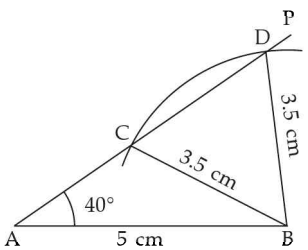
$$AB = PQ,\ BC = QR \text{ and}$$
$$\angle B = \angle Q$$
$$\therefore \quad \Delta ABC \cong \Delta PQR$$

This axiom is known as **Side-Angle-Side** criterion (or rule) of congruency.

Note. The equality of the 'included angle' is essential.

Look at the adjoining figure:

Here, AB = 5 cm and $\angle PAB = 40°$. Taking B as centre and radius 3.5 cm, we draw an arc to meet AP at points C and D (as shown in figure).



We consider two triangles ABC and ABD. In these triangles, we have

$$AB = AB,\ BC = BD \text{ and}$$
$$\angle CAB = \angle DAB \qquad (\text{each} = 40°)$$

Thus, two sides and one angle of one triangle are equal to two sides and one angle of the other triangle but the triangles are not congruent which is clear from the figure because ΔABC is a part of ΔABD. In fact, two vertices A and B are

same for both triangles but their third vertices C and D do not coincide. Here, note that ∠CAB and ∠DAB are not **included angles.**

Hence, for SAS congruence criterion the equality of the included angle is essential.

Thus, SAS congruence rule holds but not ASS or SSA rule.

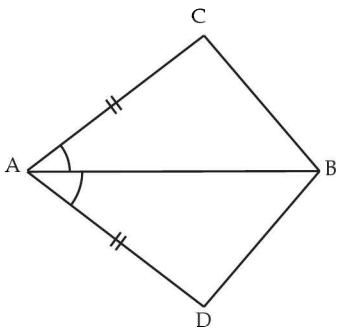
Illustrative Examples

Example 1. *If ΔPQR ≅ ΔEDF, then is it true to say that PR = EF? Give reason for your answer.* *(NCERT Exemplar)*

Solution. Given ΔPQR ≅ ΔEDF. It means that P ↔ E, Q ↔ D and R ↔ F, therefore, PR = EF (corresponding sides are equal).

Hence, the given statement is true.

Example 2. *In the adjoining quadrilateral, AC = AD and AB bisects ∠A. Show that ΔABC ≅ ΔABD. What can you say about BC and BD?* *(NCERT)*



Solution. As AB bisects ∠A, ∠CAB = ∠DAB.

In ΔABC and ΔABD,

AC = AD (given)

AB = AB (common)

∠CAB = ∠DAB (∵ AB bisects ∠A)

∴ ΔABC ≅ ΔABD

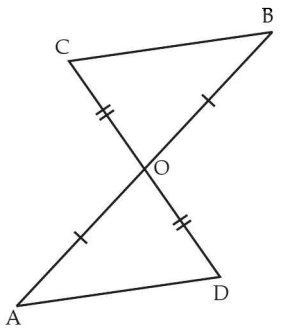
(by SAS axiom of congruency)

∴ BC = BD (c.p.c.t.)

Example 3. *In the adjoining figure, OA = OB and OD = OC. Show that*

(i) ΔAOD ≅ ΔBOC

(ii) AD ∥ BC.



Solution. (*i*) In ΔAOD and ΔBOC,

OA = OB (given)

OD = OC (given)

∠AOD = ∠BOC (vert. opp. ∠s)

∴ ΔAOD ≅ ΔBOC

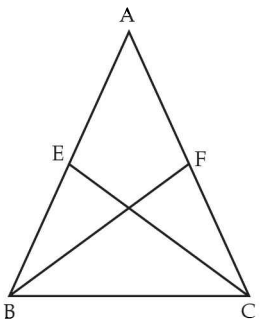
(by SAS axiom of congruency)

(*ii*) ∠OAD = ∠OBC (c.p.c.t.)

But these form a pair of alternate angles for line segments AD and BC.

Therefore, AD ∥ BC.

Example 4. *In the adjoining figure, E and F are respectively the mid-points of equal sides AB and AC of ΔABC. Show that BF = CE.* *(NCERT)*



Solution. In ΔABF and ΔACE,

AB = AC (given)

AF = AE

(Halves of equal sides AC and AB)

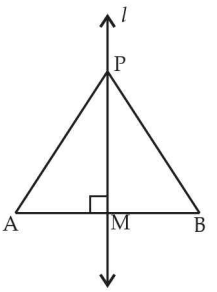
∠A = ∠A (common)

∴ ΔABF ≅ ΔACE

(by SAS axiom of congruency)

∴ BF = CE (c.p.c.t.)

Example 5. *AB is a line segment and line l is its perpendicular bisector. If P is a point on l, show that P is equidistant from A and B.* *(NCERT)*



Solution. Given line l is perpendicular bisector of AB and P is a point on l, so M is mid-point of AB and MP ⊥ AB.

In ΔAMP and ΔBMP,

AM = MB (∵ M is mid-point of AB)

MP = MP (common)

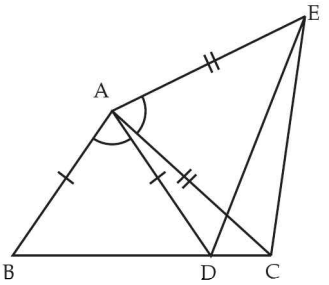
∠AMP = ∠BMP (each = 90°)

∴ ΔAMP ≅ ΔBMP (by SAS axiom of congruency)

∴ AP = BP (c.p.c.t.)

⇒ P is equidistant from A and B.

Example 6. *In the adjoining figure, AC = AE, AB = AD and ∠BAD = ∠EAC. Show that BC = DE.* *(NCERT)*



Solution. Given ∠BAD = ∠EAC

⇒ ∠BAD + ∠DAC = ∠EAC + ∠DAC

(Adding ∠DAC to both sides)

⇒ ∠BAC = ∠DAE.

In ΔABC and ΔADE,

AB = AD (given)

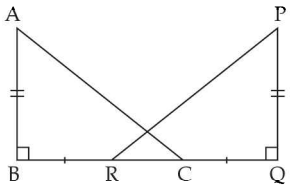
AC = AE (given)

∠BAC = ∠DAE (proved above)

∴ ΔABC ≅ ΔADE (by SAS axiom of congruency)

∴ BC = DE (c.p.c.t.)

Example 7. *In the adjoining figure, AB = PQ, BR = CQ, AB ⊥ BC and PQ ⊥ RQ. Prove that AC = PR.*



Solution. Given BR = CQ

⇒ BR + RC = CQ + RC

(Adding RC to both sides)

⇒ BC = QR

In ΔABC and ΔPQR,

AB = PQ (given)

BC = QR (proved above)

∠ABC = ∠PQR (each = 90°, ∵ AB ⊥ BC and PQ ⊥ RQ)

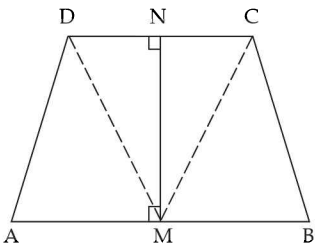
∴ ΔABC ≅ ΔPQR (by SAS axiom of congruency)

∴ AC = PR (c.p.c.t.)

Example 8. *Line segment joining the mid-points M and N of parallel sides AB and DC respectively of a trapezium ABCD is perpendicular to both the sides AB and DC. Prove that AD = BC.* *(NCERT Exemplar)*

Solution. Join CM and DM

In ΔCMN and ΔDMN,



MN = MN (common)

CN = DN (∵ N is mid-point of DC)

∠CNM = ∠DNM

(each = 90°, ∵ MN ⊥ DC)

∴ ΔCMN ≅ ΔDMN

(by SAS axiom of congruency)

$\therefore$ $CM = DM$ (c.p.c.t.)

and $\angle CMN = \angle DMN$ (c.p.c.t.)

As $MN \perp AB$, $\angle AMN = \angle BMN$ (each = 90°)

$\Rightarrow$ $\angle AMN - \angle DMN = \angle BMN - \angle CMN$ ($\because$ $\angle DMN = \angle CMN$, proved above)

$\Rightarrow$ $\angle AMD = \angle BMC$

In ΔAMD and ΔBMC,

$AM = BM$ ($\because$ M is mid-point of AB)

$DM = CM$ (proved above)

$\angle AMD = \angle BMC$ (proved above)

$\therefore$ $\Delta AMD \cong \Delta BMC$

$\therefore$ $AD = BC$ (c.p.c.t.)

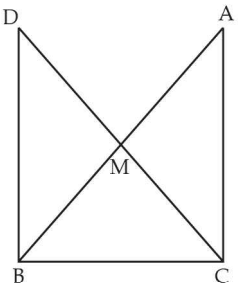
Example 9. *In right triangle ABC, right angled at C, M is mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B. Show that*

(i) $\Delta AMC \cong \Delta BMD$

(ii) $\angle DBC$ *is a right angle*

(iii) $\Delta DBC \cong \Delta ACB$

(iv) $CM = \frac{1}{2}AB$ *(NCERT)*



Solution. (*i*) In ΔAMC and ΔBMD,

$AM = BM$ ($\because$ M is mid-point of AB)

$CM = DM$ (given)

$\angle AMC = \angle BMD$ (vert. opp. $\angle$s)

$\therefore$ $\Delta AMC \cong \Delta BMD$ (by SAS axiom of congruency)

$\therefore$ $AC = DB$ (c.p.c.t.)

and $\angle CAM = \angle DBM$ (c.p.c.t.)

(*ii*) $\angle CAM = \angle DBM$ (proved above)

But these are alternate angles, therefore, AC || DB

$\therefore$ $\angle DBC + \angle ACB = 180°$ (sum of co-int. $\angle$s)

$\Rightarrow$ $\angle DBC + 90° = 180° \Rightarrow \angle DBC = 90°$

$\Rightarrow$ $\angle DBC$ is a right angle.

(*iii*) In ΔDBC and ΔACB,

$DB = AC$ (proved above)

$BC = CB$ (common)

$\angle DBC = \angle ACB$ (each = 90°)

$\therefore$ $\Delta DBC \cong \Delta ACB$ (by SAS axiom of congruency)

(*iv*) $DC = AB$ (c.p.c.t.)

$\Rightarrow$ $\frac{1}{2}DC = \frac{1}{2}AB \Rightarrow CM = \frac{1}{2}AB$ ($\because$ DM = CM)

Exercise 5.1

1. It is given that $\Delta ABC \cong \Delta RPQ$. Is it true to say that BC = QR? Why? *(NCERT Exemplar)*

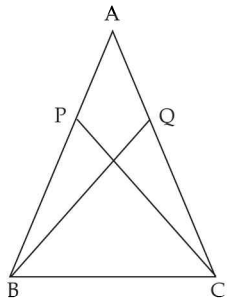
2. "If two sides and an angle of one triangle are equal to two sides and an angle of another triangle, then the two triangles must be congruent." Is the statement true? Why? *(NCERT Exemplar)*

3. In the adjoining figure, AB = AC and AP = AQ. Prove that

(*i*) $\Delta APC \cong \Delta AQB$

(*ii*) CP = BQ

(*iii*) $\angle APC = \angle AQB$.

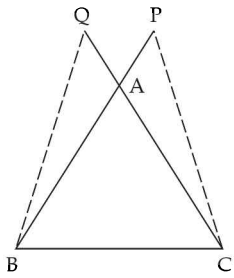


4. In the adjoining figure, AB = AC, P and Q are points on BA and CA respectively such that AP = AQ. Prove that

(*i*) $\Delta APC \cong \Delta AQB$

(*ii*) CP = BQ

(*iii*) $\angle ACP = \angle ABQ$.

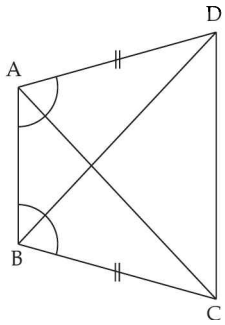


5. In the adjoining figure, ABCD is a quadrilateral in which AD = BC and $\angle DAB = \angle CBA$. Prove that

(*i*) $\Delta ABD \cong \Delta BAC$

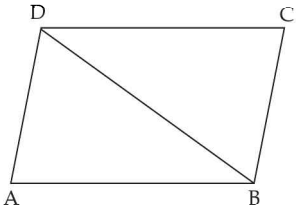
(*ii*) BD = AC

(*iii*) $\angle ABD = \angle BAC$. *(NCERT)*



6. In the adjoining figure, AB = DC and AB || DC. Prove that AD = BC.

Hint: As AB || DC, $\angle ABD = \angle CDB$ (alt. $\angle$s).
Show that $\Delta ABD \cong \Delta CDB$.



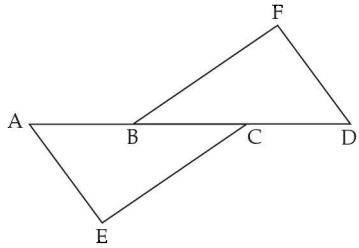
7. In the adjoining figure, AB = CD, CE = BF and $\angle ACE = \angle DBF$. Prove that

(*i*) $\Delta ACE \cong \Delta DBF$

(*ii*) AE = DF.

Hint: AB = CD $\Rightarrow$ AB + BC = CD + BC

$\Rightarrow$ AC = BD.



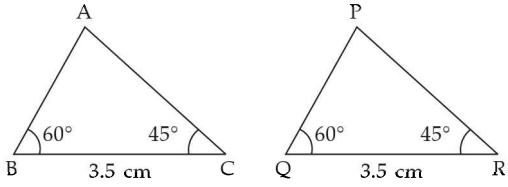
Answers

5.3 ASA CONGRUENCE RULE

Draw two triangles ABC and PQR such that BC = QR = 3.5 cm, $\angle B = \angle Q = 60°$ and $\angle C = \angle R = 45°$.

Make a trace-copy of ΔABC and place it over ΔPQR. We observe that the two triangles cover each other exactly and so they are congruent.

Repeat this activity with more pairs of triangles satisfying these conditions. We observe that the equality of two angles and the included side is sufficient for the congruence of two triangles. Let us state and prove this result with the help of SAS axiom of congruency.



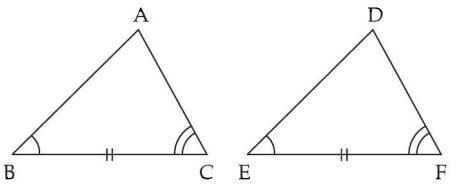
Theorem 5.1. *Two triangles are congruent if two angles and the included side of one triangle are equal to two angles and the included side of the other triangle.*

Given. Two triangles ABC and DEF such that $\angle B = \angle E$, $\angle C = \angle F$ and BC = EF.

To prove. $\Delta ABC \cong \Delta DEF$.

Proof. Three cases arise:

Case I. Let AB = DE (see the adjoining figure)



In ΔABC and ΔDEF,

$\angle B = \angle E$ (given)

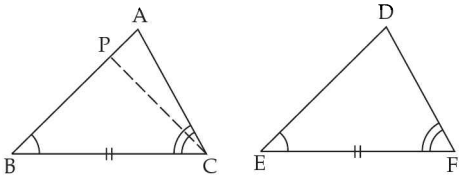
AB = DE (assumption)

BC = EF (given)

$\therefore \quad \Delta ABC \cong \Delta DEF$

(by SAS axiom of congruency)

Case II. Let AB > DE, then take point P on AB such that PB = DE (see the adjoining figure).



In ΔPBC and ΔDEF,

$\angle B = \angle E$ (given)

BC = EF (given)

PB = DE (by construction)

$\therefore \quad \Delta PBC \cong \Delta DEF$

(by SAS axiom of congruency)

$\therefore \quad \angle PCB = \angle DFE$ (c.p.c.t.)

But we are given that $\angle ACB = \angle DEF$.

So, $\angle PCB = \angle DEF$.

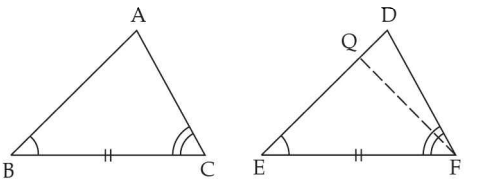
This is possible only if points P and A coincide

$\Rightarrow$ PB = AB but PB = DE

$\Rightarrow$ AB = DE.

$\therefore \quad \Delta ABC \cong \Delta DEF$ (by SAS axiom of congruency)

Case III. Let AB < DE, then take a point Q on ED such that AB = QE (see the adjoining figure).



By repeating the arguments as given in case II, we can conclude that

AB = DE

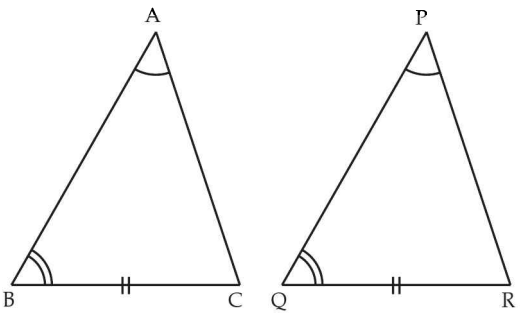
$\therefore \quad \Delta ABC \cong \Delta DEF$ (by SAS axiom of congruency)

The above rule is known as **Angle-Side-Angle** rule (or criterion) of congruency.

❑ **AAS congruency rule**

Two triangles are congruent if any two pairs of angles and a pair of corresponding sides are equal.

In the adjoining figure,



$\angle A = \angle P$, $\angle B = \angle Q$ and

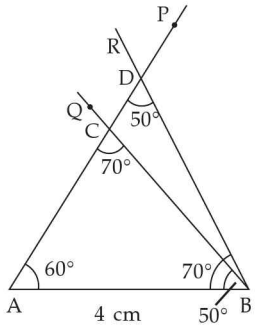
BC = QR

$\therefore \quad \Delta ABC \cong \Delta PQR$

(The result follows immediately from ASA congruence rule because if two angles of a triangle are equal to two angles of another triangle, then the third angles of both triangles are also equal.)

Note. The equality of 'corresponding sides' is essential.

Look at the adjoining figure :



Here, AB = 4 cm and ∠PAB = 60°.

Draw ∠ABQ = 50° to meet AP at C.

Also draw ∠ABR = 70° to meet AP at D.

Thus, we get two triangles ABC and ABD.

In ΔABC, ∠CAB = 60° and ∠ABC = 50° so that

∠ACB = 180° – 60° – 50° = 70°

(∵ sum of angles in a triangle = 180°)

In ΔABD, ∠DAB = 60° and ∠ABD = 70° so that

∠ADB = 180° – 60° – 70° = 50° (as shown in the figure).

We consider two triangles ABC and ABD. In these triangles, we have

∠CAB = ∠DAB (each = 60°)

∠ABC = ∠ADB (each = 50°)

and AB = AB.

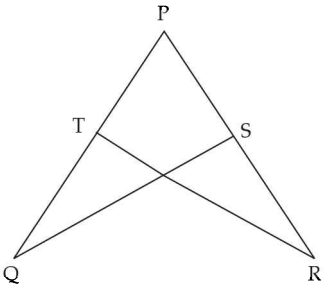
Thus, two angles and one side of one triangle is equal to two angles and one side of another triangle but the triangles are not congruent which is clear from the figure because ΔABC is a part of ΔABD. In fact, two vertices A and B are same for both triangles but their third vertices C and D do not coincide. Here, note that the side AB is not corresponding side (because in ΔABC, side AB is opposite to angle of 70° and in ΔABD, side AB is opposite to angle of 50°).

Illustrative Examples

Example 1. *In the adjoining figure, PQ = PR and ∠Q = ∠R. Prove that*

(i) $\Delta PQS \cong \Delta PRT$

(ii) $QS = RT$. *(NCERT Exemplar)*



Solution. (*i*) In ΔPQS and ΔPRT,

∠Q = ∠R (given)

∠P = ∠P (common)

PQ = PR (given)

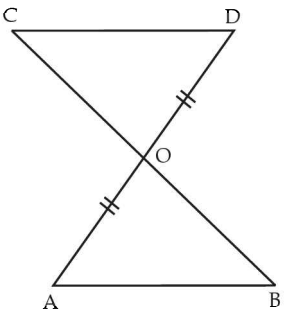
∴ ΔPQS ≅ ΔPRT (by ASA congruence rule)

(*ii*) QS = RT (c.p.c.t.)

Example 2. *In the adjoining figure, line segment AB is parallel to another line segment CD. O is mid-point of AD. Show that*

(i) $\Delta AOB \cong \Delta DOC$

(ii) O is also mid-point of BC. *(NCERT)*



Solution. (*i*) As AB || CD and BC is a transversal,

∠ABO = ∠DCO (alt. ∠s)

In ΔAOB and ΔDOC,

OA = OD (O is mid-point of AD)

∠ABO = ∠DCO, (proved above)

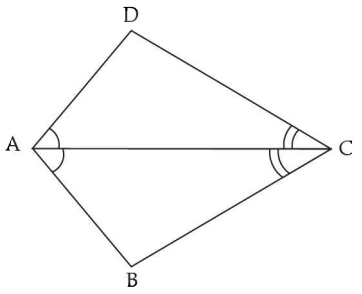
and ∠AOB = ∠DOC (vert. opp. ∠s)

∴ ΔAOB ≅ ΔDOC (by AAS rule of congruency)

(*ii*) OB = OC (c.p.c.t.)

⇒ O is mid-point of BC.

Example 3. *ABCD is a quadrilateral such that diagonal AC bisects the angles A and C. Prove that AB = AD and CB = CD.* *(NCERT Exemplar)*



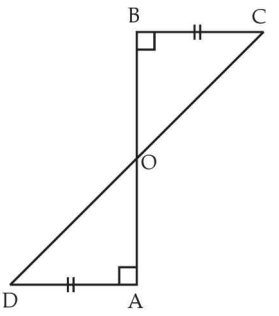
Solution. As AC is bisector of ∠A and of ∠C,

$\therefore$ ∠BAC = ∠DAB and ∠BCA = ∠DCA.

In ΔABC and ΔADC,

	∠BAC	= ∠DAB	(from given)
	∠BCA	= ∠DCA	(from given)
	AC	= AC	(common)
$\therefore$	ΔABC	≅ ΔADC	
$\therefore$	AB	= AD and CB = CD	(c.p.c.t.)

Example 4. *In the adjoining figure, AD and BC are equal perpendiculars to a line segment AB. Show that CD bisects AB.* *(NCERT)*



Solution. In ΔOBC and ΔOAD,

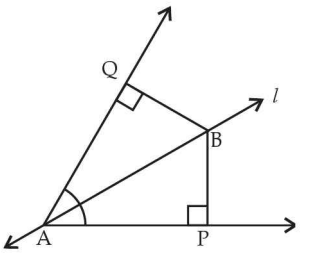
	BC	= AD	(given)
	∠OBC	= ∠OAD	(each = 90°)
	∠BOC	= ∠AOD	(vert. opp. ∠s)
$\therefore$	ΔOBC	≅ ΔOAD	(by AAS rule of congruency)
$\therefore$	OB	= OA	(c.p.c.t.)

$\Rightarrow$ CD bisects AB.

Example 5. *In the adjoining figure, line l is the bisector of an angle A and B is any point on l. BP and BQ are perpendiculars from B to the arms of ∠A. Show that*

(i) $\Delta APB \cong \Delta AQB$

(ii) B is equidistant from the arms of ∠A. *(NCERT)*



Solution. (*i*) In ΔAPB and ΔAQB,

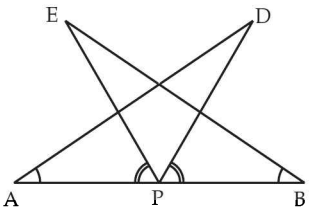
	∠BAP	= ∠BAQ	($\because$ line *l* is bisector of ∠A)
	∠APB	= ∠AQB	(each = 90°)
and	AB	= AB	(common)
$\therefore$	ΔAPB	≅ ΔAQB	(by AAS rule of congruency)
(*ii*)	BP	= BQ	(c.p.c.t.)

$\Rightarrow$ B is equidistant from the arms of ∠A.

Example 6. *In the adjoining figure, AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that ∠BAD = ∠ABE and ∠EPA = ∠DPB. Show that*

(i) $\Delta DAP \cong \Delta EBP$

(ii) AD = BE.



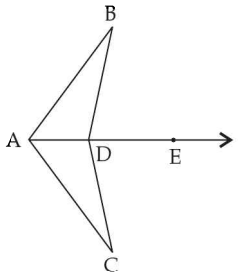
Solution. (*i*) Given ∠EPA = ∠DPB

$\Rightarrow$	∠EPA + ∠EPD	= ∠EPD + ∠DPB	(adding ∠EPD to both sides)
$\Rightarrow$	∠APD	= ∠EPB.	

In ΔDAP and ΔEBP,

	∠PAD	= ∠PBE	($\because$ ∠BAD = ∠ABE given)
	∠APD	= ∠EPB	(proved above)
	AP	= BP	($\because$ P is mid-point of AB)
$\therefore$	ΔDAP	≅ ΔEBP	(by ASA rule of congruency)
(*ii*)	AD	= BE	(c.p.c.t.)

Example 7. *In the adjoining figure, AE is bisector of ∠BAC and of ∠BDC. Show that ΔABD ≅ ΔACD and hence BD = CD.*



Solution. As AE is bisector of ∠BAC,

$\therefore \quad \angle BAD = \angle CAD \quad \ldots(i)$

Also, AE is bisector of ∠BDC,

$\therefore \quad \angle BDE = \angle CDE \quad \ldots(ii)$

Since exterior angle of a triangle is equal to sum of two interior opposite angles,

$\angle BDE = \angle BAD + \angle ABD$ and $\angle CDE = \angle CAD + \angle ACD$

$\Rightarrow \quad \angle BAD + \angle ABD = \angle CAD + \angle ACD$ (using (*ii*))

$\Rightarrow \quad \angle ABD = \angle ACD$ (using (*i*))

In ΔABD and ΔACD,

$\angle BAD = \angle CAD$ (by (*i*))

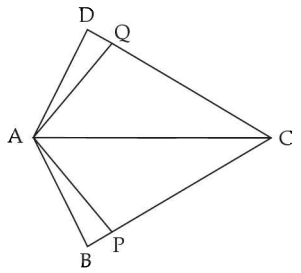
$\angle ABD = \angle ACD$ (proved above)

$AD = AD$ (common)

$\therefore \quad \Delta ABD \cong \Delta ACD$ (by AAS congruency rule)

$\Rightarrow \quad BD = CD$ (c.p.c.t.)

Example 8. *In the adjoining figure, AB = AD, ∠BAP = ∠QAD and ∠PAC = ∠CAQ. Prove that AP = AQ.*



Solution. Given $\angle BAP = \angle QAD$

and $\angle PAC = \angle CAQ$

$\Rightarrow \quad \angle BAP + \angle PAC = \angle QAD + \angle CAQ$

$\Rightarrow \quad \angle BAC = \angle CAD$

In ΔABC and ΔADC,

$AB = AD$ (given)

$AC = AC$ (common)

$\angle BAC = \angle CAD$ (proved above)

$\therefore \quad \Delta ABC \cong \Delta ADC$ (by SAS axiom of congruency)

$\Rightarrow \quad \angle ABC = \angle ADC$ (c.p.c.t.)

In ΔABP and ΔADQ,

$AB = AD$ (given)

$\angle BAP = \angle QAD$ (given)

$\angle ABC = \angle ADC$ (proved above)

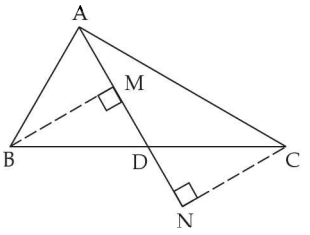
$\therefore \quad \Delta ABP \cong \Delta ADQ$ (by ASA congruence rule)

$\Rightarrow \quad AP = AQ.$

Exercise 5.2

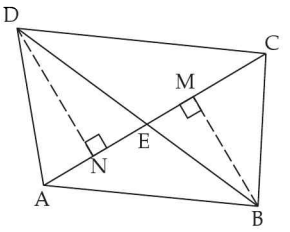
1. In triangles ABC and PQR, ∠A = ∠Q and ∠B = ∠R. Which side of ΔPQR should be equal to side AB of ΔABC so that the two triangles are congruent? Give reason for your answer. *(NCERT Exemplar)*
2. In triangles ABC and PQR, ∠A = ∠Q and ∠B = ∠R. Which side of ΔPQR should be equal to side BC of ΔABC so that the two triangles are congruent? Give reason for your answer. *(NCERT Exemplar)*
3. "If two angles and a side of one triangle are equal to two angles and a side of another triangle, then the two triangles must be congruent". Is the statement true? Why? *(NCERT Exemplar)*

4. In the adjoining figure, AD is median of ΔABC, BM and CN are perpendiculars drawn from B and C respectively on AD and AD produced. Prove that BM = CN.



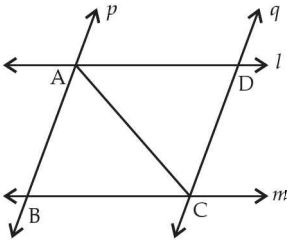
Hint: ΔBMD ≅ ΔCND by AAS.

5. In the adjoining figure, BM and DN are perpendiculars to the line segment AC. If BM = DN, prove that AC bisects BD.

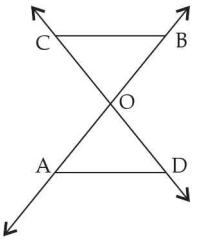


Hint: ΔBEM ≅ ΔDEN by AAS.

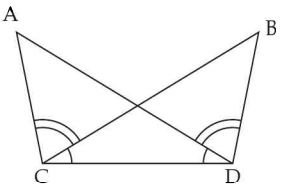
6. In the adjoining figure, l and m are two parallel lines intersected by another pair of parallel lines p and q. Show that ΔABC ≅ ΔCDA. *(NCERT)*



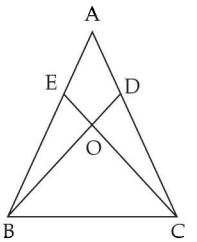
7. In the adjoining figure, two lines AB and CD intersect each other at the point O such that BC || DA and BC = DA. Show that O is the mid-point of both the line segments AB and CD. *(NCERT Exemplar)*



8. In the adjoining figure, ∠BCD = ∠ADC and ∠BCA = ∠ADB. Show that
   (*i*) ΔACD ≅ ΔBDC
   (*ii*) BC = AD
   (*iii*) ∠A = ∠B.



9. In the adjoining figure, ∠ABC = ∠ACB, D and E are points on the sides AC and AB respectively such that BE = CD. Prove that
   (*i*) ΔEBC ≅ ΔDCB
   (*ii*) ΔOEB ≅ ΔODC
   (*iii*) OB = OC.

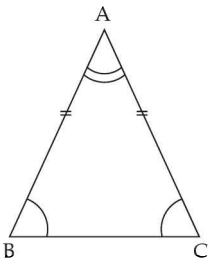


Answers

5.4 SOME PROPERTIES OF A TRIANGLE

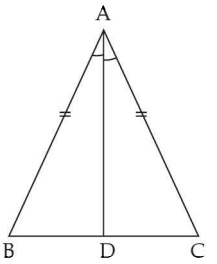
A triangle in which lengths of two sides are equal is called an isosceles triangle.

In the adjoining figure, AB = AC, so ΔABC is an isosceles triangle. The angles which are opposite to equal sides are called *base angles* and ∠A is *vertical angle.*



Make a replica of $\angle B$ and place it over $\angle C$. We observe that replica of $\angle B$ exactly covers $\angle C$. It follows that $\angle B = \angle C$ *i.e.* the base angles are equal. It is an important property of an isosceles triangle. In fact, we can prove this property by using SAS axiom of congruency. We have:

Theorem 5.2. ***The angles opposite to equal sides of an isosceles triangle are equal.***



Given. An isosceles triangle ABC in which

$AB = AC$.

To prove. $\angle B = \angle C$.

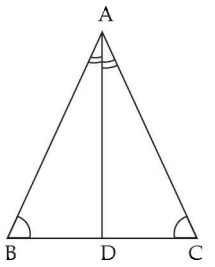
Construction. Draw AD bisector of $\angle A$ to meet BC at D.

Proof.

Statements	*Reasons*
In ΔABD and ΔACD	
1. $AB = AC$	1. Given
2. $AD = AD$	2. Common
3. $\angle BAD = \angle CAD$	3. AD is bisector of $\angle A$ by construction.
4. $\Delta ABD \cong \Delta ACD$	4. SAS axiom of congruency
5. $\angle B = \angle C$	5. c.p.c.t.

The converse of the above theorem is also true *i.e.* the sides opposite to equal angles of a triangle are equal. We can prove this property of a triangle by using AAS congruence rule. We have:

Theorem 5.3. ***The sides opposite to equal angles of a triangle are equal.***



Given. A triangle ABC in which $\angle B = \angle C$.

To prove. $AB = AC$.

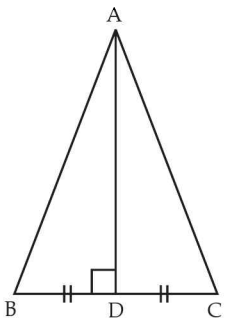
Construction. Draw AD bisector of $\angle A$ to meet BC at D.

Proof.

Statements	*Reasons*
In ΔABD and ΔACD	
1. $\angle B = \angle C$	1. Given
2. $AD = AD$	2. Common
3. $\angle BAD = \angle CAD$	3. AD is bisector of $\angle A$ by construction.
4. $\Delta ABD \cong \Delta ACD$	4. AAS axiom of congruency
5. $AB = AC$	5. c.p.c.t.

Illustrative Examples

Example 1. *In ΔABC, AD is the perpendicular bisector of BC. Show that ΔABC is isosceles in which $AB = AC$.* *(NCERT)*



Solution. In ΔABD and ΔACD,

$BD = CD$

($\because$ AD is bisector of BC *i.e.* D is mid-point of BC)

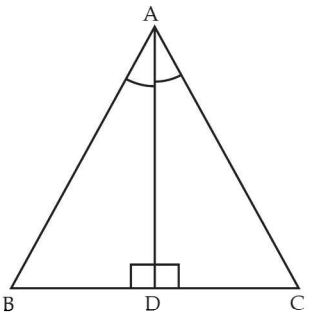
$\angle ADB = \angle ADC$ (each $= 90°$ as $AD \perp BC$)

AD = AD (common)

$\therefore$ $\Delta ABD \cong \Delta ACD$ (by SAS axiom of congruency)

$\therefore$ AB = AC $\Rightarrow$ ΔABC is isosceles.

Example 2. *In ΔABC, the bisector AD of $\angle A$ is perpendicular to the side BC. Show that AB = AC and ΔABC is isosceles.* *(NCERT)*



Solution. In ΔABD and ΔACD,

$\angle BAD = \angle CAD$ ($\because$ AD is bisector of $\angle A$)

$\angle ADB = \angle ADC$ (each = 90° as AD $\perp$ BC)

AD = AD (common)

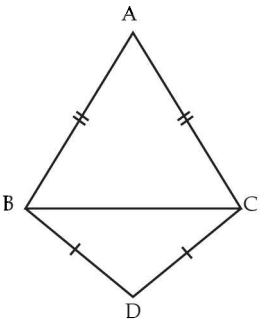
$\therefore$ $\Delta ABD \cong \Delta ACD$

(by ASA rule of congruency)

$\therefore$ AB = AC $\Rightarrow$ ΔABC is isosceles.

Example 3. *In the adjoining figure, ABC and DBC are two isosceles triangles on the same base BC.*

Show that $\angle ABD = \angle ACD$. *(NCERT)*



Solution. AC = AB (given) $\Rightarrow$ $\angle ABC = \angle ACB$ ...(*i*)

($\because$ $\angle$s opp. equal sides are equal)

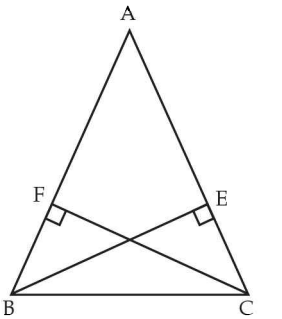
DC = DB (given) $\Rightarrow$ $\angle CBD = \angle BCD$ ...(*ii*)

On adding (*i*) and (*ii*), we get

$\angle ABC + \angle CBD = \angle ACB + \angle BCD$

$\Rightarrow$ $\angle ABD = \angle ACD$.

Example 4. *ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively. Show that these altitudes are equal.* *(NCERT)*



Solution. Given AC = AB $\Rightarrow$ $\angle B = \angle C$

($\because$ $\angle$s opp. equal sides are equal)

In ΔBEC and ΔCFB,

$\angle C = \angle B$ (from above)

$\angle BEC = \angle CFB$ (each = 90°)

BC = CB (common)

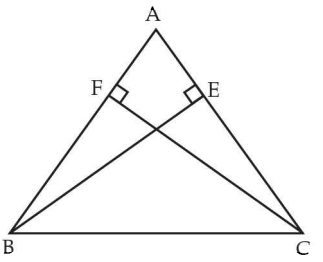
$\therefore$ $\Delta BEC \cong \Delta CFB$ (by AAS rule of congruency)

$\therefore$ BE = CF (c.p.c.t.)

Example 5. *ABC is a triangle in which altitudes BE and CF to sides AC and AB respectively are equal. Show that*

(i) $\Delta ABE \cong \Delta ACF$

(ii) AB = AC and ΔABC is isosceles. *(NCERT)*



Solution. (*i*) In ΔABE and ΔACF,

BE = CF (given)

$\angle AEB = \angle AFC$ (each = 90°)

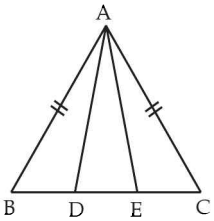
$\angle A = \angle A$ (common)

$\because$ $\Delta ABE \cong \Delta ACF$ (by AAS rule of congruency)

(*ii*) AB = AC (c.p.c.t.)

$\Rightarrow$ ΔABC is isosceles.

Example 6. *In the adjoining figure, ABC is an isosceles triangle in which AB = AC. If D and E are points on BC such that BE = DC, show that AD = AE.* *(NCERT)*



Solution. Given AB = AC ⇒ ∠B = ∠C

(∵ ∠s opp. equal sides are equal)

Also BE = DC ⇒ BE – DE = DC – DE

⇒ BD = EC.

In ΔABD and ΔACE,

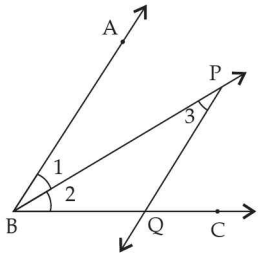
AB = AC, ∠B = ∠C and BD = EC

∴ ΔABD ≅ ΔACE (by SAS axiom of congruency)

∴ AD = AE (c.p.c.t.)

Example 7. *P is a point on the bisector of ∠ABC. If the line through P, parallel to BA meets BC at Q, prove that BPQ is an isosceles triangle.* *(NCERT Exemplar)*

Solution. Mark the angles as shown in the figure.



As BP is bisector of ∠ABC,

∠1 = ∠2 ...(*i*)

Since QP is parallel to BA and BP is a transversal,

∠1 = ∠3 ...(*ii*) (alt. int. ∠s)

From (*i*) and (*ii*), we get

∠2 = ∠3

⇒ PQ = BQ (sides opp. equal ∠s of a triangle are equal)

⇒ ΔBPQ is an isosceles triangle.

Example 8. *In an isosceles triangle ABC, with AB = AC, the bisectors of ∠B and ∠C intersect each other at O. Show that*

(i) OB = OC *(ii) OA bisects ∠A* *(NCERT)*

Solution. (*i*) Given AB = AC ⇒ ∠B = ∠C (∵ ∠s opp. equal sides are equal)

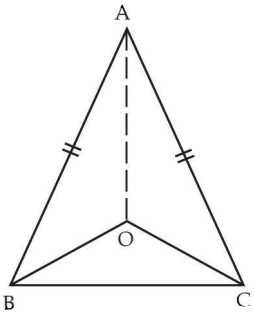
⇒ $\frac{1}{2}$∠B = $\frac{1}{2}$ ∠C

⇒ ∠OBC = ∠OCB

(∵ OB and OC are bisectors of ∠B and ∠C)

⇒ OB = OC

(sides opp. equal angles are equal)



(*ii*) Also $\frac{1}{2}$∠B = $\frac{1}{2}$ ∠C ⇒ ∠ABO = ∠ACO

(∵ OB and OC are bisectors of ∠B and ∠C respectively)

In ΔOAB and ΔOAC,

∠ABO = ∠ACO (proved above)

OB = OC (proved above)

AB = AC (given)

∴ ΔOAB ≅ ΔOAC (by SAS axiom of congruency)

∴ ∠OAB = ∠OAC (c.p.c.t.)

⇒ OA bisects ∠A.

Example 9. *Show that the angles of an equilateral triangle are 60° each.* *(NCERT)*

Solution. Let ABC be an equilateral triangle,

then AB = AC = BC.

Now $AB = AC \Rightarrow \angle C = \angle B$ ...(*i*)

($\because$ $\angle$s opp. equal sides are equal)

Also $AC = BC \Rightarrow \angle B = \angle A$ ...(*ii*)

From (*i*) and (*ii*), we get

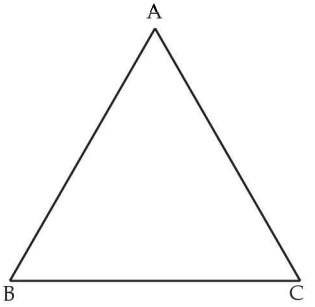
$\angle A = \angle B = \angle C$.

But $\angle A + \angle B + \angle C = 180°$ ($\angle$s of ΔABC)

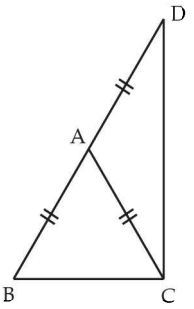
$\Rightarrow$ $\angle A + \angle A + \angle A = 180°$

$\Rightarrow$ $3\angle A = 180° \Rightarrow \angle A = 60°$.

Hence, $\angle A = \angle B = \angle C = 60°$.



Example 10. *In the adjoining figure, ABC is an isosceles triangle in which AB = AC. If side BA is produced to D such that AD = AB, then show that $\angle BCD$ is a right angle.* *(NCERT)*



Solution. In ΔABC, $AB = AC \Rightarrow \angle ABC = \angle ACB$ ...(*i*)

$AD = AB$ (given) but $AB = AC \Rightarrow AD = AC$

In ΔACD, $AD = AC \Rightarrow \angle ADC = \angle ACD$ ...(*ii*)

Adding (*i*) and (*ii*), we get

$\angle ABC + \angle ADC = \angle ACB + \angle ACD = \angle BCD$ ...(*iii*)

In ΔBCD, $\angle DBC + \angle BDC + \angle BCD = 180°$ ($\angle$s of ΔBCD)

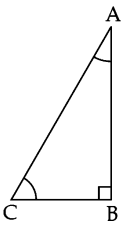
$\Rightarrow$ $\angle ABC + \angle ADC + \angle BCD = 180°$

$\Rightarrow$ $\angle BCD + \angle BCD = 180°$ [using (*iii*)]

$\Rightarrow$ $2\angle BCD = 180° \Rightarrow \angle BCD = 90°$.

Hence, $\angle BCD$ is a right angle.

Example 11. *In the adjoining figure, ABC is a right triangle and right angled at B such that $\angle BCA = 2\angle BAC$. Show that hypotenuse $AC = 2BC$.* *(NCERT Exemplar)*



Solution. Produce CB to a point D such that

$BD = BC$.

Let $\angle BAC = x$, then $\angle BCA = 2\angle BAC = 2x$.

In ΔABC and ΔABD,

$BC = BD$ (by construction)

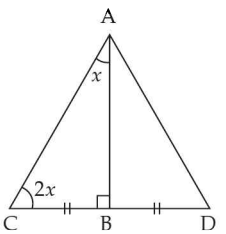
$AB = AB$ (common)

$\angle ABC = \angle ABD$ (each = 90°)

$\therefore$ $\Delta ABC \cong \Delta ABD$ (by SAS axiom of congruency)

$\therefore$ $\angle BAC = \angle BAD$ and $\angle BCA = \angle BDA$ (c.p.c.t.)

$\Rightarrow$ $\angle BAD = x$ and $\angle BDA = 2x$ *i.e.* $\angle CDA = 2x$.



Now, $\angle CAD = \angle BAC + \angle BAD = x + x = 2x$.

Thus, in ΔACD, $\angle CAD = \angle CDA$

$\Rightarrow$ $AC = CD$ (side opp. equal angles are equal)

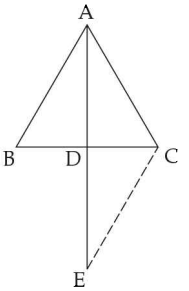
$\Rightarrow$ $AC = BC + BD \Rightarrow AC = BC + BC$ ($\because$ $BD = BC$)

$\Rightarrow$ $AC = 2BC$.

Example 12. *If the bisector of an angle of a triangle also bisects the opposite side, prove that the triangle is isosceles.* *(NCERT Exemplar)*

Solution. Let the bisector AD of $\angle A$ meet side BC of ΔABC at point D.

As the bisector AD bisects opposite side BC, so D is mid-point of BC.



Produce AD to a point E such that DE = AD. Join C and E.

In ΔABD and ΔECD,

$BD = CD$ ($\because$ D is mid-point of BC)

$AD = DE$ (by construction)

$\angle ADB = \angle EDC$ (vert. opp. $\angle$s)

$\therefore \quad \Delta ABD \cong \Delta ECD$ (by SAS axiom of congruency)

$\therefore \quad AB = EC$ and $\angle BAD = \angle CED$ (c.p.c.t.)

As AD is bisector of $\angle A$, $\angle BAD = \angle CAD$

$\Rightarrow \quad \angle CED = \angle CAD \Rightarrow \angle CEA = \angle CAE$

Thus, in ΔACE, $\angle CEA = \angle CAE$

$\Rightarrow \quad AC = EC$ (sides opp. equal $\angle$s are equal)

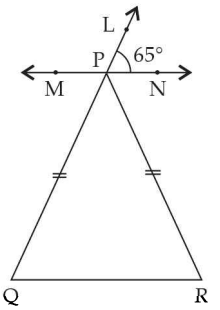
$\therefore \quad AB = EC = AC \Rightarrow AB = AC$

$\Rightarrow \quad \Delta ABC$ is an isosceles triangle.

Exercise 5.3

1. ABC is a right angled triangle in which $\angle A = 90°$ and AB = AC. Find $\angle B$ and $\angle C$. *(NCERT)*

2. In the adjoining figure, MN is parallel to QR. PQ = PR and $\angle LPN = 65°$. Find the measure of $\angle QPR$.



3. In a triangle ABC, AB = AC, D and E are points on the sides AB and AC respectively such that BD = CE. Show that:

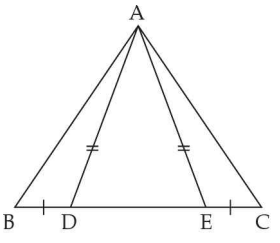
(*i*) $\Delta DBC \cong \Delta ECB$ (*ii*) $\angle DCB = \angle EBC$

(*iii*) OB = OC, where O is the point of intersection of BE and CD.

4. ABC is an isosceles triangle in which AB = AC. P is any point in the interior of ΔABC such that $\angle ABP = \angle ACP$. Prove that

(*a*) BP = CP (*b*) AP bisects $\angle BAC$.

5. In the adjoining figure, D and E are points on the side BC of ΔABC such that BD = EC and AD = AE. Show that $\Delta ABD \cong \Delta ACE$. *(NCERT Exemplar)*



Hint: AD = AE $\Rightarrow \angle ADE = \angle AED$

$\Rightarrow \quad 180° - \angle ADE = 180° - \angle AED$

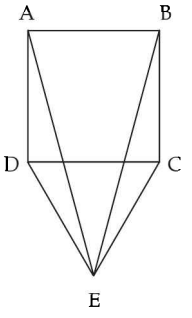
$\Rightarrow \quad \angle ADB = \angle AEC$.

$\Delta ABD \cong \Delta ACE$ (SAS).

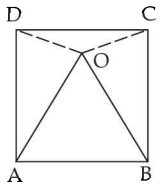
6. (*a*) In the figure (*i*) given below, CDE is an equilateral triangle formed on a side CD of a square ABCD. Show that $\Delta ADE \cong \Delta BCE$ and hence, AEB is an isosceles triangle. *(NCERT Exemplar)*

(*b*) In the figure (*ii*) given below, O is a point in the interior of a square ABCD such that OAB is an equilateral triangle. Show that OCD is an isosceles triangle.

(NCERT Exemplar)



(*i*)



(*ii*)

Hint:

(*a*) $\angle ADE = \angle ADC + \angle CDE = 90° + 60° = 150°$. Similarly, $\angle BCE = 150°$.

$\Rightarrow$ $\angle ADE = \angle BCE$. Also AD = BC and DE = EC.

$\therefore$ $\Delta ADE \cong \Delta BCE$ (SAS) $\Rightarrow$ AE = BE (c.p.c.t.)

(*b*) $\angle OAD = \angle DAB - \angle OAB = 90° - 60° = 30°$. Similarly, $\angle OBC = 30°$.

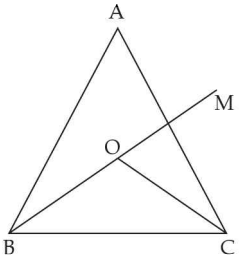
$\Rightarrow$ $\angle OAD = \angle OBC$. Also AD = BC and OA = OB.

$\therefore$ $\Delta OAD \cong \Delta OBC$ (SAS) $\Rightarrow$ OD = OC (c.p.c.t.)

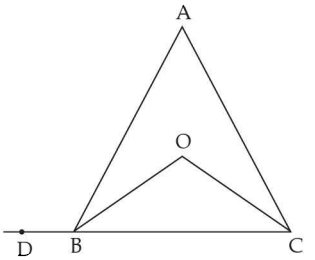
$\Rightarrow$ ΔOCD is an isosceles triangle.

7. (*a*) In the figure (*i*) given below, bisectors of $\angle B$ and $\angle C$ of an isosceles triangle ABC with AB = AC intersect each other at O. BO is produced to a point M. Prove that $\angle MOC = \angle ABC$. *(NCERT Exemplar)*

(*b*) In the figure (*ii*) given below, bisectors of $\angle B$ and $\angle C$ of an isosceles triangle ABC with AB = AC intersect each other at O. Show that the external angle adjacent to $\angle ABC$ is equal to $\angle BOC$. *(NCERT Exemplar)*



(*i*)



(*ii*)

Hint:

(*a*) AB = AC $\Rightarrow$ $\angle B = \angle C$.

As OB and OC are bisectors of $\angle B$ and $\angle C$ respectively,

$\angle OBC = \frac{1}{2}\angle B$ and $\angle OCB = \frac{1}{2}\angle C$

$\angle MOC = \angle OBC + \angle OCB$ (ext. $\angle$ = sum of two int. opp. $\angle$s)

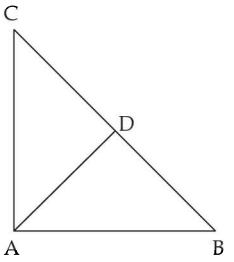
$\Rightarrow$ $\angle MOC = \frac{1}{2}\angle B + \frac{1}{2}\angle C = \frac{1}{2}\angle B + \frac{1}{2}\angle B = \angle B$.

(*b*) $\angle BOC = 180° - \angle OBC - \angle OCB = 180° - \frac{1}{2}\angle B - \frac{1}{2}\angle C$

$= 180° - \frac{1}{2}\angle B - \frac{1}{2}\angle B = 180° - \angle B = \angle ABD$.

8. In the adjoining figure, ABC is a right triangle with AB = AC. Bisector of ∠A meets BC at D. Prove that BC = 2AD.

(NCERT Exemplar)



Hint: In ΔABC, ∠A = 90° and AB = AC

⇒ ∠B = ∠C but ∠A + ∠B + ∠C = 180°

⇒ ∠B = 45°.

As AD is bisector of ∠A, ∠BAD = ∠CAD = 45°.

ΔABD ≅ ΔACD (SAS) ⇒ BD = CD (c.p.c.t.)

In ΔABD, ∠BAD = ∠ABD (each = 45°) ⇒ AD = BD.

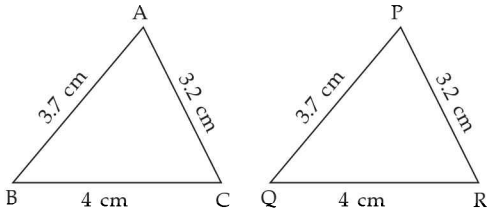
∴ BC = BD + CD = BD + BD = 2BD = 2AD.

Answers

5.5 SOME MORE CRITERIA FOR CONGRUENCE OF TRIANGLES

In earlier classes, we have learnt that three angles of one triangle equal to three angles of another triangle is not sufficient for the congruence of two triangles. Let us see whether three sides of one triangle equal to three sides of another triangle is enough for the congruence of two triangles.

Draw two triangles ABC and PQR such that AB = PQ = 3.7 cm, BC = QR = 4 cm and CA = RP = 3.2 cm.

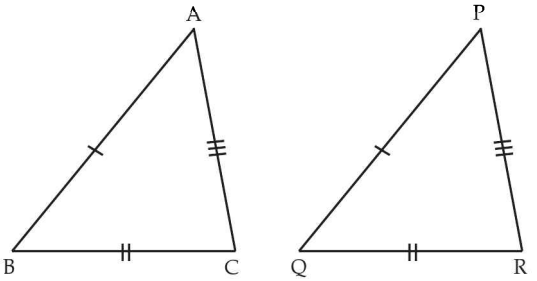


Make a trace-copy of ΔABC and place it over ΔPQR. We observe that the two triangles cover each other exactly and so they are congruent.

Repeat this activity with more pairs of triangles satisfying these conditions. We observe that the equality of three sides of two triangles is sufficient for the congruence of two triangles. In fact, this result can be proved by using a suitable construct. Thus, we get another rule of congruency. We record it as:

Theorem 5.4. (SSS congruence rule)

Two triangles are congruent if the three sides of one triangle are equal to the three sides of the other triangle.



In the adjoining diagram,

AB = PQ, BC = QR

and AC = PR

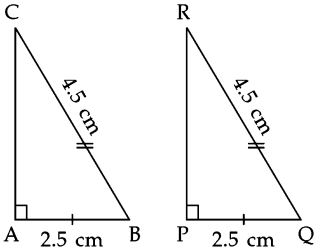
∴ ΔABC ≅ ΔPQR.

Note that SSS stands for **Side-Side-Side**.

In SAS congruence rule, we know that the pair of equal angles must be the included angle between the pairs of corresponding pair of equal sides and if this is not so, the two triangles may not be congruent.

Now perform the following activity:

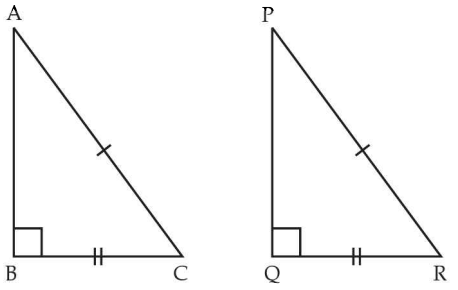
Draw two right angled triangles ABC and PQR such that hypotenuse BC = QR = 4.5 cm and one side = AB = PQ = 2.5 cm.



Make a trace-copy of ΔABC and place it over ΔPQR. We observe that the two triangles cover each other exactly and so they are congruent. Repeat this activity with more pairs of triangles satisfying these conditions. We observe that two right angled triangles are congruent if one pair of sides and the hypotenuse are equal. In fact, this result can be proved by using a suitable construction. Thus, we get another rule of congruency. We record it as:

Theorem 5.5. (RHS congruence rule)

Two right triangles are congruent if the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle.



In the adjoining figure,

$\angle B$ = a right angle = $\angle Q$,

AC = PR and BC = QR

$\therefore$ $\Delta ABC \cong \Delta PQR$.

Note that RHS stands for **Right angle-Hypotenuse-Side.**

Remark

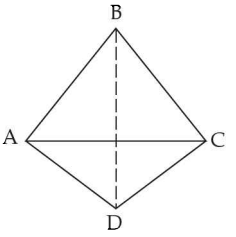
In the above congruence rule, right angle is **not** included angle between hypotenuse and side.

Illustrative Examples

Example 1. *ABCD is a quadrilateral in which AB = BC and AD = CD. Show that BD bisects both the angles ABC and ADC.* *(NCERT Exemplar)*

Solution. Given a quadrilateral ABCD in which AB = BC and AD = CD. Join B and D.

In ΔABD and ΔCBD,



	AB = BC	(given)
	AD = CD	(given)
	BD = BD	(common)
$\therefore$	$\Delta ABD \cong \Delta CBD$	(by SSS congruence rule)
$\therefore$	$\angle ABD = \angle CBD$	(c.p.c.t.)

$\Rightarrow$ BD is bisector of $\angle ABC$.

Also $\angle ADB = \angle CDB$ (c.p.c.t.)

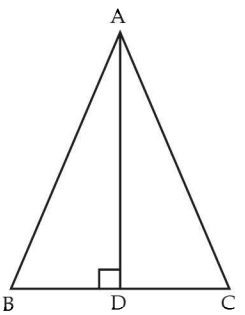
$\Rightarrow$ BD is bisector of $\angle ADC$.

Example 2. *AD is an altitude of an isosceles triangle ABC in which AB = AC. Show that*

(i) AD bisects BC *(ii) AD bisects $\angle A$.* *(NCERT)*

Solution. Given ABC is an isosceles triangle with AB = AC and AD $\perp$ BC.

In ΔABD and ΔACD,



	$\angle ADB = \angle ADC$	(each = 90° as AD $\perp$ BC)
	AB = AC	(given)
	AD = AD	(common)
$\therefore$	$\Delta ABD \cong \Delta ACD$	(by RHS rule of congruency)
$\therefore$	BD = CD	(c.p.c.t.)

$\Rightarrow$ AD bisects BC.

Also $\angle BAD = \angle CAD$ (c.p.c.t.)

$\Rightarrow$ AD bisects $\angle A$.

Example 3. *P is a point equidistant from two lines l and m intersecting at point A. Show that the line AP bisects the angle between them.* *(NCERT)*

Solution. Given lines l and m intersect each other at the point A.

Let PB $\perp$ l and PC $\perp$ m,

then PB = PC (given)

We need to show that

$\angle PAB = \angle PAC$.

In ΔAPB and ΔAPC,

$\angle ABP = \angle ACP$ (each = 90°)

$AP = AP$ (common)

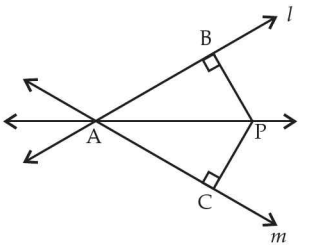
$PB = PC$ (given)

$\therefore \quad \Delta APB \cong \Delta APC$

(by RHS rule of congruency)

$\therefore \quad \angle PAB = \angle PAC.$

Hence, line AP bisects the angle between the given lines.



Example 4. *BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.*

Solution. Given $BE \perp CA$, $CF \perp AB$ and

$BE = CF.$

In ΔBEC and ΔCFB,

$\angle BEC = \angle CFB$ (each = 90°)

$BC = CB$ (common)

and $BE = CF$ (given)

$\therefore \quad \Delta BEC \cong \Delta CFB$

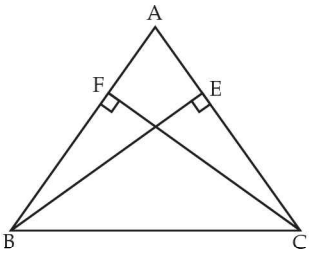
(by RHS rule of congruency)

$\therefore \quad \angle BCE = \angle CBF$ (c.p.c.t.)

$\Rightarrow \quad \angle BCA = \angle CBA$

$\Rightarrow \quad AB = AC$ ($\because$ sides opp. equal angles are equal)

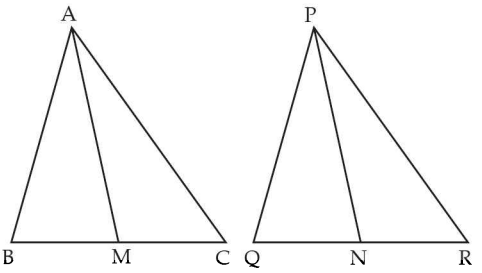
$\Rightarrow \Delta ABC$ is isosceles.



Example 5. *In the adjoining figure, two sides AB, BC and median AM of one triangle ABC are respectively equal to two sides PQ, QR and median PN of triangle PQR. Show that*

(*i*) $\Delta ABM \cong \Delta PQN$

(*ii*) $\Delta ABC \cong \Delta PQR.$ *(NCERT)*



Solution. (*i*) BC = QR (given)

$\Rightarrow \quad \frac{1}{2}BC = \frac{1}{2}QR \Rightarrow BM = QN$

In ΔABM and ΔPQN,

$AB = PQ$, $BM = QN$, $AM = PN$

$\therefore \quad \Delta ABM \cong \Delta PQN$ (by SSS rule of congruency)

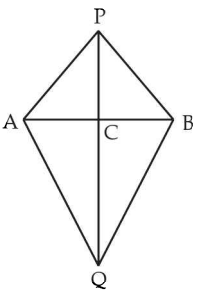
$\therefore \quad \angle B = \angle Q$ (c.p.c.t.)

(*ii*) In ΔABC and ΔPQR,

$AB = PQ$, $BC = QR$ and $\angle B = \angle Q$

$\therefore \quad \Delta ABC \cong \Delta PQR$ (by SAS axiom of congruency)

Example 6. *In the adjoining figure, AB is a line segment. P and Q are points on opposite sides of AB such that each of them is equidistant from the points A and B. Show that the line PQ is the perpendicular bisector of AB.* *(NCERT)*



Solution. Given AP = BP and AQ = BQ.

We need to show that $PQ \perp AB$ and PQ bisects AB.

In ΔAPQ and ΔBPQ,

$AP = BP$, $AQ = BQ$ and $PQ = PQ$ (common)

$\therefore \quad \Delta APQ \cong \Delta BPQ$ (by SSS rule of congruency)

$\therefore$ $\angle APC = \angle BPC$ (c.p.c.t.)

In ΔAPC and ΔBPC,

$AP = BP$, $\angle APC = \angle BPC$ and $CP = CP$ (common)

$\therefore$ $\Delta APC \cong \Delta BPC$ (by SAS axiom of congruency)

$\therefore$ $AC = CB \Rightarrow C$ is mid-point of AB

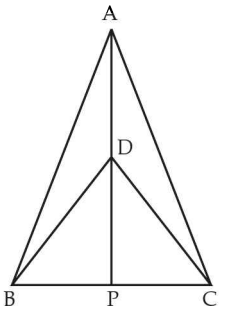
$\Rightarrow$ PQ bisects AB.

Also $\angle ACP = \angle BCP$ (c.p.c.t.)

But $\angle ACP + \angle BCP = 180°$ (linear pair)

$\Rightarrow$ $\angle ACP + \angle ACP = 180° \Rightarrow \angle ACP = 90°$

$\Rightarrow$ $PQ \perp AB$.



Example 7. *In the adjoining figure, ABC and DBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC. If AD is extended to intersect BC at P, show that*

(i) $\Delta ABD \cong \Delta ACD$ *(ii)* $\Delta ABP \cong \Delta ACP$

(iii) AP bisects $\angle A$ *as well as* $\angle D$

(iv) AP is the perpendicular bisector of BC.

Solution. *(i)* In ΔABD and ΔACD,

$AB = AC$, $BD = CD$ and $AD = AD$ (common)

$\therefore$ $\Delta ABD \cong \Delta ACD$ (SSS rule of congruency)

$\therefore$ $\angle BAD = \angle CAD$ (c.p.c.t.)

$\angle BAP = \angle CAP$

(ii) In ΔABP and ΔACP,

$AB = AC$, $\angle BAP = \angle CAP$ and $AP = AP$ (common)

$\therefore$ $\Delta ABP \cong \Delta ACP$ (SAS axiom of congruency)

(iii) $\angle BAP = \angle CAP$ (proved above)

$\Rightarrow$ AP bisects $\angle A$.

As $\Delta ABD \cong \Delta ACD$ (proved above)

$\angle ADB = \angle ADC$ (c.p.c.t.)

$\Rightarrow$ $180° - \angle ADB = 180° - \angle ADC$

$\Rightarrow$ $\angle BDP = \angle CDP \Rightarrow$ AP bisects $\angle D$.

(iv) As $\Delta ABP \cong \Delta ACP$

$BP = CP$ (c.p.c.t.)

and $\angle APB = \angle APC$ (c.p.c.t.)

But $\angle APB + \angle APC = 180°$ (linear pair)

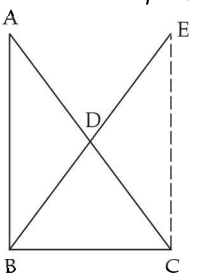
$\Rightarrow$ $\angle APB + \angle APB = 180° \Rightarrow \angle APB = 90°$.

Thus, $BP = CP$ and $\angle APB = 90°$

$\Rightarrow$ AP is the perpendicular bisector of BC.

Example 8. *In a triangle ABC, D is mid-point of side AC such that* $BD = \frac{1}{2}AC$. *Show that* $\angle ABC$ *is a right angle.* *(NCERT Exemplar)*

Solution. Given that in ΔABC, D is mid-point of AC and $BD = \frac{1}{2}AC$.



Produce BD to a point E such that DE = BD. Join E and C.

In ΔBDA and ΔEDC,

$BD = DE$ (by construction)

$AD = DC$ ($\because$ D is mid-point of AC)

$\angle BDA = \angle CDE$ (vert. opp. $\angle$s)

$\therefore$ $\Delta BDA \cong \Delta EDC$

$\therefore$ $AB = EC$ (c.p.c.t.)

and $\angle BAC = \angle ECA$ (c.p.c.t.)

Now, given that $BD = \frac{1}{2}AC \Rightarrow 2BD = AC$

$\Rightarrow$ $BD + BD = AC \Rightarrow BD + DE = AC \Rightarrow BE = AC$.

In ΔABC and ΔECB,

$AC = BE$

$AB = EC$ (proved above)

$BC = CB$ (common)

$\therefore$ $\Delta ABC \cong \Delta ECB$

$\therefore$ $\angle ABC = \angle ECB$ (c.p.c.t.)

Also $\angle BAC = \angle ECA$ (proved above), which are alternate interior angles (AB, EC are two lines and AC is a transversal),

therefore, $AB \parallel EC$

$\Rightarrow$ $\angle ABC + \angle ECB = 180°$ (sum of interior angles on the same side of transversal)

$\Rightarrow$ $\angle ABC + \angle ABC = 180°$ ($\because$ $\angle ECB = \angle ABC$)

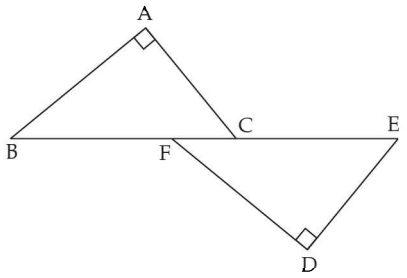
$\Rightarrow$ $2\angle ABC = 180° \Rightarrow \angle ABC = 90°$

$\Rightarrow$ $\angle ABC$ is a right angle.

Exercise 5.4

1. ABC is an isosceles triangle with AB = AC. Draw AP $\perp$ BC to show that $\angle B = \angle C$. *(NCERT)*

2. In the adjoining figure, BA $\perp$ AC, DE $\perp$ DF such that BA = DE and BF = EC. Show that $\Delta ABC \cong \Delta DEF$. *(NCERT Exemplar)*



Hint: BF = EC $\Rightarrow$ BF + FC = EC + FC

$\Rightarrow$ BC = FE.

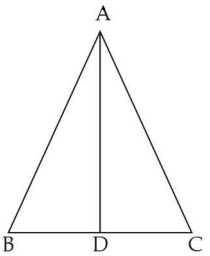
$\Delta ABC \cong \Delta DEF$ (RHS).

3. In the adjoining figure, AB = AC and D is mid-point of BC. Use SSS rule of congruency to show that

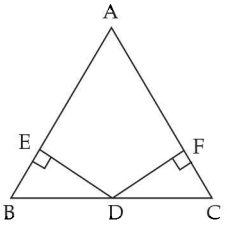
(*i*) $\Delta ABD \cong \Delta ACD$

(*ii*) AD is bisector of $\angle A$

(*iii*) AD is perpendicular to BC.



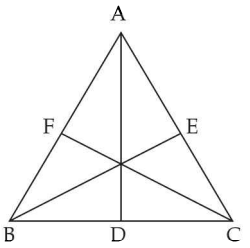
4. In the adjoining figure, D is mid-point of BC, DE and DF are perpendiculars to AB and AC respectively such that DE = DF. Prove that ABC is an isosceles triangle.



Hint: $\Delta BED \cong \Delta CFD$ (RHS).

$\Rightarrow$ $\angle B = \angle C \Rightarrow AC = AB$.

5. In the adjoining figure, AD, BE and CF are altitudes of ΔABC. If AD = BE = CF, prove that ABC is an equilateral triangle.



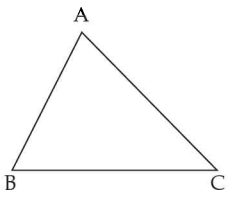
6. ABCD is a rectangle. X and Y are points on sides AD and BC respectively such that AY = BX. Prove that BY = AX and $\angle BAY = \angle ABX$.

   Hint: $\Delta ABX \cong \Delta BAY$ (RHS).

5.6 INEQUALITIES IN A TRIANGLE

In section 5.4 of this chapter, we have learnt that if two sides of triangle are equal then angles opposite to them are also equal and conversely if two angles of a triangle are equal then sides opposite to them are also equal. The question arises if two sides of a triangle are unequal then whether the angles opposite to them are also unequal, and which angle is greater? Conversely if two angles of a triangle are unequal then whether the sides opposite to them are also unequal, and which side is longer?

Look at the adjoining figure:



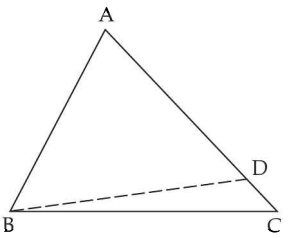
Compare sides AB and AC. We note that AC > AB. Now measure $\angle B$ and $\angle C$, we find that $\angle B > \angle C$. Repeat this activity with more triangles having unequal sides.

We observe that longer side has greater angle opposite to it.

Conversely, compare $\angle B$ and $\angle C$. We note that $\angle B > \angle C$.

Now measure sides AB and AC, we find that AC > AB.

Repeat this activity with more triangles having unequal angles. We observe that greater angle has longer side opposite to it. In fact, these results are true in general. We have:



Theorem 5.6. ***If two sides of a triangle are unequal, then the longer side has greater angle opposite to it.***

Given. A triangle ABC, AC > AB.

To prove. $\angle ABC > \angle BCA$.

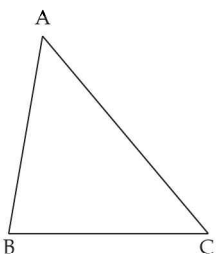
Construction. Take a point D on AC such that AD = AB. Join B and D.

Proof.

Statements	*Reasons*
1. AD = AB	1. By construction.
2. $\angle ABD = \angle BDA$	2. Angles opp. to equal sides.
3. $\angle BDA > \angle BCD$	3. $\angle BDA$ is an ext. $\angle$ of ΔBCD, and ext. $\angle$ is greater than each of int. opp. $\angle$.
4. $\angle ABD > \angle BCD$	4. From 2 and 3.
5. $\angle ABC > \angle ABD$	5. Since $\angle ABD$ is a part of $\angle ABC$.
6. $\angle ABC > \angle BCD$ Hence, $\angle ABC > \angle BCA$.	6. From 4 and 5.

Theorem 5.7. ***(Converse of theorem 5.6)***

If two angles of a triangle are unequal, the greater angle has longer side opposite to it.



Given. A triangle ABC, $\angle B > \angle C$.

To prove. AC > AB.

Proof.

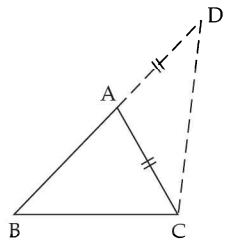
Statements	*Reasons*
If AC is not greater than AB, then either (*i*) AC = AB or (*ii*) AC < AB.	Order property of real numbers.
Case I. If AC = AB, then ∠B = ∠C which is contrary to what is given.	Angles opp. to equal sides.
Case II. If AC < AB, then ∠B < ∠C which is contrary to what is given. Hence, AC > AB.	Greater side has greater angle opposite to it.

Remarks

From theorems 5.6 and 5.7, it follows that in a triangle:

* The longest side has the greatest angle opposite to it and conversely.
* The shortest side has the smallest angle opposite to it and conversely.

Theorem 5.8. ***The sum of any two sides of a triangle is greater than the third side.***



Given. A triangle ABC.

To prove. BA + AC > BC.

Construction. Produce BA to D such that AD = AC.
Join C and D.

Proof.

Statements	*Reasons*
1. ∠ACD = ∠ADC	1. AD = AC by construction, and angles opp. to equal sides are equal.
2. ∠BCD > ∠ACD	2. Whole is greater than a part.
3. ∠BCD > ∠ADC	3. From 1 and 2.
4. BD > BC	4. Greater angle has greater side opposite to it.
5. BA + AD > BC	5. BAD is a st. line, BD = BA + AD.
6. BA + AC > BC	6. AD = AC, by construction.

Corollary. ***The difference between any two sides of a triangle is less than the third side.***

Proof. Let AC > AB.

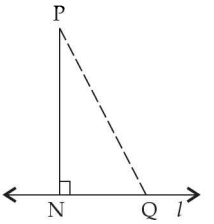
Since AB + BC > AC, so BC > AC – AB

⇒ AC – AB < BC.

Remarks

* If the lengths of three line segments are given, then identify the biggest length. In order to check whether the line segments form a triangle or not, it is sufficient to check whether the sum of two smaller lengths is greater than the biggest length or not.
* If the lengths of two sides of a triangle are given, then the length of the third side of the triangle must be greater than the difference of given lengths and it must be less than the sum of given lengths.

Theorem 5.9. ***Of all the line segments that can be drawn to a given line from a point not lying on it, the perpendicular line segment is the shortest.***



Given. A line l and a point P not lying on l. PN $\perp$ l. Q is *any point* on l.

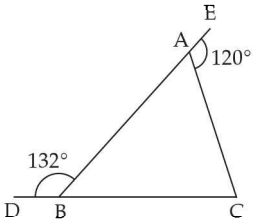
To prove. PN < PQ.

Proof.

Statements	*Reasons*
1. $\angle PNQ = 90°$	1. PN $\perp$ l.
2. $\angle PQN < 90°$	2. $\angle PQN$ is acute angle, since sum of angles in $\Delta PQN = 180°$.
3. $\angle PQN < \angle PNQ$	3. From 1 and 2.
4. PN < PQ As Q is *any point* on l, PN is the shortest of all line segments from P to line l.	4. Side opp. the smaller angle is smaller.

Illustrative Examples

Example 1. *In the adjoining figure, $\angle ABD = 132°$ and $\angle EAC = 120°$. Prove that AB > AC.*



Solution. $\angle ABC + 132° = 180°$. (linear pair)

$\Rightarrow$ $\angle ABC = 180° - 132° = 48°$.

In ΔABC, $120° = \angle ABC + \angle ACB$

(ext. $\angle$ = sum of two opp. int. $\angle$s)

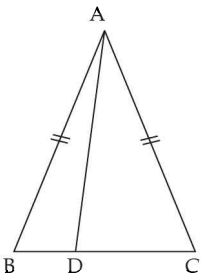
$\Rightarrow$ $120° = 48° + \angle ACB$

$\Rightarrow$ $\angle ACB = 120° - 48° = 72°$.

In ΔABC, we find that $\angle ACB > \angle ABC$

$\Rightarrow$ AB > AC (side opposite to greater angle is longer)

Example 2. *In the adjoining figure, AB = AC and D is any point on BC. Prove that AB > AD.*



Solution. Given AB = AC $\Rightarrow$ $\angle B = \angle C$.

In ΔADC, $\angle ADB > \angle C$

(ext. $\angle$ is greater than each int. opp. $\angle$)

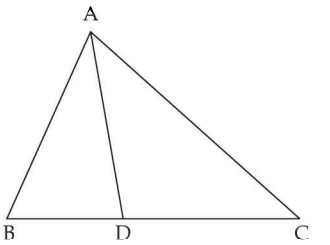
$\Rightarrow$ $\angle ADB > \angle B$ ($\because$ $\angle C = \angle B$)

$\Rightarrow$ AB > AD (side opposite to greater angle is longer)

Example 3. *In the adjoining figure, AD is bisector of $\angle BAC$. Prove that AB > BD.*

(NCERT Exemplar)

Solution. As AD is bisector of $\angle BAC$, $\angle BAD = \angle CAD$.



For ΔADC, $\angle ADB$ is an exterior angle

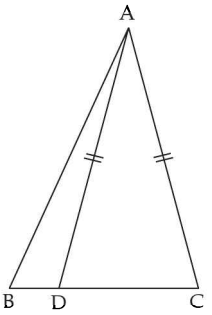
$\therefore$ $\angle ADB > \angle CAD$

(ext. $\angle$ is greater than each int. opp. $\angle$)

$\Rightarrow$ $\angle ADB > \angle BAD$ ($\because$ $\angle BAD = \angle CAD$)

$\Rightarrow$ AB > BD (side opposite to greater angle is longer)

Example 4. *In the adjoining figure, D is a point on the side BC of ΔABC such that AD = AC. Show that AB > AD.* *(NCERT)*



Solution. Given AD = AC

$\Rightarrow$ $\angle ACD = \angle ADC$ (angles opp. equal sides are equal)

For ΔABD, ∠ADC is an exterior angle,

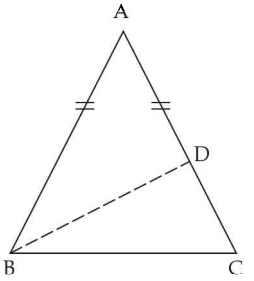
$\therefore$ $\angle ADC > \angle ABD$ (ext. ∠ is greater than each int. opp. ∠)

$\Rightarrow$ $\angle ACD > \angle ABD$ ($\because$ $\angle ACD = \angle ADC$)

$\Rightarrow$ AB > AC but AC = AD

$\therefore$ AB > AD.

Example 5. *D is any point on the side AC of a ΔABC with AB = AC. Show that CD < BD.* *(NCERT Exemplar)*



Solution. Given, in ΔABC, AB = AC

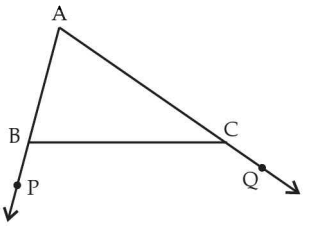
$\Rightarrow$ $\angle B = \angle C$ (∠s opp. equal sides are equal)

$\Rightarrow$ $\angle CBD < \angle B$ ($\because$ ∠CBD is a part of ∠B)

$\Rightarrow$ $\angle CBD < \angle C$ ($\because$ $\angle B = \angle C$)

$\Rightarrow$ CD < BD. (Theorem 5.7)

Example 6. *In the adjoining figure, sides AB and AC are extended to points P and Q respectively. If ∠PBC < ∠BCQ, show that AC > AB.* *(NCERT)*



Solution. $\angle PBC = \angle A + \angle ACB$ (ext. ∠ = sum of int. opp. ∠s)

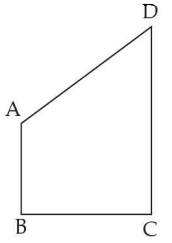
$\angle QCB = \angle A + \angle ABC$

Given $\angle PBC < \angle QCB$

$\Rightarrow$ $\angle A + \angle ACB < \angle A + \angle ABC \Rightarrow \angle ACB < \angle ABC$

$\Rightarrow$ AB < AC $\Rightarrow$ AC > AB.

Example 7. *In the adjoining figure, AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD. Show that ∠A > ∠C and ∠B > ∠D.* *(NCERT)*



Solution. Join AC and mark the angles as shown in the figure.

As AB is the smallest side,

BC > AB $\Rightarrow$ ∠1 > ∠2 ...(*i*)

As CD is the longest side,

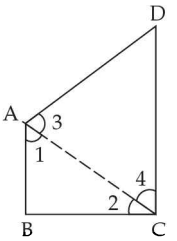
CD > AD $\Rightarrow$ ∠3 > ∠4 ...(*ii*)

From (*i*) and (*ii*), we get

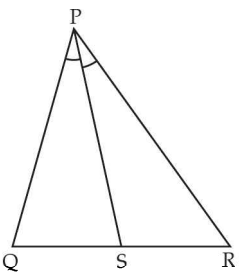
(∠1 + ∠3) > (∠2 + ∠4)

$\Rightarrow$ ∠A > ∠C.

Similarly, ∠B > ∠D. (join BD and proceed as above)



Example 8. *In the adjoining figure, PR > PQ and PS bisects ∠QPR. Prove that ∠PSR > ∠PSQ.* *(NCERT)*



Solution. Given PR > PQ $\Rightarrow$ ∠PQR > ∠PRQ

$\Rightarrow$ $\angle PQR - \angle PRQ > 0$ ...(*i*)

As PS bisects ∠QPR, ∠QPS = ∠RPS ...(*ii*)

$\angle PSR = \angle PQR + \angle QPS$ (ext. ∠ = sum of int. opp. ∠s)

$\angle PSQ = \angle PRQ + \angle RPS$

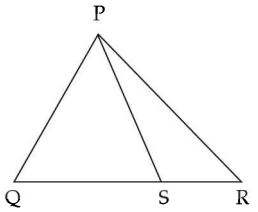
$\Rightarrow$ $\angle PSR - \angle PSQ = \angle PQR - \angle PRQ$ [using (*ii*)]

$\Rightarrow$ $\angle PSR - \angle PSQ > 0$ [using (*i*)]

$\Rightarrow$ $\angle PSR > \angle PSQ$.

Example 9. *S is any point on the side QR of ΔPQR. Show that:*

PQ + QR + RP > 2PS. *(NCERT Exemplar)*



Solution. We know that the sum of any two sides of a triangle is greater than the third side.

In ΔPQS, PQ + QS > PS ...(*i*)

In ΔPSR, SR + RP > PS ...(*ii*)

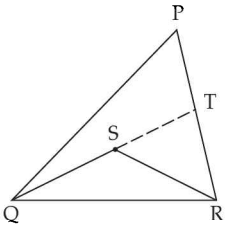
On adding (*i*) and (*ii*), we get

PQ + QS + SR + RP = 2PS

⇒ PQ + QR + RP > 2PS (∵ QS + SR = QR)

Example 10. *S is any point in the interior of ΔPQR. Show that:*

SQ + SR < PQ + PR. *(NCERT Exemplar)*



Solution. Given, S is any point in the interior of ΔPQR.

Produce QS to intersect PR at T.

In ΔPQT, PQ + PT > QT

(∵ sum of two sides of a triangle > third side)

⇒ PQ + PT > SQ + ST ...(*i*)

In ΔSTR, ST + TR > SR ...(*ii*)

(∵ sum of two sides of a triangle > third side)

On adding (*i*) and (*ii*), we get

PQ + PT + ST + TR > SQ + ST + SR

⇒ PQ + (PT + TR) > SQ + SR (∵ ST is common to both sides)

⇒ PQ + PR > SQ + SR (∵ PT + TR = PR)

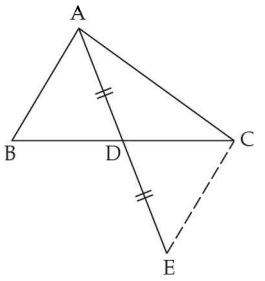
⇒ SQ + SR < PQ + PR.

Example 11. *Prove that the sum of any two sides of a triangle is greater than twice the median with respect to the third side.* *(NCERT Exemplar)*

Solution. Let ABC be any triangle and AD be median drawn from A to BC. We need to show that AB + AC > 2 AD.

Produce AD to E such that AD = DE.

Join CE.



In ΔABD and ΔECD,

BD = CD (∵ D is mid-point of BC)

AD = DE (by construction)

∠ADB = ∠EDC (vert. opp. ∠s)

∴ ΔABD ≅ ΔECD (by SAS axiom of congruency)

∴ AB = EC (c.p.c.t.)

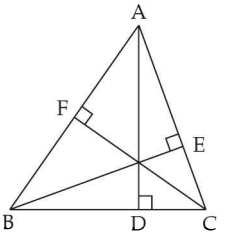
In ΔAEC, EC + AC > AE (sum of two sides of a triangle > third side)

⇒ AB + AC > AD + DE (∵ EC = AB)

⇒ AB + AC > 2AD.

Example 12. *Prove that the sum of lengths of altitudes of a triangle is less than the perimeter of the triangle.*

Solution. Let ABC be any triangle and AD ⊥ BC, BE ⊥ AC, CF ⊥ AB.



We know that of all the line segments that can be drawn to a given line from a point not lying on it, the perpendicular line segment is the shortest.

As AD ⊥ BC, therefore,

AD < AB ...(*i*)

As BE ⊥ AC, therefore,

$$BE < BC \quad \ldots(ii)$$

As CF ⊥ AB, therefore,

$$CF < AC \quad \ldots(iii)$$

On adding (*i*), (*ii*) and (*iii*), we get

$$AD + BE + CF < AB + BC + AC.$$

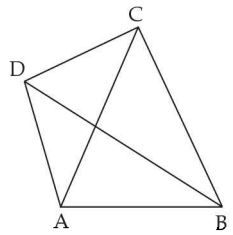
Hence, the sum of lengths of three altitudes of a triangle is less than the perimeter of the triangle.

Example 13. *Show that in a quadrilateral ABCD, AB + BC + CD + DA > AC + BD.*
(NCERT Exemplar)

Solution. In ΔABC, AB + BC > AC ...(*i*)
(sum of two sides of a triangle > third side)
In ΔBCD, BC + CD > BD ...(*ii*)
In ΔCDA, CD + DA > AC ...(*iii*)
In ΔDAB, DA + AB > BD ...(*iv*)
On adding (*i*), (*ii*), (*iii*) and (*iv*), we get

$$2(AB + BC + CD + DA) > 2(AC + BD)$$

$$\Rightarrow \quad AB + BC + CD + DA > AC + BD.$$



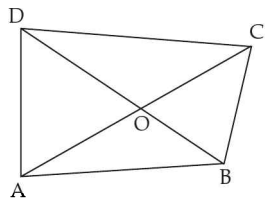
Example 14. *In a quadrilateral ABCD, show that AB + BC + CD + DA < 2(AC + BD).*
(NCERT Exemplar)

Solution. Given any quadrilateral ABCD.
Let diagonals AC and BD intersect at O.
In ΔOAB, OA + OB > AB ...(*i*)
(sum of two sides of a triangle > third side)
In ΔOBC, OB + OC > BC ...(*ii*)
In ΔOCD, OC + OD > CD ...(*iii*)
In ΔODA, OD + OA > DA ...(*iv*)
On adding (*i*), (*ii*), (*iii*) and (*iv*), we get

$$2(OA + OC + OB + OD) > AB + BC + CD + DA$$

$$\Rightarrow \quad 2(AC + BD) > AB + BC + CD + DA$$

$$\Rightarrow \quad AB + BC + CD + DA < 2(AC + BD).$$

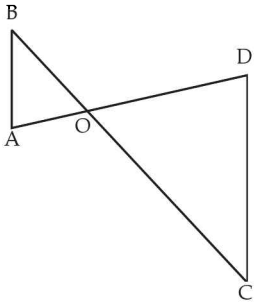


Exercise 5.5

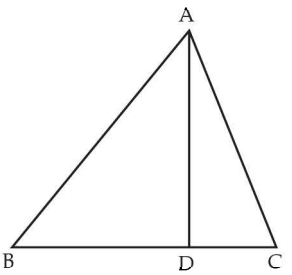
1. In ΔPQR, ∠P = 70° and ∠R = 30°. Which side of this triangle is longest? Give reason for your answer. *(NCERT Exemplar)*
2. Show that in a right angled triangle, the hypotenuse is the longest side. *(NCERT)*
3. Prove that in a triangle, other than an equilateral triangle, angle opposite the longest side is greater than $\frac{2}{3}$ of a right angle. *(NCERT Exemplar)*

   Hint: Let ABC be a triangle and AB be the longest side, then
   AB > BC and AB > AC ⇒ ∠C > ∠A and ∠C > ∠B
   ⇒ 2 ∠C > (∠A + ∠B) ⇒ 3 ∠C > (∠A + ∠B + ∠C)
   ⇒ 3 ∠C > 2 right angles.
4. (*a*) In the figure (*i*) given below, ∠B < ∠A and ∠C < ∠D. Show that AD < BC. *(NCERT)*

(*b*) In the figure (*ii*) given below, D is any point on the side BC of ΔABC. If AB > AC, show that AB > AD.



(*i*)



(*ii*)

5. (*i*) Is it possible to construct a triangle with lengths of its sides as 4 cm, 3 cm and 7 cm? Give reason for your answer. *(NCERT Exemplar)*

(*ii*) Is it possible to construct a triangle with lengths of its sides as 9 cm, 7 cm and 17 cm? Give reason for your answer. *(NCERT Exemplar)*

(*iii*) Is it possible to construct a triangle with lengths of its sides as 8 cm, 7 cm and 4 cm? Give reason for your answer. *(NCERT Exemplar)*

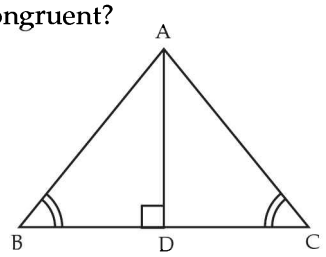
6. AD is median of the triangle ABC. Is it true that AB + BC + CA > 2 AD? Give reason for your answer. *(NCERT Exemplar)*

7. M is a point on side BC of a triangle ABC such that AM is bisector of ∠BAC. Is it true to say that perimeter of the triangle is greater than 2 AM? Give reason for your answer. *(NCERT Exemplar)*

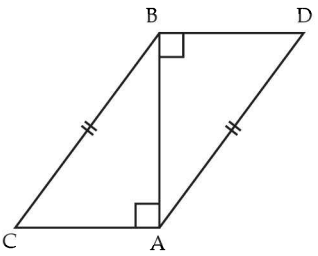
Answers

Very Short Answer Type Questions

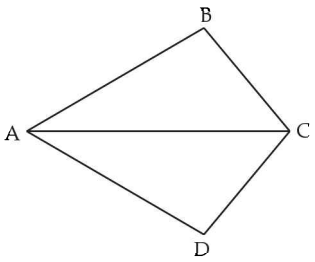
1. It is given that ΔABC ≅ ΔQRP. Is it true to say that BC = QR?
2. In triangle ABC and PQR, ∠A = ∠P and ∠B = ∠R. Which side of ΔPQR should be equal to side AC of ΔABC so that the two triangles are congruent?
3. In the adjoining figure, ∠B = ∠C and AD ⊥ BC. Write the name of rule by which ΔABD ≅ ΔACD.



4. In the adjoining figure, BC = AD, CA ⊥ AB and BD ⊥ AB. Write the name of rule by which ΔABC ≅ ΔBAD.

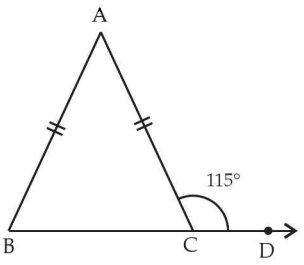


5. It is given that ΔABC ≅ ΔPRQ. If ∠A = 40° and ∠C = 60°, then find the measure of ∠R.
6. In the adjoining figure, ΔABC ≅ ΔADC. If ∠BAC = 30° and ∠ABC = 100°, then find the measure of ∠ACD.



7. In ΔABC, $\angle A = 80^\circ$ and AB = AC. Find $\angle B$ and $\angle C$.

8. In the adjoining figure, AB = AC. If $\angle ACD = 115^\circ$, then find the measure of $\angle A$.



9. In ΔABC, if $\angle B = 30^\circ$ and $\angle C = 70^\circ$, then write the lowest side of ΔABC.
10. In ΔABC, if $\angle A = 45^\circ$ and $\angle B = 70^\circ$ then write the shortest and the longest sides of ΔABC.

Answers

Objective Type Questions

(FOR FORMATIVE/SUMMATIVE ASSESSMENT)

Mental Maths

1. Fill in the blanks:
   (*i*) Two figures are congruent if they have the ..... shape and same ..... .
   (*ii*) Two squares are congruent if they have ..... sides.
   (*iii*) Two circles are congruent if their radii are ..... .
   (*iv*) Two equilateral triangles are congruent if they have ..... sides.
   (*v*) In an isosceles triangle, angles opposite to equal sides are ..... .
   (*vi*) In a triangle, sides opposite to equal angles are ..... .
   (*vii*) The sum of any two sides of a triangle is ..... the third side.
   (*viii*) The difference of any two sides of a triangle is ..... the third side.
2. State which of the following statements are true and which are false:
   (*i*) You can replace the refill of your ball point pen by a new one only if the new refill is of the same size and shape as the one you want to remove.
   (*ii*) Whenever identical objects have to be produced, the concept of congruency is used in making the cast.
   (*iii*) It is necessary to write the correspondence of vertices correctly for writing congruence of triangles in symbolic form.
   (*iv*) Two triangles are congruent if three angles of one triangle are equal to three angles of the other triangle.
   (*v*) In a triangle, the longest side has the greatest angle opposite to it.
   (*vi*) In a triangle, the greatest angle has the longest side opposite to it.
   (*vii*) In a triangle, the shortest side has the smallest angle opposite to it.
   (*viii*) In a triangle, the smallest angle has the shortest side opposite to it.
   (*ix*) In a right angled triangle, the hypotenuse is the shortest side.

Multiple Choice Questions

Choose the correct answer from the given four options (3 to 26):

3. Which of the following is not a criterion for congruency of triangles?
   (*a*) SAS (*b*) ASA (*c*) SSA (*d*) SSS

(NCERT Exemplar)

4. If AB = QR, BC = PR and CA = PQ, then

(*a*) $\Delta ABC \cong \Delta PQR$ (*b*) $\Delta CBA \cong \Delta PRQ$ (*c*) $\Delta BAC \cong \Delta RPQ$ (*d*) $\Delta PQR \cong \Delta BCA$

(NCERT Exemplar)

5. In triangles ABC and DFE, AB = FD and $\angle A = \angle D$. The two triangles will be congruent by SAS axiom if

(*a*) BC = EF (*b*) AC = DE (*c*) AC = EF (*d*) BC = DE

(NCERT Exemplar)

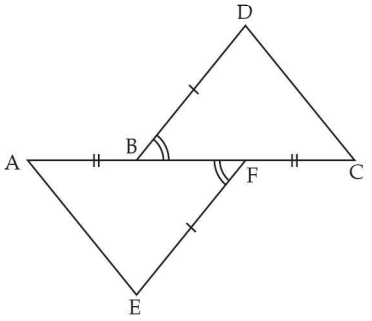
6. It is given that $\Delta ABC \cong \Delta FDE$, AB = 5 cm, $\angle B = 40°$ and $\angle A = 80°$. Then which of the following is true?

(*a*) DF = 5 cm, $\angle F = 60°$ (*b*) DF = 5 cm, $\angle E = 60°$

(*c*) DE = 5 cm, $\angle E = 60°$ (*d*) DE = 5 cm, $\angle D = 40°$

(NCERT Exemplar)

7. In the adjoining figure, AB = FC, EF = BD and $\angle AFE = \angle CBD$. Then the rule by which $\Delta AFE \cong \Delta CBD$ is



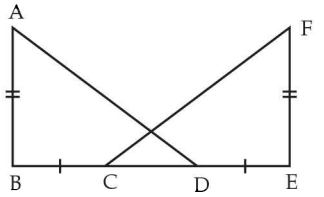
(*a*) SAS

(*b*) ASA

(*c*) SSS

(*d*) AAS

8. In the adjoining figure, AB ⊥ BE and FE ⊥ BE. If AB = FE and BC = DE, then



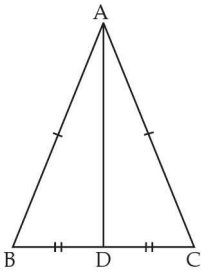
(*a*) $\Delta ABD \cong \Delta EFC$

(*b*) $\Delta ABD \cong \Delta FEC$

(*c*) $\Delta ABD \cong \Delta ECF$

(*d*) $\Delta ABD \cong \Delta CEF$

9. In the adjoining figure, AB = AC and AD is median of ΔABC, then $\angle ADC$ is equal to



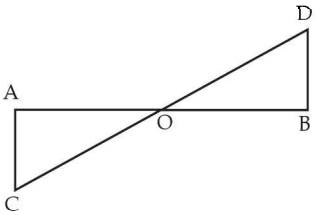
(*a*) 60°

(*b*) 120°

(*c*) 90°

(*d*) 75°

10. In the adjoining figure, O is mid-point of AB. If $\angle ACO = \angle BDO$, then $\angle OAC$ is equal to



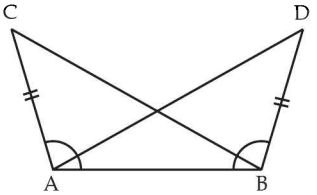
(*a*) $\angle OCA$

(*b*) $\angle ODB$

(*c*) $\angle OBD$

(*d*) $\angle BOD$

11. In the adjoining figure, AC = BD. If $\angle CAB = \angle DBA$, then $\angle ACB$ is equal to



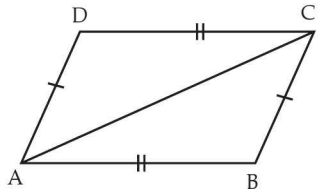
(*a*) $\angle BAD$

(*b*) $\angle ABC$

(*c*) $\angle ABD$

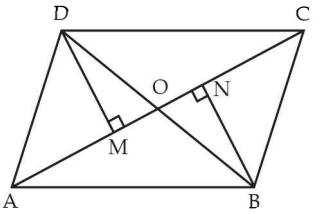
(*d*) $\angle BDA$

12. In the adjoining figure, ABCD is a quadrilateral in which AD = CB and AB = CD, then ∠ACB is equal to



(*a*) ∠ACD
(*b*) ∠BAC
(*c*) ∠CAD
(*d*) ∠BAD

13. In the adjoining figure, ABCD is a quadrilateral in which BN and DM are drawn perpendiculars to AC such that BN = DM. If OB = 4 cm, then BD is



(*a*) 6 cm
(*b*) 8 cm
(*c*) 10 cm
(*d*) 12 cm

14. In ΔABC, AB = AC and ∠B = 50°. Then ∠C is equal to

(*a*) 40° (*b*) 50° (*c*) 80° (*d*) 130°

(NCERT Exemplar)

15. In ΔABC, BC = AB and ∠B = 80°. Then ∠A is equal to

(*a*) 80° (*b*) 40° (*c*) 50° (*d*) 100°

(NCERT Exemplar)

16. In ΔPQR, ∠R = ∠P, QR = 4 cm and PR = 5 cm. Then the length of PQ is

(*a*) 4 cm (*b*) 5 cm (*c*) 2 cm (*d*) 2.5 cm

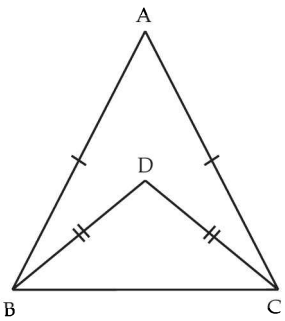
(NCERT Exemplar)

17. In ΔABC and ΔPQR, AB = AC, ∠C = ∠P and ∠B = ∠Q. The two triangles are

(*a*) isosceles but not congruent (*b*) isosceles and congruent
(*c*) congruent but isosceles (*d*) neither congruent nor isosceles

(NCERT Exemplar)

18. In the adjoining figure, AB = AC and BD = CD. The ratio ∠ABD : ∠ACD is



(*a*) 1 : 1
(*b*) 1 : 2
(*c*) 2 : 1
(*d*) 2 : 3

19. Two sides of a triangle are of lengths 5 cm and 1.5 cm. The length of the third side of the triangle can not be

(*a*) 3.6 cm (*b*) 4.1 cm (*c*) 3.8 cm (*d*) 3.4 cm

(NCERT Exemplar)

20. If *a*, *b*, *c* are the lengths of the sides of a triangle, then

(*a*) $a - b > c$ (*b*) $c > a + b$ (*c*) $c = a + b$ (*d*) $c < a + b$

21. It is not possible to construct a triangle when the lengths of its sides are

(*a*) 6 cm, 7 cm, 8 cm (*b*) 4 cm, 6 cm, 6 cm
(*c*) 5.3 cm, 2.2 cm, 3.1 cm (*d*) 9.3 cm, 5.2 cm, 7.4 cm

22. In ΔPQR, if ∠R > ∠Q, then

(*a*) QR > PR (*b*) PQ > PR (*c*) PQ < PR (*d*) QR < PR

(NCERT Exemplar)

23. If triangle PQR is right angled at Q, then

(*a*) PR = PQ (*b*) PR < PQ (*c*) PR < QR (*d*) PR > PQ

24. If triangle ABC is obtuse angled and ∠C is obtuse, then

(*a*) AB > BC (*b*) AB = BC (*c*) AB < BC (*d*) AC > AB

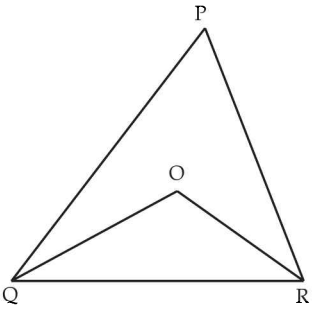
25. In the adjoining figure, PQ > PR. If OQ and OR are bisectors of ∠Q and ∠R respectively, then

(*a*) OQ = OR

(*b*) OQ < OR

(*c*) OQ ≤ OR

(*d*) OQ > OR



26. D is a point on the side BC of ΔABC such that AD bisects ∠BAC, then

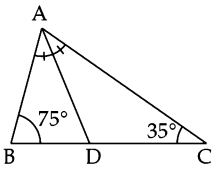
(*a*) BD = CD (*b*) BA > BD (*c*) BD > BA (*d*) CD > CA

(NCERT Exemplar)

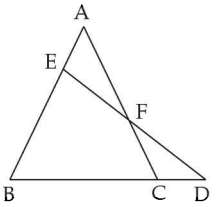
Answers

Higher Order Thinking Skills (HOTS)

1. In the adjoining figure, AD bisects ∠A. Arrange AB, BC and DC in ascending order.



2. In the adjoining figure, AB = AC. Prove that AF > AE.



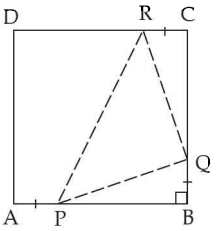
3. In a triangle ABC, AB = AC and D is a point on AB such that AD = DC = BC. Find the measure of ∠BAC.

Answers

Chapter Test

1. In triangles ABC and DEF, $\angle A = \angle D$, $\angle B = \angle E$ and AB = EF. Will the two triangles be congruent? Give reasons for your answer. *(NCERT Exemplar)*

2. In the adjoining figure, ABCD is a square. P, Q and R are points on the sides AB, BC and CD respectively such that AP = BQ = CR and $\angle PQR = 90°$. Prove that

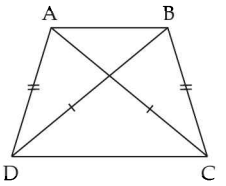


(*a*) $\Delta PBQ \cong \Delta QCR$

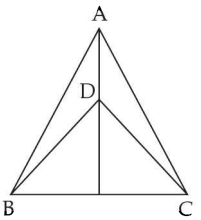
(*b*) PQ = QR

(*c*) $\angle PRQ = 45°$.

3. In the adjoining figure, AD = BC and BD = AC. Prove that $\angle ADB = \angle BCA$.



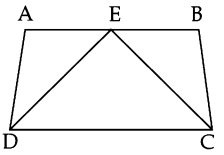
4. In the adjoining figure, AB = AC, D is a point in the interior of ΔABC such that $\angle DBC = \angle DCB$. Prove that AD bisects $\angle BAC$ of ΔABC.



Hint: $\angle DBC = \angle DCB \Rightarrow CD = BD$.

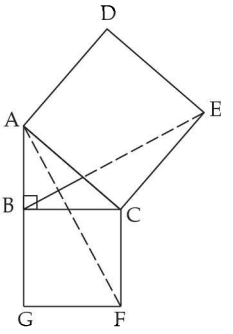
$\Delta ABD \cong \Delta ACD$ by SSS rule of congruency.

5. In the adjoining figure, AB || DC. CE and DE bisects $\angle BCD$ and $\angle ADC$ respectively. Prove that AB = AD + BC.



6. In ΔABC, D is a point on BC such that AD is the bisector of $\angle BAC$. CE is drawn parallel to DA to meet BD produced at E. Prove that ΔCAE is isosceles.

7. In the adjoining figure, ABC is a right angled triangle at B. ADEC and BCFG are squares. Prove that AF = BE.



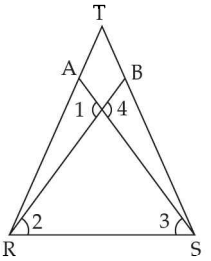
Hint: $\angle BCE = \angle BCA + 90°$,

$\angle ACF = \angle BCA + 90°$

$\Rightarrow$ $\angle BCE = \angle ACF$.

$\Delta BCE \cong \Delta FCA$ (SAS)

8. In the adjoining figure, TR = TS, $\angle 1 = 2\angle 2$ and $\angle 4 = 2\angle 3$. Prove that RB = SA.



Hint: $\angle 1 = \angle 4$ (vert. opp. $\angle$s)

$\Rightarrow$ $2\angle 2 = 2\angle 3 \Rightarrow \angle 2 = \angle 3$

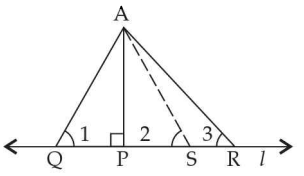
$TS = TS \Rightarrow \angle TRS = \angle TSR$

$\Rightarrow$ $\angle TRS - \angle 2 = \angle TSR - \angle 3$

$\Rightarrow$ $\angle ARB = \angle BSA$.

$\Delta RBT \cong \Delta SAT$ (SAS)

9. In the adjoining figure, AP $\perp$ l and PR > PQ. Show that AR > AQ.



Hint: Take a point S on PR such that PS = PQ.
Join A and S. Mark angles as shown.

$\Delta APQ \cong \Delta APS$ (SAS)

$\Rightarrow \quad \angle 1 = \angle 2$ (c.p.c.t.)

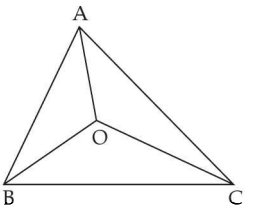
But $\angle 2 > \angle 3$ (ext. $\angle$ is greater than each int. opp. $\angle$s)

$\Rightarrow \quad \angle 1 > \angle 3$ ($\because \angle 1 = \angle 2$)

$\Rightarrow$ AR > AQ.

10. If O is any point in the interior of a triangle ABC, show that

$OA + OB + OC > \frac{1}{2}(AB + BC + CA)$.



Hint: In ΔOBC, $OB + OC > BC$ ...(*i*)

($\because$ sum of any two sides of a triangle > third side)

Similarly $OC + OA > CA$ ...(*ii*)

and $OA + OB > AB$ ...(*iii*)

On adding (*i*), (*ii*) and (*iii*), we get

$2(OA + OB + OC) > AB + BC + CA$.

Answers

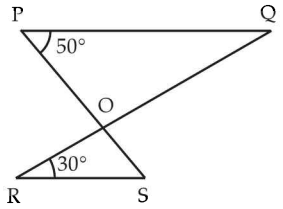
Model Question Paper 2

(FOR FORMATIVE ASSESSMENT II - TERM I)

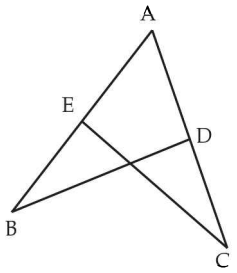
Time: 1 Hour *Maximum Marks: 30*

Note. *Questions 1-2 carry 1 mark each, 3-4 carry 2 marks each, 5-8 carry 3 marks each and 9-11 carry 4 marks each.*

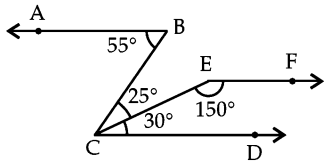
1. If an exterior angle of a triangle is 80° and the interior opposite angles are in the ratio 1 : 3, then find the measures of interior opposite angles.
2. $\Delta ABC \cong \Delta PQR$. If AB = 5 cm, $\angle B = 40°$ and $\angle A = 80°$, then find PQ and $\angle R$.
3. If a point C lies between points A and B such that AC = CB, then prove that $AC = \frac{1}{2}$ AB. Explain by drawing the figure.
4. In the adjoining figure, if PQ || RS then find $\angle SOR$.



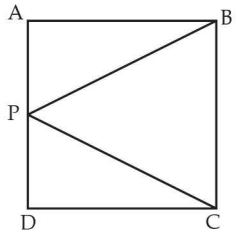
5. In the adjoining figure, if $\angle ABD = \angle ACE$ and AB = AC then prove that BD = CE.



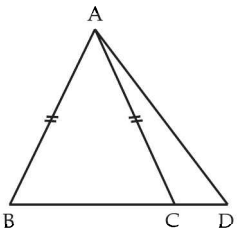
6. In the adjoining figure, prove that AB || EF.



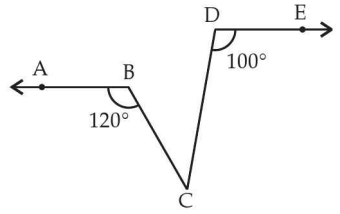
7. In the adjoining figure, ABCD is a square and P is mid-point of AD. BP and CP are joined. Prove that $\angle PCB = \angle PBC$.



8. In the adjoining figure, D is any point on the base BC produced of an isosceles triangle ABC. Prove that AD > AB.



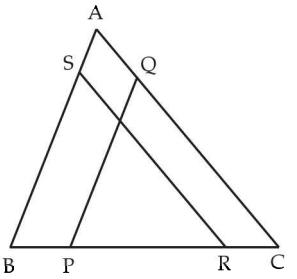
9. In the adjoining figure, if AB || DE then find the measure of $\angle BCD$.



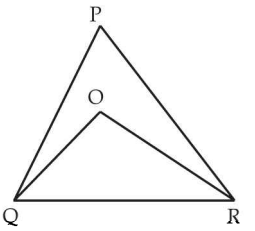
10. In the adjoining figure, BA || PQ, CA || RS and BP = RC. Prove that

(*i*) BS = PQ

(*ii*) RS = CQ.



11. In the adjoining figure, O is any point in the interior of ΔPQR. Show that OQ + OR < PQ + PR.



Answers

Coordinate Geometry



INTRODUCTION

Coordinate geometry is that branch of Mathematics which deals with the study of geometry by means of algebra. René Déscartes, a French mathematician, realised around 1637 that a line or a curve in a plane can be represented by an algebraic equation. As a result, a new branch of mathematics called **Coordinate Geometry** came into existence. In coordinate geometry, we represent a point in a plane by an ordered pair of real numbers, called coordinates of the point; and a straight line or a curve by an algebraic equation with real coefficients. Thus, we use algebra advantageously to the study of straight lines and geometric curves.

Recall that there is one and only one point on a number line associated with each real number. A similar situation exists for points in a plane and **ordered pairs** of real numbers.

6.1 ORDERED PAIR

*An **ordered pair** is a pair of objects taken in a specific order.*

An ordered pair is written by listing its two members in a specific order, separating them by a comma and enclosing the pair in parentheses. In the ordered pair (a, b), a is called the *first member* (or *component*) and b is called the *second member* (or *component*).

Equality of ordered pairs. *Two ordered pairs (a, b) and (c, d) are called **equal**, written as $(a, b) = (c, d)$, if and only if $a = c$ and $b = d$.*

Remarks

- The word 'ordered' implies that the order in which the two elements of the pair occur is meaningful. For example, if we have a sock and a shoe, the order in which they are put on does matter.
- The ordered pairs (a, b) and (b, a) are different unless $a = b$.
- The two components of an ordered pair may be equal.

6.2 COORDINATE SYSTEM

When two numbered lines perpendicular to each other (usually horizontal and vertical) are placed together so that the two origins (the points corresponding to zero) coincide then the resulting configuration is called a **cartesian coordinate system** or simply a **coordinate system** or a **cartesian plane**.

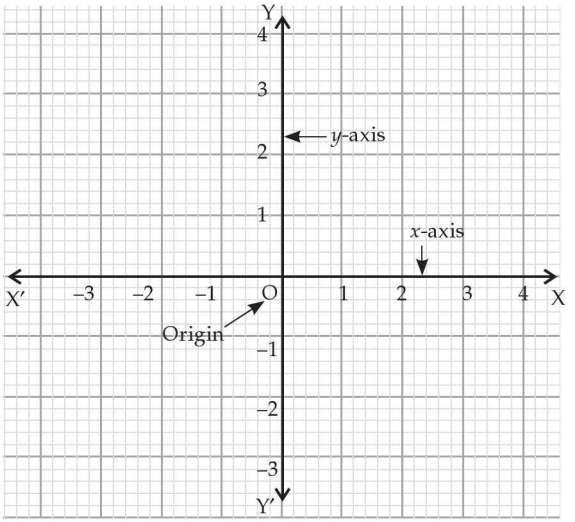
Let X'OX and Y'OY, two number lines perpendicular to each other, meet at the point O (shown in the adjoining figure), then

(*i*) X'OX is called ***x*-axis.**

(*ii*) Y'OY is called ***y*-axis.**

(*iii*) X'OX and Y'OY taken together are called **coordinate axes.**

(*iv*) the point O is called the **origin.**



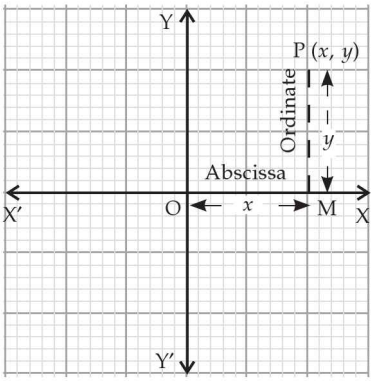
❑ **Coordinates of a point**

Let P be any point in the coordinate plane. From P, draw PM perpendicular to X'OX, then

(*i*) OM is called ***x*-coordinate** or **abscissa** of P and is usually denoted by x.

(*ii*) MP is called ***y*-coordinate** or **ordinate** of P and is usually denoted by y.

(*iii*) x and y taken together are called **cartesian coordinates** or simply coordinates of P and are denoted by (x, y).



Remarks

❍ To know the position of a point in a plane, we need two independent informations abscissa and ordinate of the point.

❍ The coordinates of a point indicate its position with reference to coordinate axes.

❍ In stating the coordinates of a point, the ***abscissa*** precedes the ***ordinate***. The two are separated by a comma and are enclosed in the bracket ( ). Thus, a point P whose abscissa is 'x' and ordinate is 'y' is written as (x, y) or P (x, y).

❑ **Convention for signs of coordinates**

(*i*) The x-coordinate (abscissa) of a point is ***positive*** if it is measured to the right of O *i.e.* along OX and is ***negative*** if it is measured to the left of O *i.e.* along OX'.

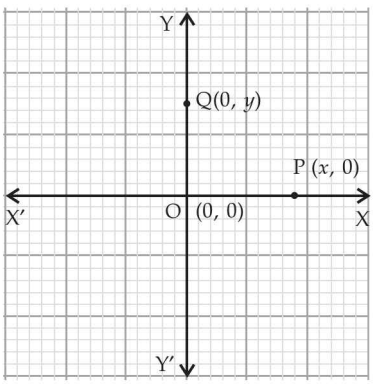
(*ii*) The y-coordinate (ordinate) of a point is ***positive*** if it is measured upwards *i.e.* along OY and is ***negative*** if it is measured downwards *i.e.* along OY'.

Remarks

❍ The coordinates of the origin O are (0, 0).

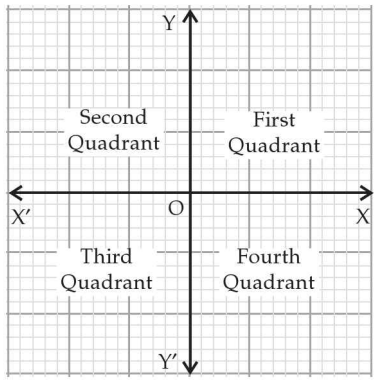
❍ Since every point on the x-axis as no distance (zero distance) from the x-axis, therefore, ordinate of every point on x-axis is always zero and so the coordinates of any point on the x-axis are $(x, 0)$, where x is the directed distance of the point from y-axis.

❍ For any point on y-axis, its abscissa is always zero and so the coordinates of any point Q on y-axis are $(0, y)$.



❑ **Quadrants**

The horizontal and the vertical number lines X′OX and Y′OY divide the coordinate plane into four parts. These four parts are called **quadrants**.



(*i*) XOY is called the **first quadrant**. In this quadrant, $x > 0$, $y > 0$ *i.e.* abscissa and ordinate are both positive.

(*ii*) YOX′ is called the **second quadrant**. In this quadrant, $x < 0$, $y > 0$ *i.e.* abscissa is negative and ordinate is positive.

(*iii*) X′OY′ is called the **third quadrant**. In this quadrant, $x < 0$, $y < 0$ *i.e.* abscissa and ordinate are both negative.

(*iv*) Y′OX is called the **fourth quadrant**. In this quadrant, $x > 0$, $y < 0$ *i.e.* abscissa is positive and ordinate is negative.

The signs of the coordinates of a point determine the quadrant in which the point lies and conversely, the signs of the coordinates of a point are determined by the quadrant in which it lies. The signs of the coordinates of a point in four quadrants can be remembered with the help of the following table:

Quadrants → *Coordinates* ↓	*1st XOY*	*2nd YOX′*	*3rd X′OY′*	*4th Y′OX*
x (abscissa)	+ve	−ve	−ve	+ve
y (ordinate)	+ve	+ve	−ve	−ve

Thus, the points (2, 3), (–2, 3), (–2, –3) and (2, –3) lie in the first, second, third and fourth quadrants respectively.

Illustrative Examples

Example 1. *Write whether the following statements are true or false. Justify your answer.*

(i) Point (3, 0) lies in the first quadrant.

(ii) Points (1, –1) and (–1, 1) lie in the same quadrant.

(iii) The coordinates of a point whose ordinate is $-\frac{1}{2}$ *and abscissa is 1 are* $\left(-\frac{1}{2}, 1\right)$.

(iv) A point lies on y-axis at a distance of 2 units from the x-axis. Its coordinates are (2, 0).

(v) (– 1, 7) is a point in the II quadrant. *(NCERT Exemplar)*

Solution. (*i*) False; because the ordinate of the given point is zero and hence it lies on x-axis.

(*ii*) False; because the point (1, –1) lies in 4th quadrant and the point (–1, 1) lies in the 2nd quadrant.

(*iii*) False; because in the coordinates of a point abscissa comes first and then ordinate, so the coordinates of the point with ordinate $-\frac{1}{2}$ and abscissa 1 are $\left(1, -\frac{1}{2}\right)$ and not $\left(-\frac{1}{2}, 1\right)$.

(*iv*) False; because a point on the y-axis has its abscissa zero. In fact, the coordinates of the point which lies on y-axis at a distance 2 units from x-axis are (0, 2).

(*v*) True; because in II quadrant the signs of the coordinates are −ve, +ve respectively.

Example 2. *Look at the figure given below and write the following :*

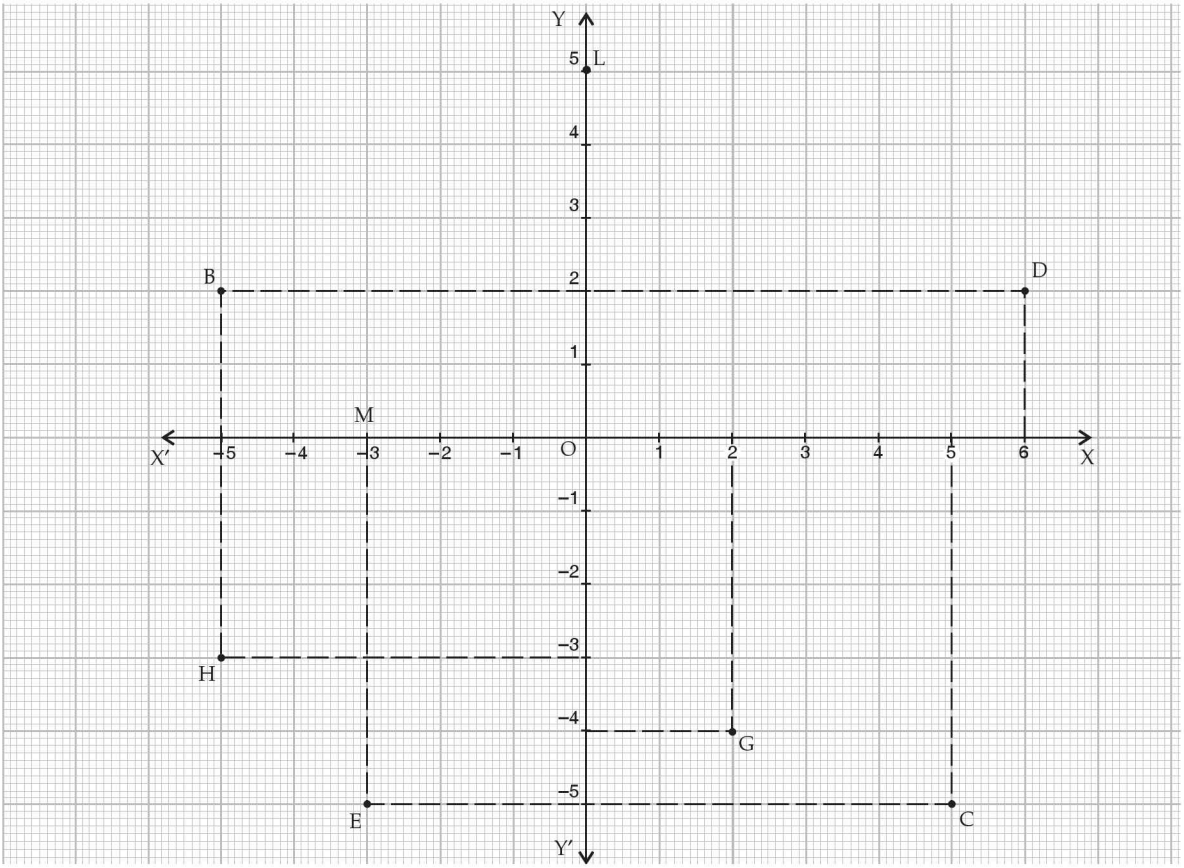
(i) The coordinates of B *(ii) The coordinates of C*

(iii) The point identified by the coordinates (–3, –5)

(iv) The point identified by the coordinates (2, –4)

(v) The abscissa of the point D *(vi) The ordinate of the point H*

(vii) The coordinates of the point L *(viii) The coordinates of the point M.* *(NCERT)*



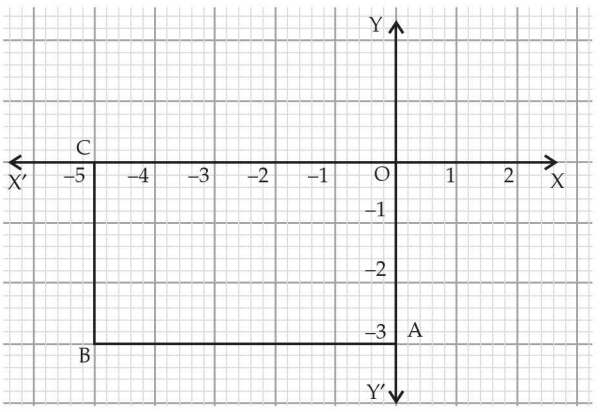
Solution. (*i*) (–5, 2) (*ii*) (5, –5) (*iii*) E (*iv*) G

(*v*) 6 (*vi*) –3 (*vii*) (0, 5) (*viii*) (–3, 0)

Example 3. *Write the coordinates of the vertices of a rectangle whose length and breadth are 5 and 3 units respectively, one vertex is at the origin, the longer side lies along x-axis and one of the vertices lies in the third quadrant.* *(NCERT Exemplar)*

Solution. Draw a rectangle OABC on a graph paper with length 5 units and breadth 3 units with one vertex at origin O, the longer side along *x*-axis and one vertex in the third quadrant as shown in the adjoining figure. The coordinates of its vertices are

O(0, 0), A(0, –3), B(–5, –3) and C(–5, 0).



Exercise 6.1

1. Write whether the following statements are True or False. Justify your answer.

(*i*) The point (0, –2) lies on *y*-axis.

(*ii*) The perpendicular distance of the point (4, 3) from the *x*-axis is 4.

(NCERT Exemplar)

2. Find the coordinates of the point

(*i*) which lies on *x* and *y* axes both.

(*ii*) whose ordinate is –4 and which lies on *y*-axis.

(*iii*) whose abscissa is 5 and which lies on *x*-axis. *(NCERT Exemplar)*

3. In which quadrant or on which axis each of the following points lie?

(–3, 5), (4, –1), (2, 0), (2, 2), (–3, –6) *(NCERT Exemplar)*

4. Which of the following points lie on *y*-axis?

A(1, 1), B(1, 0), C(0, 1), D(0, 0), E(0, –1), F(–1, 0), G(0, 5), H(–7, 0), I(3, 3)

(NCERT Exemplar)

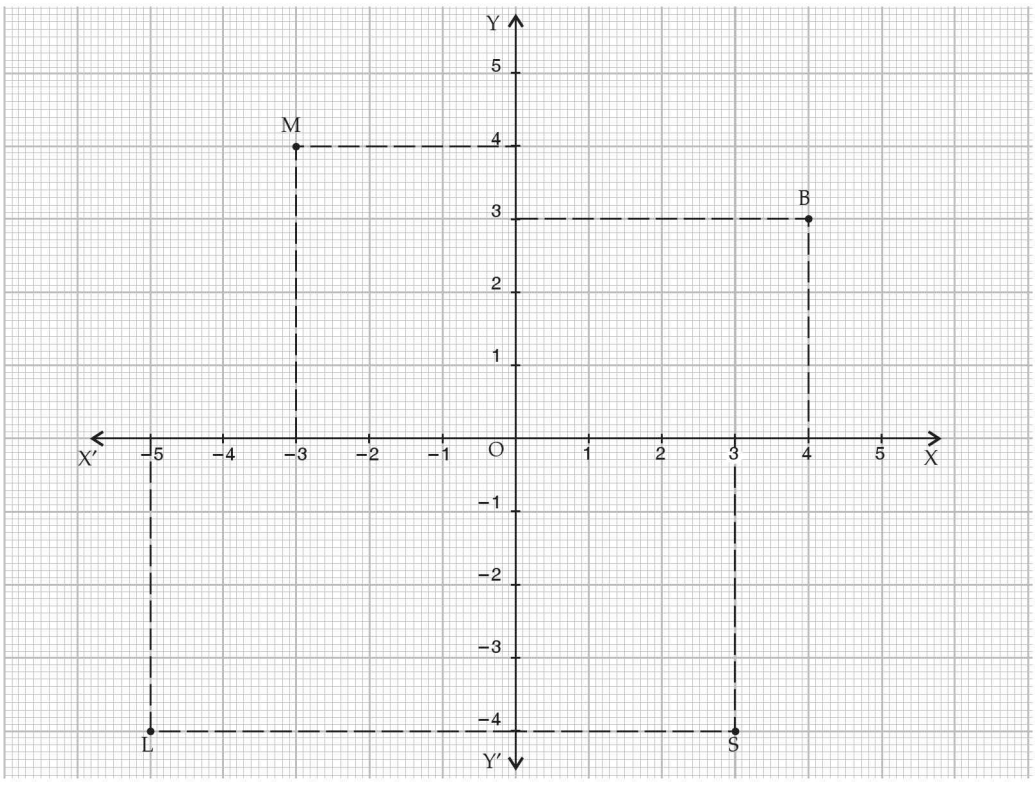
5. Look at the figure given below and complete the following statements :

(*i*) The abscissa and the ordinate of the point B are ..... and ..... respectively. Hence, the coordinates of B are (....., ...).

(*ii*) The *x*-coordinate and the *y*-coordinate of the point M are ..... and ..... respectively. Hence, the coordinates of M are (....., ...).

(*iii*) The *x*-coordinate and the *y*-coordinate of the point L are ..... and ..... respectively. Hence, the coordinates of L are (....., ...).

(*iv*) The abscissa and the ordinate of the point S are ..... and ..... respectively. Hence, the coordinates of S are (....., ...).



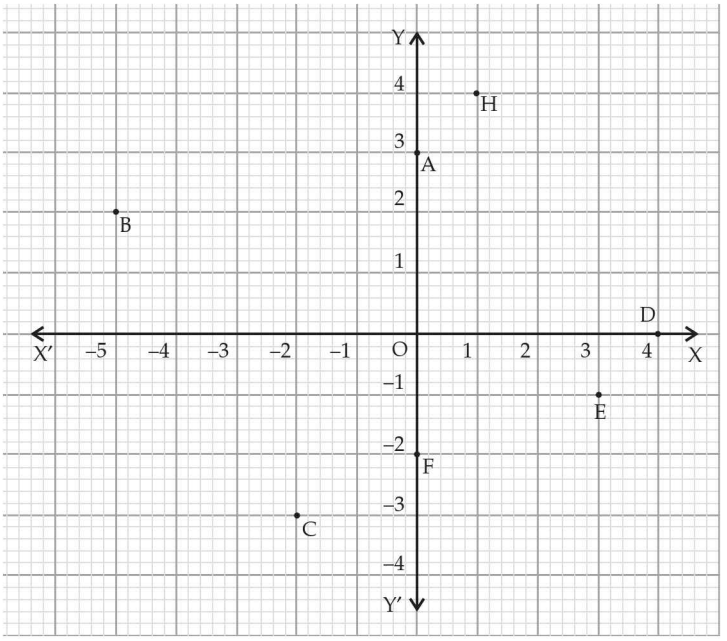
6. Look at the figure given below and write the following:

(*i*) Coordinates of B, C and E.

(*ii*) The point identified by the coordinates (0, –2).

(*iii*) The abscissa of point H.

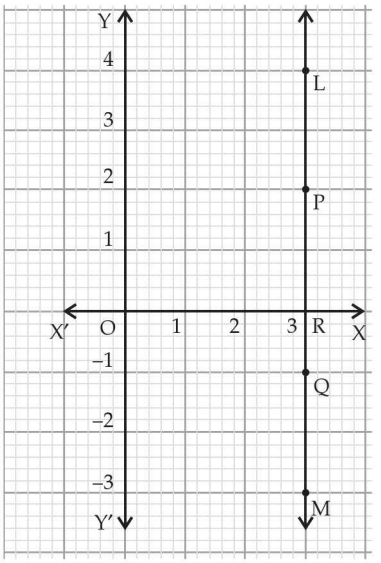
(*iv*) The coordinates of points A and D. *(NCERT Exemplar)*



7. In the adjoining figure, LM is a line parallel to y-axis at a distance of 3 units.

(*i*) What are the coordinates of the points P, R and Q?

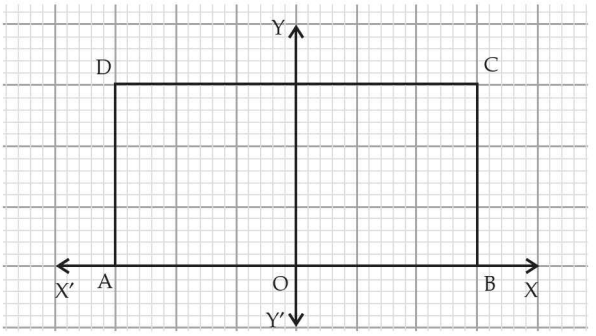
(*ii*) What is the difference between the abscissa of the points L and M? *(NCERT Exemplar)*



8. Write the coordinates of the vertices of a rectangle which is 6 units long and 4 units wide if the rectangle is in the first quadrant, its longer side lies on the x-axis and one vertex is at the origin.

9. Repeat problem 8 assuming that the rectangle is in the third quadrant with all other conditions remaining the same.

10. In the adjoining figure, ABCD is a rectangle with length 6 units and breadth 3 units. If O is mid-point of AB, find the coordinates of A, B, C and D.



Answers

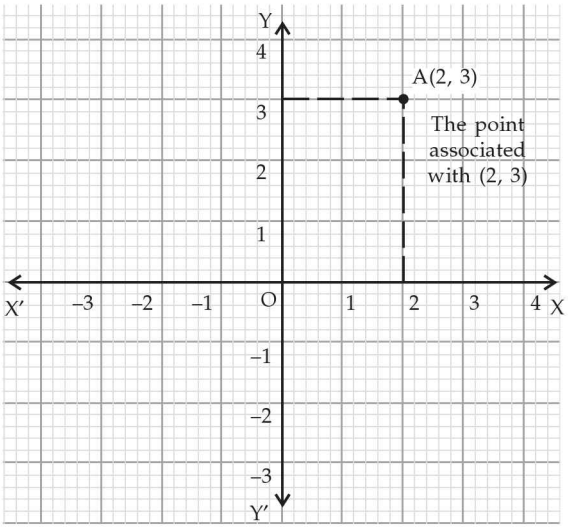
6.3 PLOTTING OF POINTS IN A PLANE

When we identify a point in the coordinate plane with a given ordered pair of real numbers, we say that we **plot** the point. For example, to plot the point (2, 3), we adopt the following steps:

(*i*) Draw the coordinates axes and choose the units such that one centimetre represents 1 unit on both the axes.

(*ii*) Start from O (origin) and move 2 units along the x-axis to the right.

(*iii*) From this place, move 3 units upwards (parallel to y-axis) and mark a **dot** at that place. This point, say A, of the coordinate plane is the point associated with the given ordered pair (2, 3). Label this point as A (2, 3).

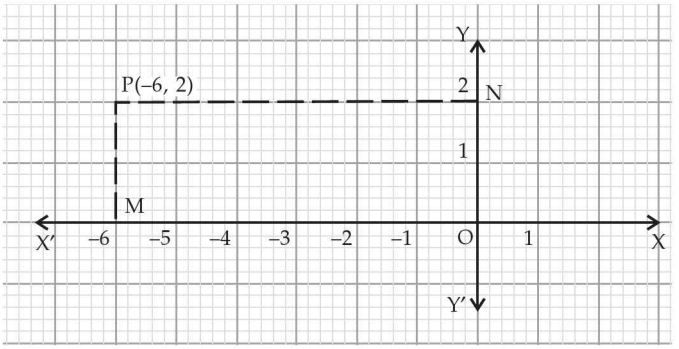


Note. Plot the point (3, 2). We observe that the points (2, 3) and (3, 2) are at different positions. Thus, if $x \neq y$ then the position of the point (x, y) in the cartesian plane is different from the position of the point (y, x). This means that the order of x and y is important in (x, y).

Illustrative Examples

Example 1. *Plot the point P(–6, 2) and from it draw PM and PN as perpendiculars to x-axis and y-axis respectively. Write the coordinates of the points M and N.* *(NCERT Exemplar)*

Solution. Draw coordinate axes on a graph paper. Take 1 cm = 1 unit on both axes. To plot the point P(–6, 2) in the coordinate plane, start from the point O(origin) and move 6 units along the x-axis to the left, and from here move 2 units upwards. Make a **dot** at this place and thus the point P(–6, 2) is plotted.



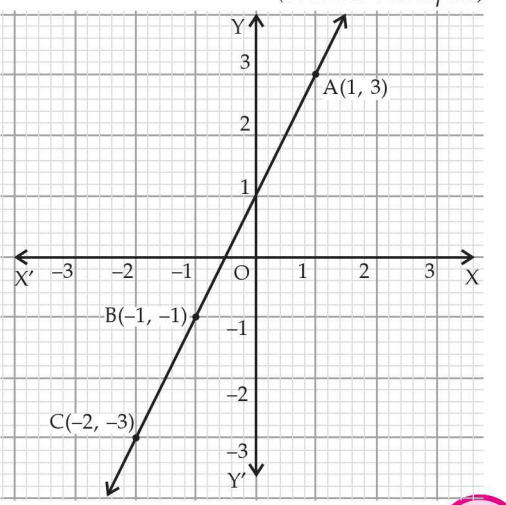
From P, draw PM perpendicular to x-axis and PN perpendicular to y-axis.

From graph, the coordinates of the points M and N are (–6, 0) and (0, 2) respectively.

Example 2. *Plot the points A(1, 3), B(–1, –1), C(–2, –3) and check whether they are collinear or not.* *(NCERT Exemplar)*

Solution. Plot the points A(1, 3), B(–1, –1) and C(–2, –3) on the graph paper as usual.

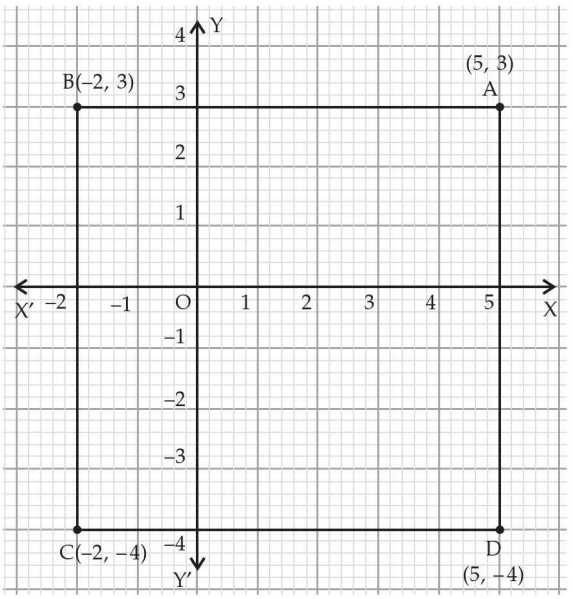
Draw a line passing through the points A and B. We find that the point C(–2, –3) lies on this line. Therefore, the given points A(1, 3), B(–1, –1) and C(–2, –3) are collinear.



Example 3. *Points A(5, 3), B(–2, 3) and D(5, –4) are three vertices of a square ABCD. Plot these points on a graph paper and hence find the coordinates of the vertex C.*

(NCERT Exemplar)

Solution. Plot the points A(5, 3), B(–2, 3) and D(5, –4) on the graph paper as usual. Join the points to complete the square ABCD as shown in the adjoining figure. Now read the coordinates of the point C from the graph paper. Clearly, the point C is (–2, –4).



Exercise 6.2

1. Locate the points (5, 0), (0, 5), (2, 5), (5, 2), (–3, 5), (–3, –5) and (6, 1) in the cartesian plane. *(NCERT)*
2. In which quadrant or on which axis do each of the points (–2, 4), (3, –1), (–1, 0), (1, 2) and (–3, –5) lie? Verify your answer by locating them on the cartesian plane. *(NCERT)*
3. Plot the following ordered pairs of numbers (x, y) as points in the cartesian plane. Use the scale 1 cm = 1 unit on both axes :

x	–3	0	–1	4	2
y	7	–3.5	–3	4	–3

(NCERT)

4. Taking 0.5 cm as 1 unit, plot the following points on the graph paper:
   A(1, 3), B(–3, –1), C(1, –4), D(–2, 3), E(0, –8), F(1, 0) *(NCERT Exemplar)*
5. Plot the following points and check whether they are collinear or not:
   (*i*) (0, 0), (2, 2), (5, 5) *(NCERT Exemplar)*
   (*ii*) (1, 1), (2, –3), (–1, –2) *(NCERT Exemplar)*
6. Plot the following points and write the name of the figure obtained by joining them in order:
   P(–3, 2), Q(–7, –3), R(6, –3), S(2, 2) *(NCERT Exemplar)*
7. Plot the points P(1, 0), Q(4, 0) and S(1, 3). Find the coordinates of the point R such that PQRS is a square. *(NCERT Exemplar)*
8. Plot the points A(1, –1) and B(4, 5).
   (*i*) Draw a line segment joining these points. Write the coordinates of a point on this line segment between the points A and B.
   (*ii*) Extend this segment and write the coordinates of a point on this line which lies outside the segment AB. *(NCERT Exemplar)*

Answers

6.4 GRAPHS OF LINEAR EQUATIONS IN TWO VARIABLES

❑ **Linear equation in two variables**

An equation of the form $Ax + By + C = 0$ where A, B, C are real numbers and $A \neq 0$, $B \neq 0$ is called a linear equation in two variables x and y.

For example, $2x + 3y - 5 = 0$, $3x - 2y + 11 = 0$ and $2x - y = 0$ are all linear equations in two variables x and y.

The equation $Ax + By + C = 0$ can be written in the form $y = mx + c$.

$Ax + By + c = 0 \Rightarrow By = -Ax - C$.

On dividing both sides by B, we get

$$y = -\frac{A}{B}x - \frac{C}{B}$$

or $y = mx + c$ where $m = -\frac{A}{B}$ and $c = -\frac{C}{B}$.

❑ **Graphs of linear equations in two variables**

The graph of a linear equation in two variables is always a straight line.

To draw the graph of a linear equation in two variables x and y, proceed as under:

(*i*) Rewrite the given equation with y as the subject *i.e.* $y = mx + c$.

(*ii*) Select any three convenient integral values (as far as possible) of x and find the corresponding values of y for each of the selected values of x.

(*iii*) Make table of values.

(*iv*) Draw the axes on the graph and choose suitable scale (same on both axes).

(*v*) Plot the points on the graph paper (co-ordinate plane).

(*vi*) Connect any two points by a straight line and check that the third point lies on it. In fact, two points are sufficient to fix the position of a straight line but we use three points for drawing the graph to check the correctness of the graph.

Illustrative Examples

Example 1. *Draw the graph of the linear equation* $2x - y = 0$.

Solution. The given equation is $2x - y = 0$.

It can be written as $y = 2x$ ($y = mx + c$ form)

Table of values:

x	0	1	2
y	0	2	4

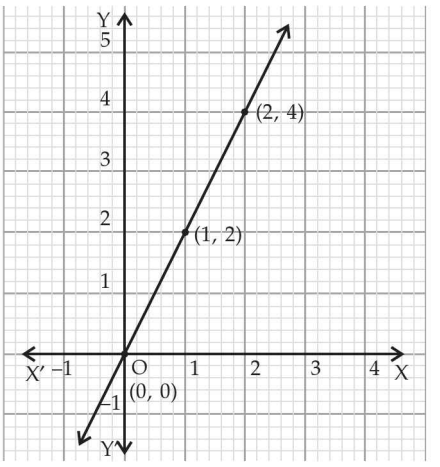
Draw the coordinate axes on a graph paper.

Take 1 cm = 1 unit on both axes.

Plot the points (0, 0), (1, 2) and (2, 4) on the graph paper (coordinate plane).

Draw a line passing through any two points. The graph of the given equation is shown in the adjoining figure.

Observe that the third point lies on the line.



Example 2. *Draw the graph of the linear equation* $2x + y - 1 = 0$.

Solution. The given equation is $2x + y - 1 = 0$.

It can be written as $y = -2x + 1$ ($y = mx + c$ form)

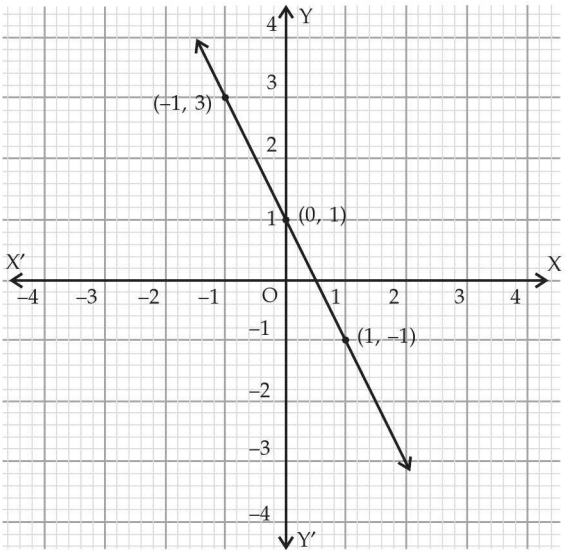
Table of values:

x	0	1	−1
y	1	−1	3

Take 1 cm = 1 unit on both axes.

Plot the points (0, 1), (1, −1) and (−1, 3) on the graph paper. Draw a line passing through any two points. The graph of the given equation is shown in the adjoining figure.

Observe that the third point lies on the line.



Example 3. *Draw the graph of the linear equation* $3x - 2y - 2 = 0$.

Solution. The given equation is

$$3x - 2y - 2 = 0,$$

it can be written as

$$2y = 3x - 2$$

or $y = \frac{3}{2}x - 1$ ($y = mx + c$ form)

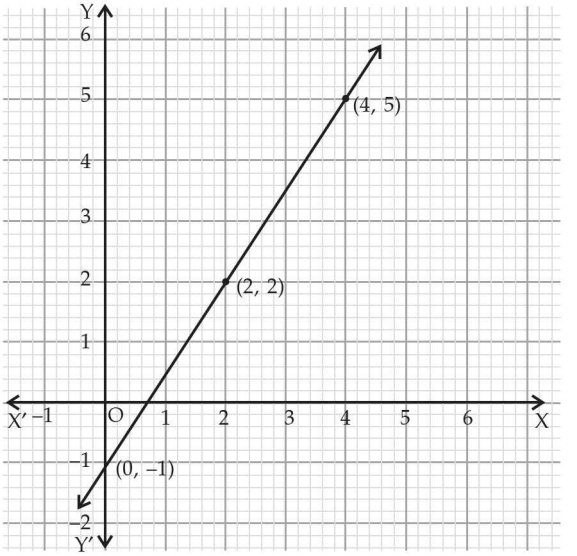
Table of values:

x	0	2	4
y	−1	2	5

Take 1 cm = 1 unit on both axes.

Plot the points (0, −1), (2, 2) and (4, 5) on the graph paper. Draw a line passing through any two points. The graph of the given equation is shown in the adjoining figure.

Observe that the third point lies on the line.



Exercise 6.3

Draw the graphs of the following linear equations:

1. $3x + y = 0$.
2. $x - y - 2 = 0$.
3. $2x - y + 1 = 0$.
4. $x + 2y + 3 = 0$.
5. $3x + y - 5 = 0$.
6. $2x + 2y - 3 = 0$.

Very Short Answer Type Questions

1. State whether the following statements are true or false. If a statement is false, write the corresponding correct statement.
   (*i*) The coordinates of any point on the x-axis is of the form $(0, k)$ where k is a real number.
   (*ii*) The coordinates of any point on the y-axis is of the form $(b, 0)$ where b is a real number.
   (*iii*) A point lies in the IInd quadrant if its abscissa is positive and ordinate is negative.
2. Write the coordinates of the point whose ordinate is $-\frac{3}{2}$ and abscissa is 5.
3. Write the coordinates of the point whose ordinate is -3 and which lies on y-axis.
4. Write the coordinates of the point which lies on x-axis and is at a distance of 4 units in the negative direction of x-axis.
5. In which quadrant does the point $(-2, 3)$ lie?
6. Write the coordinates of the points which lie on the x-axis and are at a distance of 3 units from the y-axis.

Answers

Objective Type Questions

(FOR FORMATIVE/SUMMATIVE ASSESSMENT)

Mental Maths

1. Fill in the blanks :
   (*i*) The point of intersection of the coordinate axes is called the ..... .
   (*ii*) The coordinate axes divide the plane into four parts which are called ..... .
   (*iii*) The coordinates of the origin are ..... .
   (*iv*) If the coordinates of a point are $(-3, 7)$, then its ordinate is ..... and its abscissa is ..... .
   (*v*) The point $(3, -2)$ lies in the ..... quadrant.
   (*vi*) The abscissa of any point on the y-axis is ..... .
   (*vii*) The ordinate of any point on the x-axis is ..... .
   (*viii*) The (directed) distance of a point from the y-axis is called its ..... .
   (*ix*) The (directed) distance of a point from the x-axis is called its ..... .
   (*x*) If the ordinate of a point is 3 and abscissa is -5, then its coordinates are ..... .
   (*xi*) The points with coordinates $(5, -2)$ and $(-2, 5)$ are at ..... positions of the coordinate plane.
2. State which of the following statements are true and which are false :
   (*i*) To locate the position of an object or a point in a plane, we require two independent informations.
   (*ii*) x-coordinate of a point is its (directed) distance from the y-axis.
   (*iii*) Ordinate of a point is its directed distance from the x-axis.
   (*iv*) The coordinates of a point describe the point in the plane uniquely.

(*v*) The points with coordinates (3, 4) and (4, 3) are at the same position of the plane.

(*vi*) For every point in the coordinate plane, there is unique ordered pair of real numbers.

(*vii*) For every ordered pair of real numbers, there is a unique point in the coordinate plane.

Multiple Choice Questions

Choose the correct answer from the given four options (3 to 19):

3. Point (–3, 5) lies in the

(*a*) first quadrant (*b*) second quadrant (*c*) third quadrant (*d*) fourth quadrant

(NCERT Exemplar)

4. Point (0, –7) lies

(*a*) on the *x*-axis (*b*) in the second quadrant

(*c*) on the *y*-axis (*d*) in the fourth quadrant

(NCERT Exemplar)

5. Point (–10, 0) lies

(*a*) on the negative direction of *x*-axis (*b*) in the third quadrant

(*c*) on the negative direction of *y*-axis (*d*) in the fourth quadrant

(NCERT Exemplar)

6. Signs of abscissa and ordinate of a point in the second quadrant are respectively

(*a*) +, + (*b*) –, – (*c*) –, + (*d*) +, –

(NCERT Exemplar)

7. Abscissa of a point is positive in

(*a*) I and II quadrants (*b*) I and IV quadrants

(*c*) I quadrant only (*d*) II quadrant only

(NCERT Exemplar)

8. Which of the following points lies in the IVth quadrant?

(*a*) (–2, 0) (*b*) (–3, 2) (*c*) (4, –7) (*d*) (–3, –5)

9. Which of the following points lies on the *y*-axis?

(*a*) (0, –7) (*b*) (–7, 0) (*c*) (2.7, 0) (*d*) (–1, 3)

10. The point which lies on *y*-axis at a distance of 5 units in the negative direction of *y*-axis is

(*a*) (0, 5) (*b*) (5, 0) (*c*) (0, –5) (*d*) (–5, 0)

(NCERT Exemplar)

11. If the perpendicular distance of a point P from the *x*-axis is 5 units and the foot of perpendicular lies on the negative direction of *x*-axis, then the point P has

(*a*) *x*-coordinate = –5 (*b*) *y*-coordinate = 5 only

(*c*) *y*-coordinate = –5 only (*d*) *y*-coordinate = 5 or –5

(NCERT Exemplar)

12. The points whose abscissa and ordinate have different signs will lie in

(*a*) I and II quadrants (*b*) II and III quadrants

(*c*) I and III quadrants (*d*) II and IV quadrants

(NCERT Exemplar)

13. The points (–5, 2) and (2, –5) lie in

(*a*) same quadrant (*b*) II and III quadrants respectively

(*c*) II and IV quadrants respectively (*d*) IV and II quadrants respectively

(NCERT Exemplar)

14. Points (1, –1), (2, –2), (4, –5), (–3, –4)

(*a*) lie in IInd quadrant (*b*) lie in III quadrant

(*c*) lie in IV quadrant (*d*) do not lie in the same quadrant

(NCERT Exemplar)

15. If the coordinates of the points are P(–2, 3) and Q(–3, 5), then (abscissa of P) – (abscissa of Q) is

(*a*) –5 (*b*) 1 (*c*) –1 (*d*) –2

(NCERT Exemplar)

16. If P(–1, 1), Q(3, –4), R(1, –1), S(–2, –3) and T(–4, 4) are plotted on the graph paper, then point(s) in the fourth quadrant are

(*a*) P and T (*b*) Q and R (*c*) S only (*d*) P and R

(NCERT Exemplar)

17. If P(5, 1), Q(8, 0), R(0, 4), S(0, 5) and O(0, 0) are plotted on the graph paper then point(s) on x-axis are

(*a*) P and R (*b*) R and S (*c*) only Q (*d*) Q and O

(NCERT Exemplar)

18. On plotting the points O(0, 0), A(3, 0), B(3, 4), C(0, 4) and joining OA, AB, BC and CO which of the following figure is obtained?

(*a*) Square (*b*) Rectangle (*c*) Trapezium (*d*) Rhombus

(NCERT Exemplar)

19. Which of the points P(0, 3), Q(1, 0), R(0, –1), S(–5, 0), T(1, 2) do not lie on x-axis?

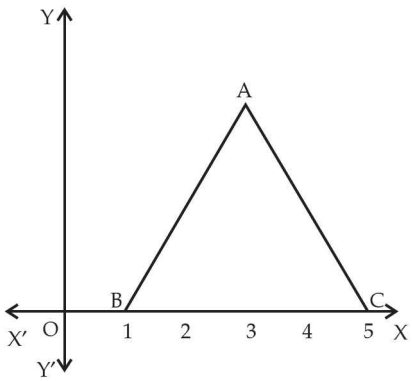
(*a*) P and R only (*b*) Q and S only (*c*) P, R and T (*d*) Q, S and T

(NCERT Exemplar)

Answers

Higher Order Thinking Skills (HOTS)

1. In the adjoining figure, ABC is an equilateral triangle with coordinates of B and C as (1, 0) and (5, 0) respectively. Find the coordinates of the vertex A.



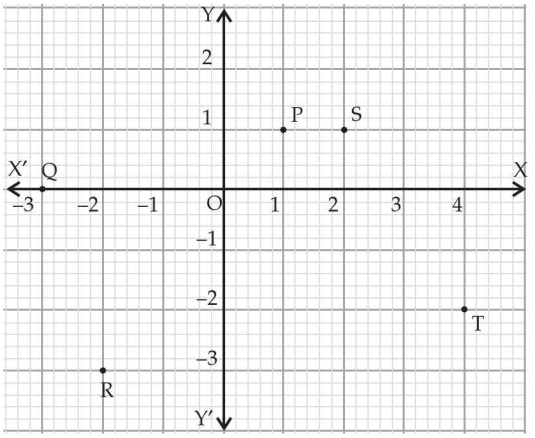
2. Three vertices of a parallelogram are A(–2, 2), B(6, 2) and C(4, –3). Plot these points on a graph paper and hence use it to find the coordinates of the fourth vertex D. Also write the coordinates of the mid-point of CD. What is the area of the parallelogram?

Answers

Chapter Test

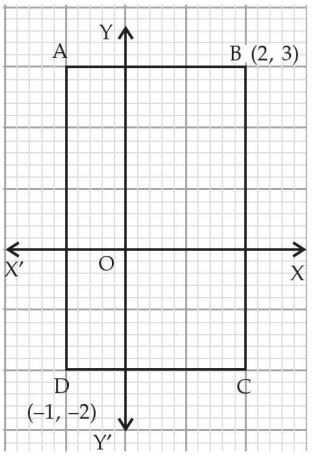
1. Without plotting the points indicate the quadrant they will lie, if
   (*i*) ordinate is 5 and abscissa is –3 (*ii*) abscissa is –5 and ordinate is –3
   (*iii*) abscissa is –5 and ordinate is 3 (*iv*) ordinate is 5 and abscissa is 3.
   (NCERT Exemplar)

2. From the adjoining figure, write the coordinates of each of the points P, Q, R, S, T and O. *(NCERT Exemplar)*



3. The perpendicular distance of a point from the x-axis is 2 units and the perpendicular distance from the y-axis is 3 units. Write the coordinates of the point if it lies in
   (*i*) I quadrant (*ii*) II quadrant (*iii*) III quadrant (*iv*) IV quadrant.

4. In the adjoining figure, ABCD is a rectangle. Find the coordinates of the points A and C.



5. Plot the point P(–3, 4). Draw PM and PN perpendiculars to x-axis and y-axis respectively. State the coordinates of the points M and N.
6. Plot the points A(1, 2), B(2, –1) and C(–1, 4) and check whether they are collinear or not.
7. Plot the points (x, y) given in the following table on the plane, choosing suitable unit of distance on the axes: *(NCERT)*

x	–2	–1	0	1	3
y	8	7	–1.25	3	–1

8. Plot the points A(1, 2), B(–4, 2), C(–4, –1) and D(1, –1). What kind of quadrilateral is ABCD? Also find the area of the quadrilateral ABCD.
9. Plot the points (0, 2), (3, 0), (0, –2) and (–3, 0) on a graph paper. Join these points. Name the figure obtained and find the area of the figure so obtained.
10. Three vertices of a square are A(2, 3), B(–3, 3) and C(–3, –2). Plot these points on a graph paper and hence use it to find the coordinates of the fourth vertex. Also find the area of the square.
11. Draw the graphs of the following linear equations:
    (*i*) $x + 2y + 1 = 0$ (*ii*) $2x + 3y - 6 = 0$

Answers

Heron's Formula



INTRODUCTION

In your earlier classes, you have already learnt about perimeters and areas of closed plane figures such as squares, rectangles, parallelograms, rhombi, trapeziums and special types of triangles. The perimeter of a closed plane figure is the length of its boundary *i.e.* the sum of lengths of its sides and the unit of measurement of perimeter is the unit of length. The area of a closed plane figure is the measurement of the region enclosed by its boundary and it is measured in square units *i.e.* square centimetres (abbreviated cm^2) or square metres (abbreviated m^2) etc.

In this chapter, we shall find areas of some more closed plane figures and shall learn a special formula known as **Heron's formula** for finding the area of any triangle (including scalene triangle). We shall also learn the applications of Heron's formula in calculating areas of quadrilaterals.

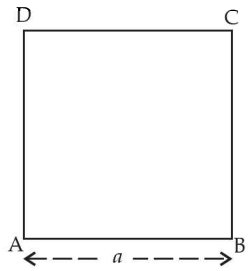
7.1 AREAS OF SOME PLANE FIGURES

We recall formulae for areas of some closed plane figures.

❑ **Area of some special quadrilaterals**

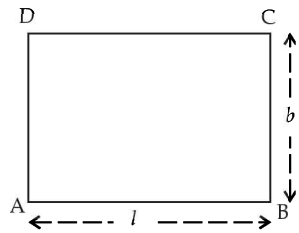
(*i*) **Square**

In square ABCD, let AB = a (side), then its ***area*** = $(AB)^2 = a^2$.



(*ii*) **Rectangle**

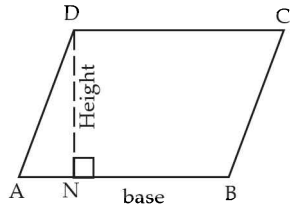
In rectangle ABCD, let AB = l (length) and BC = b (breadth), then its ***area*** = AB × BC = $l \times b$.



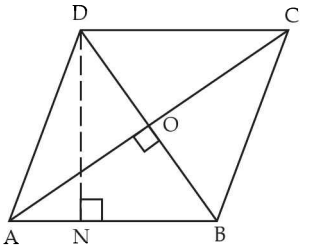
(*iii*) **Parallelogram**

Area = base × height

The height of a parallelogram is the distance between its base and the side parallel to the base.



(*iv*) **Rhombus**



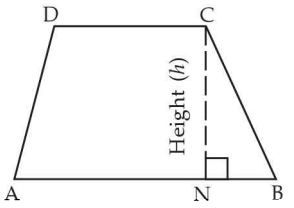
Area of rhombus ABCD = base × height.

Or

Area of rhombus = $\frac{1}{2}$ × product of diagonals

= $\frac{1}{2}$ AC × BD.

(*v*) **Trapezium**



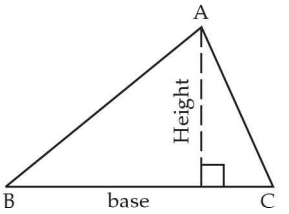
Area of trapezium ABCD

= $\frac{1}{2}$(AB + DC) × *h*

= $\frac{1}{2}$ (sum of parallel sides) × height

The height of a trapezium is the distance between its parallel sides.

❑ **Area of a triangle**

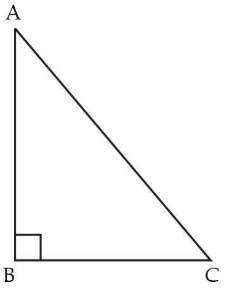


Area of triangle = $\frac{1}{2}$ ***base*** × ***height***

Any side of the triangle can be taken as its base, then the length of perpendicular from the vertex opposite to this side is called its **(corresponding) height.**

❑ **Area of some special triangles**

(*i*) **Right-angled triangle**

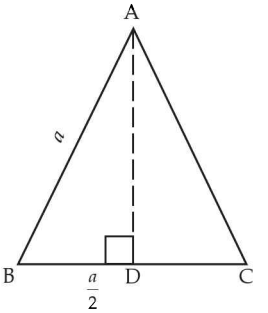


If ABC is a triangle in which ∠B = 90°,

then its area = $\frac{1}{2}$ BC × AB

= $\frac{1}{2}$ (product of sides containing right angle).

(*ii*) **Equilateral triangle**



Let ABC be an equilateral triangle with side *a*.

Draw AD ⊥ BC, then D is mid-point of BC *i.e.* BD = $\frac{a}{2}$.

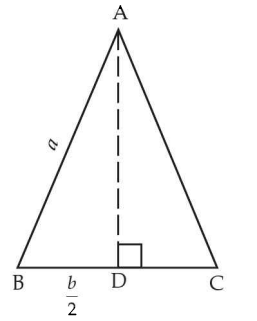
In ΔABD, $AD^2 = AB^2 - BD^2$ (Pythagoras theorem)

$\Rightarrow \quad AD^2 = a^2 - \left(\frac{a}{2}\right)^2 = a^2 - \frac{a^2}{4} = \frac{3}{4}a^2$

$\Rightarrow \quad AD = \frac{\sqrt{3}}{2}a.$

$\therefore$ Area of ΔABC = $\frac{1}{2}$BC × AD = $\frac{1}{2}a \times \frac{\sqrt{3}}{2}a = \frac{\sqrt{3}}{4}a^2$.

(*iii*) **Isosceles triangle**



Let ABC be an isosceles triangle with

AB = AC = *a* and BC = *b*. Draw AD ⊥ BC,

then D is mid-point of BC *i.e.* BD = $\frac{b}{2}$.

In ΔABD, $AD^2 = AB^2 - BD^2$ (Pythagoras theorem)

$\Rightarrow \quad AD^2 = a^2 - \left(\frac{b}{2}\right)^2 = a^2 - \frac{b^2}{4} = \frac{4a^2 - b^2}{4}$

$\Rightarrow \quad AD = \frac{\sqrt{4a^2 - b^2}}{2}.$

$\therefore$ Area of ΔABC = $\frac{1}{2}$BC × AD = $\frac{1}{2}b \times \frac{\sqrt{4a^2 - b^2}}{2} = \frac{1}{4}b\sqrt{4a^2 - b^2}$.

Illustrative Examples

Example 1. *Write whether the following statements are True or False. Justify your answer.*

(i) Area of ΔABC is 8 cm^2 in which AB = AC = 4 cm and $\angle A = 90°$. *(NCERT Exemplar)*

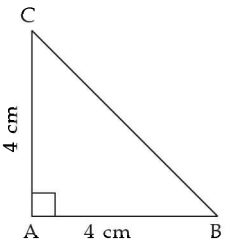
(ii) The area of the equilateral triangle is $20\sqrt{3}$ cm^2 whose each side is 8 cm. *(NCERT Exemplar)*

(iii) The area of the isosceles triangle is $\frac{5}{4}\sqrt{11}$ cm^2, if the perimeter is 11 cm and the base is 5 cm. *(NCERT Exemplar)*

Solution. *(i)* Area of $\Delta ABC = \frac{1}{2}$ base × height $= \frac{1}{2} \times 4 \times 4$ $cm^2 = 8$ cm^2.

Also area of $\Delta ABC = 8$ cm^2 (given).

Hence, the given statement is true.



(ii) Area of equilateral triangle with side 8 cm $= \frac{\sqrt{3}}{4} \times 8^2$ cm^2

$= 16\sqrt{3}$ cm^2 but area of triangle is $20\sqrt{3}$ cm^2, which is wrong.

Hence, the given statement is false.

(iii) Let length of each equal side of the isosceles triangle be a cm, then $a + a + 5 = 11$ (given) $\Rightarrow 2a = 6 \Rightarrow a = 3$.

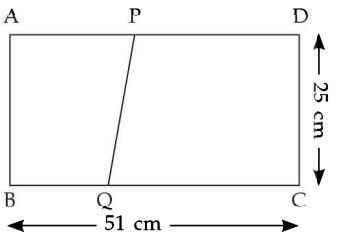
Using the formula, area $= \frac{b}{4}\sqrt{4a^2 - b^2}$, we get

area of the triangle $= \frac{5}{4}\sqrt{4 \times 3^2 - 5^2}$ $cm^2 = \frac{5}{4}\sqrt{11}$ cm^2.

Also area of triangle $= \frac{5}{4}\sqrt{11}$ cm^2 (given).

Hence, the given statement is true.

Example 2. *In the adjoining figure, ABCD is a rectangle of dimensions 51 cm × 25 cm. A trapezium PQCD with its parallel sides QC and PD in the ratio 9 : 8 is cut off from the rectangle. If the area of the trapezium PQCD is $\frac{5}{6}$th part of the area of the rectangle, find the lengths of QC and PD.* *(NCERT Exemplar)*



Solution. Area of rectangle ABCD = 51 cm × 25 cm = 1275 cm^2.

As the sides QC and PD of the trapezium PQCD are in the ratio 9 : 8, let the sides be $9x$ cm and $8x$ cm. Then, area of trapezium PQCD $= \frac{1}{2}$ (QC + PD) × height

$= \frac{5}{6}$ of area of rectangle (given)

$\Rightarrow \quad \frac{1}{2}(9x + 8x) \times 25 = \frac{5}{6} \times 1275$

$\Rightarrow \quad 17x = \frac{5}{3} \times 51 \Rightarrow x = 5$

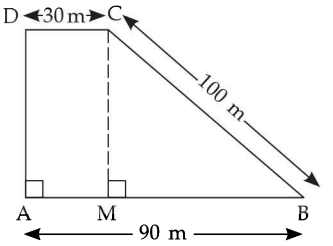
$\therefore \quad$ QC $= 9x$ cm $= (9 \times 5)$ cm $= 45$ cm

and PD $= 8x$ cm $= (8 \times 5)$ cm $= 40$ cm.

Example 3. *A field is in the shape of a trapezium having parallel sides 90 m and 30 m. These sides meet the third side at right angles. The length of the fourth side is 100 m. If it costs ₹ 4 to plough $1m^2$ of the field, find the total cost of ploughing the field.* *(NCERT Exemplar)*

Solution. Let ABCD be the trapezium with AB parallel to DC and $\angle A = 90°$. From C, draw CM ⊥ AB. Then, AMCD is a rectangle. MB = AB – AM = AB – DC = 90 m – 30 m = 60 m.

In ΔCMB, $\angle M = 90°$. By Pythagoras theorem, we get



$BC^2 = CM^2 + MB^2$

$\Rightarrow \quad 100^2 = CM^2 + 60^2$

$\Rightarrow \quad CM^2 = 100^2 - 60^2 = 10000 - 3600 = 6400$

$\Rightarrow \quad CM = 80$ m.

Area of trapezium ABCD

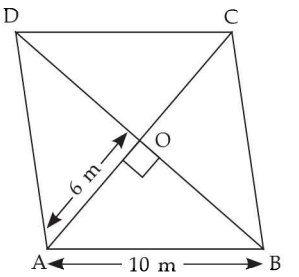
$= \frac{1}{2}$ (sum of parallel sides) × height

$= \frac{1}{2} (90 + 30) \times 80 \text{ m}^2 = 60 \times 80 \text{ m}^2 = 4800 \text{ m}^2$

As the cost of ploughing the field is ₹ 4 per m^2, therefore, the total cost of ploughing the field = ₹ (4×4800) = ₹ 19200.

Example 4. *A rhombus shaped sheet with perimeter 40 m and one diagonal 12 m, is painted on both sides at the rate of ₹ 5 per m^2. Find the cost of painting.* *(NCERT Exemplar)*

Solution. Let ABCD be the rhombus with perimeter = 40 m and diagonal AC = 12 m. Then each side of rhombus = 10 m, so AB = 10 m.



As the diagonals of a rhombus bisect each other at right angles, AO = 6 m and $\angle AOB = 90°$.

In ΔOAB, $\angle O = 90°$. By Pythagoras theorem, we get

$AB^2 = AO^2 + OB^2 \Rightarrow 10^2 = 6^2 + OB^2$

$\Rightarrow OB^2 = 10^2 - 6^2 = 100 - 36 = 64 \Rightarrow OB = 8$ m.

$\therefore$ Length of diagonal BD = (2×8) m = 16 m.

$\therefore$ Area of rhombus ABCD = $\frac{1}{2}$ × product of diagonals

$= \frac{1}{2} \times 12 \times 16 \text{ m}^2 = 96 \text{ m}^2$

As the rhombus is to be painted on both sides, area to be painted

$= (2 \times 96) \text{ m}^2 = 192 \text{ m}^2$

Since the rate of painting is ₹ 5 per m^2, therefore, the total cost of painting

= ₹ (5×192) = ₹ 960.

Example 5. *The perimeter of an isosceles triangle is 32 cm. If the ratio of its equal side to its base is 3 : 2, find the area of the triangle.* *(NCERT Exemplar)*

Solution. As the ratio of equal side of an isosceles triangle to its base is 3 : 2, let the length of each equal side be $3x$ cm, then the length of base = $2x$ cm.

$\therefore$ Perimeter of the triangle = $(3x + 3x + 2x)$ cm = 32 cm (given)

$\Rightarrow \quad 8x = 32 \Rightarrow x = 4$

$\therefore$ Length of each equal side = (3×4) cm = 12 cm

and length of base = (2×4) cm = 8 cm.

Using the formula for the area of an isosceles triangle = $\frac{b}{4}\sqrt{4a^2 - b^2}$, we get

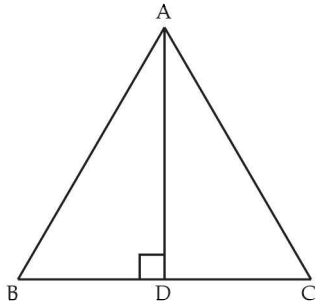
area of given triangle = $\frac{8}{4}\sqrt{4 \times 12^2 - 8^2}$ cm^2 = $2\sqrt{576 - 64}$ cm^2

$= 2\sqrt{512}$ cm^2 = $32\sqrt{2}$ cm^2.

Example 6. *The base of an isosceles triangle is 24 cm and its area is 192 sq. cm. Find its perimeter.*

Solution. Let ABC be an isosceles triangle with base BC = 24 cm and area = 192 cm^2.

Let h cm be the height and AB = AC = a cm.



then, area of $\Delta ABC = \frac{1}{2}$ base × height

$\Rightarrow \quad 192 = \frac{1}{2} \times 24 \times h \Rightarrow 16.$

In ΔABD, $\angle D = 90°$. By Pythagoras theorem, we have

$AB^2 = AD^2 + BD^2 \Rightarrow a^2 = 16^2 + 12^2$

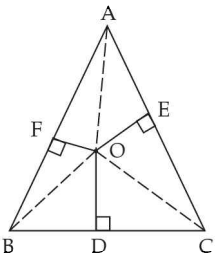
$(\because \text{BD} = \frac{1}{2} \text{BC} = 12 \text{ cm})$

$\Rightarrow \quad a^2 = 400 \Rightarrow a = 20$

$\therefore$ Perimeter of $\Delta ABC = 2a + b = (2 \times 20 + 24)$ cm = 64 cm.

Example 7. *From a point in the interior of an equilateral triangle, perpendiculars are drawn on the three sides. If the lengths of the perpendiculars are 14 cm, 10 cm and 6 cm, find the area of the triangle.* *(NCERT Exemplar)*

Solution. Let ABC be an equilateral triangle with length of each side = a cm.



O is a point in the interior of ΔABC, OD $\perp$ BC, OE $\perp$ CA and OF $\perp$ AB such that OD = 14 cm, OE = 10 cm and OF = 6 cm.

Area of equilateral triangle ABC = $\frac{\sqrt{3}}{4} a^2$ cm^2.

Also area of ΔABC = area of ΔOBD + area of ΔOCA + area of ΔOAB

$= \frac{1}{2}$ BC × OD + $\frac{1}{2}$ CA × OE + $\frac{1}{2}$ AB × OF

$\Rightarrow \quad \frac{\sqrt{3}}{4} a^2 \text{ cm}^2 = \frac{1}{2} (a \times 14 + a \times 10 + a \times 6) \text{ cm}^2$

$\Rightarrow \quad \frac{\sqrt{3}}{2} a^2 = 30a \Rightarrow \sqrt{3}\, a = 60 \qquad (\because a \neq 0)$

$\Rightarrow \quad a \Rightarrow 20\sqrt{3}$.

$\therefore \quad$ Area of $\Delta ABC = \frac{\sqrt{3}}{4} \times (20\sqrt{3})^2 \text{ cm}^2 = 300\sqrt{3} \text{ cm}^2$.

Exercise 7.1

1. Write whether the following statements are True or False. Justify your answer.

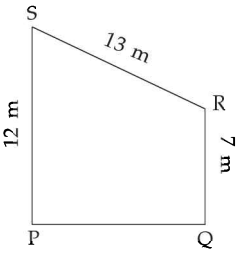
(*i*) The area of a triangle with base 4 cm and height 6 cm is 24 cm^2. *(NCERT Exemplar)*

(*ii*) If the base and the corresponding altitude of a parallelogram are 10 cm and 3.5 cm respectively, then the area of the parallelogram is 30 cm^2. *(NCERT Exemplar)*

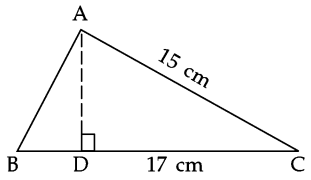
(*iii*) If the side of a rhombus is 10 cm and one diagonal is 16 m, then the area of the rhombus is 96 cm^2. *(NCERT Exemplar)*

(*iv*) The area of a regular hexagon of side 'a' is the sum of the areas of the five equilateral triangles with side a. *(NCERT Exemplar)*

2. Find the area of the trapezium PQRS with height PQ given in the adjoining figure. *(NCERT Exemplar)*



3. The area of a trapezium is 475 cm^2 and the height is 19 cm. Find the lengths of its two parallel sides if one side is 4 cm greater than the other. *(NCERT Exemplar)*
4. A square and an equilateral triangle have equal perimeters. If a diagonal of the square is $24\sqrt{2}$ cm, then find the area of the triangle.
5. In the adjoining figure, ABC is a right angled triangle at A and AD is perpendicular to BC. If BC = 17 cm and AC = 15 cm, find:
   (*i*) the area of ΔABC.
   (*ii*) the length of altitude AD.

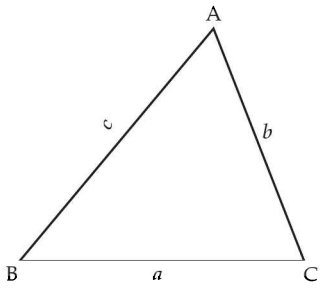


Answers

7.2 HERON'S FORMULA

Till now, we were finding the areas of some special triangles—right triangles, equilateral and isosceles triangles. The question arises! Can we find the area of a scalene triangle? Suppose you know the lengths of three sides of a scalene triangle and not the height. Can you still find its area? To find the area of the triangle, we need its height and there is no method to find out the height of such a triangle.

Heron derived a formula for finding the area of any triangle (including scalene triangle) which is known as ***Heron's formula***.



❑ **Heron's formula**

If ABC is any triangle with sides a, b and c,

then its area = $\sqrt{s(s-a)(s-b)(s-c)}$

where s = semi-perimeter = $\frac{a+b+c}{2}$.

Illustrative Examples

Example 1. *Find the area of a triangular park whose sides are 40 m, 32 m and 24 m.*

Solution. Let the sides of ΔABC be a = 40 m, b = 32 m and c = 24 m.

Then, s = semi-perimeter of ΔABC = $\frac{40+32+24}{2}$ m = 48 m.

$\therefore$ $s - a$ = (48 – 40) m = 8 m, $s - b$ = (48 – 32) = 16 m

and $s - c$ = (48 – 24) m = 24 m.

By Heron's formula,

area of ΔABC = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{48 \times 8 \times 16 \times 24}$ m^2 = 384 m^2.

Example 2. *Find the area of a triangle whose two sides are 18 cm and 10 cm and the perimeter is 42 cm.* *(NCERT)*

Solution. Let a, b and c be the sides of the given triangle and a = 18 cm, b = 10 cm.

As the perimeter of the triangle is 42 cm,

so the third side c = 42 cm – 18 cm – 10 cm = 14 cm,

s = semi-perimeter = $\frac{42}{2}$ cm = 21 cm.

Using Heron's formula, area of triangle

= $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{21(21-18)(21-10)(21-14)}$ cm^2

= $\sqrt{21 \times 3 \times 11 \times 7}$ cm^2 = $21\sqrt{11}$ cm^2.

Example 3. *Sides of a triangle are in the ratio 12 : 17 : 25 and its perimeter is 540 cm. Find its area.* *(NCERT)*

Solution. As the sides of the triangle are in the ratio 12 : 17 : 25, let the sides be $12x$, $17x$ and $25x$ (in centimetres).

Since the perimeter of the triangle is 540 cm, therefore,

$12x + 17x + 25x = 540$

$\Rightarrow \quad 54x = 540 \Rightarrow x = 10.$

$\therefore$ The sides of the triangle are (12 × 10) cm, (17 × 10) cm and (25 × 10) cm

i.e. 120 cm, 170 cm and 250 cm.

s = semi-perimeter = $\frac{540}{2}$ cm = 270 cm.

Using Heron's formula,

area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{270(270-120)(270-170)(270-250)}\ \text{cm}^2$

$= \sqrt{270 \times 150 \times 100 \times 20}\ \text{cm}^2 = 9000\ \text{cm}^2.$

Example 4. *A traffic signal board, indicating 'SCHOOL AHEAD', is an equilateral triangle with side 'a'. Find the area of the signal board, using Heron's formula. If its perimeter is 180 cm, what will be the area of the signal board?*

What value is depicted in the question? ***(Value Based)***

Solution. s = semi-perimeter of triangle = $\frac{a+a+a}{2} = \frac{3a}{2}$

Using Heron's formula, area of the equilateral triangle

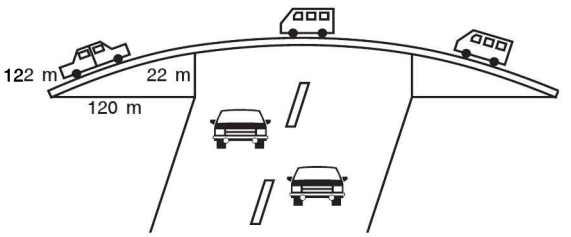
$= \sqrt{\frac{3a}{2}\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)} = \sqrt{\frac{3a}{2}\cdot\frac{a}{2}\cdot\frac{a}{2}\cdot\frac{a}{2}} = \frac{\sqrt{3}}{4}a^2.$

Given perimeter = 180 cm, so each side of equilateral triangle = $\frac{180}{3}$ cm = 60 cm.

$\therefore$ Area of triangle = $\frac{\sqrt{3}}{4} \times 60^2\ \text{cm}^2 = 900\sqrt{3}\ \text{cm}^2.$

Value : It is a precautionary measure for the safety of students.

Example 5. *The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 122 m, 22 m and 120 m (shown in the adjoining figure). The advertisements yield an earning of ₹ 5000 per m^2 per year. A company hired one of its walls for 3 months. How much rent did it pay?*



Solution. Here, s = $\frac{122+22+120}{2}$ m = 132 m.

Using Heron's formula,

area of one triangular wall = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{132(132-122)(132-22)(132-120)}\ \text{m}^2$

$= \sqrt{132 \times 10 \times 110 \times 12}\ \text{m}^2 = 1320\ \text{m}^2.$

Rent = ₹ 5000 per m^2 per year.

$\therefore$ Rent of one wall for 3 months = ₹ $\frac{1320 \times 5000 \times 3}{12}$ = ₹ 1650000.

Example 6. *The perimeter of a triangle is 50 cm. One side of a triangle is 4 cm longer than the smallest side and the third side is 6 cm less than twice the smallest side. Find the area of the triangle.* *(NCERT Exemplar)*

Solution. Let the smallest side of the triangle be x cm, then the other two sides are $(x + 4)$ cm and $(2x - 6)$ cm.

Given, perimeter of the triangle = 50 cm

$\Rightarrow \quad x + (x + 4) + (2x - 6) = 50$

$\Rightarrow \quad 4x = 52 \Rightarrow x = 13$

$\therefore$ The lengths of three sides of the triangle are 13 cm, (13 + 4) cm and (2 × 13 – 6) cm *i.e.* 13 cm, 17 cm and 20 cm.

Here, s = semi-perimeter = $\frac{50}{2}$ cm = 25 cm.

Using Heron's formula,

area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

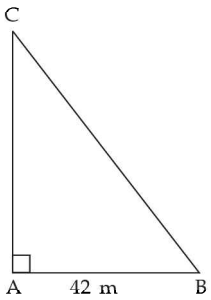
$= \sqrt{25(25-13)(25-17)(25-20)}$ cm^2

$= \sqrt{25 \times 12 \times 8 \times 5}$ cm^2 = $20\sqrt{30}$ cm^2.

Example 7. *In a right angled triangle, one side is 42 m and the difference of hypotenuse and the other side is 14 m. Find the lengths of both unknown sides. Calculate its area by using the formula for finding the area of a right triangle. Verify your result by using Heron's formula.*

Solution. Let ABC be a right angled triangle with $\angle A = 90°$ and c = 42 m. Let the other two sides be a metres and b metres, then

$a - b = 14$ ...(*i*)



In ΔABC, $\angle A = 90°$. By Pythagoras theorem,

$a^2 = b^2 + 42^2 \Rightarrow a^2 - b^2 = 42^2$

$\Rightarrow \quad (a - b)(a + b) = 1764$

$\Rightarrow \quad 14(a + b) = 1764$ (using (*i*))

$\Rightarrow \quad a + b = 126$ ...(*ii*)

Adding (*i*) and (*ii*), we get

$2a = 140 \Rightarrow a = 70$

From (*ii*), $70 + b = 126 \Rightarrow b = 56$

Hence, the other two sides are 70 m and 56 m.

$\therefore$ Area of $\Delta ABC = \frac{1}{2}$ base × height = $\frac{1}{2} \times 42 \times 56$ m^2

$= (21 \times 56)$ m^2 = 1176 m^2 ...(*iii*)

Verification:

Here, $s = \frac{42 + 56 + 70}{2}$ m = 84 m.

Using Heron's formula,

area of ABC = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{84(84-42)(84-56)(84-70)}$ m^2

$= \sqrt{84 \times 42 \times 28 \times 14}$ m^2 = 1176 m^2.

Hence, area of triangle = 1176 m^2 which is equal to area obtained in (*iii*).

Example 8. *If each side of a triangle is doubled, then find the ratio of areas of the new triangle thus formed and the given triangle.* *(NCERT Exemplar)*

Solution. Let a, b, c be the sides of given triangle and s be its semi-perimeter, then

$s = \frac{a + b + c}{2}$

$\Rightarrow \quad 2s = a + b + c$

Area of the given triangle = $\sqrt{s(s-a)(s-b)(s-c)}$ ...(*i*)

As the sides of the given triangle are doubled, the sides of new triangle are $2a$, $2b$ and $2c$. Let S be the semi-perimeter of new triangle, then

$$S = \frac{2a + 2b + 2c}{2} = a + b + c = 2s \quad \text{...(ii)}$$

Area of new triangle $= \sqrt{S(S - 2a)(S - 2b)(S - 2c)}$

$= \sqrt{2s(2s - 2a)(2s - 2b)(2s - 2c)}$ (using (*ii*))

$= \sqrt{16s(s - a)(s - b)(s - c)}$

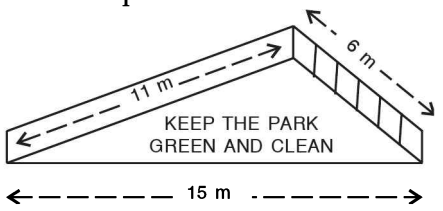
$= 4\sqrt{s(s - a)(s - b)(s - c)}$

$= 4 \times$ area of given triangle (using (*i*))

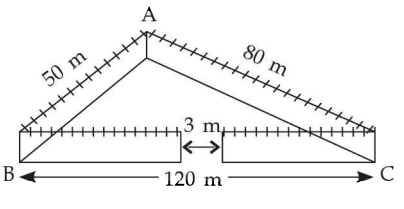
Therefore, the required ratio is 4 : 1.

Exercise 7.2

1. Find the area of a triangle whose sides are 40 cm, 30 cm and 20 cm.
2. Find the cost of laying grass in a triangular field of sides 50 m, 65 m and 65 m at the rate of ₹ 7 per m^2. *(NCERT Exemplar)*
3. Find the area of a triangle whose two sides are 8 cm and 11 cm long and the perimeter is 32 cm. *(NCERT)*
4. The perimeter of an isosceles triangle is 30 cm and each of the equal sides measures 12 cm. Find the area of the triangle by using Heron's formula. *(NCERT)*
5. If the sides of a triangle are 13 cm, 5 cm and 12 cm, then find the length of the altitude corresponding to the longest side as base by using Heron's formula.
6. The sides of a triangular plot are in the ratio 3 : 5 : 7 and its perimeter is 300 m. Find its area. *(NCERT)*
7. There is a slide in a park. One of its side walls has been painted in some colour with a message "KEEP THE PARK GREEN AND CLEAN" (shown in the adjoining figure). If the sides of the wall are 15 m, 11 m and 6 m, find the area painted in colour. *(NCERT)*



8. The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 13 m, 14 m and 15 m. The advertisements yield an earning of ₹ 2000 per m^2 a year. A company hired one of its walls for 6 months. How much rent did it pay? *(NCERT Exemplar)*
9. A triangular park ABC has sides 120 m, 80 m and 50 m (as shown in the adjoining figure). A gardner Dhania has to put a fence around it and also plant grass inside. How much area does she need to plant? Find the cost of fencing it with barbed wire at the rate of ₹ 20 per metre leaving a space 3 m wide for a gate on one side. *(NCERT)*



Answers

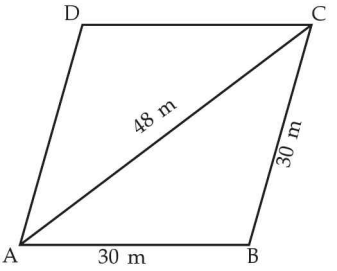
7.3. APPLICATIONS OF HERON'S FORMULA

In this section, we shall find areas of quadrilaterals and some plane figures by using Heron's formula.

Illustrative Examples

Example 1. *A rhombus shaped field has green grass for 18 cows to graze. If each side of the rhombus is 30 m and its longer diagonal is 48 m, how much area of the grass field will each cow be getting?* *(NCERT)*

Solution. Diagonal AC divides the rhombus ABCD into two congruent triangles of equal area.



For ΔABC, $s = \frac{30 + 30 + 48}{2}$ m = 54 m,

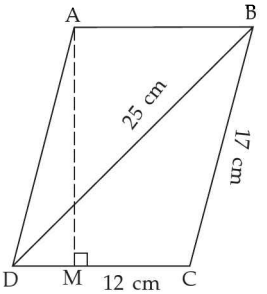
area of $\Delta ABC = \sqrt{54(54 - 30)(54 - 30)(54 - 48)}$ m^2

$= \sqrt{54 \times 24 \times 24 \times 6}$ $m^2 = 432$ m^2.

$\therefore$ Area of field = 2 × area of $\Delta ABC = (2 \times 432)$ $m^2 = 864$ m^2.

$\therefore$ Area of grass field which each cow will be getting = $\frac{864}{18}$ $m^2 = 48$ m^2.

Example 2. *Find the area of the parallelogram given in the adjoining figure. Also find the length of altitude from vertex A on the side DC.* *(NCERT Exemplar)*



Solution. Diagonal DB divides the parallelogram ABCD into two congruent triangles of equal area.

For ΔBCD, $s = \frac{12 + 17 + 25}{2}$ cm = 27 cm.

Using Heron's formula,

area of $\Delta BCD = \sqrt{27(27 - 12)(27 - 17)(27 - 25)}$ cm^2

$= \sqrt{27 \times 15 \times 10 \times 2}$ $cm^2 = 90$ cm^2.

$\therefore$ Area of parallelogram ABCD = 2 × area of ΔBCD

$= (2 \times 90)$ $cm^2 = 180$ cm^2.

Let AM = h cm.

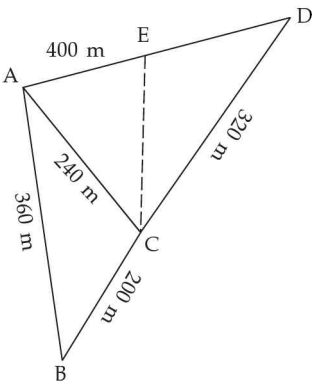
Area of parallelogram ABCD = base × height = DC × AM

$\Rightarrow$ $12 \times h = 180 \Rightarrow h = 15$.

$\therefore$ The length of altitude from vertex A to side DC = 15 cm.

Example 3. *Kamla has a triangular field with sides 240 m, 200 m and 300 m, where she grew wheat. In another triangular field with sides 240 m, 320 m and 400 m adjacent to the previous field, she wanted to grow potatoes and onions. She divided the field into two parts by joining the mid-point of the longest side to the opposite vertex and grew potatoes in one part and onions in the other part. How much area (in hectares) has been used for wheat, potatoes and onions? (1 hectare = 10000 m^2).* *(NCERT)*

Solution. Let ABC be the triangular field where wheat is grown and ACD be the adjacent triangular field which has been divided into two parts by joining vertex C to the mid-point E of the longest side AD. For ΔABC, $s = \frac{240 + 200 + 360}{2}$ m = 400 m.



$\therefore$ Area for growing wheat

= area of ΔABC

$= \sqrt{400(400 - 240)(400 - 200)(400 - 360)}$ m^2

$= \sqrt{400 \times 160 \times 200 \times 40}$ m

$= 16000\sqrt{2}$ $m^2 = \frac{16000\sqrt{2}}{10000}$ hectares

= 2.26 hectares (approx.)

For ΔACD, $s = \frac{240 + 400 + 320}{2}$ m = 480 m.

Area of $\Delta ACD = \sqrt{480(480 - 240)(480 - 400)(480 - 320)}$ m^2

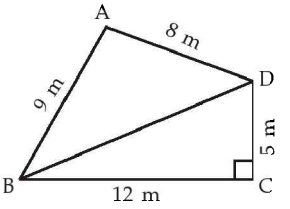
$= \sqrt{480 \times 240 \times 80 \times 160}$ $m^2 = 38400$ $m^2 = 3.84$ hectares.

Note that the line segment EC divides ΔACD into two triangles of equal area because ΔACE and ΔDCE have equal bases AE, DE and have equal heights.

∴ Area for growing potatoes = area for growing onions = $\frac{1}{2}$ of 3.84 hectares = 1.92 hectares.

Example 4. *A park, in the shape of a quadrilateral ABCD, has ∠C = 90°, AB = 9 m, BC = 12 m, CD = 5 m and AD = 8 m. How much area does it occupy?* *(NCERT)*

Solution. In ΔDBC, ∠C = 90°



By Pythagoras theorem, we get

$BD^2 = BC^2 + CD^2 = 12^2 + 5^2$

$\Rightarrow$ $BD^2 = 169 \Rightarrow BD = 13$ m.

Area of ΔDBC = $\frac{1}{2}$ base × height

$= \frac{1}{2} \times 12 \times 5 \text{ m}^2 = 30 \text{ m}^2$.

For ΔABD, $s = \frac{9 + 13 + 8}{2}$ m = 15 m.

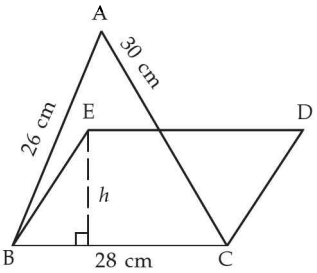
∴ Area of ΔABD = $\sqrt{15(15 - 9)(15 - 13)(15 - 8)}$ m^2

$= \sqrt{15 \times 6 \times 2 \times 7}$ $\text{m}^2 = 6\sqrt{35}$ $\text{m}^2 = 35.5 \text{ m}^2$ (approx.)

∴ Area of quad. ABCD = (30 + 35.5) m^2 approx. = 65.5 m^2 approx.

Example 5. *A triangle and a parallelogram have the same base and same area. If the sides of the triangle are 26 cm, 28 cm and 30 cm, and parallelogram stands on the base 28 cm, find the height of the parallelogram.* *(NCERT)*

Solution. Let ABC be a triangle and BCDE be a parallelogram having same base BC (= 28 cm) and equal area.



Let h cm be the height (corresponding to base BC) of the || gm BCDE.

For ΔABC, $s = \frac{26 + 28 + 30}{2}$ cm = 42 cm.

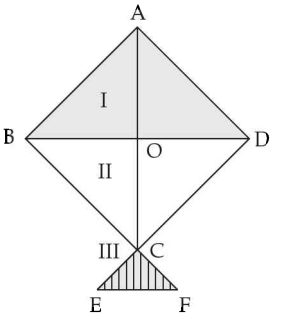
∴ Area of ΔABC = $\sqrt{42(42 - 26)(42 - 28)(42 - 30)}$ cm^2

$= \sqrt{42 \times 16 \times 14 \times 12}$ $\text{cm}^2 = 336 \text{ cm}^2$

∴ Area of || gm BCDE = base × height $\Rightarrow 28 \times h = 336 \Rightarrow h = 12$.

∴ The height of parallelogram = 12 cm.

Example 6. *A kite in the shape of a square with diagonal 32 cm and an isosceles triangle of base 8 cm and sides 6 cm each is to be made of three different shades as shown in the adjoining figure. How much paper of each shade has been used in it?* *(NCERT)*



Solution. As the diagonals of a square bisect each other at right angles,

AO = $\frac{1}{2} \times 32$ cm = 16 cm.

Area of shade I = $\frac{1}{2} \times BD \times AO = \frac{1}{2} \times 32 \times 16 \text{ cm}^2 = 256 \text{ cm}^2$,

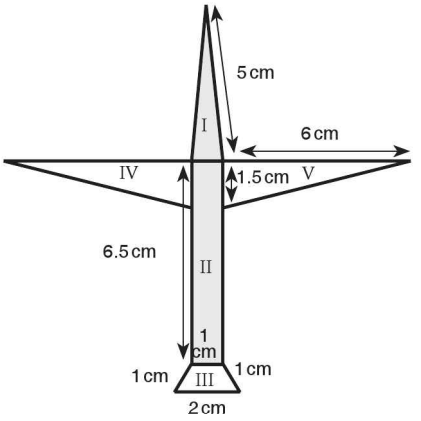
Area of shade II = $\frac{1}{2} \times BD \times OC = \frac{1}{2} \times 32 \times 16 \text{ cm}^2 = 256 \text{ cm}^2$.

For ΔCEF, $s = \frac{6 + 8 + 6}{2} = 10$ cm

$\therefore$ Area of $\Delta CEF = \sqrt{10(10 - 6)(10 - 8)(10 - 6)}$ cm^2 $= \sqrt{10 \times 4 \times 2 \times 4}$ cm^2

$= 8\sqrt{5}$ cm^2 $= 17.9$ cm^2 (approx.)

$\therefore$ Area of shade III $= 17.9$ cm^2 (approx.)



Example 7. *Radha cut an aeroplane of coloured paper as shown in the adjoining figure. Find the total area of the paper used. What values students learn by making articles from paper cutting?* ***(Value Based)***

Solution. Region I consists of an isosceles triangle with sides 5 cm, 5 cm and 1 cm.

For region I, $s = \frac{5 + 5 + 1}{2} = \frac{11}{2}$ cm.

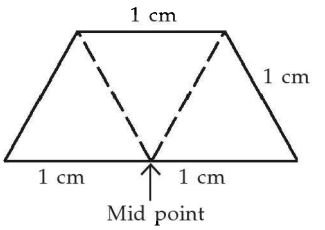
Its area $= \sqrt{\frac{11}{2}\left(\frac{11}{2} - 5\right)\left(\frac{11}{2} - 5\right)\left(\frac{11}{2} - 1\right)}$ cm^2

$= \frac{3}{4}\sqrt{11}$ cm^2 $= 2.5$ cm^2 (approx)

Region II is a rectangle with sides 6.5 cm and 1 cm,

its area $= 6.5 \times 1$ cm^2 $= 6.5$ cm^2

Region III is an isosceles trapezium, which can be divided into the three equilateral triangles each of side 1 cm (as shown in the adjoining figure).



Its area $= 3.\frac{\sqrt{3}}{4} \times 1^2$ cm^2 $= \frac{3}{4}\sqrt{3}$ cm^2 $= 1.3$ cm^2 (approx.)

Region IV and V are two congruent right triangles with base 6 cm and height 1.5 cm.

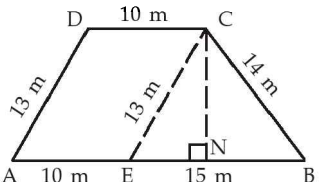
Area of region IV and V $= 2 \times \frac{1}{2} \times 6 \times 1.5$ cm^2 $= 9$ cm^2.

$\therefore$ The total area of the paper used $= (2.5 + 6.5 + 1.3 + 9)$ cm^2 approx.

$= 19.3$ cm^2 approximately.

Students learn art of drawing and cutting. They get innovative ideas and utilise their free time in good activities.

Example 8. *A field is in the shape of a trapezium whose parallel sides are 25 m and 10 m, and the non-parallel sides are 14 m and 13 m. Find the area of the field.*



Solution. Let ABCD be the given trapezium in which AB = 25 m, BC = 14 m, DC = 10 m, AD = 13 m and AB $\parallel$ DC.

Draw CE $\parallel$ DA, then AECD is a $\parallel$ gm with AE = 10 m and AD = 13 m. Draw CN $\perp$ EB.

Now EB = AB – AE = 25 m – 10 m = 15 m

For ΔEBC, $s = \frac{15 + 14 + 13}{2}$ m $= 21$ m,

$\therefore$ area of $\Delta EBC = \sqrt{21(21 - 15)(21 - 13)(21 - 14)}$ m^2

$= \sqrt{21 \times 6 \times 8 \times 7}$ m^2 $= 84$ m^2.

Also area of $\Delta EBC = \frac{1}{2} \times$ base $\times$ height

$\Rightarrow \quad 84 = \frac{1}{2} \times 15 \times CN \Rightarrow CN = \frac{56}{5}$ m.

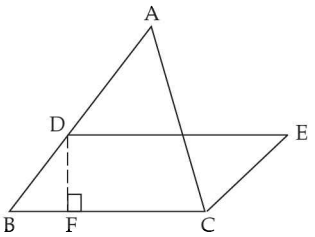
Area of || gm AECD = base × height = AE × CN = $10 \times \frac{56}{5}$ m^2 = 112 m^2.

$\therefore$ Area of trapezium ABCD = area of || gm AECD + area of ΔEBC

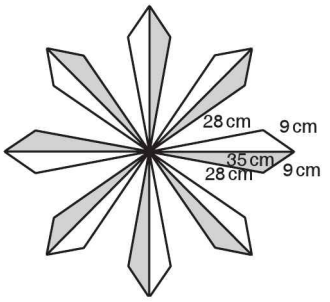
= 112 m^2 + 84 m^2 = 196 m^2.

Exercise 7.3

1. A field in the form of a parallelogram has sides 60 m, 40 m and one of its diagonal is 80 m long. Find the area of the parallelogram. *(NCERT Exemplar)*
2. If the perimeter of a rhombus is 400 m and its one diagonal is 160 m long, find the area of the rhombus. *(NCERT)*
3. Find the area of a quadrilateral ABCD in which AB = 3 cm, BC = 4 cm, CD = 4 cm, DA = 5 cm and AC = 5 cm. *(NCERT)*
4. The sides of a quadrilateral ABCD are 6 cm, 8 cm, 12 cm and 14 cm (taken in order) respectively and the angle between the first two sides is a right angle. Find its area. *(NCERT Exemplar)*
5. In the adjoining figure, ΔABC has sides AB = 7.5 cm, AC = 6.5 cm and BC = 7 cm. On the base BC, a parallelogram DBCE of same area as that of ΔABC is constructed. Find the height DF of the parallelogram. *(NCERT Exemplar)*



6. A floral design on a floor is made up of 16 tiles which are triangular, the sides of the triangle being 9 cm, 28 cm and 35 cm (shown in the adjoining figure). Find the cost of polishing the tiles, at the rate of 50 paise per cm^2. *(NCERT)*



7. An umbrella is made by stitching 10 triangular pieces of cloth of two different colours (shown in the adjoining figure), each piece measuring 20 cm, 50 cm and 50 cm. How much cloth of each colour is required for the umbrella? *(NCERT)*



Answers

Very Short Answer Type Questions

1. Find the area of a triangle with base 8 cm and height 5.5 cm.
2. If one side of a parallelogram is 8 cm and its corresponding altitude is 6 cm, then find its area.
3. If the diagonals of a rhombus are 10 cm and 8 cm, then find its area.
4. If the area of an equilateral triangle is $25\sqrt{3}$ cm^2, then find the perimeter of the triangle.
5. If the side of a rhombus is 10 cm and one diagonal is 12 cm, then find the area of the rhombus.
6. If the area of an isosceles right angled triangle is 12.5 cm^2, then find the length of its hypotenuse.
7. If the altitude of an equilateral triangle is $\sqrt{6}$ cm, then find its area.
8. If one diagonal of a parallelogram is 6.6 cm and the perpendicular distance of this diagonal from an opposite vertex is 2.5 cm, then find its area.
9. Find the area of a triangle whose sides are 10 cm, 24 cm and 26 cm.
10. Find the area of a triangle whose sides are 12 cm, 14 cm and 16 cm.

Answers

Objective Type Questions

(FOR FORMATIVE/SUMMATIVE ASSESSMENT)

Mental Maths

1. Fill in the blanks :
   (*i*) If side of an equilateral triangle is a, then its area is ..... .
   (*ii*) If side of an equilateral triangle is a, then its altitude is ..... .
   (*iii*) If ABC is an isosceles triangle with AB = AC = a and BC = b, then its area is ..... .
   (*iv*) If the sides of a triangle are a, b and c and its semi-perimeter is s, then the area of the triangle is ..... .
   (*v*) The area of a rhombus = $\frac{1}{2}$ × product of its ..... .
   (*vi*) The area of a parallelogram = base × ..... .
   (*vii*) The area of a triangle = $\frac{1}{2}$ × base × ..... .
   (*viii*) The area of a right triangle = $\frac{1}{2}$ product of its sides containing the ..... .
2. State whether the following statements are true or false :
   (*i*) A diagonal of a parallelogram divides it into two congruent triangles of equal area.
   (*ii*) A diagonal of a rhombus divides it into two congruent triangles of equal area.
   (*iii*) Area of a quadrilateral whose sides and one diagonal are given can be calculated by dividing the quadrilateral (along the given diagonal) into two triangles and using Heron's formula.

Multiple Choice Questions

Choose the correct answer from the given four options (3 to 13) :

3. An isosceles right triangle has area 8 cm^2. The length of its hypotenuse is

(*a*) $\sqrt{32}$ cm (*b*) $\sqrt{16}$ cm (*c*) $\sqrt{48}$ cm (*d*) $\sqrt{24}$ cm

(NCERT Exemplar)

4. If the perimeter of an equilateral triangle is 60 m, then the area is

(*a*) $10\sqrt{3}$ m^2 (*b*) $15\sqrt{3}$ m^2 (*c*) $20\sqrt{3}$ m^2 (*d*) $100\sqrt{3}$ m^2

(NCERT Exemplar)

5. The length of each side of an equilateral triangle having area of $9\sqrt{3}$ cm^2 is

(*a*) 8 cm (*b*) 36 cm (*c*) 4 cm (*d*) 6 cm

(NCERT Exemplar)

6. If the area of equilateral triangle is $16\sqrt{3}$ cm^2, then the perimeter of the triangle is

(*a*) 48 cm (*b*) 24 cm (*c*) 12 cm (*d*) 36 cm

(NCERT Exemplar)

7. If the sides of a parallelogram are 9 cm and 4 cm, then the ratio of their corresponding altitudes is

(*a*) 2 : 3 (*b*) 3 : 2 (*c*) 9 : 4 (*d*) 4 : 9

8. If the perimeter of a rhombus is 80 cm and one of its diagonals is 24 cm, then the length of the other diagonal is

(*a*) 16 cm (*b*) 20 cm (*c*) 32 cm (*d*) 48 cm

9. If the sides of a triangle are 56 cm, 60 cm and 52 cm long, then the area of the triangle is

(*a*) 1322 cm^2 (*b*) 1311 cm^2 (*c*) 1344 cm^2 (*d*) 1392 cm^2

(NCERT Exemplar)

10. The area of an isosceles triangle having base 2 cm and length of one of equal sides 4 cm is

(*a*) $\sqrt{15}$ cm^2 (*b*) $\sqrt{\frac{15}{2}}$ cm^2 (*c*) $2\sqrt{15}$ cm^2 (*d*) $4\sqrt{15}$ cm^2

(NCERT Exemplar)

11. The edges of a triangular board are 6 cm, 8 cm and 10 cm. The cost of painting it at the rate of 9 paise per cm^2 is

(*a*) ₹ 2.00 (*b*) ₹ 2.16 (*c*) ₹ 2.48 (*d*) ₹ 3.00

(NCERT Exemplar)

12. Two adjacent sides of a parallelogram are 9 cm and 8 cm. If one of its diagonal is 13 cm, then its area is

(*a*) $24\sqrt{35}$ cm^2 (*b*) $12\sqrt{35}$ cm^2 (*c*) $6\sqrt{35}$ cm^2 (*d*) 150 cm^2

13. The sides of a triangle are 35 cm, 54 cm and 61 cm. The length of its longest altitude is

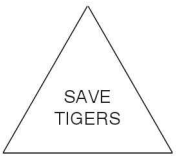
(*a*) $16\sqrt{5}$ cm (*b*) $10\sqrt{5}$ cm (*c*) $24\sqrt{5}$ cm (*d*) 28 cm

(NCERT Exemplar)

Answers

Value Based Questions

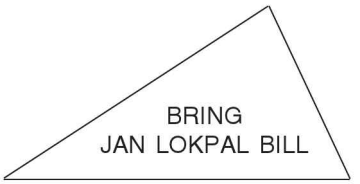
1. Some social workers started a campaign to protect animals. They prepared 5 cardboard banners in the shape of equilateral triangles as shown in the adjoining figure. The perimeter of a banner is 120 cm. If cardboard costs ₹ 1 per 10 cm^2, find the total cost of banners. Take $\sqrt{3} = 1.73$.



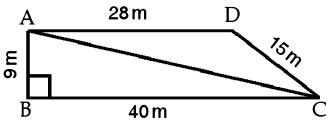
   What value is being promoted in this question?

2. Hoarding indicating "BE HONEST BE TRUTHFUL" is in triangular shape with sides 120 cm, 170 cm and 250 cm. Find its area. What values are being promoted in the question?

3. Anna Hazare started a movement to eradicate social evils of society. His workers prepared 100 posters of equal size in triangular shape as shown in the adjoining figure. The sides of a poster are 130 cm, 120 cm and 50 cm. If the posters cost ₹ 25 per sq. metre, find the total cost of the posters. What values are being promoted in the question?



4. Students of a school staged a rally for cleanliness campaign. They walked through the lanes in two groups. One group walked through the lanes AB, BC and CA; while the other through AC, CD and DA (shown in the adjoining figure). They cleaned the area enclosed within their lanes.

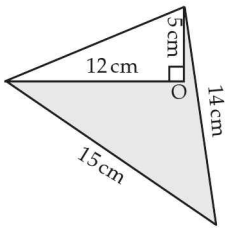


   If AB = 9 m, BC = 40 m, CD = 15 m, DA = 28 m and $\angle B = 90°$, which group cleaned more area and by how much? Find the total area cleaned by the students (neglecting the width of the lanes). What value is reflected in the question?

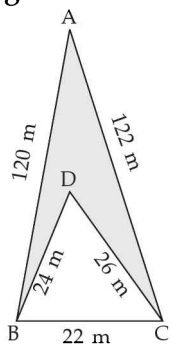
Answers

Chapter Test

1. The perimeter of a triangular field is 420 m and its sides are in the ratio 6 : 7 : 8. Find the area of the triangular field. *(NCERT Exemplar)*
2. The base of an isosceles triangle is 24 cm and its area is 60 cm^2. Find its perimeter (use Heron's formula).
3. (*a*) Find the area of the shaded region in figure (*i*) given below.
   (*b*) Find the area of the shaded region in figure (*ii*) given below. *(NCERT Exemplar)*

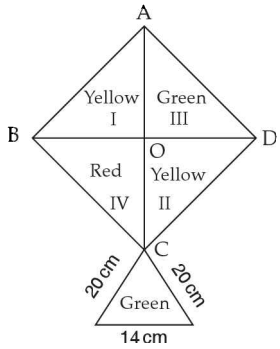


(*i*)

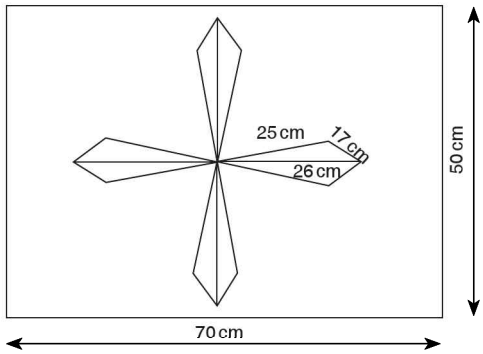


(*ii*)

4. How much paper of each shade is needed to make a kite given in the adjoining figure, in which ABCD is a square with diagonal 44 cm? *(NCERT Exemplar)*



5. A design is made on a rectangular tile of dimensions 70 cm × 50 cm as shown in the adjoining figure. The design shows 8 triangles, each of sides 26 cm, 17 cm and 25 cm. Find the total area of the design and the remaining area of the tile.
   (NCERT Exemplar)



Answers

Model Question Paper 3

(FOR SUMMATIVE ASSESSMENT - FIRST TERM)

Time: 3 Hours *Maximum Marks: 90*

General Instructions:

(*i*) All questions are compulsory.

(*ii*) The question paper consists of 31 questions divided into four sections A, B, C and D. Section A comprises of 4 questions of 1 mark each, Section B comprises of 6 questions of 2 marks each, Section C comprises of 10 questions of 3 marks each and Section D comprises of 11 questions of 4 marks each.

(*iii*) Use of calculator is not permitted.

Section A

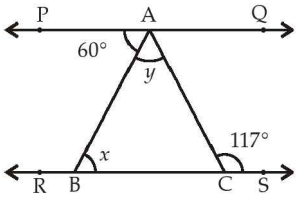
Question numbers 1 to 4 are of one mark each.

1. Simplify: $\left(3\ \frac{51}{64}\right)^{-\frac{2}{5}}$.
2. If two interior angles on the same side of a transversal intersecting two parallel lines are in the ratio 2 : 3, then find the smaller of the two angles.
3. Find the coordinates of the points which lie on y-axis and are at a distance of 4 units from the x-axis.
4. If the area of an equilateral triangle is $16\sqrt{3}$ m^2, then find its perimeter.

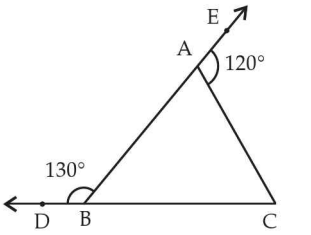
Section B

Question numbers 5 to 10 carry 2 marks each.

5. If $x = 3 + 2\sqrt{2}$, then find whether $x - \frac{1}{x}$ is rational or irrational.
6. Factorise : $7\sqrt{2}\,x^2 - 10x - 4\sqrt{2}$
7. Using suitable identity, find the value of $(42)^3 - (24)^3 - (18)^3$.
8. Plot the points A(3, 0), B(3, 3) and C(0, 3) in the cartesian plane. Join OA, AB, BC and CO. Name the figure so formed.
9. In the adjoining figure, if PQ $\|$ RS, $\angle$PAB = 60° and $\angle$ACS = 117° then find $x - y$.



10. In the adjoining figure, $\angle$ABD = 130° and $\angle$EAC = 120°. Prove that AB > AC.

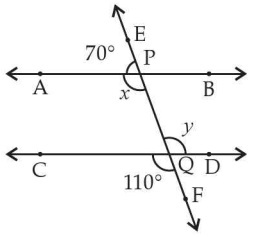


Section C

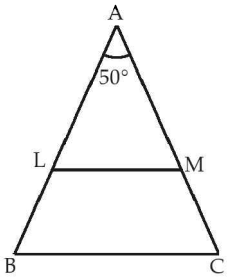
Question numbers 11 to 20 carry 3 marks each.

11. Express $2.3\overline{57}$ in the form $\frac{p}{q}$ where p, q are integers, $q \neq 0$.
12. If $\frac{7 + 3\sqrt{5}}{3 + \sqrt{5}} - \frac{7 - 3\sqrt{5}}{3 - \sqrt{5}} = a + b\sqrt{5}$, find the values of a and b.

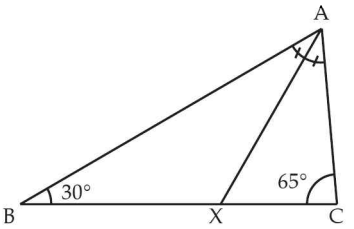
13. Factorise : $2\sqrt{2}\,a^3 + 8b^3 - 27c^3 + 18\sqrt{2}\,abc$.
14. If remainder is same when the polynomial $p(x) = x^3 + 8x^2 + 17x + ax$ is divided by $(x + 2)$ and $(x + 1)$, then find the value of a.
15. If both $x - 2$ and $2x - 1$ are factors of $px^2 + 5x + r$, show that $p = r$.
16. In the adjoining figure, find the values of x and y. Hence, show that AB ∥ CD.



17. In the adjoining figure, ABC is an isosceles triangle in which AB = AC and LM ∥ BC. If ∠A = 50°, then find ∠LMC.



18. AD is an altitude of an isosceles triangle ABC in which AB = AC. Show that AD bisects BC.
19. In the adjoining figure, ∠B = 30°, ∠C = 65° and the bisector of ∠A meets BC in X. Arrange AX, BX and CX in ascending order of magnitude.

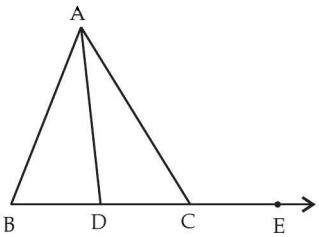


20. Draw the graph of the linear equation: $x - 2y - 3 = 0$.

Section D

Question numbers 21 to 31 carry 4 marks each.

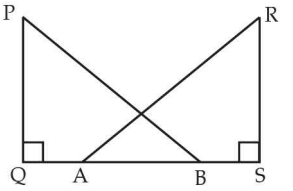
21. Simplify: $\frac{1}{2+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{6}} + \frac{1}{\sqrt{6}+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{8}}$.
22. If $x = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ and $y = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, then find the value of $x^2 + y^2 - 5xy$.
23. If $x = 3 + 2\sqrt{2}$, then find the value of $x^3 - \frac{1}{x^3}$.
24. The polynomials $ax^3 - 3x^2 + 4$ and $2x^3 - 5x + a$ when divided by $(x - 2)$ leave the remainders p and q respectively. If $p - 2q = 4$, find the value of a.
25. Using factor theorem, factorise the polynomial : $2x^3 + 9x^2 + 10x + 3$.
26. In the adjoining figure, E is a point on the side BC produced of ΔABC. If the bisector of ∠A meets BC at D, prove that ∠ABC + ∠ACE = 2 ∠ADC.



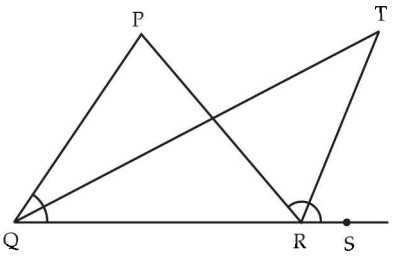
27. If the bisector of the vertical angle of a triangle bisects the base of the triangle, then prove that the triangle is isosceles.

28. In the adjoining figure, PQ and RS are perpendicular to QS, QA = BS and PB = RA. Prove that

(*i*) PQ = RS (*ii*) $\angle$QPB = $\angle$SRA.

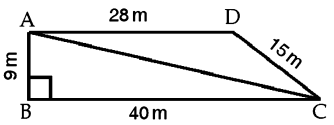


29. In the adjoining figure, the side QR of ΔPQR is produced to a point S. If the bisectors of $\angle$PQR and $\angle$PRS meet at the point T, then prove that $\angle QTR = \frac{1}{2} \angle QPR$.



30. If AD is the median of ΔABC, prove that AB + AC > 2AD.

31. Students of a school staged a rally for cleanliness campaign. They walked through the lanes in two groups. One group walked through the lanes AB, BC and CA; while the other through AC, CD and DA (shown in the adjoining figure). They cleaned the area enclosed within their lanes.



If AB = 9 m, BC = 40 m, CD = 15 m, DA = 28 m and $\angle$B = 90°, which group cleaned more area and by how much? Find the total area cleaned by the students (neglecting the width of the lanes). What value is reflected in the question?

Answers

Term - II

Linear Equations in Two Variables

8

INTRODUCTION

In earlier classes, we have already read about linear equations in one variable. An equation of the form $ax + b = 0$, where a, b are real numbers and $a \neq 0$ is a linear equation in one variable x. For example, $2x + 3 = 0$, $5y - 7 = 0$, $\sqrt{5}\,x + \pi = 0$ are linear equations in one variable. We have also learnt that a linear equation in one variable has a unique solution and it can be represented by a point on a number line.

In this chapter, we shall extend our knowledge of linear equations in one variable to linear equations in two variables and we shall see that a linear equation in two variables has infinitely many solutions and these solutions can be represented in the cartesian plane by ordered pairs of real numbers.

8.1 LINEAR EQUATIONS

❑ **Linear equation in one variable**

An equation of the form $ax + b = 0$, where a, b are real numbers and $a \neq 0$ is called **a linear equation in one variable x.**

For example, $2x + 3 = 0$, $3y - 5 = 0$, $\sqrt{2}\,t + 7 = 0$ are linear equations in one variable.

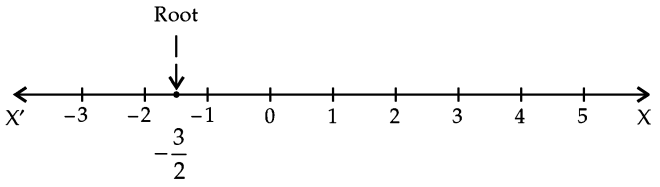
❑ **Solution (or root) of an equation**

Any real value (or values) of the variable (or variables) which when substituted in an equation makes its both sides equal is called a **solution** (or **root**) of the equation.

Thus, a real number α is a solution (or root) of the equation $ax + b = 0$ if and only if $a\alpha + b = 0$ *i.e.* if and only if $\alpha = -\frac{b}{a}$.

For example, $-\frac{3}{2}$ is a root of the equation $2x + 3 = 0$ because when we substitute $x = -\frac{3}{2}$ in this equation, we get $2\left(-\frac{3}{2}\right) + 3 = 0$ *i.e.* $0 = 0$, which is true.

The root $-\frac{3}{2}$ of the equation $2x + 3 = 0$ can be represented on the number line as shown below:



Every linear equation in one variable has one and only one solution (or root).

❑ **Permissible rules for solving equations**

(*i*) The same number or expression can be added to (or subtracted from) both sides of an equation.

(*ii*) Both sides of an equation can be multiplied (or divided) by the same non-zero real number or expression.

(*iii*) A term may be transposed from one side of the equation to the other side, but its sign will change.

❑ **Linear equation in two variables**

An equation of the form $ax + by + c = 0$, where a, b, c are real numbers, and a and b are not both zero, is called **a linear equation in two variables x and y.**

For example, $2x + 3y - 12 = 0$, $5p - 6q = 11$, $\sqrt{2}\,u + 5v = 7$ are linear equations in two variables.

Note that a and b are not both zero means that a and b cannot be both zero simultaneously *i.e.* any one of them can be zero.

Any linear equation in one variable can be expressed as a linear equation in two variables.

For example, $2x + 3 = 0$ can be expressed as $2x + 0y + 3 = 0$, and $3y - 5 = 0$ can be expressed as $0x + 3y - 5 = 0$.

❑ **Solution of a linear equation in two variables**

Consider the equation $3x + 2y = 12$, we note that $x = 2$ and $y = 3$ is a solution of this equation because when we substitute $x = 2$ and $y = 3$ in the equation $3x + 2y = 12$, we get $3 \times 2 + 2 \times 3 = 12$ *i.e.* $12 = 12$, which is true.

This solution can be written as the ordered pair (2, 3) which represents a point in the cartesian plane. Thus, the point (2, 3) is a solution of the linear equation $3x + 2y = 12$. Further, we note that (3, 2) is not a solution of this equation because when we substitute $x = 3$ and $y = 2$ in the equation $3x + 2y = 12$, we get $3 \times 3 + 2 \times 2 = 12$ *i.e.* $13 = 12$, which is not true.

Similarly, it is easy to check that (4, 0) is a solution but (0, 4) is not a solution of this equation.

Thus, we find that the equation $3x + 2y = 12$ has atleast two solutions (2, 3) and (4, 0).

In fact, we can find many more solutions of the equation $3x + 2y = 12$. On putting $x = k$ (any real number), we get $3k + 2y = 12 \Rightarrow 2y = 12 - 3k \Rightarrow y = \frac{12 - 3k}{2}$. Thus, $(k, \frac{12 - 3k}{2})$, where k is any real number, is a solution of the equation $3x + 2y = 12$. It follows that the linear equation $3x + 2y = 12$ has infinitely many solutions. In fact, this is true for every linear equation in two variables.

Thus, every linear equation in two variables has infinitely many solutions.

Illustrative Examples

Example 1. *Write the following linear equations in the form $ax + by + c = 0$ and indicate the values of a, b and c in each case* :

(*i*) $2x + 3y = 4.37$ (*ii*) $x - 4 = \sqrt{3}\,y$ (*iii*) $x = 3y$ *(NCERT)*

Solution.

(*i*) Given equation is $2x + 3y = 4.37$

It can be written as $2x + 3y - 4.37 = 0$, which is of the form $ax + by + c = 0$.

Here, $a = 2$, $b = 3$ and $c = -4.37$.

(*ii*) Given equation is $x - 4 = \sqrt{3}\,y$.

It can be written as $x - \sqrt{3}\,y - 4 = 0$, which is of the form $ax + by + c = 0$.

Here, $a = 1$, $b = -\sqrt{3}$ and $c = -4$.

(*iii*) Given equation is $x = 3y$.

It can be written as $x - 3y + 0 = 0$, which is of the form $ax + by + c = 0$.

Here, $a = 1$, $b = -3$ and $c = 0$.

Example 2. *Write each of the following linear equation as a linear equation in two variables:*

(*i*) $x = -5$ (*ii*) $5y = 2$ (*iii*) $3x = -2$ *(NCERT)*

Solution. (*i*) Given equation is $x = -5$.

It can be written as $x + 0y = -5$ or $1.x + 0.y + 5 = 0$, which is a linear equation in two variables.

(*ii*) Given equation is $5y = 2$.

It can be written as $0.x + 5y - 2 = 0$, which is a linear equation in two variables.

(*iii*) Given equation is $3x = -2$.

It can be written as $3x + 0.y + 2 = 0$, which is a linear equation in two variables.

Example 3. *The cost of a notebook is twice the cost of a pen. Write a linear equation in two variables to represent this statement.* *(NCERT)*

Solution. Let the cost of a notebook be ₹ x and the cost of a pen be ₹ y. As the cost of a notebook is twice the cost of a pen, so we have $x = 2y$ or $x - 2y = 0$, which is the required linear equation.

Example 4. *Check which of the following are solutions of the equation $x - 2y = 4$ and which are not:*

(*i*) (0, 2) (*ii*) (2, 0) (*iii*) (4, 0) (*iv*) $(\sqrt{2}, 4\sqrt{2})$ (*v*) (1, 1) *(NCERT)*

Solution. The given equation is $x - 2y = 4$ ...(1)

(*i*) Putting $x = 0$ and $y = 2$ in (1), we get

$0 - 2 \times 2 = 4$ *i.e.* $-4 = 4$, which is wrong.

Hence, (0, 2) is not a solution of the given equation.

(*ii*) Putting $x = 2$ and $y = 0$ in (1), we get

$2 - 2 \times 0 = 4$ *i.e.* $2 = 4$, which is wrong.

Hence, (2, 0) is not a solution of the given equation.

(*iii*) Put $x = 4$ and $y = 0$ in (1), we get

$4 - 2 \times 0 = 4$ *i.e.* $4 = 4$, which is true.

Hence, (4, 0) is a solution of the given equation.

(*iv*) Put $x = \sqrt{2}$ and $y = 4\sqrt{2}$ 0 in (1), we get

$\sqrt{2} - 2 \times 4\sqrt{2} = 4$ *i.e.* $-7\sqrt{2} = 4$, which is wrong.

Hence, $(\sqrt{2}, 4\sqrt{2})$ is not a solution of the given equation.

(*v*) Putting $x = 1$ and $y = 1$ in (1), we get

$1 - 2 \times 1 = 4$, *i.e.* $-1 = 4$, which is wrong.

Hence, (1, 1) is not a solution of the given equation.

Example 5. *Find the value of k, if $x = 2$, $y = 1$ is a solution of the equation $2x + 3y = k$.* *(NCERT)*

Solution. Since $x = 2$, $y = 1$ is a solution of the equation

$2x + 3y = k$, we have $2 \times 2 + 3 \times 1 = k \Rightarrow 4 + 3 = k \Rightarrow k = 7$

Hence, the value of k is 7.

Example 6. *Find two solutions for each of the following equations:*

(*i*) $4x + 3y = 12$ (*ii*) $2x + 5y = 0$ (*iii*) $3y + 4 = 0$ *(NCERT)*

Solution. (*i*) The given equation is $4x + 3y = 12$.

On taking $x = 0$, we get $3y = 12$ *i.e.* $y = 4$. So, (0, 4) is a solution of the given equation.

Similarly, on taking $y = 0$, we get $4x = 12$ *i.e.* $x = 3$. So (3, 0) is a solution of the given equation.

(*ii*) The given equation is $2x + 5y = 0$.

On taking $x = 0$, we get $5y = 0$ *i.e.* $y = 0$. So, (0, 0) is a solution of the given equation.

On taking $y = 0$, we get $2x = 0$ *i.e.* $x = 0$. So, (0, 0) is a Solution. Note that this solution is the same as obtained earlier *i.e.* we do not get a different Solution.

On taking $x = 1$, we get $2 \times 1 + 5y = 0$ *i.e.* $y = -\frac{2}{5}$. So, $(1, -\frac{2}{5})$ is a solution of the given equation.

(*iii*) The given equation is $3y + 4 = 0$. It can be written as $0.x + 3y + 4 = 0$, which is a linear equation in two variables.

Now $0.x + 3y + 4 = 0 \Rightarrow y = -\frac{4}{3}$ for all real values of x. Thus, the two solutions are $(0, -\frac{4}{3})$, $(1, -\frac{4}{3})$.

In fact, $(k, -\frac{4}{3})$ for all real values of k are solutions of the given equation.

Example 7. *Write four different solutions for each of the following equations:*

(*i*) $x + 2y = 6$ (*ii*) $\pi x + y = 9$ *(NCERT)*

Solution. (*i*) Given $x + 2y = 6 \Rightarrow 2y = 6 - x$

$\Rightarrow \quad y = \frac{6 - x}{2}$

On putting arbitrary (real) values of x, we can find corresponding values of y.

Values of x	*Values of y*
2	$y = \frac{6-2}{2} = \frac{4}{2} = 2$
4	$y = \frac{6-4}{2} = \frac{2}{2} = 1$
6	$y = \frac{6-6}{2} = \frac{0}{2} = 0$
-2	$y = \frac{6-(-2)}{2} = \frac{8}{2} = 4$

Hence, the four different solutions of the given equation are (2, 2), (4, 1), (6, 0) and (–2, 4).

In fact, there is no end to different solutions of a linear equation in two variables.

(*ii*) The given equation is $\pi x + y = 9 \Rightarrow y = 9 - \pi x$.

On putting arbitrary (real) values of x, we can find corresponding values of y.

Values of x	*Values of y*
$x = 0$	$y = 9 - \pi \times 0 = 9$
$x = 1$	$y = 9 - \pi \times 1 = 9 - \pi$
$x = 2$	$y = 9 - \pi \times 2 = 9 - 2\pi$
$x = -1$	$y = 9 - \pi \times (-1) = 9 + \pi$

Hence, the four different solutions of the given equation are (0, 9), $(1, 9 - \pi)$, $(2, 9 - 2\pi)$, $(-1, 9 + \pi)$.

Example 8. *Which one of the following options is true, and why? The equation $y = 3x + 5$ has*

(i) a unique solution (ii) only two solutions (iii) infinitely many solution. *(NCERT)*

Solution. The given equation is $y = 3x + 5$, which is a linear equation in two variables.

On putting arbitrary (real) values of x, we can find corresponding values of y.

Values of x	*Values of y*
0	$y = 3 \times 0 + 5 = 5$
1	$y = 3 \times 1 + 5 = 8$
2	$y = 3 \times 2 + 5 = 11$
–1	$y = 3 \times (-1) + 5 = 2$ etc.

All these pairs *i.e.* (0, 5), (1, 8), (2, 11), (–1, 2) etc. are solutions of the given equation.

Hence, the given equation has infinitely many solutions.

Therefore, the correct option is (*iii*).

Example 9. *Let y vary directly as x. If y = 12 when x = 4, then write a linear equation. What is the value of y when x = 5?* *(NCERT Exemplar)*

Solution. Since y varies directly as x, let $y = kx$ ...(*i*) where k is constant *i.e.* some real number.

Given that $y = 12$ when $x = 4$,

$\therefore \quad 12 = k \times 4 \Rightarrow k = 3$

Substituting $k = 3$ in (*i*), the required linear equation is $y = 3x$.

Now, when $x = 5$, $y = 3 \times 5 = 15$

Hence, when $x = 5$, $y = 15$.

Exercise 8.1

1. Express the following equations in the form $ax + by + c = 0$ and indicate the values of a, b and c in each case:

(*i*) $4 = 5x - 3y$ (*ii*) $2x + 3y = 9.3\overline{5}$ (*iii*) $-2x + 3y = 6$ (*iv*) $2x = y$

(*v*) $x - \frac{y}{5} = 10$ (*vi*) $2x = -5y$ *(NCERT)*

2. Write each of the following linear equation as a linear equation in two variables:

(*i*) $y = 2$ (*ii*) $2x = 3$ (*iii*) $5y = -\sqrt{7}$ *(NCERT)*

3. Express the following linear equations in the form $ax + by + c = 0$ and indicate the values of a, b and c in each case :

(*i*) $7x + 3 = 0$ (*ii*) $y - 2 = 0$ (*iii*) $5 = 2x$ *(NCERT)*

4. The cost of a pen is ₹ 5 less than half the cost of a notebook. Write this statement as a linear equation in two variables.

5. Check which of the following are solutions of the equation $2x - y = 5$ and which are not:

(*i*) (1, 3) (*ii*) (3, 1) (*iii*) (0, 5)

(*iv*) $(2\sqrt{3}, -\sqrt{3})$ (*v*) $\left(\frac{7}{4}, -\frac{3}{2}\right)$

6. If $x = -1$, $y = 2$ is a solution of the equation $2x + 5y = k$, then find the value of k.

7. Find the value of k for each of the following equations if $x = 1$, $y = 1$ is its solution:

(*i*) $x - y = k$ (*ii*) $5x + 2ky = 3k$ (*iii*) $9kx + 7ky = 48$

8. Check whether $x = 2$, $y = 1$ is a solution of the following equations or not:

(*i*) $2x - 3y = 1$ (*ii*) $5x - 3y + 7 = 0$ (*iii*) $3x + 5y - 11 = 0$

9. Find two solutions for each of the following equations:

(*i*) $3x + 5y = 0$ (*ii*) $2x + y = 11$ (*iii*) $3x + 5 = 0$

10. Find four different solutions for each of the following equation:

(*i*) $2x + y = 7$ *(NCERT)* (*ii*) $x = 4y$ *(NCERT)* (*iii*) $3(x - 1) = 2(y + 3) + 3$

Answers

8.2 GRAPH OF A LINEAR EQUATION IN TWO VARIABLES

So far, we have obtained the solutions of a linear equation in two variables. Now, let us look at their geometric representation. We know that every linear equation in two variables has infinitely many solutions. The question arises! How can we show them in the coordinate plane?

Consider the linear equation $3x+2y=12$ ... (1) in two variables x and y. It has infinitely many solutions. These solutions can be expressed in the form of a table as shown below by writing the values of y below the corresponding values of x.

x	0	2	4	6	...
y	6	3	0	–3	...

We know that these solutions can be represented by the points (0, 6), (2, 3), (4, 0) and (6, –3).

Plot these points on the graph paper. Join any two points, say (0, 6) and (4, 0), and obtain the straight line AB. We observe that the other two points also lie on this line AB. Now, pick up any other point on the line AB, say $(1, \frac{9}{2})$. Is this a solution? We note that $3 \times 1 + 2 \times \frac{9}{2} = 12$ *i.e.* $12 = 12$, which is true. So, $(1, \frac{9}{2})$ is a solution of the equation (1).

In fact, every point on the line AB is a solution of the equation (1).

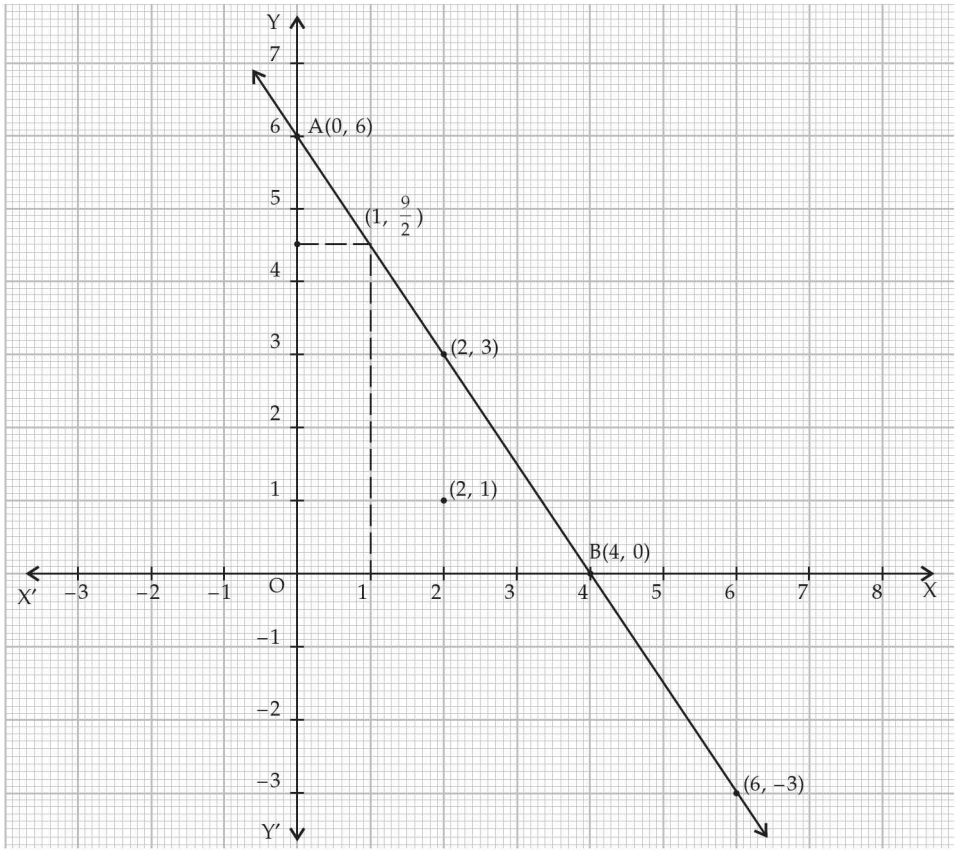
Now take any other point not lying on the line AB, say (2, 1). We note that $3 \times 2 + 2 \times 1 = 12$ *i.e.* $8 = 12$, which is wrong. So, (2, 1) is not a solution of the equation (1).

We list our observations as :

(*i*) Every point whose coordinates satisfy equation (1) lies on the line AB.

(*ii*) Every point (*a*, *b*) on the line AB gives a solution $x = a$ and $y = b$ of the equation (1).

(*iii*) Any point which does not lie on the line AB, is not a solution of equation (1).



Thus, we can conclude that a linear equation in two variables is represented geometrically by a line whose points make up the collection of solutions of the equation. This line is the **graph** of the linear equation.

Hence, the graph of every linear equation in two variables is a straight line.

Every point on the graph of a linear equation in two variables is a solution of the linear equation. Conversely, every solution of the linear equation is a point on the graph of the linear equation.

Remark

The reason that a polynomial equation $ax + by + c = 0$ of degree one is called a linear equation is that its geometrical representation is a straight line.

❑ **Drawing graph of a linear equation in two variables**

To draw the graph of a linear equation in two variables x and y, proceed as under:

(*i*) Rewrite the given equation with y as the subject *i.e.* $y = mx + c$.

(*ii*) Select any three convenient integral values (as far as possible) of x and find the corresponding values of y for each of the selected value of x.

(*iii*) Make table of values.

(*iv*) Draw the coordinate axes on the graph paper and choose suitable scale (same on both axes).

(*v*) Plot the points on the graph paper (coordinate plane).

(*vi*) Connect any two points by a straight line and check that the third point lies on it. In fact, two points are sufficients to draw the straight line but we use three points for drawing the graph to check the correctness of the graph.

Illustrative Examples

Example 1. *Draw the graph of* $2x + y = 3$. *(NCERT)*

Solution. The given equation is $2x + y = 3$.

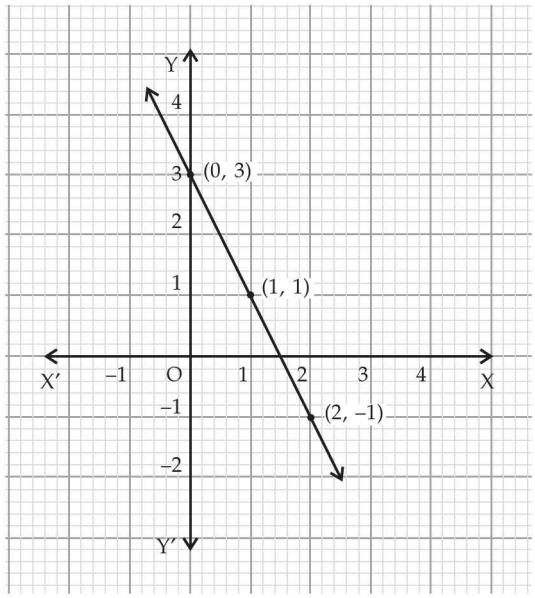
It can be written as $y = 3 - 2x$.

Table of values

x	0	1	2
y	3	1	–1

Draw the coordinate axes on a graph paper.

Take 1 cm = 1 unit on both axes. Plot the points (0, 3), (1, 1) and (2, – 1). Draw a line passing through any two points. The graph of the given equation is shown in the adjoining figure. Observe that the third point lies on the line.



Example 2. *Draw the graph of the linear equation $2x + 3y = 12$. At what points, the graph of the equation cuts the x-axis and the y-axis?* *(NCERT Exemplar)*

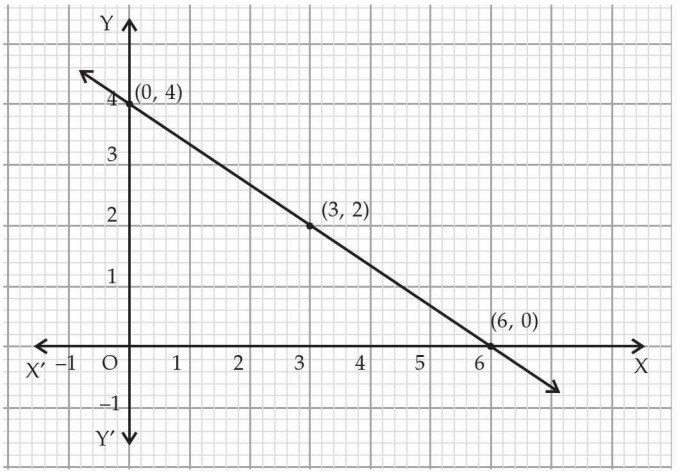
Solution. The given equation is $2x + 3y = 12$.

It can be written as $3y = 12 - 2x$.

or $y = \frac{12 - 2x}{3}$

Table of values

x	0	3	6
y	4	2	0



Draw coordinate axes on a graph paper.

Take 1 cm = 1 unit on both axes. Plot the points (0, 4), (3, 2) and (6, 0). Draw a line passing through any two points. The graph of the given equation is shown in the adjoining figure.

From graph, we find that the line cuts the x-axis at the point (6, 0) and it cuts the y-axis at the point (0, 4).

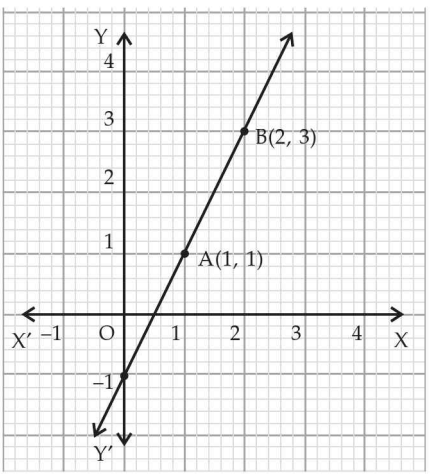
Example 3. *The following values of x and y are thought to satisfy a linear equation:*

x	1	2
y	1	3

Draw the graph, using the values of x and y as given in the above table. At what point the graph of the linear equation

(*i*) *cuts the x-axis* (*ii*) *cuts the y-axis?* *(NCERT Exemplar)*

Solution. From the given table, we get two points A (1, 1) and B (2, 3) which lie on the graph of linear equation. As we are required to draw the graph of a linear equation, it must be a straight line. So, plot the points A (1, 1) and B (2, 3). Join these points by a line. The graph is shown in the adjoining figure.



From graph, we see that the line cuts the x-axis at the point $(\frac{1}{2}, 0)$ and it cuts the y-axis at the point $(0, -1)$.

Example 4. *If the point (3, 4) lies on the graph of the equation $3y = ax + 7$, find the value of a.* *(NCERT)*

Solution. Since every point that lies on the graph of a linear equation is a solution of the equation, so the point (3, 4) is a solution of the given equation $3y = ax + 7$.

$\therefore \quad 3 \times 4 = a \times 3 + 7$

$\Rightarrow \quad 12 = 3a + 7$

$\Rightarrow \quad 3a = 5 \Rightarrow a = \frac{5}{3}$.

Hence, the value of a is $\frac{5}{3}$.

Example 5. *Determine the point on the graph of the equation $2x + 5y = 20$ whose x-coordinate is $\frac{5}{2}$ times its ordinate.* *(NCERT Exemplar)*

Solution. As the x-coordinate of the point is $\frac{5}{2}$ times its ordinate, therefore, $x = \frac{5}{2}y$. Thus, the point $(\frac{5}{2}y, y)$.

The point $(\frac{5}{2}y, y)$ lies on the graph of the equation $2x + 5y = 20$,

$\therefore 2 \times \frac{5}{2}y + 5y = 20 \Rightarrow 10y = 20 \Rightarrow y = 2$,

$\therefore x = \frac{5}{2}y = \frac{5}{2} \times 2 = 5$

Hence, the required point is (5, 2).

Example 6. *Find the points where the graph of the equation $3x + 4y = 12$ cuts the x-axis and the y-axis.* *(NCERT Exemplar)*

Solution. The graph of the linear equation $3x + 4y = 12$ cuts the x-axis at the point where its ordinate is zero *i.e.* $y = 0$. On putting $y = 0$ in the given linear equation, we get

$3x + 4 \times 0 = 12 \Rightarrow 3x = 12 \Rightarrow x = 4$.

$\therefore$ The graph of the given equation cuts the x-axis at the point (4, 0). Similarly, the graph cuts the y-axis at the point where its abscissa is zero *i.e.* $x = 0$. On putting $x = 0$ in the given equation, we get $3 \times 0 + 4y = 12 \Rightarrow 4y = 12 \Rightarrow y = 3$.

$\therefore$ The required point is (0, 3).

Example 7. *Give the equations of two lines passing through the point (2, 14). How many more such lines are there, and why?* *(NCERT)*

Solution. Two lines are:

$x + y = 16$ $(\because x + y = 2 + 14 = 16)$

$2x + y = 18$ $(\because 2x + y = 2 \times 2 + 14 = 18)$

Infinitely many; because through a given point infinitely many lines can be drawn. In fact, all the lines $y - 14 = k(x - 2)$, where k is any real number, pass through the point (2, 14).

Example 8. *Alka and Noori, two students of class IX, together contribute ₹ 500 towards Prime Minister's Relief Fund to help the earthquake victims. Write a linear equation which satisfies this data and draw the graph of the same.*

What value is being promoted by Alka and Noori in helping the earthquake victims?

(Value Based)

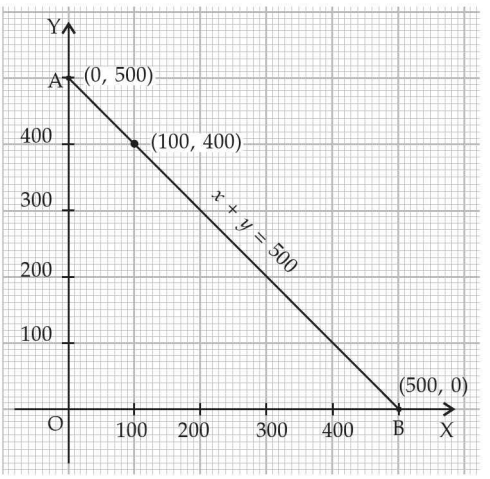
Solution. Let Alka and Noori contribute ₹ x and ₹ y respectively, then the linear equation satisfying the given data is $x + y = 500$.

It can be written as $y = 500 - x$.

Table of values

x	0	100	500
y	500	400	0

Let 1 cm represent ₹ 100 on both axes. Plot the points (0, 500), (100, 400) and (500, 0). Join any two points by a line. The graph of the linear equation is shown in the adjoining figure.



Value: Students are kind hearted and well wishers of helpless people.

Note. As the values of x or y cannot be negative, the graph of the given data is the line segment AB.

Example 9. *If the work done by a body on application of a constant force is directly proportional to the distance travelled by the body, express this in the form of an equation in two variables and draw the graph of the same by taking constant force as 3 units. Also read from the graph the work done when the distance travelled by the body is*

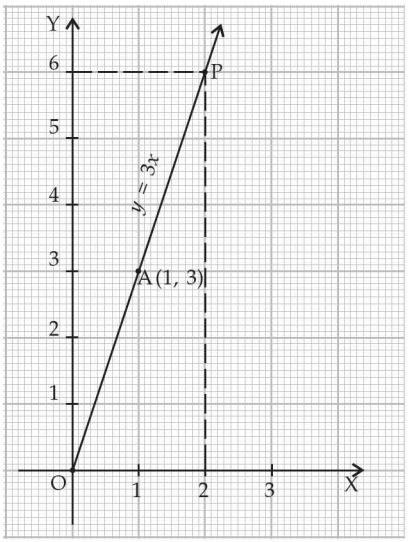
(i) 0 unit *(ii) 2 units* *(NCERT Exemplar)*

Solution. Let the distance travelled be x units and work done be y units. As the work done by the application of a constant force is directly proportional to the work done, we can express this fact as $y = kx$ where k is the constant force.

As the constant force is 3 units *i.e.* $k = 3$, the equation is $y = 3x$.

Table of values

x	0	1
y	0	3



Take 1 cm = 1 unit distance travelled on x-axis and 1 cm = 1 unit work done on y-axis.

Plot the points (0, 0) and (1, 3).

Draw a line passing through these points.

The graph of the equation $y = 3x$ is shown in the adjoining figure.

(*i*) When distance travelled is zero *i.e.* $x = 0$, from the graph we find that $y = 0$

$\Rightarrow$ work done = 0 units.

(*ii*) When distance travelled is 2 units *i.e.* $x = 2$, from the graph we find that $y = 6$

$\Rightarrow$ work done = 6 units.

Note. As the values of x or y cannot be negative, the graph is the ray OP.

Example 10. *The taxi fare in a city is as follows:*

For the first kilometre, the fare is ₹ 8 and for the subsequent distance, it is ₹ 5 per km. Taking the distance covered as x km and the total fare as ₹ y, write a linear equation for this information, and draw its graph. *(NCERT)*

Solution. Total distance travelled = x km.

Fare for the first kilometre = ₹ 8.

Fare for the next $(x - 1)$ km = ₹ $5(x - 1)$

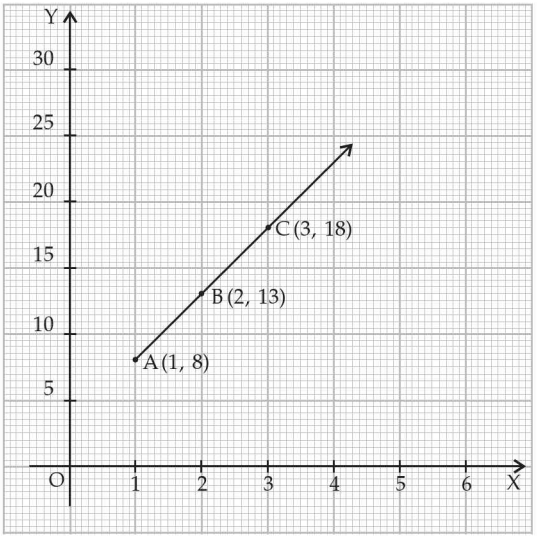
As the total fare is ₹ y, we have

$y = 8 + 5(x - 1)$ *i.e.* $y = 5x + 3$.

Note that this equation is meaningful only for $x \geq 1$.

Table of values

x	1	2	3
y	8	13	18



Scale: Take 1 cm on x-axis = 1 km and 1 cm on y-axis = ₹ 5.

Plot the points (1, 8), (2, 13) and (3, 18).

The graph is shown in the adjoining figure.

Note. As $x \geq 1$, the graph of $y = 5x + 3$ is the ray AB.

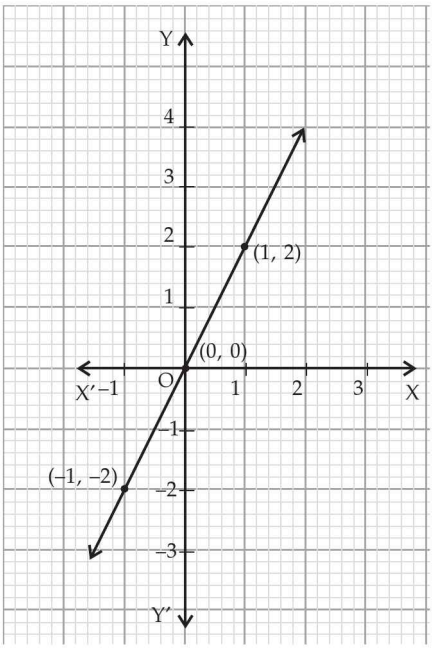
Example 11. *From the choices given below, choose the equation whose graph has been Given.*

(a) For figure (1)

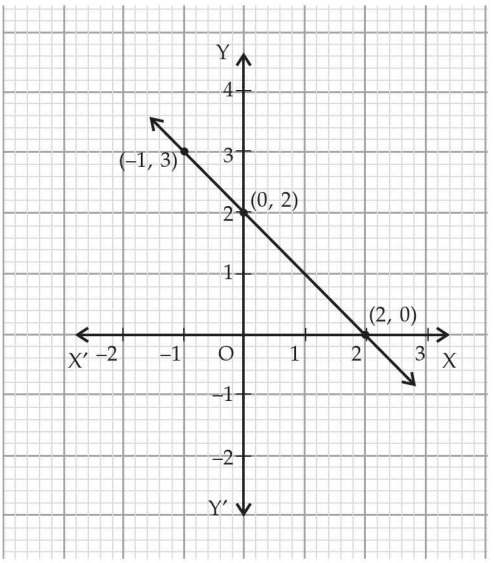
(i) $y = x$ *(ii)* $y = 2x$ *(iii)* $x + y = 0$ *(iv)* $y = 2x + 1$

(b) For figure (2)

(i) $y = x + 2$ *(ii)* $y = x - 2$ *(iii)* $x + 2y = 6$ *(iv)* $x + y = 2$



(1)



(2)

Solution. (*a*) In figure (1), the graph of the linear equation passes through the points (0, 0), (1, 2) and (–1, –2). (1, 2) does not satisfy the equation $y = x$.

All the three points (0, 0), (1, 2) and (–1, 2) satisfy the equation $y = 2x$. Therefore, for figure (1) the correct choice is $y = 2x$ *i.e.* (*ii*).

(*b*) In figure (2), the graph of the linear equation passes through the points (2, 0), (0, 2) and (–1, 3).

(2, 0) does not satisfy the equation $y = x + 2$.

(0, 2) does not satisfy the equation $y = x - 2$.

(–1, 3) does not satisfy the equation $x + y = 2$.

We note that all the three points (2, 0), (0, 2) and (–1, 3) satisfy the equation $x + y = 2$. Therefore, for figure (2) the correct choice is $x + y = 2$ *i.e.* (*iv*).

Remark. An equation of the form $y = mx$, where m is any real number, represents a line passing through origin.

Exercise 8.2

1. Draw the graphs of the following linear equations in two variables :

   (*i*) $x + y = 7$ *(NCERT)* (*ii*) $x - y = 2$ *(NCERT)* (*iii*) $2(x - 1) + 3y = 4$

2. Draw the graph of the linear equation $3x + 4y = 6$. At what points, the graph cuts the x-axis and the y-axis? *(NCERT Exemplar)*

3. The following values of x and y are thought to satisfy a linear equation:

x	6	-6
y	-2	6

Draw the graph, using the values of x and y given in the adjoining table. At what points the graph of the linear equation.

(i) cuts the x-axis (ii) cuts the y-axis? *(NCERT Exemplar)*

4. Show that the points A(1, 2), B (–1, –16) and C (0, – 7) lie on the graph of the linear equation $y = 9x - 7$. *(NCERT Exemplar)*
5. If the point (–3, 2) lies on the graph of the equation $5x + 2ky = 3$, find the value of k.
6. Determine the point on the graph of the linear equation $2x + 5y = 19$, whose ordinate is $1\frac{1}{2}$ times its abscissa. *(NCERT Exemplar)*
7. Given the point (1, 2), find the equation of a line on which it lies. How many such lines are there? *(NCERT)*
8. Find the solution of the linear equation $x + 2y = 8$ which represents a point on

(i) x-axis (ii) y-axis. *(NCERT Exemplar)*

9. Draw the graph of the linear equation whose solutions are represented by the points having the sum of coordinates as 10 units. *(NCERT Exemplar)*

Hint: $x + y = 10$.

10. Draw the graphs of the linear equations $y = x$ and $y = -x$ on the same coordinate plane. What do you observe? *(NCERT Exemplar)*
11. The autorickshaw fare in a city is charged ₹ 10 for first kilometre and at the rate of ₹ 4 per kilometre for subsequent distance covered. Write the linear equation to express the above statement. Draw the graph of the linear equation. *(NCERT Exemplar)*
12. The force exerted to pull a cart is directly proportional to the acceleration produced. Express the statement as a linear equation in two variables and draw the graph of the same by taking constant mass equal to 6 kg. Read from the graph, the force required when the acceleration produced is

(i) 5 m/sec^2 (ii) 6 m/sec^2 *(NCERT Exemplar)*

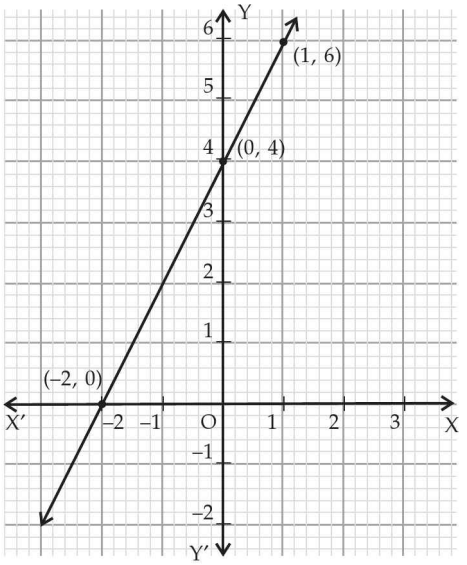
13. From the choices given below, choose the equation whose graph has been Given.

(a) For figure (1)

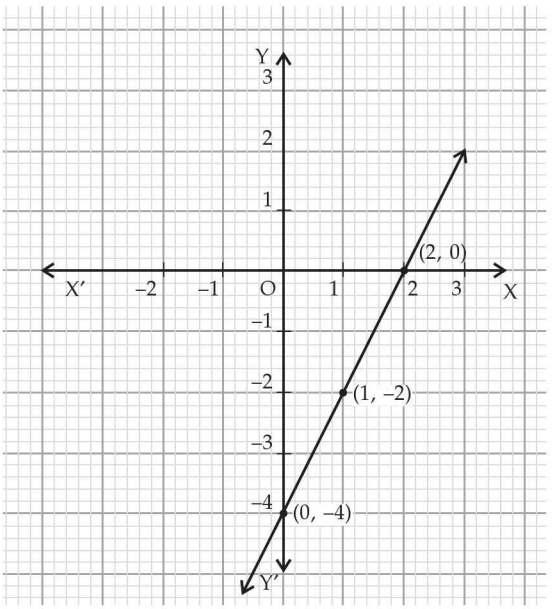
(i) $x + y = 0$ (ii) $y = 2x$ (iii) $y = 2x + 4$ (iv) $y = x + 4$

(b) For figure (2)

(i) $x + y = 0$ (ii) $y = 2x$ (iii) $y = 2x + 1$ (iv) $y = 2x - 4$



(1)



(2)

8.3 EQUATIONS OF LINES PARALLEL TO COORDINATE AXES

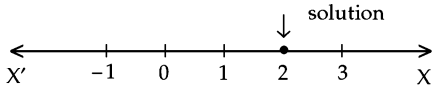
You have already learnt how to write the coordinates of a given point in the cartesian plane. You know that the points (2, 0), (4, 0), (–3, 0) and (x, 0) for any real number x lie on the x-axis. But do you know why? Because on the x-axis, y-coordinate of every point is zero. In fact, every point on the x-axis is of the form (x, 0). So, the equation of the x-axis is $y = 0$.

Similarly, every point on the y-axis is of the form (0, y) and the equation of the y-axis is $x = 0$.

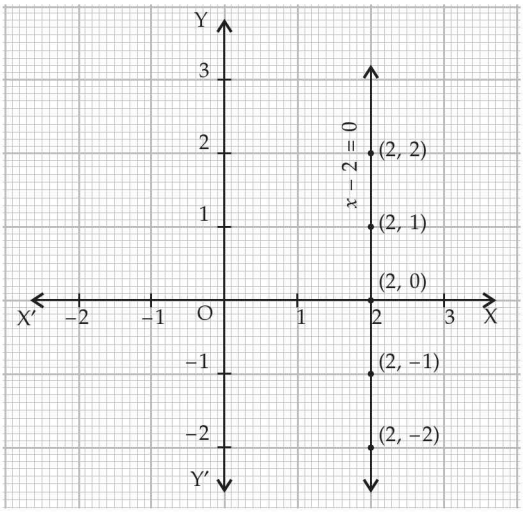
❑ **Equation of a line parallel to y-axis**

Consider the equation $x - 2 = 0$.

(i) When the equation $x - 2 = 0$ is considered as an equation in one variable then it has a unique solution $x = 2$, which can be represented by a point on the number line (shown in the adjoining figure).



(ii) The equation $x - 2 = 0$ can be written as $x + 0y - 2 = 0$, so it can be considered as a linear equation in two variables. It has infinitely many solutions (2, k), where k is any real number. Conversely, we can check that every point of the form (2, k) is a solution of this equation. Therefore, $x - 2 = 0$, when considered as an equation in two variables, represents a straight line parallel to y-axis (shown in the adjoining figure).



Thus, the graph of $x = a$ is a straight line parallel to x-axis. It follows that :

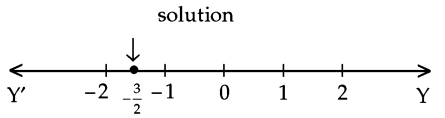
The equation of a line parallel to y-axis is x = a.

The line $x = a$ lies to the right of y-axis if a is positive and it lies towards the left of y-axis if a is negative.

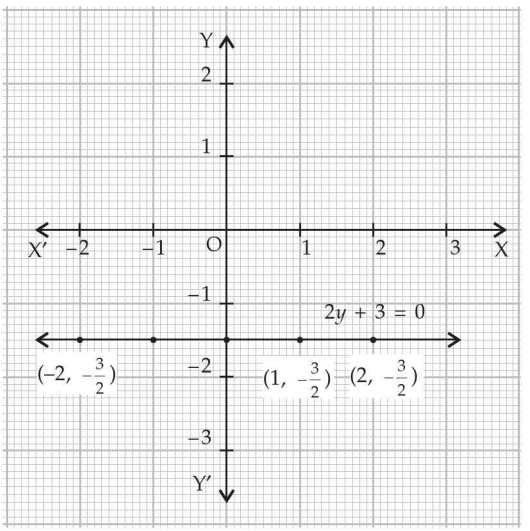
❑ **Equation of a line parallel to x-axis**

Consider the equation $2y + 3 = 0$

(i) When the equation $2y + 3 = 0$ is considered as an equation in one variable, then it has a unique solution $y = -\frac{3}{2}$, which can be represented by a point on the number line (shown in the adjoining figure).



(ii) The equation $2y + 3 = 0$ can be written as $0x + 2y + 3 = 0$, so it can be considered as a linear equation in two variables. It has infinitely many solutions (k, $-\frac{3}{2}$), where k is any real number. Conversely, we can check



that every point of the form $(k, -\frac{3}{2})$ is a solution of this equation. Therefore, the equation $2y + 3 = 0$, when considered as an equation in two variables, represents a straight line parallel to x-axis (shown in the adjoining figure).

It follows that the graph of $y = a$ is a straight line parallel to x-axis. Thus:

The equation of a line parallel to x-axis is y = a.

The line $y = a$ lies above the x-axis if a is positive and it lies below the x-axis if a is negative.

Remarks

- The equation of the line parallel to y-axis at a distance of a units from it towards the right of y-axis is $x = a$ and towards the left of y-axis is $x = -a$.
- The equation of the line parallel to x-axis at a distance of a units from it above the x-axis is $y = a$ and below the x-axis is $y = -a$.

Illustrative Examples

Example 1. *Draw the graph of the equation represented by the line which is parallel to x-axis and is 4 units above it.* *(NCERT Exemplar)*

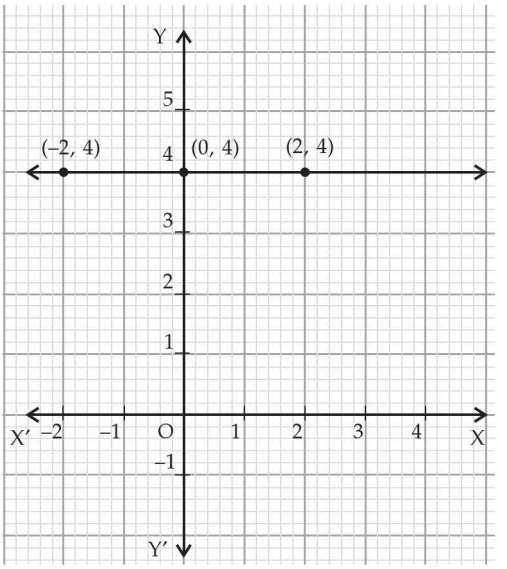
Solution. The equation of the line parallel to x-axis and at of 4 units from it above the x-axis is $y = 4$.

It can be written as $0.x + 1.y - 4 = 0$, which is a linear equation in two variables. It has infinitely many solutions.

Table of values

x	0	2	–2
y	4	4	4

Draw coordinate axes on a graph paper. Take 1 cm = 1 unit distance on both axes. Plot the points (0, 4), (2, 4) and (–2, 4). Draw a line passing through any two points. The graph of the equation is a line parallel to x-axis as shown in the adjoining figure.



Example 2. *Solve the equation $2x + 1 = x - 3$, and represent the solution(s) on:*

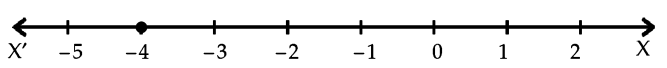
(i) the number line *(ii) the cartesian plane* *(NCERT)*

Solution. The given equation is $2x + 1 = x - 3$

$\Rightarrow$ $2x - x = -3 - 1 \Rightarrow x = -4$.

Hence, the solution is $x = -4$.

(*i*) The solution is represented by a point on the number line shown in the figure given below.

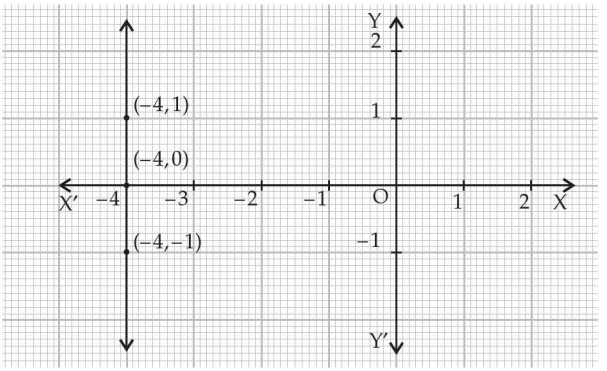


(*ii*) The equation $x = -4$ can be written as $1.x + 0.y + 4 = 0$, which is a linear equation in two variables. It has infinitely many solutions.

Table of values

x	-4	-4	-4
y	0	1	-1

Plot the points $(-4, 0)$, $(-4, 1)$ and $(-4, -1)$. Draw a line passing through any two points. The graph of the equation is a line parallel to y-axis as shown in the adjoining figure.



Exercise 8.3

1. Draw the graph of the equation represented by a line which is parallel to x-axis and at a distance of 3 units below it. *(NCERT Exemplar)*

   Hint: The equation of the line is $y = -3$.
2. Give the geometrical representation of $y - 3 = 0$ as an equation:
   (*i*) in one variable (*ii*) in two variables. *(NCERT)*
3. Give the geometrical representation of $2x + 9 = 0$ as an equation:
   (*a*) in one variable (*b*) in two variables. *(NCERT)*

Answers

Very Short Answer Type Questions

1. If $x = -3$ and $y = 5$ is a solution of the equation $2x - 3y + k = 0$, then find the value of k.
2. If $(3, -2)$ is a solution of the linear equation $3x - py - 7 = 0$, then find the value of p.
3. Find the value of y for which $x = -2$ is a solution of the linear equation $5x - 3y = 5$.
4. Find the value of x for which $y = -4$ is a solution of the linear equation $5x - 8y = 47$.
5. Write two solutions of the equation $3x + 4y + 12 = 0$.
6. Find the solution of the linear equation $5x - 3y + 10 = 0$ which represents a point on the x-axis.
7. If the point $(-2, 3)$ lies on the graph of $3x = ky + 2$, then find the value of k.
8. Find the equations of the lines parallel to x-axis at a distance of $\frac{5}{2}$ units from the y-axis.

Answers

Objective Type Questions

(FOR FORMATIVE/SUMMATIVE ASSESSMENT)

Mental Maths

1. Fill in the blanks :
   (*i*) The equation of the x-axis is ..... .
   (*ii*) The equation of the y-axis is ..... .
   (*iii*) The equation of a line parallel to x-axis is ..... $= a$, where a is any non-zero real number.

(*iv*) The equation of a line parallel to y-axis is ..... = a, where a is any non-zero real number.

(*v*) The equation of a line passing through origin is of the form ..... .

(*vi*) The graph of every linear equation in two variables is a ..... .

(*vii*) An equation of the form $ax + b = 0$, where a, b are real numbers and $a \neq 0$, in the variable x geometrically represents ..... .

2. State whether the following statements are true or false. Justify your answer.

(*i*) The polynomial equation $ax + by + c = 0$, where a, b, c are real numbers, of degree one is called a linear equation because it represents a straight line.

(*ii*) The linear equation $2x + 3y = 5$ has a unique Solution. *(NCERT Exemplar)*

(*iii*) The point (0, 3) lies on the graph of linear equation $3x + 4y = 12$. *(NCERT Exemplar)*

(*iv*) The graph of the linear equation $x + 2y = 7$ passes through the point (0, 7). *(NCERT Exemplar)*

(*v*) All the points (2, 0), (–3, 0), (4, 2) and (0, 5) lie on the x-axis. *(NCERT Exemplar)*

(*vi*) The graph of the equation $y = mx + c$ passes through origin. *(NCERT Exemplar)*

(*vii*) The coordinates of points in the table

x	0	1	2	3	4
y	2	3	4	–5	6

represent some of solutions of the equation $x - y + 2 = 0$. *(NCERT Exemplar)*

(*viii*) Every point on the graph of a linear equation in two variables does not represent a solution of the linear equation. *(NCERT Exemplar)*

(*ix*) The graph of every linear equation in two variables need not be a line. *(NCERT Exemplar)*

(*x*) The line parallel to y-axis at a distance of 4 units to the left of y-axis is given by the equation $x = -4$. *(NCERT Exemplar)*

(*xi*) The linear equation $ax + b = 0$, where a, b are real numbers and $a \neq 0$, when considered as an equation in one variable has a unique solution but when considered as an equation in two variables then it has infinitely many solutions.

Multiple Choice Questions

Choose the correct answer from the given four options (3 to 21) :

3. The coefficient of the variable y in the linear equation $5(2x - y) + 3x + 4y - 7 = 0$ is

(*a*) 13 (*b*) –9 (*c*) –1 (*d*) 9

4. The linear equation $2x - 5y = 7$ has

(*a*) a unique solution (*b*) two solutions

(*c*) infinitely many solutions (*d*) no solution *(NCERT Exemplar)*

5. The equation $2x + 5y = 7$ has a unique solution if x, y are:

(*a*) natural numbers (*b*) positive real numbers

(*c*) real numbers (*d*) rational numbers *(NCERT Exemplar)*

6. Any solution of the linear equation $2x + 0y + 9 = 0$ in two variables is of the form

(*a*) $\left(-\frac{9}{2}, m\right)$ (*b*) $\left(n, -\frac{9}{2}\right)$ (*c*) $\left(0, -\frac{9}{2}\right)$ (*d*) $(-9, 0)$

(NCERT Exemplar)

7. The graph of the linear equation $2x + 3y = 6$ cuts the y-axis at the point

(*a*) (2, 0) (*b*) (0, 3) (*c*) (3, 0) (*d*) (0, 2)

(NCERT Exemplar)

8. $x = 5$, $y = 2$ is a solution of the linear equation

(*a*) $x + 2y = 7$ (*b*) $5x + 2y = 7$ (*c*) $x + y = 7$ (*d*) $5x + y = 7$

(NCERT Exemplar)

9. A solution of the equation $2x + 5y - 3 = 0$ is

(*a*) (5, –2) (*b*) (–5, 2) (*c*) (–1, 1) (*d*) (1, –1)

10. Any point on the line $y = x$ is of the form

(*a*) (a, a) (*b*) $(0, a)$ (*c*) $(a, 0)$ (*d*) $(a, -a)$

(NCERT Exemplar)

11. The point of the form (a, a) always lies on

(*a*) x-axis (*b*) y-axis (*c*) the line $y = x$ (*d*) the line $x + y = 0$

(NCERT Exemplar)

12. The graph of the linear equation $y = x$ passes through the point

(*a*) $\left(\frac{3}{2}, -\frac{3}{2}\right)$ (*b*) $\left(0, \frac{3}{2}\right)$ (*c*) (1, 1) (*d*) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

(NCERT Exemplar)

13. If a linear equation has solutions (–2, 2), (0, 0) and (2, –2), then its equation is

(*a*) $y - x = 0$ (*b*) $x + y = 0$ (*c*) $-2x + y = 0$ (*d*) $-x + 2y = 0$

(NCERT Exemplar)

14. The number of linear equations in x and y which can be satisfied by $x = 1$ and $y = 2$ is (are)

(*a*) only one (*b*) two (*c*) three (*d*) infinitely many

(NCERT Exemplar)

15. The point $(a, -a)$ does not lie on the graph of

(*a*) $x = a$ (*b*) $y = -a$ (*c*) $y = x$ (*d*) $x + y = 0$

16. Which of the following equations represents a line parallel to x-axis?

(*a*) $3x + 2 = 0$ (*b*) $3y + 2 = 0$ (*c*) $3x + 2y = 0$ (*d*) $3x - 2y = 0$

17. Which of the following equations represents a line parallel to y-axis?

(*a*) $2y = 5x$ (*b*) $2y = 5$ (*c*) $2x = 5$ (*d*) $2x + 3y = 5$

18. Which of the following equations represents a line passing through the origin?

(*a*) $2x + 3 = 0$ (*b*) $3y + 2 = 0$

(*c*) $2x + 3y + 5 = 0$ (*d*) $2x + 3y = 0$

19. The equation $y + 3 = 0$ represents a line

(*a*) parallel to y-axis (*b*) parallel to x-axis

(*c*) passing through origin (*d*) passing through the point (–3, 0)

20. The equation $x = 2y$ represents a line

(*a*) parallel to x-axis (*b*) parallel to y-axis

(*c*) passing through origin (*d*) passing through the point (1, 2)

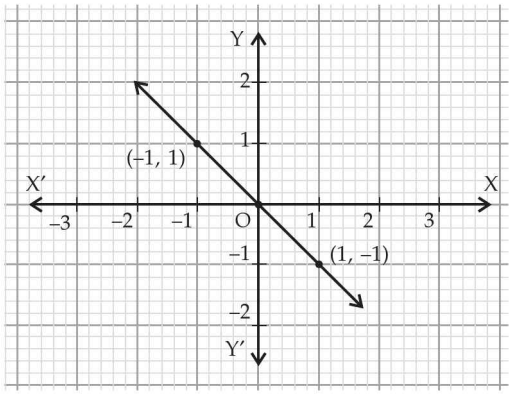
21. Which of the following equations has a graph shown in the adjoining figure?

(*a*) $y = x$

(*b*) $x + y = 0$

(*c*) $y = 2x$

(*d*) $2 + 3y = 7x$



Value Based Question

The cost of a notebook is thrice the cost of a pen. Write a linear equation in two variables to represent this statement.

What is the role of pens and notebooks in promoting education?

Answers

Chapter Test

1. Express the following linear equations in the form $ax + by + c = 0$ and write the values of a, b and c in each case:

   (*a*) $3x = \sqrt{2}\,y - 5$ (*b*) $2x = -5y$ (*c*) $2y = \frac{5}{7}$.

2. Write four solutions of the linear equation : $\sqrt{2}\,x + y = 5$.
3. What can you say about the number of solutions of the equation $3x + 7 = 0$, when considered as an equation

   (*i*) in one variable? (*ii*) in two variables?

4. Draw the graph of $2x + 3y = 9$. Using the graph, check whether (3, 1) and (−1, 2) are solutions of the given equation.
5. Write four solutions of the equation $x + 2y = 6$. Also draw its graph.
6. Draw the graph of $4x - 3y + 12 = 0$. From the graph, find the value of

   (*i*) y when $x = 3$ (*ii*) x when $y = -4$.

7. Draw the graph of the linear equation $2x - y + 1 = 0$. From your graph, find the values of h and k if the graph passes through the points $(h, 4)$ and $(\frac{1}{2}, k)$.
8. The linear equation which converts temperature in degrees in Celsius (C) to degrees in Fahrenheit (F) is given by $F = \frac{9}{5}C + 32$.

   (*i*) Draw the graph of the given linear equation using Celsius for x-axis and Fahrenheit for y-axis.

   (*ii*) If the temperature is 30°C, what is the temperature in Fahrenheit?

   (*iii*) If the temperature is 95°F, what is the temperature in Celsius?

   (*iv*) If the temperature is 0°C, what is the temperature in Fahrenheit and if the temperature is 0°F, what is the temperature in Celsius?

   (*v*) Is there a temperature which is numerically the same in both Celsius and Fahrenheit? If yes, find it. *(NCERT)*

Hint: To draw graph : take 1 cm = 8°C on x-axis and 1 cm = 8°F on y-axis.

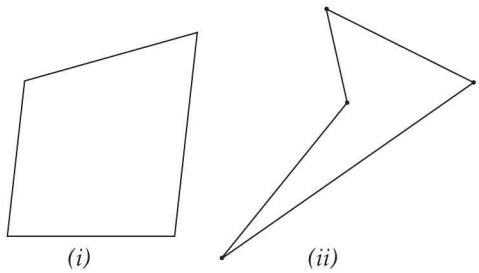
Answers

Quadrilaterals



INTRODUCTION

We know that when four points in a plane, no three points out of them are collinear, are joined in pairs in an order by line segments we obtain a closed figure with sides (shown in figures (*i*) and (*ii*)).



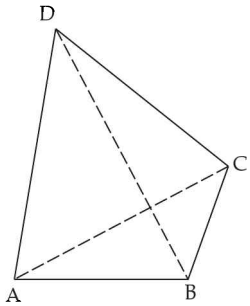
Such a figure is called a *quadrilateral*. Quadrilateral of type shown in figure (*i*) is called a *convex quadrilateral* (a quadrilateral in which each angle is less than 180°) and quadrilateral of type shown in figure (*ii*) is called a *concave* or *re-entrant quadrilateral* (a quadrilateral which has a reflex angle *i.e.* an angle of measure > 180°).

In this chapter, we shall consider convex quadrilaterals *i.e.* quadrilaterals of type shown in figure (*i*) only. We shall study about different types of quadrilaterals and their properties especially those of parallelograms. We shall also learn mid-point theorem and its converse.

9.1 QUADRILATERALS

A simple closed figure bounded by four line segments is called a ***quadrilateral.***

In the adjoining figure, ABCD is a quadrilateral.



It has

four sides — AB, BC, CD and DA

four (interior) angles — ∠A, ∠B, ∠C and ∠D

four vertices — A, B, C and D

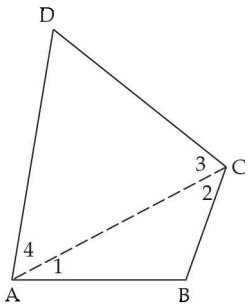
two diagonals — AC and BD.

In quadrilateral ABCD, sides AB, BC; BC, CD; CD, DA ; DA, AB are pairs of **adjacent sides.**

Sides AB, CD; BC, DA are pairs of **opposite sides.**

Angles ∠A, ∠B; ∠B, ∠C; ∠C, ∠D; ∠D, ∠A are pairs of **adjacent angles.**

Angles ∠A, ∠C; ∠B, ∠D are pairs of **opposite angles.**



❑ **Angle sum property of a quadrilateral**

Sum of (interior) angles of a quadrilateral is 360°.

In the adjoining figure, ABCD is any quadrilateral. Divide it into two triangles ΔABC and ΔACD by drawing the diagonal AC. Mark the angles as shown in adjoining figure.

We know that the sum of angles of a triangle is 180°,

in ΔABC, $\angle 1 + \angle B + \angle 2 = 180°$ ...(*i*)

in ΔACD, $\angle 4 + \angle D + \angle 3 = 180°$ ...(*ii*)

On adding (*i*) and (*ii*), we get

$\angle 1 + \angle 4 + \angle B + \angle D + \angle 2 + \angle 3 = 360°$

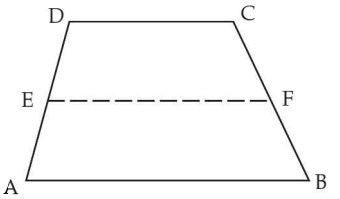
$\Rightarrow \quad \angle A + \angle B + \angle D + \angle C = 360°$ (from figure)

Hence, the sum of (interior) angles of a quadrilateral is 360°.

❑ **Types of quadrilaterals**

1. Trapezium

A quadrilateral in which one pair of opposite sides is parallel is called a ***trapezium*** *(abbreviated trap.).*

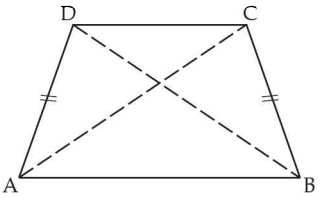


The parallel sides are called bases of the trapezium. The line segment joining mid-points of non-parallel sides is called its median.

In the adjoining quadrilateral, AB || DC, so ABCD is a trapezium, AB and CD are its bases, and EF is its median where E and F are mid-points of the sides AD and BC respectively.

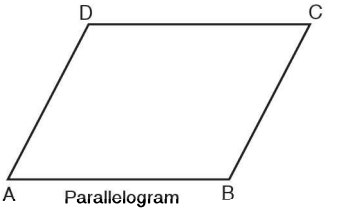
Isosceles trapezium

If two non-parallel sides of a trapezium are equal then it is called an ***isosceles trapezium.***



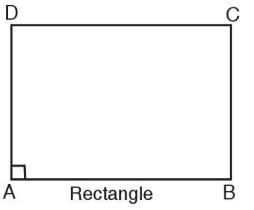
In the adjoining quadrilateral, AB || DC and AD = BC, so ABCD is an isosceles trapezium.

2. Parallelogram. *A quadrilateral in which both pairs of opposite sides are parallel is called a* ***parallelogram****. It is usually written as '||gm'.*



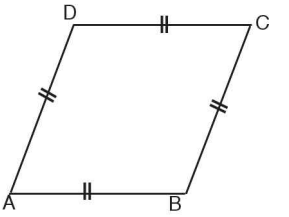
In the adjoining quadrilateral, AB || DC and AD || BC, so ABCD is a parallelogram.

3. Rectangle. *If one of the angles of a parallelogram is a right angle, then it is called a* ***rectangle****.*



In the adjoining parallelogram, $\angle A = 90°$, so ABCD is a rectangle. Of course, the remaining angles will also be right angles.

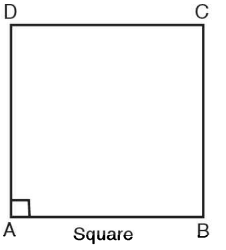
4. Rhombus. *If all the sides of a quadrilateral are equal, then it is called a* ***rhombus****.*



In the adjoining quadrilateral,

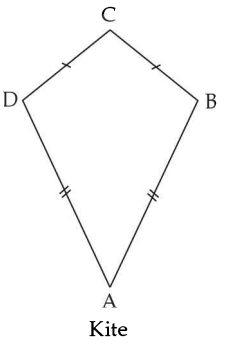
AB = BC = CD = DA, so ABCD is a rhombus.

5. Square. *If two adjacent sides of a rectangle are equal, then it is called a* ***square****.* Alternatively, if one angle of a rhombus is a right angle, it is called a ***square****.*



In the adjoining rectangle, AB = AD, so ABCD is a square.

Of course, the remaining sides are also equal.



Kite

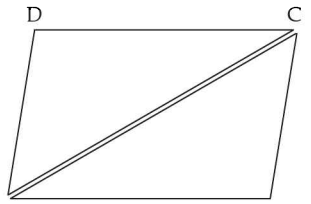
6. **Kite.** *A quadrilateral in which two pairs of adjacent sides are equal is called a* ***kite (or diamond).***

In the adjoining quadrilateral, AD = AB and DC = BC, so ABCD is a kite.

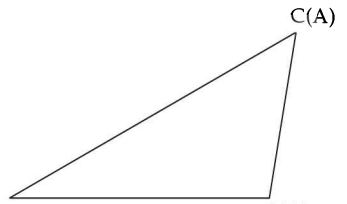
9.2 PROPERTIES OF PARALLELOGRAMS

Let us perform the following activity:

Draw a parallelogram ABCD on a sheet of paper. Cut out the parallelogram ABCD (region) and then cut it along the diagonal AC as shown in figure (*i*). Now take ΔCDA (region) and turn it around (through 180°) and place it over ΔABC (region) as shown in figure (*ii*).



(*i*)



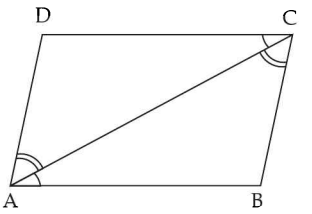
(*ii*)

We observe that ΔCDA covers ΔABC exactly. C falls on A, D falls on B and A falls on C. Thus, the two triangles ΔCDA and ΔABC are congruent *i.e.* ΔCDA ≅ ΔABC.

Repeat this activity with more parallelograms. In fact, the result can be proved. We have:

Theorem 9.1

A diagonal of a parallelogram divides it into two congruent triangles.



Given. A parallelogram ABCD and diagonal AC divides it into two triangles ΔABC and ΔCDA.

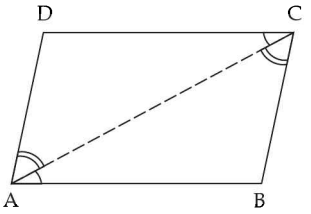
To prove. ΔABC ≅ ΔCDA

Proof.

Statements	*Reasons*
In ΔABC and ΔCDA	
1. ∠BAC = ∠ACD	1. Alt. ∠s, AB ‖ DC and AC is transversal.
2. ∠BCA = ∠CAD	2. Alt. ∠s, BC ‖ AD and AC is transversal.
3. AC = CA	3. Common
4. ΔABC ≅ ΔCDA	4. ASA rule of congruency.

Theorem 9.2

In a parallelogram, opposite sides are equal.



Given. A parallelogram ABCD.

To prove. AB = DC and BC = AD.

Construction. Join AC.

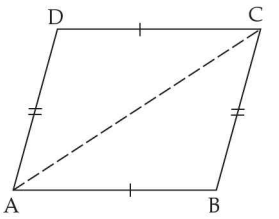
Proof.

Statements	*Reasons*
In ΔABC and ΔCDA	
1. $\angle BAC = \angle ACD$	1. Alt. $\angle$s, AB $\parallel$ DC and AC is transversal.
2. $\angle BCA = \angle CAD$	2. Alt. $\angle$s, BC $\parallel$ AD and AC is transversal.
3. AC = CA	3. Common
4. $\Delta ABC \cong \Delta CDA$	4. ASA rule of congruency.
5. AB = DC and BC = AD	5. c.p.c.t.

The converse of the above theorem is also true. In fact we have:

Theorem 9.3

If each pair of opposite sides of a quadrilateral is equal, then it is a parallelogram.



Given. A quadrilateral ABCD in which AB = DC and BC = AD.

To prove. ABCD is a parallelogram.

Construction. Join AC.

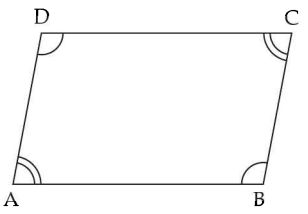
Proof.

Statements	*Reasons*
In ΔABC and ΔCDA	
1. AB = DC	1. Given.
2. BC = AD	2. Given.
3. AC = AC	3. Common.
4. $\Delta ABC \cong \Delta CDA$	4. SSS (Axiom of congruency)
5. $\angle BAC = \angle ACD$ $\Rightarrow$ AB $\parallel$ DC	5. c.p.c.t. alt. $\angle$s are equal formed by lines AB, BC and transversal AC.
6. $\angle ACB = \angle CAD$ $\Rightarrow$ BC $\parallel$ AD	6. c.p.c.t. alt. $\angle$s are equal formed by lines AD, BC and transversal AC.

Hence, ABCD is a parallelogram.

Theorem 9.4

In a parallelogram, opposite angles are equal.



Given. A parallelogram ABCD.

To prove. $\angle A = \angle C$ and $\angle B = \angle D$.

Proof.

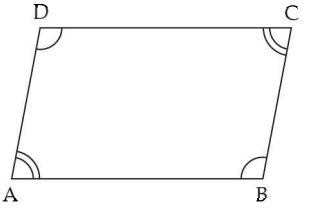
Statements	*Reasons*
1. $\angle A + \angle B = 180°$	1. AD $\parallel$ BC and AB is a transversal, sum of co-interior angles = 180°
2. $\angle B + \angle C = 180°$	2. AB $\parallel$ DC and BC is a transversal, sum of co-interior angles = 180°

3. $\angle A + \angle B = \angle B + \angle C$ $\Rightarrow \angle A = \angle C$ Similarly, $\angle B = \angle D$.	3. From 1 and 2

The converse of the above theorem is also true. In fact, we have:

Theorem 9.5

If each pair of opposite angles of a quadrilateral is equal, then it is a parallelogram.



Given. A quadrilateral ABCD in which $\angle A = \angle C$ and $\angle B = \angle D$.

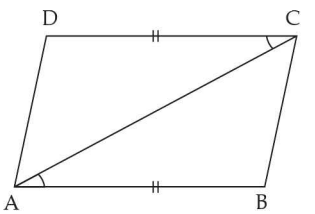
To prove. ABCD is a parallelogram.

Proof.

Statements	*Reasons*
1. $\angle A = \angle C$	1. Given
2. $\angle B = \angle D$	2. Given
3. $\angle A + \angle B = \angle C + \angle D$	3. Adding 1 and 2
4. $\angle A + \angle B + \angle C + \angle D = 360°$	4. Sum of angles of a quadrilateral.
5. $2(\angle A + \angle B) = 360°$ $\Rightarrow \angle A + \angle B = 180°$ $\Rightarrow$ BC $\parallel$ AD	5. Using 3 Sum of co-interior angles = 180°, formed by lines BC, AD and transversal AB.
Similarly, AB $\parallel$ DC Hence, ABCD is a parallelogram.	

Theorem 9.6

If a pair of opposite sides of a quadrilateral is equal and parallel, then it is a parallelogram.



Given. A quadrilateral ABCD in which AB $\parallel$ DC and AB = DC.

To prove. ABCD is a parallelogram.

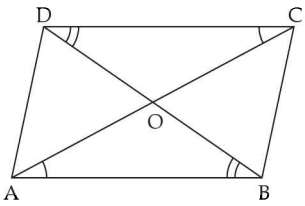
Construction. Join AC.

Proof.

Statements	*Reasons*
In ΔABC and ΔCDA	
1. $\angle BAC = \angle ACD$	1. Alt. $\angle$s, AB $\parallel$ DC and AC is a transversal.
2. AB = DC	2. Given
3. AC = CA	3. Common
4. $\Delta ABC \cong \Delta CDA$	4. SAS axiom of congruency.
5. $\angle ACB = \angle CAD$ $\Rightarrow$ AD $\parallel$ BC Hence, ABCD is a parallelogram.	5. c.p.c.t. Alt. $\angle$s are equal formed by lines AD, BC and transversal AC.

Theorem 9.7

The diagonals of a parallelogram bisect each other.



Given. A parallelogram ABCD whose diagonals AC and BD intersect at O.

To prove. OA = OC and OB = OD.

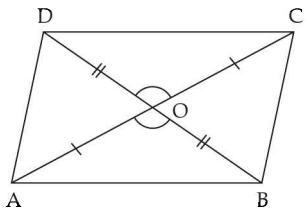
Proof.

Statements	*Reasons*
In ΔOAB and ΔOCD	
1. ∠BAC = ∠ACD	1. Alt. ∠s, AB ‖ DC and AC is a transversal.
2. ∠ABD = ∠CDB	2. Alt. ∠s, AB ‖ DC and BD is a transversal.
3. AB = CD	3. Opp. sides of a parallelogram are equal.
4. ΔOAB ≅ ΔOCD ∴ OA = OC and OB = OD	4. ASA rule of congruency c.p.c.t.

The converse of the above theorem is also true. In fact, we have:

Theorem 9.8

If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.



Given. A quadrilateral ABCD whose diagonals AC and BD intersect at O such that OA = OC and OB = OD.

To prove. ABCD is a parallelogram.

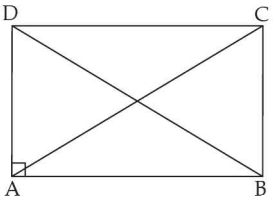
Proof.

Statements	*Reasons*
In ΔOAB and ΔOCD	
1. OA = OC	1. Given
2. OB = OD	2. Given
3. ∠AOB = ∠COD	3. Vert. opp. ∠*s*
4. ΔOAB ≅ ΔOCB	4. SAS axiom of congruency
5. ∠OAB = ∠OCD ∠CAB = ∠ACD ⇒ AB ‖ DC	5. c.p.c.t. Alt. ∠s are equal formed by lines AB, DC and transversal AC.
6. AB = CD	6. c.p.c.t.
7. ABCD is a parallelogram.	7. In quadrilateral ABCD, AB ‖ DC and AB = DC (Theorem 9.6)

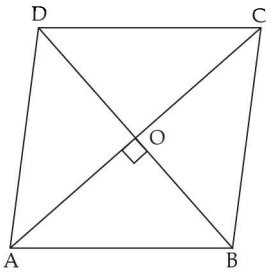
❑ **Properties of a rectangle**

Since every rectangle is a parallelogram, therefore, it has all the properties of a parallelogram. Additional properties of a rectangle are:

- *All the (interior) angles of a rectangle are right angles.*
  In the adjoining figure, ∠A = ∠B = ∠C = ∠D = 90°.
- *The diagonals of a rectangle are equal.*
  In the adjoining figure, AC = BD.



❑ **Properties of a rhombus**

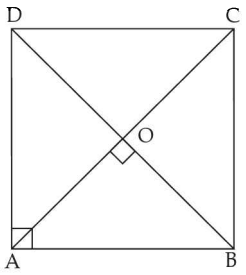


Since every rhombus is a parallelogram, therefore, it has all the properties of a parallelogram. Additional properties of a rhombus are :

- *All the sides of a rhombus are equal.*
  In the adjoining figure, AB = BC = CD = DA.
- *The diagonals of a rhombus intersect at right angles.*
  In the adjoining figure, AC ⊥ BD.
- *The diagonals bisect the angles of a rhombus.*
  In the adjoining figure, diagonal AC bisects ∠A as well as ∠C and diagonal BD bisects ∠B as well as ∠D.

❑ **Properties of a square**

Since every square is a parallelogram, therefore, it has all the properties of a parallelogram. Additional properties of a square are :



- *All the interior angles of a square are right angles.*
  In the adjoining figure, ∠A = ∠B = ∠C = ∠D = 90°.
- *All the sides of a square are equal.*
  In the adjoining figure, AB = BC = CD = DA.
- *The diagonals of a square are equal.*
  In the adjoining figure, AC = BD.
- *The diagonals of a square intersect at right angles.*
  In the adjoining figure, AC ⊥ BD.
- *The diagonals bisect the angles of a square.*
  In the adjoining figure, diagonal AC bisects ∠A as well as ∠C and diagonal BD bisects ∠B as well as ∠D.

In fact, a square is a rectangle as well as a rhombus, so it has all the properties of a rectangle as well as that of a rhombus.

Illustrative Examples

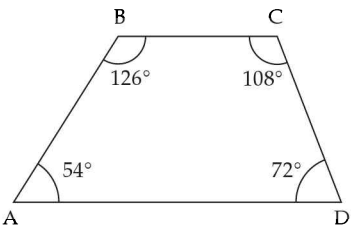
Example 1. *If angles A, B, C and D of a quadrilateral ABCD, taken in order, are in the ratio 3 : 7 : 6 : 4 then ABCD is a trapezium. Is this statement true? Give reason for your answer.* *(NCERT Exemplar)*

Solution. As the angles are in the ratio 3 : 7 : 6 : 4,

let these angles be $3x$, $7x$, $6x$ and $4x$.

Since sum of angles of a quadrilateral is 360°, $3x + 7x + 6x + 4x = 360°$

$\Rightarrow 20x = 360° \Rightarrow x = 18°$.



$\therefore$ The angles are: $\angle A = 3 \times 18° = 54°$, $\angle B = 7 \times 18° = 126°$, $\angle C = 6 \times 18° = 108°$ and

$\angle D = 4 \times 18° = 72°$.

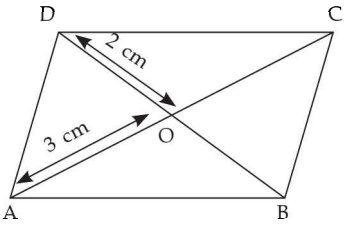
We note that $\angle A + \angle B = 54° + 126° = 180°$.

Thus, the sum of co-interior angles is 180° formed by lines AD, BC and transversal AB, therefore, AD ∥ BC. So, ABCD is a trapezium. Hence, the given statement is true.

Example 2. *Three angles of a quadrilateral ABCD are equal. Is it a parallelogram? Why or why not?* *(NCERT Exemplar)*

Solution. It need not be a parallelogram; because we may have $\angle A = \angle B = \angle C = 80°$, then $\angle D = 360° - 3 \times 80° = 120°$, so $\angle B \neq \angle D$ (opposite angles are not equal).

Example 3. *Diagonals AC and BD of a parallelogram ABCD intersect each other at O. If OA = 3 cm and OD = 2 cm, determine the lengths of AC and BD.*



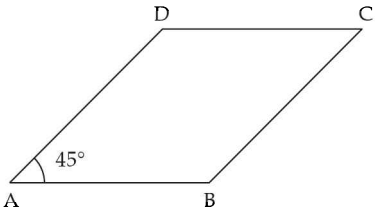
Solution. As the diagonals of a parallelogram bisect each other,

AC = 2 OA = (2 × 3) cm = 6 cm

and BD = 2 OD = (2 × 2) cm = 4 cm.

Example 4. *Diagonals of a quadrilateral ABCD bisect each other. If ∠A = 45°, determine ∠B.* *(NCERT Exemplar)*

Solution. Since the diagonals AC and BD of quadrilateral ABCD bisect each other, ABCD is a parallelogram.



AD ∥ BC and AB is a transversal,

∠A + ∠B = 180° (sum of co-interior angle = 180°)

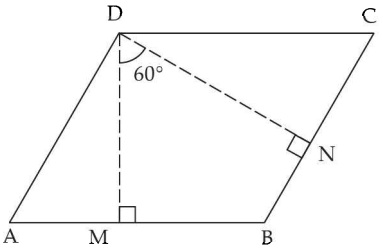
⇒ 45° + ∠B = 180°

⇒ ∠B = 135°.

Example 5. *The angle between two altitudes of a parallelogram through the vertex of an obtuse angle of the parallelogram is 60°. Find the angles of the parallelogram.* *(NCERT Exemplar)*

Solution. Let ABCD be parallelogram such that

DM ⊥ AB, DN ⊥ BC and ∠MDN = 60°.



In quadrilateral DMBN,

∠MDN + ∠M + ∠N + ∠B = 360°

⇒ 60° + 90° + 90° + ∠B = 360°

⇒ ∠B = 120°.

AD ∥ BC and AB is a transversal,

∠A + ∠B = 180°

⇒ ∠A + 120° = 180° ⇒ ∠A = 60°.

∠C = ∠A and ∠D = ∠B (opp. ∠s of a ∥ gm are equal.)

⇒ ∠C = 60° and ∠D = 120°.

Hence, the angles of ∥ gm ABCD are 60°, 120°, 60°, 120°.

Example 6. *In a rectangle ABCD, diagonals intersect at O. If ∠OAB = 30°, find*

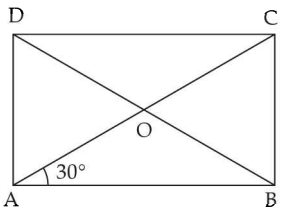
(i) ∠ACB (ii) ∠ABO (iii) ∠COD (iv) ∠BOC.

Solution. (*i*) ∠ABC = 90° (each angle of a rectangle = 90°)

∠ACB + 30° + 90° = 180° (sum of angles in ΔABC)

⇒ ∠ACB = 180° − 30° − 90° = 60°.

(*ii*) AC = BD (diagonals are equal)



⇒ 2AO = 2BO (diagonals bisect each other)

⇒ AO = BO

⇒ ∠ABO = ∠OAB

(angles opp. equal sides in ΔOAB)

⇒ ∠ABO = 30° (∵ ∠OAB = 30° given)

(*iii*) ∠AOB + 30° + 30° = 180° (sum of angles in ΔOAB)

⇒ ∠AOB = 180° − 30° − 30° = 120°.

But ∠COD = ∠AOB (vert. opp. ∠s)

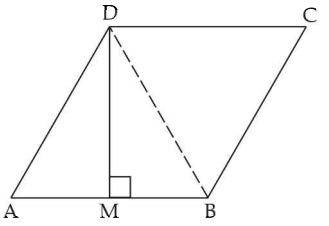
⇒ ∠COD = 120°

(*iv*) ∠BOC + 120° = 180° (linear pair)

⇒ ∠BOC = 180° − 120° = 60°.

Example 7. *ABCD is a rhombus in which altitude from D to side AB bisects AB. Find the angles of the rhombus.* *(NCERT Exemplar)*

Solution. ABCD is a rhombus in which DM ⊥ AB such that M is mid-point of AB. Join BD.



In ΔDAM and ΔDBM,

$\angle AMD = \angle BMD$ (each = 90°)

$AM = BM$ (M is mid-point of AB)

$DM = DM$ (common)

$\therefore \quad \Delta DAM \cong \Delta DBM$

(SAS axiom of congruency)

$\Rightarrow \quad AD = BD$

Also, $AD = AB$ ($\because$ ABCD is a rhombus)

$\Rightarrow \quad AD = BD = AB$

$\Rightarrow$ ΔABD is an equilateral triangle

$\Rightarrow \quad \angle A = 60°$

Now, AD ∥ BC and AB is a transversal,

$\therefore \quad \angle A + \angle B = 180°$ (sum of co-interior ∠s)

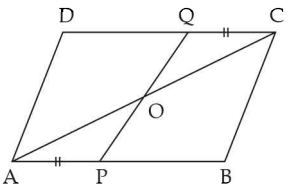
$\Rightarrow \quad 60° + \angle B = 180° \Rightarrow \angle B = 120°$

$\angle C = \angle A$ and $\angle D = \angle B$ (opp. ∠s in a ∥ gm are equal)

$\Rightarrow \quad \angle C = 60°$ and $\angle D = 120°$

Hence, the angles of the rhombus are 60°, 120°, 60°, 120°.

Example 8. *In the adjoining figure, points P and Q have been taken on opposite sides AB and CD respectively of a parallelogram ABCD such that AP = CQ. Show that AC and PQ bisect each other.* *(NCERT Exemplar)*



Solution. In ΔAOP and ΔCOQ,

$AP = CQ$ (given)

$\angle AOP = \angle COQ$ (vert. opp. ∠s)

$\angle OAP = \angle OCQ$

(alt. ∠s, AB ∥ DC and AC is a transversal)

$\therefore \quad \Delta AOP \cong \Delta COQ$ (AAS rule of congruency)

$\Rightarrow \quad OA = OC$ and $OP = OQ$ (c.p.c.t.)

$\Rightarrow$ AC and PQ bisect each other.

Example 9. *In parallelogram ABCD, AB = 10 cm and AD = 6 cm. The bisector of ∠A meets DC in E. AE and BC produced meet at F. Find the length of CF.* *(NCERT Exemplar)*

Solution. Mark the angles as shown in the figure.

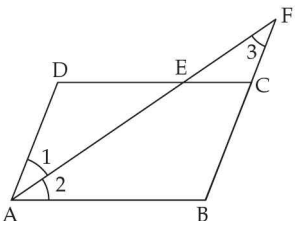
As AE is bisector of ∠A,

$\angle 1 = \angle 2$ ...(*i*)

Since ABCD is a parallelogram, AD ∥ BC *i.e.* AD ∥ BF

$\angle 1 = \angle 3$ (alt. ∠s are equal)

$\Rightarrow \quad \angle 2 = \angle 3$ (using (*i*))



In ΔABF, $\angle 2 = \angle 3$

$\Rightarrow \quad BF = AB$

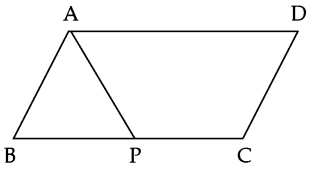
(sides opp. equal angles are equal)

$\Rightarrow \quad BC + CF = 10$ cm

$\Rightarrow \quad AD + CF = 10$ cm (BC = AD, opp. sides of a ∥ gm)

$\Rightarrow \quad 6$ cm $+ CF = 10$ cm $\Rightarrow CF = 4$ cm.

Example 10. *In the adjoining figure, P is mid-point of the side BC of a parallelogram ABCD such that ∠BAP = ∠DAP. Prove that AD = 2CD.* *(NCERT Exemplar)*



Solution. Given ∠BAP = ∠DAP, so AP is bisector of ∠A

$\Rightarrow \quad \angle A = 2\angle BAP \quad ...(i)$

As ABCD is a parallelogram, BC || AD

$\angle A + \angle B = 180°$ (sum of co-interior angles)

$\Rightarrow \quad 2\angle BAP + \angle B = 180° \quad ...(ii)$ (using (i))

$\angle BAP + \angle B + \angle APB = 180°$ (sum of ∠s in a Δ)

$\Rightarrow \quad 2\angle BAP + \angle B = \angle BAP + \angle B + \angle APB$ (using (ii))

$\Rightarrow \quad \angle BAP = \angle APB$

$\Rightarrow \quad AB = BP$ (in ΔABP, sides opp. equal angles are equal)

$\Rightarrow \quad AB = \frac{1}{2}BC$ ($\because$ P is mid-point of BC)

$\Rightarrow \quad 2AB = BC \Rightarrow 2\,CD = AD$

($\because$ In || gm ABCD, AB = CD and BC = AD, opp. sides are equal).

Example 11. *Prove that each angle of a rectangle is 90°.* *(NCERT)*

Solution. Let ABCD be a rectangle in which ∠A = 90°. We need to prove that each of ∠B, ∠C and ∠D is 90°. As every rectangle is a parallelogram, AD || BC and AB is a transversal,

so $\angle A + \angle B = 180°$ (sum of co-int. ∠s)

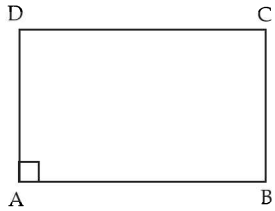
But $\angle A = 90°$

so $90° + \angle B = 180° \Rightarrow \angle B = 90°$

Now $\angle C = \angle A$ and $\angle D = \angle B$ (opp. angles of || gm)

$\Rightarrow \quad \angle C = 90°$ and $\angle D = 90°$

Thus, each of ∠B, ∠C and ∠D is 90°.



Example 12. *If the diagonals of a parallelogram are equal, then prove that it is a rectangle.* *(NCERT)*

Solution. Let ABCD be a parallelogram in which AC = BD. We need to prove that ∠A = 90°.

In ΔABC and ΔBAD,

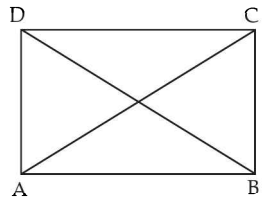
$BC = AD$ (opp. sides of a || gm)

$AB = AB$ (common)

$AC = BD$ (given, diagonals are equal)

$\therefore \quad \Delta ABC \cong \Delta BAD$

$\therefore \quad \angle B = \angle A$ (c.p.c.t.)



As AD || BC and AB is a transversal,

$\angle A + \angle B = 180°$ (sum of co-int. ∠s)

$\Rightarrow \quad \angle A + \angle A = 180° \Rightarrow 2\angle A = 180° \Rightarrow \angle A = 90°.$

$\therefore$ ABCD is a rectangle.

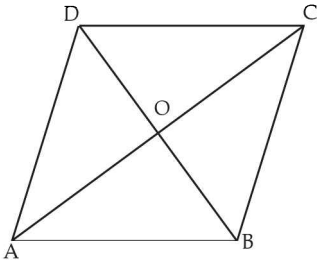
Example 13. *Show that the diagonals of a rhombus bisect each other at right angles.* *(NCERT)*

Solution. Let ABCD be a rhombus and let its diagonals AC and BD meet at O.

We need to prove that OA = OC, OB = OD and ∠AOB = 90°. As every rhombus is a || gm, therefore, the diagonals bisect each other *i.e.* OA = OC and OB = OD.

Thus, the diagonals of a rhombus bisects each other.

In ΔOAB and ΔOCB,



$OA = OC$ (proved above)

$OB = OB$ (common)

$AB = BC$ (sides of a rhombus)

$\therefore \quad \Delta OAB \cong \Delta OCB$ (SSS rule of congruency)

$\therefore \quad \angle AOB = \angle BOC$ (c.p.c.t.)

But $\angle AOB + \angle BOC = 180°$ (linear pair)

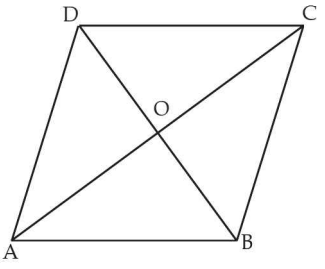
$\Rightarrow \quad \angle AOB + \angle AOB = 180° \Rightarrow \angle AOB = 90°$

Hence, the diagonals of a rhombus bisect each other at right angles.

Example 14. *If the diagonals of a quadrilateral bisect each other at right angles, then prove that the quadrilateral is a rhombus.* *(NCERT)*

Solution. Let ABCD be a quadrilateral in which the diagonals AC and BD bisect each other at O and are at right angles *i.e.* $OA = OC$, $OB = OD$ and $AC \perp BD$. We need to prove that ABCD is a rhombus.

As the diagonals of the quadrilateral ABCD bisect each other, ABCD is a parallelogram.



In ΔOAB and ΔOCB,

$OA = OC$ (from given)

$OB = OB$ (common)

$\angle AOB = \angle COB$

(each = 90°, because $AC \perp BD$ given)

$\therefore \quad \Delta OAB \cong \Delta OCB$ (by SAS rule of congruency)

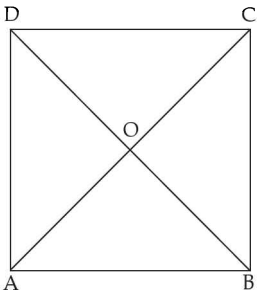
$\therefore \quad AB = BC$ (c.p.c.t.)

Thus, ABCD is a parallelogram in which two adjacent sides are equal, therefore, ABCD is a rhombus.

Example 15. *Prove that the diagonals of a square are equal and bisect each other at right angles.* *(NCERT)*

Solution. Let ABCD is a square and let its diagonals AC and BD meet at O. We need to prove that $OA = OC$, $OB = OD$, $\angle AOB = 90°$ and $AC = BD$.

As ABCD is a square, so it is a parallelogram. Therefore, its diagonals bisect each other



i.e. $OA = OC$ and $OB = OD$.

Thus, the diagonals of a square bisect each other.

In ΔOAB and ΔOCB,

$OA = OC$ (proved above)

$AB = BC$ (sides of a square)

$OB = OB$ (common)

$\therefore \quad \Delta OAB \cong \Delta OCB$ (by SSS rule of congruency)

$\therefore \quad \angle AOB = \angle BOC$ (c.p.c.t.)

But $\angle AOB + \angle BOC = 180°$ (linear pair)

$\Rightarrow \quad \angle AOB + \angle AOB = 180° \Rightarrow \angle AOB = 90°$

As ABCD is a square, so one of its angle is 90°. Let $\angle A = 90°$.

Now $AD \parallel BC$ and AB is a transversal

$\angle A + \angle B = 180°$ (sum of co-int. $\angle$s)

$\Rightarrow \quad 90° + \angle B = 180° \Rightarrow \angle B = 90°$.

In ΔABC and ΔBAD,

$BC = AD$ (sides of a square)

$AB = BA$ (common)

$\angle B = \angle A$ (each = 90°)

$\therefore \quad \Delta ABC \cong \Delta BAD$

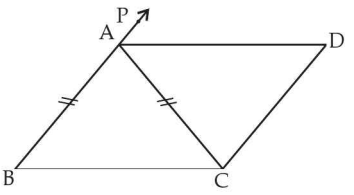
$\therefore \quad AC = BD$ (c.p.c.t.)

Hence, the diagonals of a square are equal and bisect each other at right angles.

Example 16. *In the adjoining figure, ABC is an isosceles triangle in which AB = AC. AD bisects exterior angle PAC and CD ∥ BA. Show that*

(*i*) $\angle DAC = \angle BCA$

(*ii*) *ABCD is a parallelogram.* *(NCERT)*



Solution. (*i*) $AB = AC \Rightarrow \angle ABC = \angle BCA$ ($\angle$s opp. equal sides)

Also, $\angle PAC = \angle ABC + \angle BCA$ (ext. $\angle$ = sum of two int. opp. $\angle$s)

$\Rightarrow \quad \angle PAC = 2\angle BCA$ ...(1)

As AD bisects $\angle PAC$,

$\angle PAC = 2\angle DAC$ ...(2)

From (1) and (2), we get

$2\angle DAC = 2\angle BCA \Rightarrow \angle DAC = \angle BCA.$

(*ii*) We note that the line segments BC and AD are intersected by the transversal AC, so $\angle DAC$ and $\angle BCA$ form a pair of alternate interior angles and these angles are equal (from (*i*)),

$\therefore \quad BC \parallel AD$

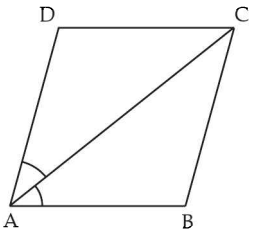
Also $BA \parallel CD$ (given)

Thus, both pairs of opposite sides of a quadrilateral ABCD are parallel, therefore, ABCD is a parallelogram.

Example 17. *In the adjoining figure, the diagonal AC of a parallelogram ABCD bisects ∠A. Show that*

(*i*) *AC bisects ∠C also*

(*ii*) *ABCD is a rhombus.* *(NCERT)*



Solution. (*i*) As ABCD is a ∥ gm, AB ∥ DC and AC is a transversal,

$\angle BAC = \angle DCA$ (alt. $\angle$s)

Similarly, $\angle DAC = \angle BCA$ (alt. $\angle$s)

But $\angle BAC = \angle DAC$ ($\because$ AC bisects $\angle A$)

$\Rightarrow \quad \angle DCA = \angle BCA \Rightarrow$ AC bisects $\angle C$.

(*ii*) $\angle DCA = \angle BCA$ (from above)

But $\angle DAC = \angle BAC$ ($\because$ AC bisects $\angle A$)

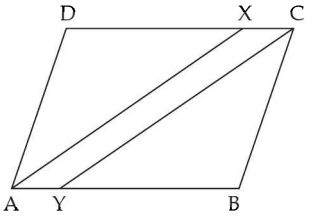
$\therefore \quad \angle BCA = \angle BAC.$

In ΔABC, $\angle BCA = \angle BAC$

$\Rightarrow \quad AB = BC$ (sides opp. equal angles are equal)

Therefore, ABCD is a rhombus.

Example 18. *In the adjoining figure, AX and CY are respectively the bisectors of ∠A and ∠C of a parallelogram ABCD. Show that AX ∥ CY.* *(NCERT Exemplar)*



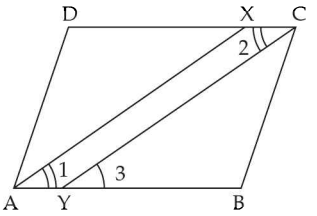
Solution. Given ABCD is a parallelogram.

$\Rightarrow \quad \angle A = \angle C$ (opp. ∠s are equal)

$\Rightarrow \quad \frac{1}{2}\angle A = \frac{1}{2}\angle C$

$\Rightarrow \quad \angle 1 = \angle 2 \quad \ldots(i)$

($\because$ AX is bisector of ∠A and CY is bisector of ∠C)



AB ∥ CD and CY is a transversal.

$\angle 2 = \angle 3$ (alt. ∠s are equal)

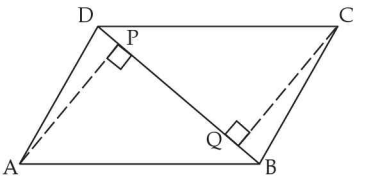
$\Rightarrow \quad \angle 1 = \angle 3$ (using (*i*))

But ∠1 and ∠3 are a pair of corresponding angles, therefore, AX ∥ CY.

Example 19. *In the adjoining figure, ABCD is a parallelogram and AP, CQ are perpendiculars from the vertices A, C respectively on the diagonal BD. Show that*

(i) $\Delta APB \cong \Delta CQD$

(ii) $AP = CQ$. *(NCERT)*



Solution. (*i*) In ΔAPB and ΔCQD,

AB = CD (opp. sides of a ∥ gm)

∠P = ∠Q (each = 90°)

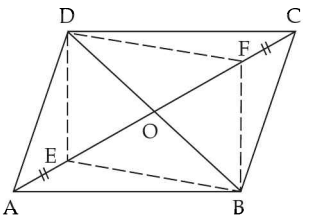
∠ABP = ∠CDQ (alt. ∠s, AB ∥ DC and BD is a transversal)

$\therefore$ ΔAPB ≅ ΔCQD (by AAS rule of congruency)

(*ii*) AP = CQ (c.p.c.t.)

Example 20. *If E and F are points on diagonal AC of a parallelogram ABCD such that AE = CF, then show that BFDE is a parallelogram.* *(NCERT Exemplar)*

Solution. As ABCD is a parallelogram, its diagonals bisect each other *i.e.* OA = OC and OB = OD.



Given AE = CF

$\therefore$ OA – AE = OC – CF

$\Rightarrow$ OE = OF

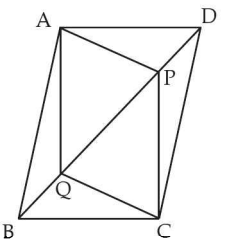
Thus, in quadrilateral BFDE, OE = OF and OB = OD *i.e.* its diagonals EF and BD bisect each other, therefore, BFDE is a parallelogram.

Example 21. *In the adjoining figure, ABCD is a parallelogram. Two points P and Q are taken on the diagonal BD such that DP = BQ. Show that*

(i) $\Delta APD \cong \Delta CQB$ *(ii)* $AP = CQ$

(iii) $\Delta AQB \cong \Delta CPD$ *(iv)* $AQ = CP$

(v) APCQ is a parallelogram. *(NCERT)*



Solution. (*i*) In ΔAPD and ΔCQB,

AD = BC (opp. sides of a ∥ gm)

DP = BQ (given)

∠ADP = ∠CBQ (alt. ∠s, BC ∥ AD and BD is a transversal)

$\therefore$ ΔAPD ≅ ΔCQB (by SAS rule of congruency)

(*ii*) AP = CQ (c.p.c.t.)

(*iii*) In ΔAQB and ΔCPD,

$AB = CD$ (opp. sides of a $\parallel$ gm)

$BQ = DP$ (given)

$\angle ABQ = \angle CDP$ (alt. $\angle$s, $AB \parallel DC$ and BD is a transversal)

$\therefore \quad \Delta AQB \cong \Delta CPD$ (by SAS rule of congruency)

(*iv*) $AQ = CP$ (c.p.c.t.)

(*v*) From (*ii*) and (*iv*), we have $AP = CQ$ and $AQ = CP$.

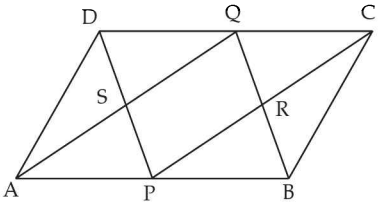
Thus, in quadrilateral APCQ, each pair of opposite sides is equal, therefore, APCQ is a parallelogram.

Example 22. *In the adjoining figure, ABCD is a parallelogram in which P and Q are mid-points of opposite sides AB and CD respectively. If AQ intersects DP at S and BQ intersects CP at R, show that*

(i) APCQ is a parallelogram

(ii) DPBQ is a parallelogram

(iii) PSQR is a parallelogram. *(NCERT)*



Solution. (*i*) As ABCD is a parallelogram, $AB \parallel DC$

$\Rightarrow \quad AP \parallel QC$

Also $AB = DC$ (opp. sides of a $\parallel$ gm are equal)

$\Rightarrow \quad \frac{1}{2}AB = \frac{1}{2}DC$

$\Rightarrow \quad AP = QC$ ($\because$ P and Q are mid-points of AB and CD respectively)

Thus, in the quadrilateral APCQ, $AP \parallel QC$ and $AP = QC$ *i.e.* two opposite sides are equal and parallel

$\Rightarrow$ APCQ is a parallelogram.

(*ii*) As $AB \parallel DC \Rightarrow PB \parallel DQ$.

Also $AB = DC \Rightarrow \frac{1}{2}AB = \frac{1}{2}DC \Rightarrow PB = DQ$.

Thus, in the quad. DPBQ, $PB \parallel DQ$ and $PB = DQ$

$\Rightarrow$ DPBQ is a parallelogram.

(*iii*) As APCQ is a $\parallel$ gm, $AQ \parallel PC \Rightarrow SQ \parallel PR$.

Also as DPBQ is a parallelogram, $PD \parallel BQ \Rightarrow PS \parallel RQ$.

Thus, in quad. PSQR, $SQ \parallel PR$ and $PS \parallel RQ$

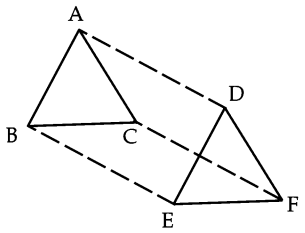
$\Rightarrow$ PSQR is a parallelogram.

Example 23. *In the adjoining figure, in ΔABC and ΔDEF, $AB = DE$, $AB \parallel DE$, $BC = EF$ and $BC \parallel EF$. Vertices A, B and C are joined to vertices D, E and F respectively. Show that:*

(i) ABED is a parallelogram *(ii) BEFC is a parallelogram*

(iii) $AD \parallel CF$ and $AD = CF$ *(iv) ACFD is a parallelogram*

(v) $AC = DF$ *(vi) $\Delta ABC \cong \Delta DEF$* *(NCERT)*



Solution. (*i*) In quadrilateral ABED, $AB = DE$ and $AB \parallel DE$ (given), therefore, ABED is a parallelogram.

(*ii*) In quadrilateral BEFC, $BC = EF$ and $BC \parallel EF$ (given),

therefore, BEFC is a parallelogram.

(*iii*) As ABED is a parallelogram (from (*i*)), AD = BE and AD ∥ BE.

Also, BEFC is a parallelogram (from (*ii*)), CF = BE and CF ∥ BE.

⇒ AD = CF and AD ∥ CF.

(*iv*) In quadrilateral ACFD, AD = CF and AD ∥ CF, therefore, ACFD is a parallelogram.

(*v*) As ACFD is a parallelogram (from (*iv*)), AC = DF.

(*vi*) In ΔABC and ΔDEF,

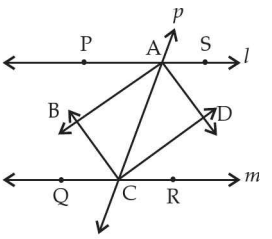
AB = DE (given)

BC = EF (given)

AC = DF (from (*v*))

∴ ΔABC ≅ ΔDEF (by SSS rule of congruency)

Example 24. *In the adjoining figure, two parallel lines l and m are intersected by a transversal p. Show that the quadrilateral formed by the bisectors of interior angles is a rectangle.* *(NCERT)*



Solution. It is given that the lines *l* and *m* are parallel *i.e.* PS ∥ QR and transversal *p* intersects them at points A and C respectively.

The bisectors of ∠PAC and ∠QCA intersect at B, and the bisectors of ∠ACR and ∠CAS intersect at D. We need to show that quadrilateral ABCD is a rectangle.

Now, ∠PAC = ∠ACR (alt. ∠s, *l* ∥ *m* and *p* is a transversal)

$\Rightarrow \quad \frac{1}{2}\angle PAC = \frac{1}{2}\angle ACR \Rightarrow \angle BAC = \angle ACD.$

But these angles form a pair of alternate angles for lines AB and DC with AC as transversal and these angles are equal,

therefore, AB ∥ DC.

Similarly, BC ∥ AD (∠ACB = ∠CAD, why?)

Therefore, quadrilateral ABCD is a parallelogram.

Also, ∠PAC + ∠CAS = 180° (linear pair)

$\Rightarrow \frac{1}{2}\angle PAC + \frac{1}{2}\angle CAS = \frac{1}{2} \times 180° = 90°$

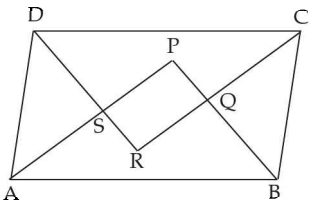
⇒ ∠BAC + ∠CAD = 90° ⇒ ∠BAD = 90°.

Thus, ABCD is a parallelogram in which one angle is 90°.

Therefore, ABCD is a rectangle.

Example 25. *Show that the bisectors of the angles of a parallelogram form a rectangle.* *(NCERT)*

Solution. Let ABCD be a parallelogram and let P, Q, R and S be the points of intersection of the bisectors of ∠A and ∠B, ∠B and ∠C, ∠C and ∠D, and ∠D and ∠A respectively. We need to show that PQRS is a rectangle.



As ABCD is a ∥ gm, AD ∥ BC and AB is a transversal.

∴ ∠A + ∠B = 180° (sum of co-int. ∠s = 180°)

$\Rightarrow \quad \frac{1}{2}\angle A + \frac{1}{2}\angle B = 90°$

⇒ ∠PAB + ∠PBA = 90° (∵ AP is bisector of ∠A and BP is bisector of ∠B)

In ΔPAB, $\angle APB + \angle PAB + \angle PBA = 180°$ (sum of angles in a Δ)

$\Rightarrow$ $\angle APB + 90° = 180° \Rightarrow \angle APB = 90° \Rightarrow \angle SPQ = 90°$

Similarly, $\angle PQR = 90°$, $\angle QRS = 90°$ and $\angle RSP = 90°$.

So, PQRS is a quadrilateral in which each angle is 90°.

Now, $\angle SPQ = \angle QRS$ (each = 90°)

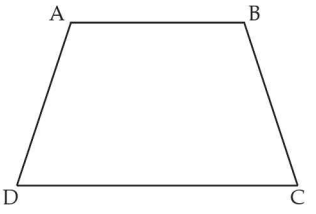
and $\angle PQR = \angle RSP$ (each = 90°)

Thus, PQRS is a quadrilateral in which both pairs of opposite angles are equal, therefore, PQRS is a parallelogram. Also, in this parallelogram one angle (in fact all angles) is 90°. Therefore, PQRS is a rectangle.

Example 26. *In the adjoining figure, ABCD is a trapezium in which AB || DC and AD = BC. Show that*

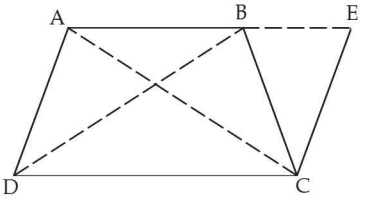
(i) $\angle A = \angle B$ *(ii)* $\angle C = \angle D$

(iii) $\Delta ABC \cong \Delta BAD$ *(iv) diagonal AC = diagonal BD.* *(NCERT)*



Solution. Given ABCD is a trapezium in which AB || DC and AD = BC.

Through C, draw a line parallel to DA intersecting AB produced at E.



(*i*) As DA || CE (by construction)

and AB || DC (given)

So, AECD is a parallelogram, therefore, AD = EC.

Also AD = BC (given)

$\Rightarrow$ BC = EC

$\Rightarrow$ $\angle CBE = \angle BEC$ ($\because$ in a Δ, angles opp. equal sides are equal)

Now $\angle A + \angle BEC = 180°$ (sum of co-int. $\angle$s, AD || EC and AE is transversal)

Also $\angle B + \angle CBE = 180°$ (linear pair)

$\Rightarrow$ $\angle A + \angle BEC = \angle B + \angle CBE$ but $\angle CBE = \angle BEC$

$\angle A = \angle B$.

(*ii*) As AB || DC, $\angle A + \angle D = 180°$ and $\angle B + \angle C = 180°$ (sum of co-int. $\angle$s)

$\Rightarrow \angle A + \angle D = \angle B + \angle C$ but $\angle A = \angle B$ (from (*i*))

$\Rightarrow$ $\angle D = \angle C$.

(*iii*) In ΔABC and ΔBAD,

BC = AD (given)

AB = AB (common)

$\angle B = \angle A$ (from (*i*))

$\therefore$ $\Delta ABC \cong \Delta BAD$ (by SAS rule of congruency)

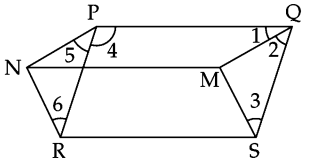
(*iv*) AC = BD (c.p.c.t.)

Example 27. *PQ and RS are equal and parallel line segments. Any point M not lying on PQ or RS is joined to Q and S and lines through P parallel to QM and through R parallel to SM meet at N. Prove that the line segments MN and PQ are equal and parallel to each other.*

(NCERT Exemplar)

Solution. We draw the figure as per the given conditions.

Mark the angles as shown in the figure.



Given, PQ = RS and PQ || RS, therefore, PQSR is a parallelogram

$\Rightarrow$ PR = QS and PR || QS

Now, as PR || SQ, $\angle 4 + \angle PQS = 180°$(sum of co-interior $\angle$s)

$\Rightarrow \quad \angle 4 + \angle 1 + \angle 2 = 180°$ ...(*i*)

Also, as NP || MQ (given), $\angle NPQ + \angle 1 = 180°$ (sum of co-int. $\angle$s)

$\Rightarrow \quad \angle 5 + \angle 4 + \angle 1 = 180°$

$\Rightarrow \quad \angle 5 + \angle 4 + \angle 1 = \angle 4 + \angle 1 + \angle 2$ (using (*i*))

$\Rightarrow \quad \angle 5 = \angle 2$

Similarly, $\angle 6 = \angle 3$

In ΔNPR and ΔMQS,

$\angle 5 = \angle 2$ (proved above)

$\angle 6 = \angle 3$ (proved above)

PR = QS (proved above)

$\therefore \quad \Delta NPR \cong \Delta MQS$

$\Rightarrow \quad PN = QM.$

Now, as PN || QM and PN = QM, therefore, PQMN is a parallelogram

$\Rightarrow$ PQ = MN and PQ || MN, as required.

Exercise 9.1

1. (*i*) One angle of a quadrilateral is 108° and the remaining three angles are equal. Find each of the three equal angles. *(NCERT Exemplar)*

(*ii*) The angles of a quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the quadrilateral. *(NCERT)*

2. (*i*) Can the angles 110°, 80°, 70° and 95° be the angles of a quadrilateral? Why or why not? *(NCERT Exemplar)*

(*ii*) Can all the four angles of a quadrilateral be obtuse? Give reason for your answer. *(NCERT Exemplar)*

(*iii*) Can all the four angles of a quadrilateral be acute? Give reason for your answer. *(NCERT Exemplar)*

(*iv*) Can all the angles of a quadrilateral be right angles? Give reason for your answer. *(NCERT Exemplar)*

3. (*i*) In quadrilateral ABCD, $\angle A + \angle D = 180°$. What special name can be given to this quadrilateral? *(NCERT Exemplar)*

(*ii*) All the four angles of a quadrilateral are equal. What special name can be given to this quadrilateral? *(NCERT Exemplar)*

(*iii*) All the four sides of a quadrilateral are equal. What special name can be given to this quadrilateral?

4. (*i*) Diagonals of a parallelogram are perpendicular to each other. Is this statement true? Give reason for your answer. *(NCERT Exemplar)*

(*ii*) Diagonals of a rectangle are equal and perpendicular. Is this statement true? Give reason for your answer. *(NCERT Exemplar)*

(*iii*) Diagonals of a rhombus are equal and perpendicular to each other. Is this statement true? Give reason for your answer. *(NCERT Exemplar)*

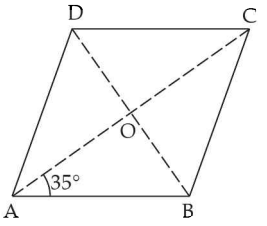
(*iv*) Diagonals AC and BD of a quadrilateral intersect each other at O such that OA : OC = 3 : 2. Is ABCD a parallelogram? Why or why not? *(NCERT Exemplar)*

5. (*i*) ABCD is a parallelogram. If its diagonals are equal, then find the value of $\angle ABC$. *(NCERT Exemplar)*

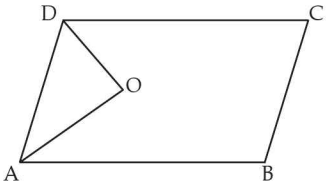
(*ii*) Opposite sides of a quadrilateral are equal. If AB = 4 cm, then determine CD. *(NCERT Exemplar)*

(*iii*) ABCD is a trapezium in which AB || DC and $\angle A = \angle B = 45°$. Find $\angle C$ and $\angle D$ of the trapezium. *(NCERT Exemplar)*

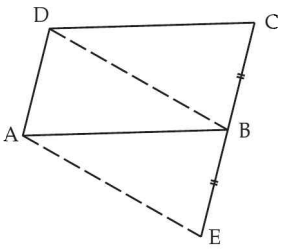
6. In the adjoining rhombus ABCD, $\angle BAC = 35°$. Find:
   (*i*) $\angle ABO$
   (*ii*) $\angle BCD$
   (*iii*) $\angle CDA$.



7. In the adjoining figure, ABCD is a parallelogram. If the bisector of $\angle A$ and $\angle D$ meet at O, then find $\angle AOD$.

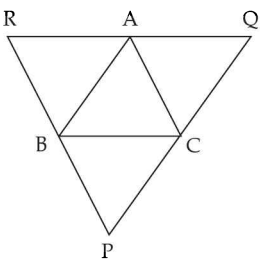


8. In the adjoining figure, ABCD is a parallelogram. CB is produced to E such that BE = BC. Prove that AEBD is a parallelogram.



9. ABCD is a rhombus. Show that diagonal AC bisects $\angle A$ as well as $\angle C$ and diagonal BD bisects $\angle B$ as well as $\angle D$. *(NCERT)*
10. P and Q are points on opposite sides AD and BC of a parallelogram ABCD such that PQ passes through the point of intersection O of its diagonals AC and BD. Show that PQ is bisected at O. *(NCERT Exemplar)*

   Hint: Show that $\Delta OAP \cong \Delta OCQ$.
11. ABCD is a rectangle in which diagonal BD bisects $\angle B$. Show that ABCD is a square. *(NCERT Exemplar)*
12. ABCD is a rectangle in which diagonal AC bisects $\angle A$ as well as $\angle C$. Show that:
   (*i*) ABCD is a square
   (*ii*) diagonal BD bisects $\angle B$ as well as $\angle D$. *(NCERT)*
13. Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square. *(NCERT)*
14. Diagonals of a rhombus are of lengths 16 cm and 12 cm respectively, find the perimeter of the rhombus.
15. In the adjoining figure, through A, B and C, lines RQ, PR and QP have been drawn respectively parallel to sides BC, CA and AB of a triangle ABC. Show that $BC = \frac{1}{2} RQ$. *(NCERT Exemplar)*

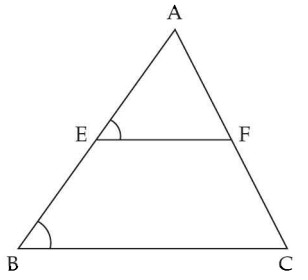


   Hint: Show that BCAR is a $\parallel$ gm $\Rightarrow$ BC = RA.
   Also, show that BCQA is a $\parallel$ gm $\Rightarrow$ BC = AQ.
   Now $RQ = RA + AQ = BC + BC \Rightarrow BC = \frac{1}{2} RQ$.

Answers

9.3 MID-POINT THEOREM

We have learnt many properties of a triangle as well as of a quadrilateral. Now, let us learn another result which is related to the mid-points of sides of a triangle.



Perform the following activity:

Draw any triangle ABC. Mark mid-points E and F of sides AB and AC respectively. Join points E and F. Measure EF and BC. Also measure ∠AEF and ∠ABC.

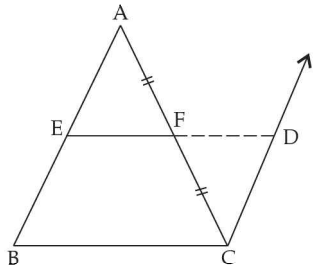
We observe that EF = $\frac{1}{2}$ BC and ∠AEF = ∠ABC.

But these angles are a pair of corresponding angles formed by lines EF, BC and transversal AB, therefore, EF ‖ BC.

Repeat this activity with more triangles. Every time we get the same result. In fact, we have:

❑ **Mid-point theorem**

The line segment joining the mid-points of any two sides of a triangle is parallel to the third side and is equal to half of it.



Given. A triangle ABC, E and F are the mid-points of sides AB and AC respectively.

To prove. EF ‖ BC and EF = $\frac{1}{2}$ BC.

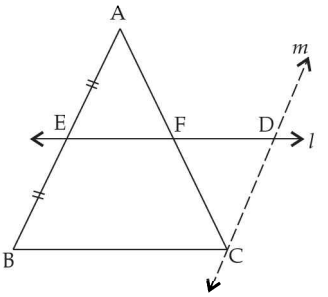
Construction. Through C, draw a line parallel to BA to meet EF produced at D.

Proof.

Statements	*Reasons*
In ΔAEF and ΔCDF	
1. AF = CF	1. F is mid-point of AC (given).
2. ∠AFE = ∠CFD	2. Vert. opp. ∠s
3. ∠EAF = ∠DCF	3. Alt. ∠s, BA ‖ CD (by construction) and AC is a transversal.
4. ΔAEF ≅ ΔCDF	4. ASA rule of congruency.
5. EF = FD and AE = CD	5. c.p.c.t.
6. AE = BE	6. E is mid-point of AB (given).
7. BE = CD	7. From 5 and 6
8. EBCD is a parallelogram.	8. BA ‖ CD (const.) and BE = CF (from 7)
9. EF ‖ BC and ED = BC	9. Since EBCD is a parallelogram.
10. EF = $\frac{1}{2}$ ED	10. Since EF = FD, from 5
11. EF = $\frac{1}{2}$ BC	11. Since ED = BC, from 9
Hence, EF ‖ BC and EF = $\frac{1}{2}$ BC.	

The converse of the above theorem is also true. In fact, we have

❑ **Converse of mid-point theorem**



The line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side.

Given. A triangle ABC, E is mid-point of AB. Line l drawn through E and parallel to BC meeting AC at F.

To prove. AF = FC.

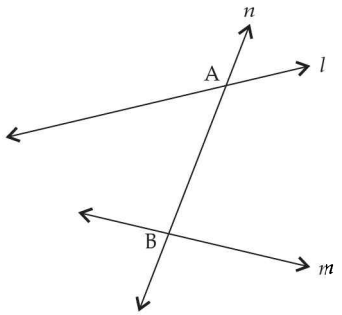
Construction. Through C, draw a line m parallel BA to meet line l at D.

Proof.

Statements	*Reasons*
1. EBCD is a parallelogram	1. EF ‖ BC (given), BA ‖ CD (const.).
2. BE = CD	2. Opp. sides of a parallelogram are equal.
3. EA = BE	3. E is mid-point of AB (given).
4. EA = CD	4. From 2 and 3.
In ΔAEF and ΔCDF	
5. ∠EAF = ∠DCF	5. Alt. ∠s, CD ‖ BA and AC is a transversal.
6. ∠EFA = ∠DFC	6. Vert. opp. ∠s
7. EA = CD	7. From 4
8. ΔAEF ≅ ΔCDF	8. AAS rule of congruency
9. AF = FC	9. c.p.c.t.
Hence, F is mid-point of AC.	

❑ **Intercepts**

If a line n intersects two lines l and m (drawn in a plane) at points A and B respectively, then the line segment AB is called the *intercept* made on line n by the lines l and m.



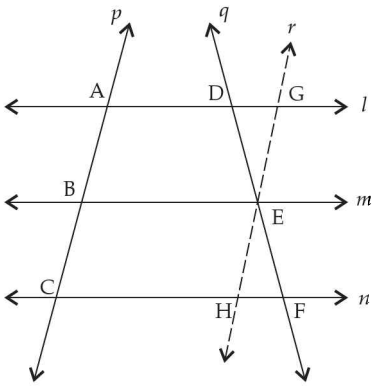
❑ **Theorem of intercepts**

If a transversal makes equal intercepts on three (or more) parallel lines, then any other line cutting them also makes equal intercepts.

Given. Three parallel lines l, m and n. A transversal p cutting them at points A, B and C respectively such that AB = BC. Any other line q cuts them at points D, E and F respectively.

To prove. DE = EF.

Construction. Through E, draw a line r parallel to line p to meet line l at G and line n at H.

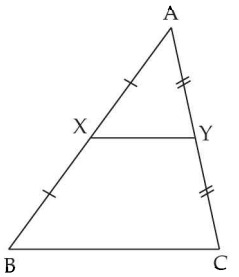


Proof.

Statements	*Reasons*
1. ABEG is a parallelogram.	1. AG ‖ BE (given), AB ‖ GE (const.).
2. AB = GE	2. Opp. sides of a parallelogram are equal.
3. BCHE is a parallelogram.	3. BE ‖ CH (given), BC ‖ EH (const.).
4. BC = EH	4. Opp. sides of a parallelogram are equal.
5. GE = EH	5. From 2 and 4; also AB = BC (given)
In ΔDEG and ΔFEH	
6. ∠EGD = ∠FHE	6. Alt. ∠s, AG ‖ CH (given) and GH is a transversal.
7. ∠DEG = ∠FEH	7. Vert. opp. ∠s are equal.
8. GE = EH	8. From 5 (proved above)
9. ΔDEG ≅ ΔFEH	9. ASA rule of congruency.
10. DE = EF	10. c.p.c.t.

Illustrative Examples

Example 1. *In the adjoining figure, X and Y are mid-points of sides AB and AC respectively of ΔABC. If BC = 6 cm, AB = 7.4 cm and AC = 6.4 cm, then find the perimeter of trapezium XBCY.*



Solution. Since X is mid-point of AB and Y is mid-point of AC,

therefore, XY is parallel to BC and $XY = \frac{1}{2} BC = (\frac{1}{2} \times 6)$ cm

$= 3$ cm.

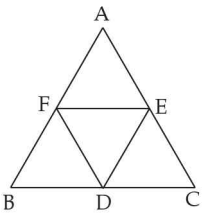
Also $XB = \frac{1}{2} AB = (\frac{1}{2} \times 7.4)$ cm $= 3.7$ cm and

$YC = \frac{1}{2} AC = (\frac{1}{2} \times 6.4)$ cm $= 3.2$ cm

∴ Perimeter of trapezium XBCY = (3.7 + 6 + 3.2 + 3) cm = 15.9 cm.

Example 2. *D, E and F are mid-points of the sides BC, CA and AB respectively of an equilateral triangle ABC. Show that ΔDEF is also an equilateral triangle.* *(NCERT Exemplar)*

Solution. As F and E are mid-points of AB and CA respectively of ΔABC.



$$FE = \frac{1}{2} BC \quad \text{(by Mid-point theorem)}$$

Similarly, $FD = \frac{1}{2} AC$ and $DE = \frac{1}{2} AB$

Given, ΔABC is equilateral triangle.

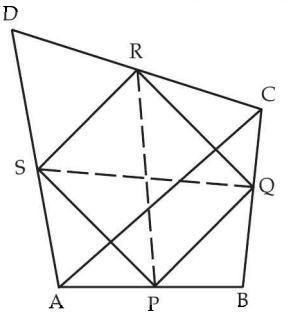
$\therefore \quad AB = BC = CA$

$\Rightarrow \quad \frac{1}{2} AB = \frac{1}{2} BC = \frac{1}{2} CA$

$\Rightarrow \quad DE = FE = FD$

Hence, ΔDEF is also an equilateral triangle.

Example 3. *Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.* *(NCERT)*



Solution. Let ABCD be a quadrilateral and P, Q, R, S be mid-points of sides AB, BC, CD, AD respectively. We need to show that the line segments PR and SQ bisect each other.

In ΔABC, P and Q are mid-points of sides AB and BC respectively, therefore,

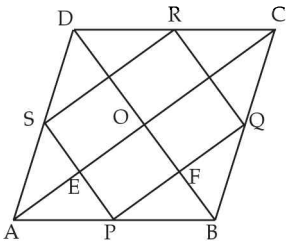
$$PQ \parallel AC \text{ and } PQ = \frac{1}{2}AC.$$

Similarly, $SR \parallel AC$ and $SR = \frac{1}{2}AC$

$\Rightarrow$ $PQ \parallel SR$ and $PQ = SR \Rightarrow$ PQRS is a parallelogram.

As the diagonals of a parallelogram bisect other, therefore, the line segments PR and SQ bisect each other.

Example 4. *ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.* *(NCERT)*



Solution. Given ABCD is a rhombus and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. We need to show that PQRS is a rectangle.

Let the diagonals AC and BD of the rhombus ABCD intersect at O.

In ΔABC, P and Q are the mid-points of the sides AB and BC respectively, therefore, by mid-point theorem,

$$PQ \parallel AC \text{ and } PQ = \frac{1}{2}AC.$$

Similarly, $SR \parallel AC$ and $SR = \frac{1}{2}AC$

$\Rightarrow$ $PQ \parallel SR$ and $PQ = SR$

Thus, in quadrilateral PQRS, $PQ \parallel SR$ and $PQ = SR$,

therefore, PQRS is a parallelogram.

In ΔABD, P and S are mid-points of AB and AD respectively, therefore,

$PS \parallel BD$

Thus, $EP \parallel OF$ and $PF \parallel EO$, therefore, OEPF is a parallelogram

But $\angle EOF = 90°$ ($\because$ diagonals of a rhombus intersect at right angles)

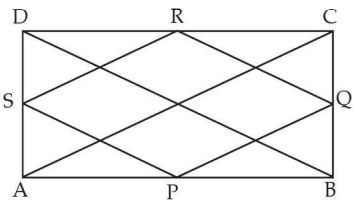
Also, in a parallelogram opposite angles are equal

$\therefore$ $\angle EPF = \angle EOF = 90°$.

Thus, PQRS is a parallelogram in which one angle *i.e.* $\angle P = 90°$,

therefore, PQRS is a rectangle.

Example 5. *ABCD is a rectangle and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.* *(NCERT)*



Solution. Given ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. We need to prove that PQRS is a rhombus.

In ΔABC, P and Q are mid-points of the sides AB and BC respectively, therefore, by mid-point theorem.

$$PQ \parallel AC \text{ and } PQ = \frac{1}{2}AC.$$

Similarly, $SR \parallel AC$ and $SR = \frac{1}{2}AC$

$\Rightarrow$ $PQ \parallel SR$ and $PQ = SR$

Thus, in quadrilateral PQRS, $PQ \parallel SR$ and $PQ = SR$, therefore, PQRS is a parallelogram.

In ΔABD, P and S are mid-points of the sides AB and AD respectively, therefore,

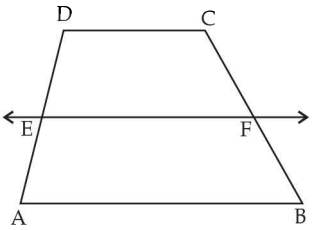
$PS \parallel BD$ and $PS = \frac{1}{2}BD$.

But $AC = BD$ ($\because$ diagonals of a rectangle are equal)

$\therefore$ $PQ = PS$.

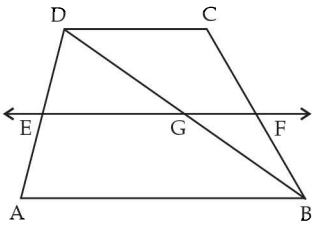
Thus, PQRS is a parallelogram in which two adjacent sides are equal, therefore, PQRS is a rhombus.

Example 6. *In the adjoining figure, ABCD is a trapezium in which AB ‖ DC and E is mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F. Show that F is mid-point of BC.* *(NCERT)*



Solution. Join BD and let BD intersect EF at G.

In ΔDAB, E is mid-point of AD and $EF \parallel AB$ *i.e.* $EG \parallel AB$, therefore, by the converse of mid-point theorem, G is mid-point of BD.



Now, $DC \parallel AB$ (given) and $EF \parallel AB$

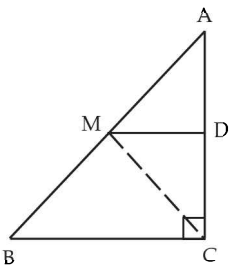
$\Rightarrow$ $DC \parallel EF \Rightarrow DC \parallel GF$.

In ΔBCD, G is mid-point of BD and $GF \parallel DC$, therefore, by the converse of mid-point theorem, F is mid-point of BC.

Example 7. *ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that*

(i) D is mid-point of AC (ii) $MD \perp AC$ (iii) $CM = MA = \frac{1}{2}AB$. *(NCERT)*

Solution. (*i*) In ΔABC, M is mid-point of AB and MD is drawn parallel to BC, therefore, by the converse of mid-point theorem, D is mid-point of AC.



(*ii*) As $MD \parallel BC$ and AC is a transversal,

$\angle BCD + \angle CDM = 180°$ (sum of co-int. $\angle$s)

$\Rightarrow$ $90° + \angle CDM = 180°$ ($\because$ $\angle BCA = 90°$, given)

$\Rightarrow$ $\angle CDM = 90° \Rightarrow MD \perp AC$.

(*iii*) In ΔAMD and ΔCMD,

$AD = DC$ ($\because$ D is mid-point of AC from part (*i*))

$\angle ADM = \angle CDM$ (each = 90°, $MD \perp AC$)

$MD = MD$ (common)

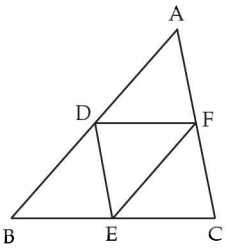
$\therefore$ $\Delta ADM \cong \Delta CMD$ (by SAS rule of congruency)

$\therefore$ $AM = CM$ (c.p.c.t.)

Also, as M is mid-point of AB, $AM = \frac{1}{2}AB$.

$\therefore$ $CM = AM = \frac{1}{2}AB$.

Example 8. *In the adjoining figure, D, E and F are the mid-points of the sides AB, BC and CA respectively of a triangle ABC. Show that ΔABC is divided into four congruent triangles by joining the points D, E and F.* *(NCERT)*



Solution. In ΔABC, D and E are mid-points of the sides AB and BC respectively, therefore, by mid-point theorem, DE ∥ AC.

Similarly, FE ∥ AB and DF ∥ BC

⇒ ADEF, BEFD and CFDE are all parallelograms.

Now, as DE is a diagonal of the parallelogram BEFD, so DE divides the parallelogram BEFD into two congruent triangles

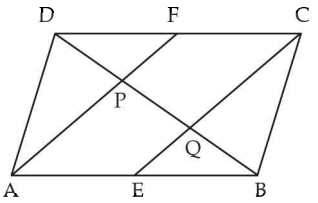
i.e. $\Delta BDE \cong \Delta FED$.

Similarly, $\Delta DAF \cong \Delta FED$

and $\Delta EFC \cong \Delta FED$.

Therefore, all the four triangles are congruent.

Example 9. *In the adjoining figure, ABCD is a parallelogram and E, F are the mid-points of the sides AB, CD respectively. Show that the line segments AF and EC trisect the diagonal BD.* *(NCERT)*



Solution. As ABCD is a parallelogram, AB ∥ DC

⇒ AE ∥ FC.

Also AB = DC (opp. sides of a ∥ gm are equal)

⇒ $\frac{1}{2}AB = \frac{1}{2}DC$

⇒ AE = FC (∵ E is mid-point of AB and F is mid-point of CD)

Thus, in the quadrilateral AECF, AE = FC and AE ∥ FC

⇒ AECF is a parallelogram ⇒ AF ∥ EC.

In ΔDQC, PF ∥ QC (∵ AF ∥ EC proved above)

and F is mid-point of CD

∴ P is mid-point of DQ (using converse of mid-point theorem)

⇒ DP = PQ ...(*i*)

In ΔABP, EQ ∥ AP (∵ EC ∥ AF, from above)

and E is mid-point of AB

∴ Q is mid-point of BP (using converse of mid-point theorem)

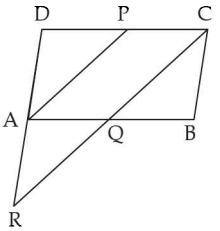
⇒ BQ = PQ ...(*ii*)

From (*i*) and (*ii*), we get DP = PQ = BQ

⇒ the line segments AF and EC trisect the diagonal BD.

Example 10. *P is the mid-point of the side CD of a parallelogram. A line through C parallel to PA intersects AB at Q and DA produced at R. Prove that DA = AR and CQ = QR.* *(NCERT Exemplar)*

Solution. Draw the figure according to given conditions.



In ΔDCR, P is mid-point of side CD and PA ∥ CR, therefore, by converse of mid-point theorem, A is mid-point of DR and $AP = \frac{1}{2}CR$.

As A is mid-point of DR, so DA = AR.

Now AB ∥ DC (∵ ABCD is a ∥ gm)

⇒ AQ ∥ PC.

Also, PA ∥ CR *i.e.* PA ∥ CQ, therefore, AQCP is a parallelogram

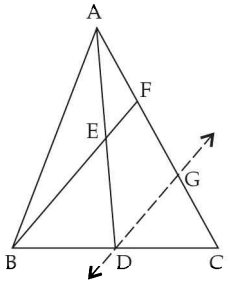
$\Rightarrow$ $AP = CQ \Rightarrow CQ = \frac{1}{2} CR$ $(\because AP = \frac{1}{2} CR)$

$\Rightarrow$ $2CQ = CR \Rightarrow 2CQ = CQ + QR$

$\Rightarrow$ $CQ = QR$.

Example 11. *E is mid-point of the median AD of ΔABC and BE is produced to meet AC at F. Show that* $AF = \frac{1}{3}AC$. *(NCERT Exemplar)*

Solution. Through D, draw a line parallel to BF to meet AC at G.



In ΔADG, E is mid-point of side AD and DG ∥ BF *i.e.* DG ∥ EF, therefore by converse of mid-point theorem, F is mid-point of AG *i.e.* AF = FG ...(*i*)

In ΔBCF, D is mid-point of side BC and DG ∥ BF, therefore, by converse of mid-point theorem, G is mid-point of CF *i.e.* FG = GC ... (*ii*)

From (*i*) and (*ii*), we get

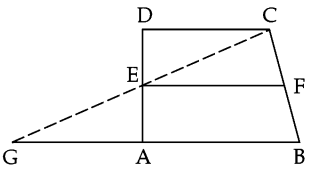
$AF = FG = GC$ ...(*iii*)

Now $AC = AF + FG + GC = AF + AF + AF$ (using (*iii*))

$\Rightarrow$ $AC = 3AF \Rightarrow AF = \frac{1}{3} AC$.

Example 12. *E and F are respectively the mid-points of non-parallel sides AD and BC of a trapezium ABCD. Prove that EF ∥ AB and* $EF = \frac{1}{2}(AB + CD)$. *(NCERT Exemplar)*

Solution. ABCD is a trapezium in which AB ∥ DC and E, F are mid-points of AD, BC respectively.



Join CE and produce it to meet BA produced at G.

In ΔEDC and ΔEAG,

$ED = EA$ $(\because$ E is mid-point of AD)

$\angle CED = \angle GEC$ (Vert. opp. ∠s)

$\angle ECD = \angle EGA$ (Alt. ∠s, DC ∥ AB *i.e.* DC ∥ GB and CG is transversal)

$\therefore$ $\Delta EDC \cong \Delta EAG$

$\Rightarrow$ $CD = GA$ and $EC = EG$ (c.p.c.t.)

In ΔCGB,

E is mid-point of CG $(\because$ EC = EG proved above)

F is mid-point of BC (given)

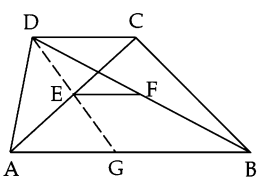
Therefore, by mid-point theorem, EF ∥ AB and $EF = \frac{1}{2} GB$.

But GB = GA + AB = CD + AB. $(\because$ GA = CD proved above)

Hence, EF ∥ AB and $EF = \frac{1}{2}(AB + CD)$.

Example 13. *Prove that the line segment joining the mid-points of the diagonals of a trapezium is parallel to the parallel sides of trapezium and is equal to half the difference of these sides.* *(NCERT Exemplar)*

Solution. Let ABCD be a trapezium in which AB ∥ CD. E and F are mid-points of its diagonals AC and BD respectively. We want to prove that EF ∥ AB and $EF = \frac{1}{2}(AB - DC)$.



Join DE and produce it to meet AB at G.

In ΔECD and ΔEAG

$EC = EA$ $(\because$ E is mid-point of AC)

$\angle ECD = \angle EAG$ (Alt. $\angle$s, AB $\parallel$ DC *i.e.* AG $\parallel$ DC and AC is transversal)

$\angle CED = \angle AEG$ (Vert. opp. $\angle$s)

$\therefore$ $\Delta ECD \cong \Delta EAG$ (ASA rule of congruency)

$\Rightarrow$ AG = DC and DE = EG.

In ΔDGB,

E is mid-point of DG ($\because$ DE = EG, proved above)

F is mid-point of BD (given)

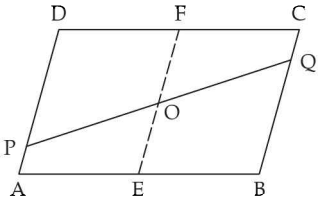
Therefore, by mid-point theorem,

EF $\parallel$ GB *i.e.* EF $\parallel$ AB and EF = $\frac{1}{2}$GB.

But GB = AB – AG = AB – DC ($\because$ AG = DC, proved above)

Hence, EF $\parallel$ AB and EF = $\frac{1}{2}$(AB – DC).

Example 14. *In the adjoining figure, ABCD is a parallelogram. E and F are mid-points of sides AB and CD respectively. PQ is any line that meets AD, EF and BC in points P, O and Q respectively. Prove that PO = OQ.*



Solution. AE = $\frac{1}{2}$AB ($\because$ E is mid-point of AB)

and DF = $\frac{1}{2}$ DC ($\because$ F is mid-point of CD)

$\Rightarrow$ AE = DF ($\because$ AB $\parallel$ DC, opp. sides of $\parallel$ gm)

Also AE $\parallel$ DF (AB $\parallel$ DC, ABCD is a $\parallel$ gm)

$\Rightarrow$ AEFD is a $\parallel$ gm ($\because$ opp. sides of quad. AEFD are equal and parallel)

$\Rightarrow$ AD $\parallel$ EF

Also AD $\parallel$ BC ($\because$ ABCD is a $\parallel$ gm)

$\Rightarrow$ AD, EF and BC are parallel lines.

Now, as the transversal AB makes equal intercepts AE = EB on the three parallel lines AD, EF and BC, therefore, the transversal PQ also makes equal intercepts on these parallel lines (theorem 8)

$\Rightarrow$ PO = OQ, as required.

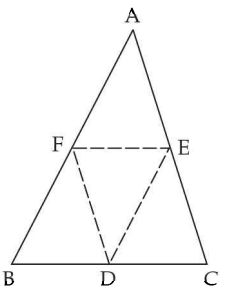
Exercise 9.2

1. In ΔABC, AB = 5 cm, BC = 8 cm and CA = 7 cm. If D and E are respectively the mid-points of AB and BC, determine the length of DE. *(NCERT Exemplar)*

2. In the adjoining figure, D, E and F are mid-points of the sides BC, CA and AB respectively of ΔABC. If AB = 6 cm, BC = 4.8 cm and CA = 5.6 cm, find the perimeter of

(*i*) the trapezium FBCE

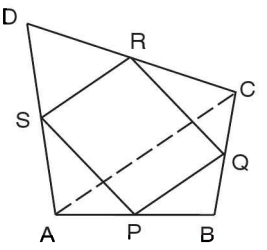
(*ii*) the triangle DEF.



3. The diagonals AC and BD of a parallelogram ABCD intersect at O. If P is the mid-point of AD, prove that

(*i*) PO $\parallel$ AB (*ii*) PO = $\frac{1}{2}$CD.

4. In the adjoining figure, ABCD is a quadrilateral in which P, Q, R and S are mid-points of AB, BC, CD and DA respectively. AC is its diagonal. Show that

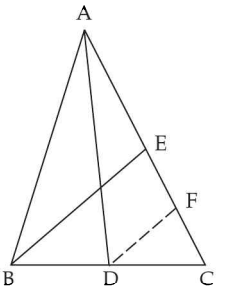


(*i*) SR || AC and SR = $\frac{1}{2}$AC

(*ii*) PQ = SR

(*iii*) PQRS is a parallelogram.

5. In the adjoining figure, AD and BE are medians of ΔABC. If DF || BE, prove that CF = $\frac{1}{4}$ AC.



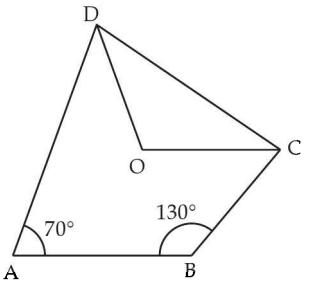
Hint: In ΔBCE, D is mid-point of BC and DF || BE, therefore, F is mid-point of CE (converse of mid-point theorem)

$\Rightarrow$ CF = $\frac{1}{2}$ CE = $\frac{1}{2}$ ($\frac{1}{2}$ AC).

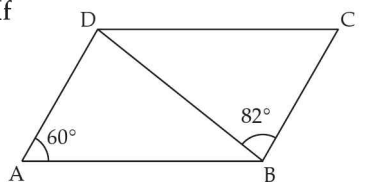
Answers

Very Short Answer Type Questions

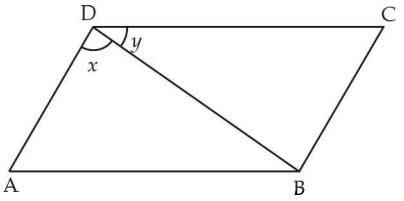
1. The diagonals AC and BD of a parallelogram ABCD intersect each other at the point O. If OC = 3.5 cm and OD = 5 cm, then find the lengths of AC and BD.
2. If one angle of a quadrilateral is 102° and the other three angles are equal. Find each of the three equal angles.
3. If the angles of a quadrilateral are $x°$, $(x + 20)°$, $2x°$ and $(3x - 80)°$, then find the greatest angle.
4. In a quadrilateral, the angles are in a ratio 2 : 4 : 5 : 7. Find the difference between the greatest and the smallest angle.
5. In the adjoining quadrilateral, ∠A = 70°, ∠B = 130° and the bisectors of ∠C and ∠D meet at O. Find ∠COD.



6. In the adjoining figure, ABCD is a parallelogram. If ∠A = 60° and ∠CBD = 82°, then find ∠CDB.

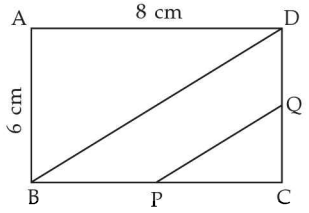


7. In a trapezium ABCD, AB || DC. If ∠A = 83°, then find ∠D.
8. In the adjoining figure, ABCD is a parallelogram. If ∠ABD = 35° and ∠ABC = 113°, then find the values of x and y.



9. If ΔABC, AB = 7.4 cm, BC = 6.5 cm and CA = 9 cm. If D and E are mid-points of AB and BC respectively, then find the length of DE.

10. In the adjoining figure, ABCD is a rectangle in which AB = 6 cm and AD = 8 cm. If P and Q are mid-points of the sides BC and CD respectively, then find the length of PQ.



Answers

Objective Type Questions

(FOR FORMATIVE/SUMMATIVE ASSESSMENT)

Mental Maths

1. Fill in the blanks:
   (*i*) The sum of interior angles of a quadrilateral is ..... right angles.
   (*ii*) If one pair of opposite sides of a quadrilateral is parallel, then it is called a ..... .
   (*iii*) If both pairs of opposite sides of a quadrilateral are parallel, then it is called a ..... .
   (*iv*) If one angle of a parallelogram is a right angle, then it is called a ..... .
   (*v*) If all the four sides of a parallelogram are equal, then it is called a ..... .
   (*vi*) If two pairs of adjacent sides of a quadrilateral are equal, then it is called a ..... .
   (*vii*) A diagonal of a parallelogram divides it into ..... congruent ..... .
   (*viii*) In a parallelogram, both pairs of opposite sides are ..... .
   (*ix*) If both pairs of opposite sides of a quadrilateral are equal, then it is a ..... .
   (*x*) In a parallelogram, both pairs of opposite angles are ..... .
   (*xi*) If both pairs of opposite angles of a quadrilateral are equal, then it is a ..... .
   (*xii*) In a parallelogram, the diagonals ..... each other.
   (*xiii*) If the diagonals of a quadrilateral bisect each other, then it is a ..... .
   (*xiv*) The line segment joining the mid-points of any two sides of a triangle is ..... to the third side and is equal to ..... of it.
   (*xv*) The line drawn through the mid-point of one side of a triangle parallel to another side ..... the third side.
2. State whether the following statements are true or false.
   (*i*) Every trapezium is a parallelogram.
   (*ii*) Every parallelogam is a trapezium.
   (*iii*) Every rhombus is a square.
   (*iv*) Every square is a rhombus.
   (*v*) Every kite is a parallelogram.
   (*vi*) Every parallelogram is a kite.
   (*vii*) Every rectangle is a square.
   (*viii*) Every square is a rectangle.
   (*ix*) The diagonals of a parallelogram are equal.

(*x*) The diagonals of a rhombus are equal.

(*xi*) The diagonals of a rectangle intersect at a right angles.

(*xii*) The diagonals of a rhombus intersect at right angles.

(*xiii*) The diagonals bisect the angles of a parallelogram.

(*xiv*) The diagonals bisect the angles of a rhombus.

(*xv*) The diagonals bisect the angles of a rectangle.

Multiple Choice Questions

Choose the correct answer from the given four options (3 to 21):

3. Three angles of a quadrilateral are 75°, 90° and 75°. The fourth angle is

(*a*) 90° (*b*) 95° (*c*) 105° (*d*) 120°

(NCERT Exemplar)

4. A quadrilateral ABCD is a trapezium if

(*a*) AB = DC (*b*) AD = BC

(*c*) $\angle A + \angle C = 180°$ (*d*) $\angle B + \angle C = 180°$

5. If PQRS is a parallelogram, then $\angle Q - \angle S$ is equal to

(*a*) 90° (*b*) 120° (*c*) 0° (*d*) 180°

6. If two adjacent angles of a parallelogram are in the ratio 3 : 2, then the angles are

(*a*) 100°, 80° (*b*) 144°, 36° (*c*) 120°, 60° (*d*) 108°, 72°

7. A diagonal of a rectangle is inclined to one side of the rectangle at 25°. The acute angle between the diagonals is

(*a*) 55° (*b*) 50° (*c*) 40° (*d*) 25°

(NCERT Exemplar)

8. ABCD is a rhombus such that $\angle ACB = 40°$. Then $\angle ADB$ is

(*a*) 40° (*b*) 45° (*c*) 50° (*d*) 60°

(NCERT Exemplar)

9. The diagonals AC and BD of a parallelogram ABCD intersect each other at the point O. If $\angle DAC = 32°$ and $\angle AOB = 70°$, then $\angle DBC$ is equal to

(*a*) 24° (*b*) 86° (*c*) 38° (*d*) 32°

(NCERT Exemplar)

10. If the diagonals of a square ABCD intersect each other at O, then ΔOAB is

(*a*) an equilateral triangle

(*b*) a right angled but not an isosceles triangle

(*c*) an isosceles but not right angled triangle

(*d*) an isosceles right angled triangle.

11. If the diagonals of a quadrilateral PQRS bisect each other, then the quadrilateral PQRS must be a

(*a*) parallelogram (*b*) rhombus (*c*) rectangle (*d*) square

12. If the diagonals of a quadrilateral PQRS bisect each other at right angles, then the quadrilateral PQRS must be a

(*a*) parallelogram (*b*) rectangle (*c*) rhombus (*d*) square

13. Which of the following statement is true for a parallelogram?

(*a*) Its diagonals are equal.

(*b*) Its diagonals are perpendicular to each other.

(*c*) The diagonals divide the parallelogram into four congruent triangles.

(*d*) The diagonals bisect each other.

14. Which of the following is not true for a parallelogram?

(*a*) opposite sides are equal

(*b*) opposite angles are equal

(*c*) opposite angles are bisected by the diagonals

(*d*) diagonals bisect each other. *(NCERT Exemplar)*

15. A quadrilateral in which the diagonals are equal and bisect each other at right angles is a

(*a*) rectangle which is not a square

(*b*) rhombus which is not a square

(*c*) kite which is not a square

(*d*) square

16. Choose the correct statement:

(*a*) The diagonals of a parallelogram are equal.

(*b*) The diagonals of a rectangle are perpendicular to each other.

(*c*) Every quadrilateral is either a parallelogram or a kite or a trapezium.

(*d*) If the diagonal of a quadrilateral intersect at right angles, then it is not necessarily a rhombus.

17. If APB and CQD are two parallel lines, then the bisectors of $\angle APQ$, $\angle BPQ$, $\angle CQP$ and $\angle PQD$ form

(*a*) a square (*b*) a rhombus

(*c*) a rectangle (*d*) any other parallelogram *(NCERT Exemplar)*

18. D and E are mid-points of the sides AB and AC of ΔABC and O is any point on the side BC. O is joined to A. If P and Q are mid-points of OB and OC respectively, then DEQP is

(*a*) a square (*b*) a rectangle (*c*) a rhombus (*d*) a parallelogram *(NCERT Exemplar)*

19. The quadrilateral formed by joining the mid-points of the sides of a quadrilateral PQRS, taken in order, is a rectangle if

(*a*) PQRS is a parallelogram (*b*) PQRS is a rectangle

(*c*) the diagonals of PQRS are perpendicular to each other

(*d*) the diagonals of PQRS are equal. *(NCERT Exemplar)*

20. The quadrilateral formed by joining the mid-points of the sides of a quadrilateral ABCD, taken in order, is a rhombus if

(*a*) ABCD is a parallelogram (*b*) ABCD is a rhombus

(*c*) The diagonals of ABCD are equal

(*d*) The diagonals of ABCD are perpendicular to each other. *(NCERT Exemplar)*

21. The figure formed by joining the mid-points of the sides of a quadrilateral ABCD, taken in order, is a square only if

(*a*) ABCD is a rhombus

(*b*) diagonals of ABCD are equal

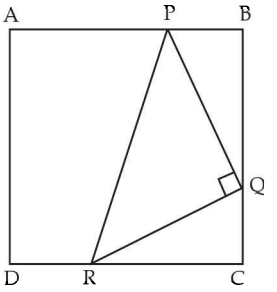
(*c*) diagonals of ABCD are perpendicular to each other

(*d*) diagonals of ABCD are equal and perpendicular to each other.

(NCERT Exemplar)

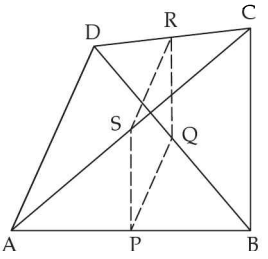
Answers

Higher Order Thinking Skills (HOTS)



1. In the adjoining figure, ABCD is a square. If $\angle PQR = 90°$ and PB = QC, then find $\angle QPR$.

2. ABCD is a rhombus in which $\angle A = 60°$. Find the ratio AC : BD.



3. In the adjoining figure, AD = BC. P, Q, R and S are mid-points of AB, BD, CD and AC respectively. Prove that PQRS is a rhombus.

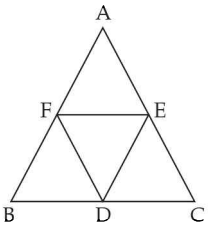
Answers

Chapter Test

1. In a quadrilateral ABCD, AO and BO are bisectors of $\angle A$ and $\angle B$ respectively. Prove that $\angle AOB = \frac{1}{2}(\angle C + \angle D)$.

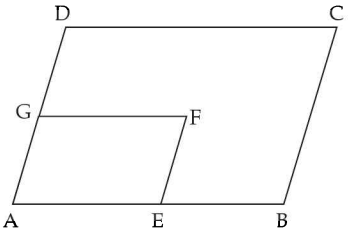
2. In the adjoining figure, it is given that BDEF and FDCE are parallelograms. Can you say that BD = CD? Why or Why not?

(NCERT Exemplar)

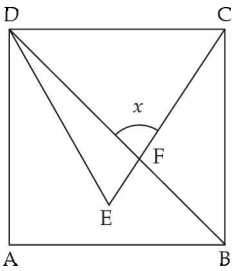


3. In the adjoining figure, ABCD and AEFG are two parallelograms. If $\angle C = 55°$, determine $\angle F$.

(NCERT Exemplar)



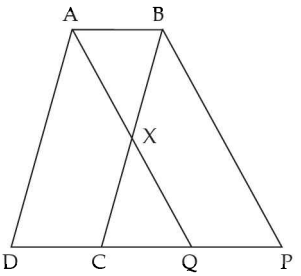
4. In the adjoining figure, CED is an equilateral triangle in the square ABCD. Find the value of x.



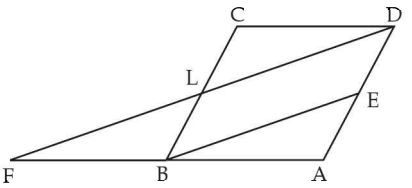
5. In the adjoining figure, ABCD is a parallelogram and X is mid-point of BC. Produce the line segment AX to meet DC produced at Q. The parallelogram ABPQ is completed. Prove that:

(*i*) $\Delta ABX \cong \Delta QCX$

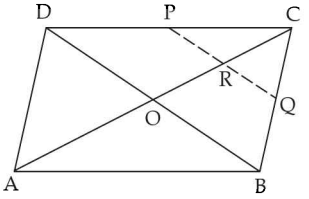
(*ii*) DC = CQ = QP.



6. In the adjoining figure, ABCD is a parallelogram and E is mid-point of AD. DL ‖ EB meets AB produced at F. Prove that B is mid-point of AF and EB = LF.



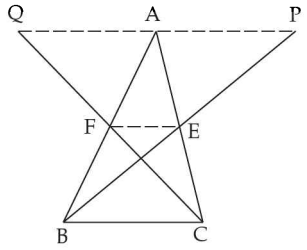
7. In the adjoining figure, ABCD is a parallelogram. If P and Q are mid-points of sides CD and BC respectively. Show that $CR = \frac{1}{4}AC$.



8. In the adjoining figure, the medians BE and CF of ΔABC are produced to points P and Q respectively such that EP = BE and FQ = CF. Prove that:

   (*i*) points Q, A and P are collinear

   (*ii*) A is mid-point of QP.



9. P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively of a quadrilateral ABCD in which AC = BD. Prove that PQRS is a rhombus. *(NCERT Exemplar)*

10. P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively of a quadrilateral ABCD in which AC $\perp$ BD. Prove that PQRS is a rectangle. *(NCERT Exemplar)*

11. Show that the quadrilateral formed by joining the mid-points of the consecutive sides of a square is also a square. *(NCERT Exemplar)*

12. Points A and B are on the same side of a line *l*. AM and BN are perpendiculars to the line *l*. If C is the mid-points of AB, prove that CM = CN.

    Hint: From C, draw CD perpendicular to line *l*, then AM, CN and BN are parallel line. By using theorem of intercepts, MD = ND.

    Show that $\Delta CMD \cong \Delta CND$.

Answers

Areas of Parallelograms and Triangles



INTRODUCTION

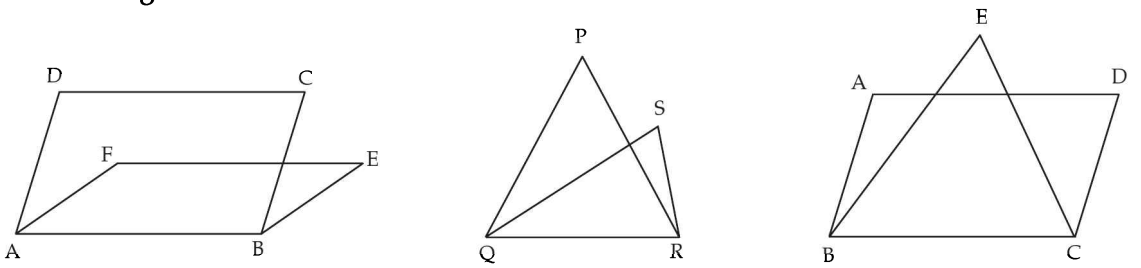
In chapter 3, we have seen that Geometry originated from the need of measuring land or recasting/refixing its boundaries in the process of distribution of certain land or field among different people. You may recall that the part of the plane enclosed by a simple closed curve is called a ***planar region*** corresponding to that figure and the magnitude (or measure) of this plane region is called the ***area*** of that figure. This magnitude of measure is always expressed with the help of a number (in some unit) such as 15 cm^2, 32 m^2, 3.5 hectares etc. So, we can say that the area of a simple closed plane figure is a number (in some unit) associated with the part of the plane enclosed by the figure.

In earlier classes, we have learnt some formulae for finding the areas of some different simple closed plane figures such as triangle, rectangle, square, parallelogram etc. In this chapter, we shall consolidate the knowledge about these formulae by studying some relationship between the areas of these geometric figures under the condition when they lie on the same (or equal) base and between the same parallel lines.

10.1 BASIC CONCEPTS

❑ Figures on the same base and between the same parallel lines

1. Figures on the same base



(i) *(ii)* *(iii)*

Fig. 10.1

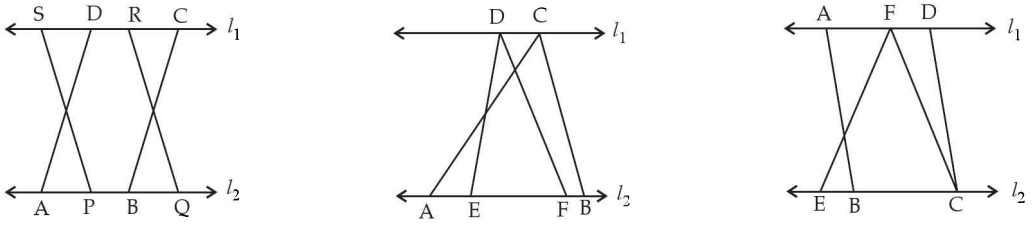
In fig. 10.1 (*i*), parallelograms ABCD and ABEF are on the same base AB.

In fig. 10.1 (*ii*), triangles PQR and SQR are on the same base QR.

In fig. 10.1 (*iii*), parallelogram ABCD and ΔEBC are on the same base BC.

2. **Figures between the same parallel lines**

Let lines l_1 and l_2 be parallel, then:



(i) *(ii)* *(iii)*

Fig. 10.2

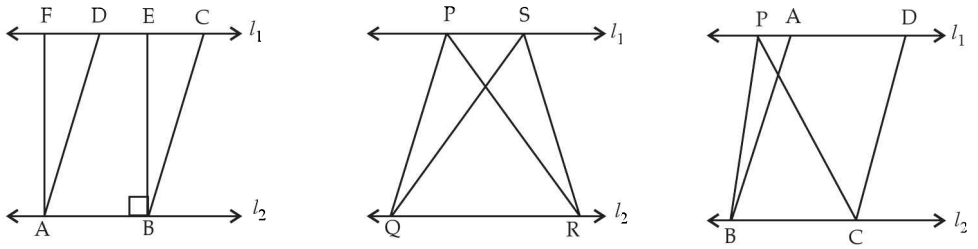
In fig. 10.2 (*i*), parallelograms ABCD and PQRS are between the same parallel lines l_1 and l_2.

In fig. 10.2 (*ii*), triangles ABC and EFD are between the same parallel lines l_1 and l_2.

In fig. 10.2 (*iii*), parallelogram ABCD and ΔECF are between the same parallel lines l_1 and l_2.

3. **Figures on the same base and between the same parallel lines**

Let lines l_1 and l_2 be parallel, then:



(i) *(ii)* *(iii)*

Fig. 10.3

In fig. 10.3 (*i*), parallelograms ABCD and rectangle ABEF are on the same base AB and between the same parallel lines l_1 and l_2.

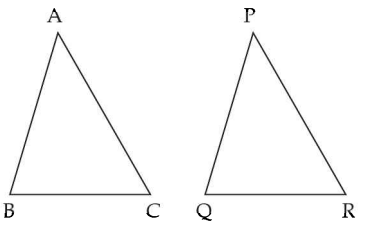
In fig. 10.3 (*ii*), triangles PQR and SQR are on the same base QR and between the same parallel lines l_1 and l_2.

In fig. 10.3 (*iii*), parallelogram ABCD and ΔPBC are on the same base BC and between the same parallel lines l_1 and l_2.

❑ **Axioms of areas**

1. ***If two figures are congruent, then the areas enclosed by these figures are equal.***

Thus, if two figures A and B are congruent, then area enclosed by A = area enclosed by B.



This is known as **congruence area axiom**.

In the adjoining figure, ΔABC ≅ ΔPQR.

So, area of ΔABC = area of ΔPQR.

Note that by area of ΔABC we mean the area of the region enclosed by the triangle ABC.

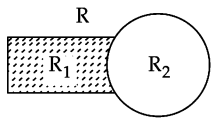
Notation. Area of ΔABC is denoted by ar(ΔABC).

2. ***If a planar region R consists of two non-overlapping planar regions R_1 and R_2, then area of region R = area of region R_1 + area of region R_2.***

This is known as **addition area axiom**.

In the adjoining figure, planar regions R_1 and R_2 are non-overlapping. If R is the total region *i.e.* the region made up of regions R_1 and R_2, then

area of region R = area of region R_1 + area of region R_2.

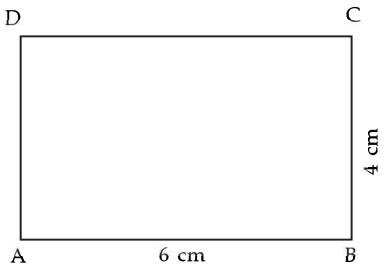


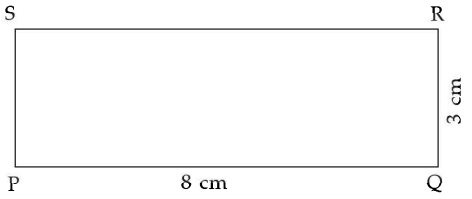
❑ Equal figures

Two figures are called equal if they have equal area.

As two congruent figures have equal area, therefore, two congruence figures are always equal figures. However the converse may not be true *i.e.* two equal figures may not be congruent.

For example, consider the two rectangles ABCD and PQRS given below:





Area of rectangle ABCD = (6×4) cm^2 = 24 cm^2 and

area of rectangle PQRS = (8×3) cm^2 = 24 cm^2

$\therefore$ Area of rectangle ABCD = area of rectangle PQRS

$\Rightarrow$ rectangles ABCD and PQRS are equal figures.

Clearly, these figures are not congruent.

❑ **A diagonal of a parallelogram divides it into two triangles of equal area.**

As a diagonal of a parallelogram divides it into two congruent triangles and congruent triangles have equal area, therefore, a diagonal of a parallelogram divides it into two triangles of equal area.

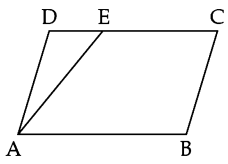
10.2 RELATIONSHIPS BETWEEN AREAS OF PARALLELOGRAMS AND TRIANGLES

❑ **Relationship between areas of two parallelograms on the same base and between the same parallel lines**

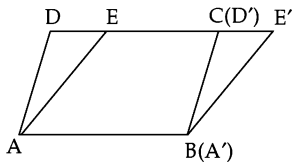
Let us find the relationship, if any, between areas of two parallelograms on the same base and between same parallel lines.

Perform the following activity:

Draw any parallelogram ABCD and draw any line AE as shown in fig. 10.4 (*i*).



(*i*)



(*ii*)

Fig. 10.4

Make a replica, say $\Delta A'D'E'$, of ΔADE with the help of a tracing paper. Paste the replica *i.e.* $\Delta A'D'E'$ in such a way that A'D' coincides with BC as shown in fig. 10.4 (*ii*). Thus, we get two parallelograms ABCD and ABE'E on the same base AB and between the same parallel lines DC and EE'.

As $\Delta ADE \cong \Delta A'D'E'$, so area of ΔADE = area $\Delta A'D'E'$.

Now, area of $\parallel$ gm ABCD = area of ΔADE + area of quad. (EABC)

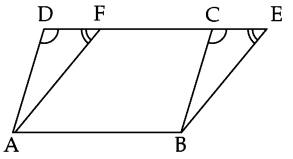
= area of $\Delta A'D'E'$ + area of quad. (EABC)

= area of $\parallel$ gm ABE'E

Thus, the two parallelograms ABCD and ABE'E are equal in area. Repeat this activity with more parallelograms. Every time we get the same result. In fact, we have:

Theorem 10.1

Parallelograms on the same base and between the same parallel lines are equal in area.



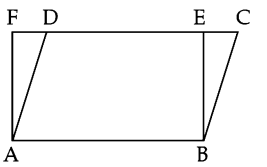
Given. Two parallelograms ABCD and ABEF on the same base AB and between the same parallel lines AB and DE.

To prove. Area of $\parallel$ gm ABCD = area of $\parallel$ gm ABEF.

Proof.

Statements	*Reasons*
In ΔADF and ΔBCE	
1. $\angle ADF = \angle BCE$	1. Corres. $\angle$s, AD $\parallel$ BC and DE is a transversal.
2. $\angle AFD = \angle BEC$	2. Corres. $\angle$s, AF $\parallel$ BE and DE is a transversal.
3. AD = BC	3. Opp. sides of $\parallel$ gm ABCD.
4. $\Delta ADF \cong \Delta BCE$	4. AAS rule of congruency.
5. area of ΔADF = area of ΔBCE	5. Congruent figures have equal area.
6. area of ΔADF + area of quad. ABCF = area of ΔBCF + area of quad. ABCF	6. Adding same area on both sides.
7. area of $\parallel$ gm ABCD = area of $\parallel$ gm ABEF	7. Addition area axiom.

Corollary 1. *A parallelogram and a rectangle on the same base and between the same parallel lines are equal in area.*



Proof. Let a parallelogram ABCD and a rectangle ABEF be on the same base AB and between the same parallel lines AB and FC (as shown in the adjoining figure).

We want to prove that area of $\parallel$ gm ABCD = area of rect. ABEF.

Since a rectangle is also a parallelogram, therefore,

area of $\parallel$ gm ABCD = area of rect. ABEF (Theorem 10.1).

Hence, a parallelogram and a rectangle on the same base and between the same parallel lines are equal in area.

Corollary 2. *Area of a parallelogram = base × height.*

Proof. By corollary 1, we have

area of parallelogram ABCD = area of rectangle ABEF ...(*i*)

(see figure of corollary 1)

Also area of rectangle ABEF = AB × BE ...(*ii*)

From (*i*) and (*ii*), we get

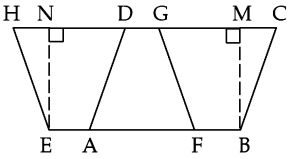
area of parallelogram ABCD = AB × BE = base × height.

Hence, the area of a parallelogram = base × height.

Corollary 3. *Parallelograms with equal bases and between the same parallels are equal in area.*

Proof. Let ABCD and EFGH be two parallelograms with equal bases *i.e.* AB = EF and between the same parallel lines EB and HC.

From B, draw BM ⊥ DC and from E, draw EN ⊥ HG, then BM = EN



(∵ EBMN is a rectangle (why?)),
so opp. sides are equal *i.e.* BM = EN)

By corollary 2,

area of parallelogram ABCD = base × height = AB × BM
= EF × EN (∵ AB = EF and BM = EN)
= area of parallelogram EFGH.

Hence, parallelograms with equal bases and between the same parallel lines are equal in area.

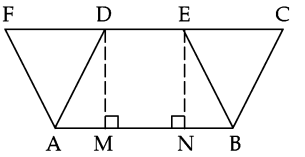
❑ **Converse of theorem 10.1**

The converse of the above theorem 10.1 is also true. In fact, we have:

Theorem 10.2

Parallelograms on the same base and having equal areas lie between same parallel lines.

Given. Two parallelograms ABCD and ABEF on the same base AB and area of ‖ gm ABCD = area of ‖ gm ABEF.



To prove. AB ‖ FC.

Construction. From D, draw DM ⊥ AB and from E, draw EN ⊥ AB.

Proof.

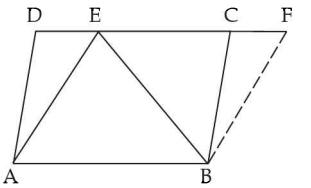
Statements	*Reasons*
1. Area of ‖ gm ABCD = AB × DM	1. Area of a ‖ gm = base × height.
2. Area of ‖ gm ABEF = AB × EN	2. Area of a ‖ gm = base × height.
3. AB × DM = AB × EN ⇒ DM = EN	3. Area of ‖ gm ABCD = area of ‖ gm ABEF (given)
4. DM ‖ EN	4. DM and EN are both perpendicular to the same line AB.
5. DMNE is a parallelogram.	5. Two sides DM and EN of quad. DMNE are equal and parallel.
6. MN ‖ DE *i.e.* AB ‖ FC	6. By definition of a ‖ gm.

Corollary. *Parallelograms on equal bases and having equal areas lie between same parallel lines.*

Theorem 10.3

Area of a triangle is half that of a parallelogram on the same base and between the same parallel lines.

Given. A triangle ABE and a parallelogram ABCD on the same base AB and between the same parallel lines AB and DC.



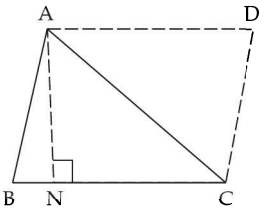
To prove. Area of ΔABE = $\frac{1}{2}$ area of ‖ gm ABCD.

Construction. Through B, draw BF ‖ AE to meet DC (produced) at F.

Proof.

Statements	*Reasons*
1. ABFE is a parallelogram	1. By construction.
2. Area of $\Delta ABE = \frac{1}{2}$ area of $\parallel$ gm ABFE	2. BE is a diagonal of $\parallel$ gm ABFE, and a diagonal divides it into two triangles of equal areas.
3. Area of $\parallel$ gm ABCD = area of $\parallel$ gm ABFE	3. Parallelograms on the same base and between the same parallels are equal in area.
4. Area of $\Delta ABE = \frac{1}{2}$ area of $\parallel$ gm ABCD	4. From 2 and 3.

Corollary 1. *Area of a triangle* $= \frac{1}{2}$ *base* $\times$ *height*



Proof. Let ABC be a triangle with base BC and AN $\perp$ BC, then height of ΔABC = AN. Through A and C draw lines parallel to BC and BA to meet at D, then ABCD is a parallelogram.

Thus, ΔABC and parallelogram ABCD are on the same base BC and between same parallel lines BC and AD.

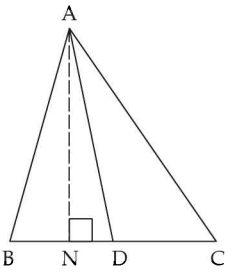
Area of $\Delta ABC = \frac{1}{2}$ area of parallelogram ABCD (Theorem 10.3)

$= \frac{1}{2}$ BC $\times$ AN (area of $\parallel$ gm = base $\times$ height)

$= \frac{1}{2}$ base $\times$ height.

Corollary 2. *A median of a triangle divides it into two triangles of equal areas.*

Proof. Let ABC be any triangle and AD be one of its medians (shown in the adjoining figure).



We need to show that ar (ΔABD) = ar (ΔACD).

From A, draw AN $\perp$ BC.

Now, ar (ΔABD) $= \frac{1}{2}$ base $\times$ corresponding height

$= \frac{1}{2}$ BD $\times$ AN $= \frac{1}{2}$ DC $\times$ AN

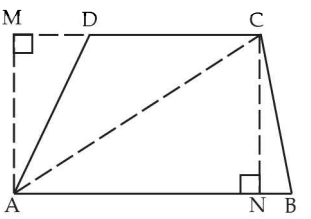
($\because$ D is mid-point of BC, so BD = DC)

= ar (ΔACD).

Hence, a median of a triangle divides it into two triangles of equal area.

Corollary 3. *Area of a trapezium* $= \frac{1}{2}$ *(sum of parallel sides)* $\times$ *height.*

Proof. Let ABCD be a trapezium in which AB $\parallel$ DC. Join AC. From C, draw CN $\perp$ AB and from A, draw AM $\perp$ CD (produced).



Then CN = AM = height of trapezium

$= h$ (say).

Area of trapezium ABCD = ar (ΔABC) + ar (ΔACD)

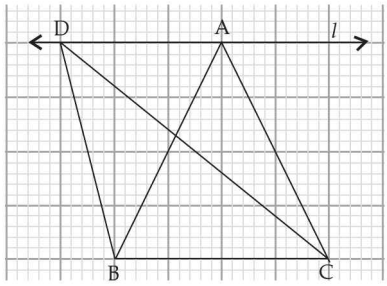
$= \frac{1}{2}$ AB $\times$ CN + $\frac{1}{2}$ DC $\times$ AM

$= \frac{1}{2}$ (AB $\times h$ + DC $\times h$) $= \frac{1}{2}$ (AB + DC) $\times h$

$= \frac{1}{2}$ (sum of parallel sides) $\times$ height.

❑ **Relationship between areas of two triangles on the same base and between the same parallel lines.**

To know the relationship between the areas of two triangles on the same base and between same parallel lines, perform the following activity:

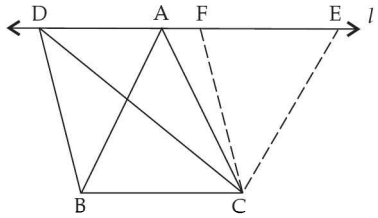


On a graph paper, draw any two triangles ABC and BCD on the same base BC and between same parallel lines BC and l (as shown in the adjoining figure). Find the areas of these two triangles by counting squares (using process of estimating area learnt in class VI). We observe that these triangles are equal in area *i.e.* area of ΔABC = area of ΔBCD.

Repeat this activity with more such triangles. Every time we get the same result. In fact, we have:

Theorem 10.4

Triangles on the same base (or equal bases) and between the same parallel lines are equal in area.



Given. Two triangles ABC and BCD on the same BC and between the same parallel lines BC and l.

To Prove. Area of ΔABC = area of ΔBCD.

Construction. Through C, draw CE || BA and CF || BD to meet line l at E and F respectively.

Proof.

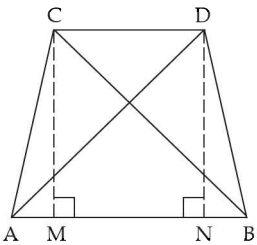
Statements	*Reasons*
1. ABCE is a parallelogram.	1. BC || AE (given), CE || BA (Const.)
2. BCFD is a parallelogram	2. BC || DF (given), CF || BD (Const.)
3. Area of || gm ABCE = area of || gm BCFD.	3. Parallelogram on same base and between same parallel lines.
4. Area of $\Delta ABC = \frac{1}{2}$ area of || gm ABCE	4. Area of a Δ is half that of a || gm on the same base and between same parallels.
5. Area of $\Delta BCD = \frac{1}{2}$ area of || gm BCFD	5. Area of a Δ is half that of a || gm on the same base and between same parallels.
6. Area of ΔABC = area of ΔBCD	6. From 3, 4 and 5.

❑ **Converse of theorem 10.4**

The converse of the above theorem 10.4 is also true. In fact, we have:

Theorem 10.5

Triangles on the same base (or equal bases) and having equal areas lie between the same parallel lines.



Given. Two triangles ABC and ABD on the same base AB, and area of ΔABC = area of ΔABD.

To prove. CD || AB.

Construction. From C and D, draw perpendiculars CM and DN on AB respectively.

Proof.

Statements	*Reasons*
1. Area of $\Delta ABC = \frac{1}{2}$ AB × CM	1. Area of a triangle = $\frac{1}{2}$ base × height.
2. Area of $\Delta ABD = \frac{1}{2}$ AB × DN	2. Same as above.
3. $\frac{1}{2}$ AB × CM = $\frac{1}{2}$ AB × DN	3. Area of ΔABC = area of ΔABD (given)
4. CM = DN	4. From 3, cancelling $\frac{1}{2}$ AB.
5. CM ‖ DN	5. CM and DN are both perpendiculars to the same line AB.
6. CMND is a parallelogram.	6. Two sides CM and DN of quad. CMND are equal and parallel.
7. CD ‖ MN *i.e.* CD ‖ AB	7. By definition of a ‖ gm.

Corollary 1. Triangles on the same base (or equal bases) and having equal areas have equal corresponding altitudes.

(In the above proof of the theorem, we obtained CM = DN. So, ΔABC and ΔABD have equal corresponding altitudes.)

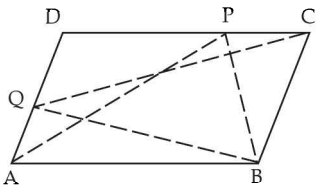
Corollary 2. If two triangles lie between the same parallels (*i.e.* have equal altitudes), then the ratio of their areas equals the ratio of their bases.

Corollary 3. If two triangles have equal bases, then the ratio of their areas equals the ratio of their altitudes.

Illustrative Examples

Example 1. *P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD. Show that ar(ΔAPB) = ar(ΔBQC).* *(NCERT)*

Solution. Given a parallelogram ABCD, and P and Q are points lying on the sides DC and AD respectively as shown in the adjoining figure.



As ΔAPB and ‖ gm ABCD are on the same base and between the same parallels AB and DC,

$$\text{ar}(\Delta APB) = \frac{1}{2} \text{ ar } (\| \text{ gm ABCD}) \qquad ...(i)$$

Also, as ΔBQC and ‖ gm ABCD are on the same BC and between the same parallels AD and BC,

$$\text{ar}(\Delta BQC) = \frac{1}{2} \text{ ar } (\| \text{ gm ABCD}) \qquad ...(ii)$$

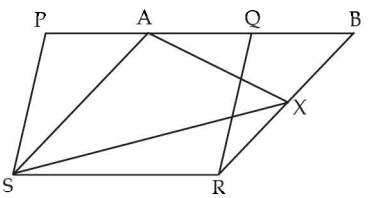
From (*i*) and (*ii*), we get

$$\text{ar}(\Delta APB) = \text{ar } (\Delta BQC).$$

Example 2. *In the adjoining figure, PQRS and ABRS are parallelograms and X is any point on the side BR. Show that*

(i) ar(‖ gm PQRS) = ar(‖ gm ABRS)

(ii) $ar(\Delta AXS) = \frac{1}{2} ar(\| gm\ PQRS)$. *(NCERT)*



Solution. (*i*) As parallelogram PQRS and ABRS are on the same base SR and between the same parallels SR and PB,

$$\text{ar}(\parallel \text{gm PQRS}) = \text{ar}(\parallel \text{gm ABRS}).$$

(*ii*) As ΔAXS and $\parallel$ gm ABRS are on the same base AS and between same parallels AS and BR,

$$\text{ar}(\Delta AXS) = \frac{1}{2}\text{ar}(\parallel \text{gm ABRS})$$

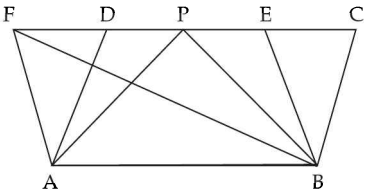
$$= \frac{1}{2}\text{ar}(\parallel \text{gm PQRS}) \qquad \text{(using part } (i))$$

Example 3. *In the adjoining figure, ABCD and ABEF are parallelograms and P is any point on DC. If ar ($\parallel$ gm ABCD) = 90 cm^2, find:*

(i) ar ($\parallel$ gm ABEF)

(ii) ar (ΔABP)

(iii) ar (ΔBEF). *(NCERT Exemplar)*



Solution. (*i*) As parallelograms ABCD and ABEF are on the same base AB and between same parallels AB and FC,

$$\text{ar}(\parallel \text{gm ABEF}) = \text{ar}(\parallel \text{gm ABCD}) = 90 \text{ cm}^2 \qquad \text{(given)}$$

(*ii*) As ΔABP and $\parallel$ gm ABCD are on the same base AB and between same parallels AB and DC,

$$\text{ar}(\Delta ABP) = \frac{1}{2}\text{ar}(\parallel \text{gm ABCD}) = \left(\frac{1}{2} \times 90\right) \text{cm}^2 = 45 \text{ cm}^2$$

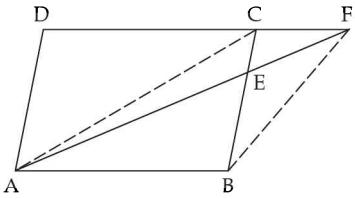
(*iii*) As a diagonals of a $\parallel$ gm divides it into two congruent triangles of equal area,

$$\text{ar}(\Delta BEF) = \frac{1}{2}\text{ar}(\parallel \text{gm ABEF}) = \left(\frac{1}{2} \times 90\right) \text{cm}^2 = 45 \text{ cm}^2. \qquad \text{(using part } (i))$$

Example 4. *A point E is taken on the side BC of a parallelogram ABCD. AE and DC are produced to meet at F. Prove that*

ar (ΔADF) = ar (quad. ABFC). *(NCERT Exemplar)*

Solution. Join AC and BF.



As triangles ACF and BCF have same base CF and are between the same parallels AB and CF ($\because$ AB $\parallel$ DC),

$$\text{ar}(\Delta ACF) = \text{ar}(\Delta BCF) \qquad \ldots(i)$$

As diagonal AC divides $\parallel$ gm ABCD into two triangles of equal area,

$$\text{ar}(\Delta DAC) = \text{ar}(\Delta ABC) \qquad \ldots(ii)$$

On adding (*i*) and (*ii*), we get

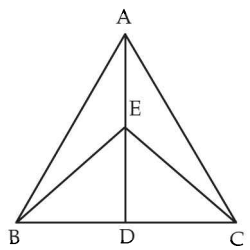
$$\text{ar}(\Delta ACF) + \text{ar}(\Delta DAC) = \text{ar}(\Delta BCF) + \text{ar}(\Delta ABC)$$

$$\Rightarrow \quad \text{ar}(\Delta ADF) = \text{ar}(\text{quad. ABFC}).$$

Example 5. *In a triangle ABC, E is mid-point of median AD.*

Show that ar (ΔBED) = $\frac{1}{4}$ ar (ΔABC). *(NCERT)*

Solution. We know that a median of a triangle divides it into two triangles of equal area.



As AD is median of ΔABC,

$$\therefore \qquad \text{ar}(\Delta ABD) = \text{ar}(\Delta ACD)$$

$$\Rightarrow \qquad \text{ar}(\Delta ABD) = \frac{1}{2}\text{ar}(\Delta ABC) \qquad \ldots(i)$$

Also E is mid-point of AD *i.e.* BE is a median of ΔABD,

$\therefore$ $\quad \text{ar}(\Delta BED) = \text{ar}(\Delta ABE)$

$\Rightarrow$ $\quad \text{ar}(\Delta BED) = \frac{1}{2}\text{ar}(\Delta ABD)$

$= \frac{1}{2} \times \frac{1}{2}\text{ar}(\Delta ABC)$ (using (*i*))

$\Rightarrow$ $\quad \text{ar}(\Delta BED) = \frac{1}{4}\text{ar}(\Delta ABC).$

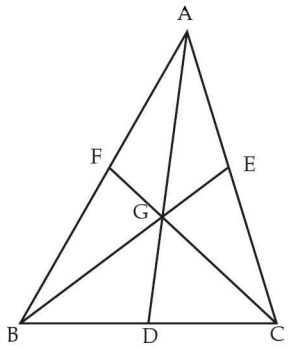
Example 6. *If the medians of a triangle ABC intersect at G, prove that*

$ar\ (\Delta AGB) = ar\ (\Delta BGC) = ar\ (\Delta CGA) = \frac{1}{3}\ ar\ (\Delta ABC).$ *(NCERT Exemplar)*

Solution. A triangle ABC such that its medians AD, BE and CF intersect at G.

We want to prove that $\text{ar}(\Delta AGB) = \text{ar}(\Delta BGC) = \text{ar}(\Delta CGA)$

$= \frac{1}{3}\text{ar}(\Delta ABC).$



We know that a median of a triangle divides it into two triangles of equal area.

In ΔABC, AD is median

$\therefore$ $\quad \text{ar}(\Delta ABD) = \text{ar}(\Delta ACD)$ ...(*i*)

In ΔBGC, GD is median

$\therefore$ $\quad \text{ar}(\Delta GBD) = \text{ar}(\Delta GDC)$ ...(*ii*)

Subtracting (*ii*) from (*i*), we get

$\text{ar}(\Delta ABD) - \text{ar}(\Delta GBD) = \text{ar}(\Delta ACD) - \text{ar}(\Delta GDC)$

$\Rightarrow$ $\text{ar}(\Delta AGB) = \text{ar}(\Delta CGA)$ ...(*iii*)

Similarly, $\text{ar}(\Delta AGB) = \text{ar}(\Delta BGC)$ ...(*iv*)

From (*iii*) and (*iv*), we get

$\Rightarrow$ $\text{ar}(\Delta AGB) = \text{ar}(\Delta BGC) = \text{ar}(\Delta CGA)$ ...(*v*)

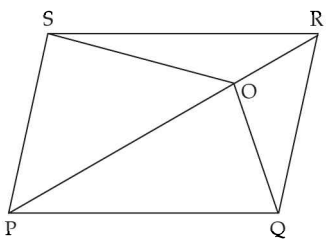
From fig., $\text{ar}(\Delta AGB) + \text{ar}(\Delta BGC) = \text{ar}(\Delta CGA) = \text{ar}(\Delta ABC)$

$\Rightarrow$ $3\ \text{ar}(\Delta AGB) = \text{ar}(\Delta ABC) \Rightarrow \text{ar}(\Delta AGB) = \frac{1}{3}\text{ar}(\Delta ABC).$

Hence, $\text{ar}(\Delta AGB) = \text{ar}(\Delta BGC) = \text{ar}(\Delta CGA) = \frac{1}{3}\text{ar}(\Delta ABC).$

Example 7. *In the adjoining figure, O is any point on the diagonal PR of parallelogram PQRS. Prove that*

ar (ΔPSO) = ar (ΔPQO). *(NCERT Exemplar)*



Solution. Join SQ. Let diagonals PR and SQ of parallelogram intersect at the point M. As diagonals of a parallelogram bisect each other, M is mid-point of SQ.

In ΔPQS, PM is median

$\therefore$ $\text{ar}(\Delta PSM) = \text{ar}(\Delta PQM)$ ...(*i*)

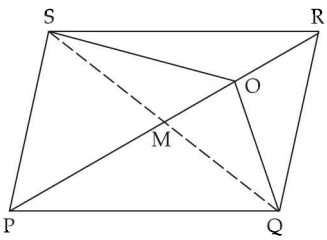
In ΔSQO, OM is median

$\therefore$ $\text{ar}(\Delta SMO) = \text{ar}(\Delta QMO)$ ...(*ii*)

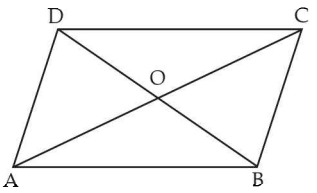
On adding (*i*) and (*ii*), we get

$\text{ar}(\Delta PSM) + \text{ar}(\Delta SMO) = \text{ar}(\Delta PQM) + \text{ar}(\Delta QMO)$

$\Rightarrow$ $\text{ar}(\Delta PSO) = \text{ar}(\Delta PQO).$



Example 8. *Show that the diagonals of a parallelogram divide it into four triangles of equal area.* *(NCERT)*



Solution. Let ABCD be a parallelogram and its diagonals intersect at O. As the diagonals of a parallelogram bisect each other OA = OC and OB = OD.

We know that a median of a triangle divides it into two triangles of equal area.

As OA = OC, OB is median of ΔABC,

$\therefore$ $\quad ar(\Delta OAB) = ar(\Delta OBC)$ $\quad ...(i)$

As OB = OD, OC is median of ΔBCD,

$\therefore$ $\quad ar(\Delta OBC) = ar(\Delta OCD)$ $\quad ...(ii)$

As OA = OC, OD is median of ΔACD,

$\therefore$ $\quad ar(\Delta OCD) = ar(\Delta OAD)$ $\quad ...(iii)$

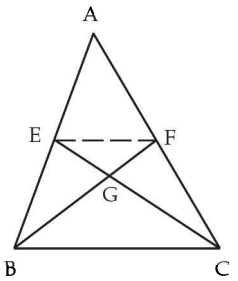
From (*i*), (*ii*) and (*iii*), we get

$ar(\Delta OAB) = ar(\Delta OBC) = ar(\Delta OCD) = ar(\Delta OAD)$.

Therefore, the diagonals of a parallelogram divide it into four triangles of equal area.

Example 9. *The medians BE and CF of a triangle ABC intersect at G. Prove that ar (ΔGBC) = ar (quad. AFGE).* *(NCERT Exemplar)*

Solution. Join EF.



As E and F are mid-points of sides AB and AC respectively,

$EF \parallel BC$.

Thus, triangles EBC and FBC are on the same base BC and between the same parallels, therefore

$ar(\Delta EBC) = ar(\Delta FBC)$

$\Rightarrow$ $ar(\Delta EBC) - ar(\Delta GBC) = ar(\Delta FBC) - ar(\Delta GBC)$

$\Rightarrow$ $ar(\Delta BGE) = ar(\Delta CGF)$ $\quad ...(i)$

As F is mid-point of AC, BF is a median of ΔABC,

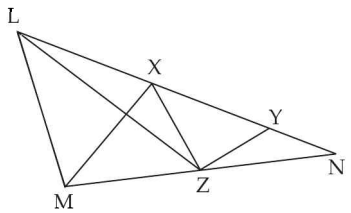
$\therefore$ $\quad ar(\Delta ABF) = ar(\Delta FBC)$ $\quad ...(ii)$

Subtracting (*i*) from (*ii*), we get

$ar(\Delta ABF) - ar(\Delta BGE) = ar(\Delta FBC) - ar(\Delta CGF)$

$\Rightarrow$ $ar(\text{quad. } AEGF) = ar(\Delta GBC)$, as required.

Example 10. *In the adjoining figure, X and Y are points on the side LN of triangle LMN. Through X, a line is drawn parallel to LM to meet MN at Z. Prove that ar (ΔLZY) = ar (quad. MZYX).* *(NCERT Exemplar)*



Solution. Triangles LZX and MZX are on the same base ZX and are between the same parallel lines XZ and LM,

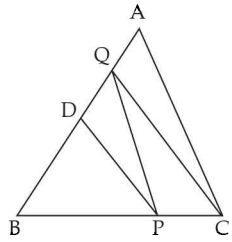
$\therefore$ $\quad ar(\Delta LZX) = ar(\Delta MZX)$

Adding same $ar(\Delta XZY)$ on both sides, we get

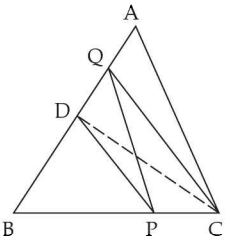
$ar(\Delta LZX) + ar(\Delta XZY) = ar(\Delta MZX) + ar(\Delta XYZ)$

$\Rightarrow$ $ar(\Delta LZY) = ar$ (quad. MZYX).

Example 11. *In the adjoining figure, D is mid-point of AB and P is any point on side BC of ΔABC. If CQ $\parallel$ PD meets AB in Q, then prove that $ar(\Delta BPQ) = \frac{1}{2} ar(\Delta ABC)$.* *(NCERT Exemplar)*



Solution. Join CD.



Triangles DPQ and DPC are on the same base PD and between same parallels CQ and PD,

$\therefore$ ar(ΔPDQ) = ar(ΔDPC) ...(*i*)

As D is mid-point of AB, so CD is a median of ΔABC. Since a median divides a triangle into two triangles of equal area,

$$\text{ar}(\Delta BCD) = \frac{1}{2}\text{ar}(\Delta ABC)$$

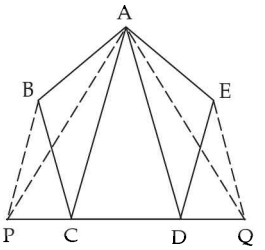
$$\Rightarrow \quad \text{ar}(\Delta BPD) + \text{ar}(\Delta DPC) = \frac{1}{2}\text{ar}(\Delta ABC)$$

$$\Rightarrow \quad \text{ar}(\Delta BPD) + \text{ar}(\Delta DPQ) = \frac{1}{2}\text{ar}(\Delta ABC) \qquad \text{(using } (i)\text{)}$$

$$\Rightarrow \quad \text{ar}(\Delta BPQ) = \frac{1}{2}\text{ar}(\Delta ABC).$$

Example 12. *In the adjoining figure, ABCDE is any pentagon. BP drawn parallel to AC meets DC produced at P and EQ drawn parallel to AD meets CD produced at Q. Prove that ar(ABCDE) = ar(ΔAPQ).*

(NCERT Exemplar)



Solution. ΔPCA and ΔBCA are on the same base CA and between same parallels BP ∥ AC.

$\therefore$ ar(ΔBCA) = ar(ΔPCA) ...(*i*)

ΔEAD and ΔQAD are on the same base AD and between same parallels EQ ∥ AD,

$\therefore$ ar(ΔEAD) = ar(ΔQAD) ...(*ii*)

Also, ar(ΔACD) = ar(ΔACD) ...(*iii*)

On adding (*i*), (*iii*) and (*ii*), we get

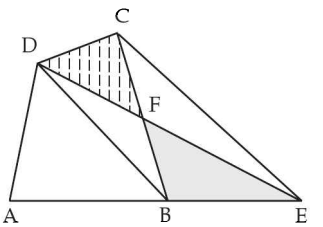
ar (ΔBCA) + ar(ΔACD) + ar(ΔEAD) = ar(ΔPCA) + ar(ΔACD) + ar(ΔQAD)

$\Rightarrow$ ar(ABCDE) = ar(ΔAPQ).

Example 13. *A villager Itwaari has a plot of land in the shape of a quadrilateral. The Gram Panchayat of the village decided to take over some portion of his plot from one of the corners to construct a Health Centre. Itwaari agrees to the above proposal with the condition that he should be given equal area of land in lieu of his land adjoining to his plot so as to form a triangular plot. Explain how this proposal will be implemented.*

What value will be promoted by establishing a health centre in the village? ***(Value Based)***

Solution. Let Itwaari have a plot of land in the shape of quad. ABCD. Join BD.



Through C, draw a line parallel to DB to meet AB produced at E. Let DE and BC intersect at F.

Now, triangles DBE and DBC have the same base BD and are between the same parallels DB ∥ CE (by construction)

$\therefore$ ar(ΔDBC) = ar(ΔDBE)

$\Rightarrow$ ar(ΔDBC) − ar(ΔDBF) = ar(ΔDBE) − ar(ΔDBF)

(subtracting same area from both sides)

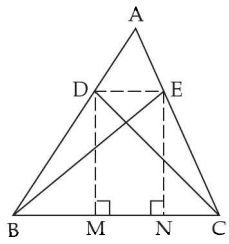
$\Rightarrow$ ar(ΔDCF) = ar(ΔBEF).

Thus, we observe that if Itwaari gives triangular plot DCF to the Gram Panchayat for Health Centre and in lieu of that takes triangular plot BEF adjoining to his remaining plot then his land will be in the shape of triangular plot DAE.

Health Centre is needed in every village to take care of the health of villagers.

Example 14. *If D and E are points on sides AB and AC respectively of ΔABC such that ar (ΔDBC) = ar (ΔEBC), then prove that DE ∥ BC.* *(NCERT)*

Solution. Join DE. From D, draw DM ⊥ BC and from E, draw EN ⊥ BC.



Given $\quad ar(\Delta DBC) = ar(\Delta EBC)$

$\Rightarrow \quad \frac{1}{2} BC \times DM = \frac{1}{2} BC \times EN$

$\Rightarrow \quad DM = EN$

Also, as DM and EN are both perpendicular to AB, therefore, DM ∥ EN.

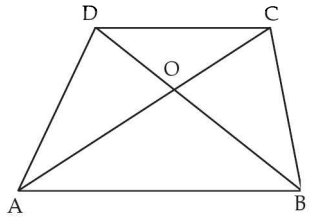
Thus, in quadrilateral DMNE, DM = EN and DM ∥ EN, therefore, DMNE is a parallelogram.

$\Rightarrow \quad DE \parallel MN$ (by def. of a parallelogram)

$\Rightarrow \quad DE \parallel BC.$

Example 15. *Diagonals AC and BD of a trapezium ABCD with AB ∥ DC intersect each other at O. Prove that ar (ΔAOD) = ar (ΔBOC).* *(NCERT)*

Solution. As the triangle ABC and ABD are on the same base AB and are between the same parallels AB ∥ DC,



$\therefore \quad ar(\Delta ABC) = ar(\Delta ABD)$

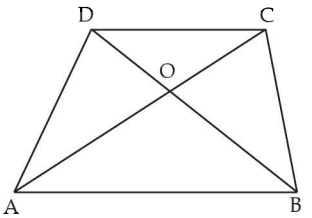
$\Rightarrow \quad ar(\Delta ABC) - ar(\Delta OAB) = ar(\Delta ABD) - ar(\Delta OAB)$

(subtracting equal area from both sides)

$\Rightarrow \quad ar(\Delta BOC) = ar(\Delta AOD)$, as required.

Example 16. *Diagonals AC and BD of a quadrilateral ABCD intersect at O in such a way that ar (ΔAOD) = ar (ΔBOC). Prove that ABCD is a trapezium.* *(NCERT)*

Solution. Given ar (ΔAOD) = ar (ΔBOC).



Adding ar (ΔOAB) to both sides, we get

$ar(\Delta AOD) + ar(\Delta OAB) = ar(\Delta BOC) + ar(\Delta OAB)$

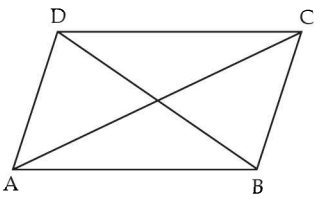
$\Rightarrow \quad ar(\Delta ABD) = ar(\Delta ABC).$

Thus, the triangles ABD and ABC have equal areas and have the same base AB, therefore, these triangles lie between the same parallels, so AB ∥ DC (see example 14).

Therefore, ABCD is a trapezium.

Example 17. *If each diagonal of a quadrilateral divides it into two triangles of equal area, then prove that the quadrilateral is a parallelogram.*

Solution. Let ABCD be a quadrilateral such that each of its diagonal divides it into two triangles of equal area *i.e.*



$ar(\Delta ABC) = ar(\Delta ACD) \quad ...(i)$

and $\quad ar(\Delta ABD) = ar(\Delta BCD) \quad ...(ii)$

Now, $ar(\Delta ABC) + ar(\Delta ACD) = ar$ (quad. ABCD)

$\Rightarrow \quad 2\ ar\ (\Delta ABC) = ar$ (quad. ABCD) [using (*i*)]

$\Rightarrow \quad ar(\Delta ABC) = \frac{1}{2} ar$ (quad. ABCD) ...(*iii*)

Also, $ar(\Delta ABD) + ar(\Delta BCD) = ar$ (quad. ABCD)

$\Rightarrow \quad 2\ ar(\Delta ABD) = ar$ (quad. ABCD) [using (*ii*)]

$\Rightarrow \quad ar(\Delta ABD) = \frac{1}{2} ar$ (quad. ABCD) ...(*iv*)

From (*iii*) and (*iv*), we get

$ar(\Delta ABC) = ar(\Delta ABD).$

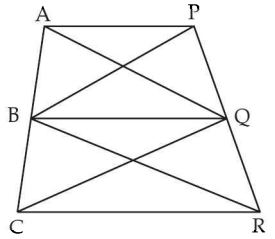
Thus, the two triangles ABC and ABD have equal areas and are on the same base AB, therefore, these triangles lie between the same parallels *i.e.* DC || AB.

Similarly, AD || BC.

Hence, ABCD is a parallelogram.

Example 18. *In the adjoining figure, AP || BQ || CR.*

Prove that ar (ΔAQC) = ar (ΔPBR). *(NCERT)*



Solution. As triangles ABQ and PBQ have the same base BQ and are between the same parallel AP || BQ (given),

$\therefore$ $\quad ar\,(\Delta ABQ) = ar\,(\Delta PBQ) \quad \ldots(i)$

Also, as triangles BQC and BQR have the same base BQ and are between the same parallels BQ || CR (given),

$\therefore$ $\quad ar\,(\Delta BQC) = ar\,(\Delta BQR) \quad \ldots(ii)$

Adding (*i*) and (*ii*), we get

$ar\,(\Delta ABQ) + ar\,(\Delta BQC) = ar\,(\Delta PBQ) + ar\,(\Delta BQR)$

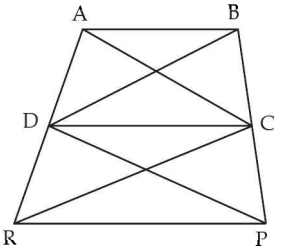
$\Rightarrow$ $\quad ar\,(\Delta AQC) = ar\,(\Delta PBR).$

Example 19. *In the adjoining figure, if*

ar (ΔDRC) = ar (ΔDPC) and

ar (ΔBDP) = ar (ΔARC), then

show that both the quadrilaterals ABCD and DCPR are trapeziums. *(NCERT)*



Solution. Given $\quad ar\,(\Delta ARC) = ar\,(\Delta BDP) \quad \ldots(i)$

and $\quad ar\,(\Delta DRC) = ar\,(\Delta DPC) \quad \ldots(ii)$

Subtracting (*ii*) from (*i*), we get

$ar\,(\Delta ARC) - ar\,(\Delta DRC) = ar\,(\Delta BDP) - ar\,(\Delta DPC)$

$\Rightarrow$ $\quad ar\,(\Delta ADC) = ar\,(\Delta BDC) \quad \ldots(iii)$

Now, triangles ADC and BDC have equal areas (from (*iii*)) and are on the same base DC, therefore, these triangles lie between the same parallels *i.e.* AB || DC

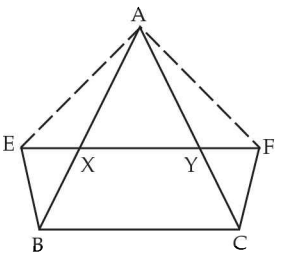
$\Rightarrow$ ABCD is a trapezium.

Also, triangles DRC and DPC have equal areas (from (*i*)) and are on the same base DC, therefore, these triangles lie between the same parallels *i.e.* DC || RP

$\Rightarrow$ DCPR is a trapezium.

Example 20. *XY is a line parallel to side BC of a triangle ABC. If BE || CA and FC || AB meet XY at E and F respectively, show that ar (ΔABE) = ar (ΔACF).* *(NCERT)*

Solution. As ΔABE and || gm EBCY have the same base BE and are between the same parallels BE || CA (given),



$\therefore$ $\quad ar\,(\Delta ABE) = \frac{1}{2}\,ar\,(\|\text{ gm EBCY}) \quad \ldots(i)$

As ΔACF and || gm XBCF have the same base CF and are between the same parallels FC || AB (given),

$\therefore$ $\quad ar\,(\Delta ACF) = \frac{1}{2}\,ar\,(\|\text{ gm XBCF}) \quad \ldots(ii)$

But || gm EBCY and || gm XBCF have the same base BC and are between the same parallels (XY || BC given),

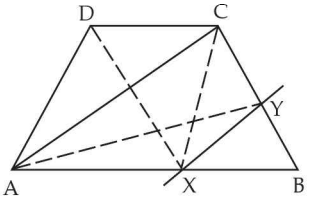
$\therefore$ $\quad ar\,(\|\text{ gm EBCY}) = ar\,(\|\text{ gm XBCF})$

$\Rightarrow \quad \frac{1}{2}\text{ar}(\|\text{ gm EBCY}) = \frac{1}{2}\text{ar}(\|\text{ gm XBCF})$

$\Rightarrow \quad \text{ar}(\Delta\text{ABE}) = \text{ar}(\Delta\text{ACF})$ (using (*i*) and (*ii*))

Example 21. *ABCD is a trapezium with AB ‖ DC. A line parallel to AC intersects AB at X and BC at Y. Prove that ar(ΔADX) = ar(ΔACY).* *(NCERT)*

Solution. Join CX.



As triangles ADX and ACX have same base AX and are between the same parallels (AB ‖ DC given, so, AX ‖ DC),

$\therefore \quad \text{ar}(\Delta\text{ADX}) = \text{ar}(\Delta\text{ACX}) \quad \ldots(i)$

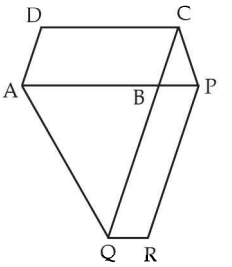
As triangles ACY and ACX have same base AC and are between the same parallels (XY ‖ AC given),

$\therefore \quad \text{ar}(\Delta\text{ACY}) = \text{ar}(\Delta\text{ACX}) \quad \ldots(ii)$

From (*i*) and (*ii*), we get

$\text{ar}(\Delta\text{ADX}) = \text{ar}(\Delta\text{ACY})$.

Example 22. *In the adjoining figure, the side AB of a parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed. Show that ar(‖ gm ABCD) = ar(‖ gm PBQR).* *(NCERT)*



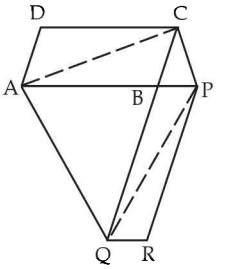
Solution. Join AC and PQ.

As AC is a diagonal of ‖ gm ABCD,

$\text{ar}(\|\text{ gm ABCD}) = 2\,\text{ar}(\Delta\text{ABC}) \quad \ldots(i)$

As PQ is a diagonal of ‖ gm PBQR,

$\text{ar}(\|\text{ gm PBQR}) = 2\,\text{ar}(\Delta\text{PBQ}) \quad \ldots(ii)$



Now, triangles CAQ and PAQ have the same base AQ and are between the same parallels AQ ‖ CP,

$\therefore \quad \text{ar}(\Delta\text{CAQ}) = \text{ar}(\Delta\text{PAQ})$

$\Rightarrow \quad \text{ar}(\Delta\text{CAQ}) - \text{ar}(\Delta\text{BAQ}) = \text{ar}(\Delta\text{PAQ}) - \text{ar}(\Delta\text{BAQ})$

(subtracting same area from both sides)

$\Rightarrow \quad \text{ar}(\Delta\text{ABC}) = \text{ar}(\Delta\text{PBQ})$

$\Rightarrow \quad 2\,\text{ar}(\Delta\text{ABC}) = 2\,\text{ar}(\Delta\text{PBQ})$

$\Rightarrow \quad \text{ar}(\|\text{ gm ABCD}) = \text{ar}(\|\text{ gm PBQR})$ (using (*i*) and (*ii*))

Example 23. *If E, F, G and H are the mid-points of the sides AB, BC, CD and DA respectively of a parallelogram ABCD, show that ar(quad. EFGH) = $\frac{1}{2}$ar(‖ gm ABCD).* *(NCERT)*

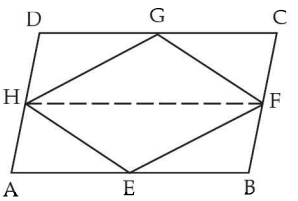
Solution. Join HF.

Now, $\text{AD} = \text{BC}$ (opp. sides of ‖ gm ABCD)

$\Rightarrow \quad \frac{1}{2}\text{AD} = \frac{1}{2}\text{BC}$

$\Rightarrow \quad \text{AH} = \text{BF}$

($\because$ H and F are mid-points of AD and BC respectively)



Also $\text{AD} \parallel \text{BC}$ *i.e.* $\text{AH} \parallel \text{BF}$.

Thus, in quad. HABF, AH = BF and AH ‖ BF

$\Rightarrow$ HABF is a parallelogram.

We note that ΔHEF and ‖ gm HABF are on the same base HF and between the same parallels AB and HF,

$\therefore \quad \text{ar}(\Delta\text{HEF}) = \frac{1}{2}\text{ar}(\|\text{ gm HABF}) \quad \ldots(i)$

Similarly, $\text{ar}(\Delta\text{HFG}) = \frac{1}{2}\text{ar}(\|\text{ gm HFCD}) \quad \ldots(ii)$

Adding (*i*) and (*ii*), we get

$$\text{ar}(\Delta HEF) + \text{ar}(\Delta HFG) = \frac{1}{2}[\text{ar}(\parallel \text{gm } HABF) + \text{ar}(\parallel \text{gm } HFCD)]$$

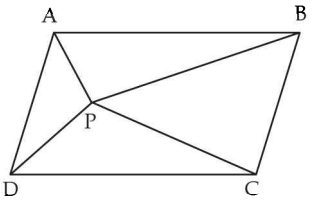
$$\Rightarrow \quad \text{ar}(\text{quad. } EFGH) = \frac{1}{2}\text{ar}(\parallel \text{gm } ABCD).$$

Example 24. *In the adjoining figure, P is a point in the interior of a parallelogram ABCD. Show that*

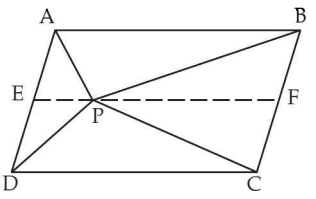
(i) $ar(\Delta APB) + ar(\Delta PDC) = \frac{1}{2}ar(\parallel gm\ ABCD)$

(ii) $ar(\Delta APD) + ar\ (\Delta PBC) = ar(\Delta APB) + ar(\Delta PDC)$.

(NCERT)



Solution. (*i*) Through P, draw a line parallel to AB to meet AD at E and BC at F.



Then AEFB is a parallelogram.

As ΔAPB and ∥ gm AEFB are on the same base AB and between the same parallels AB and EF,

$$\therefore \quad \text{ar}(\Delta APB) = \frac{1}{2}\text{ar}(\parallel \text{gm } AEFB) \qquad \ldots(1)$$

Since EF ∥ AB and AB ∥ DC, so EF ∥ DC

$\Rightarrow$ EDCF is a parallelogram.

As ΔPDC and ∥ gm EDCF are on the same base DC and between the same parallels DC and EF,

$$\therefore \quad \text{ar}(\Delta PDC) = \frac{1}{2}\text{ar}(\parallel \text{gm } EDCF) \qquad \ldots(2)$$

From (1) and (2), we get

$$\text{ar}(\Delta APB) + \text{ar}(\Delta PDC) = \frac{1}{2}[\text{ar}(\parallel \text{gm } AEFB) + \text{ar}(\parallel \text{gm } EDCF)]$$

$$= \frac{1}{2}\text{ar}(\parallel \text{gm } ABCD).$$

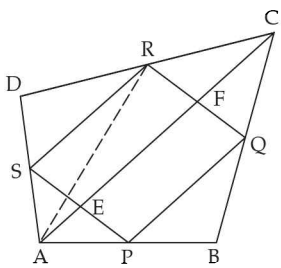
(*ii*) Similarly, by drawing a line through P and parallel to AD, we get

$$\text{ar}(\Delta APD) + \text{ar}(\Delta PBC) = \frac{1}{2}\text{ar}(\parallel \text{gm } ABCD)$$

$$= \text{ar}(\Delta APB) + \text{ar}(\Delta PDC) \qquad \text{[using part (}i\text{)]}$$

Example 25. *If the mid-points of the sides of a quadrilateral are joined in order, prove that the area of the parallelogram so formed will be half of the area of the quadrilateral.* *(NCERT Exemplar)*

Solution. Let ABCD be a quadrilateral and P, Q, R and S be the mid-points of the sides AB, BC, CD and DA respectively. Then PQRS is a parallelogram. We need to show that



$$\text{ar}(\parallel \text{gm } PQRS) = \frac{1}{2}\text{ar}(\text{quad. } ABCD)$$

Join AR. Let diagonal AC meet SP and QR at points E and F respectively.

Now ΔASR and parallelogram SEFR are on the base SR and between same parallels SR and AC,

$$\therefore \quad \text{ar}(\Delta ASR) = \frac{1}{2}\text{ar}(\parallel \text{gm } SEFR) \qquad \ldots(i)$$

As S is mid-point of DA, RS is median of ΔRDA.

$$\therefore \quad \text{ar}(\Delta DRS) = \text{ar}(\Delta ASR)$$

$$\Rightarrow \quad \text{ar}(\Delta ASR) = \frac{1}{2}\text{ar}(\Delta RDA) \qquad \ldots(ii)$$

In ΔACD, AR is median.

$$\therefore \quad \text{ar}(\Delta RDA) = \frac{1}{2}\text{ar}(\Delta ACD) \qquad \ldots(iii)$$

From (*i*) and (*ii*), we get

$$\text{ar}(\parallel \text{gm SEFR}) = 2\ \text{ar}(\Delta\text{ASR}) = 2 \times \frac{1}{2}\ \text{ar}(\Delta\text{RDA}) = \text{ar}(\Delta\text{RDA})$$

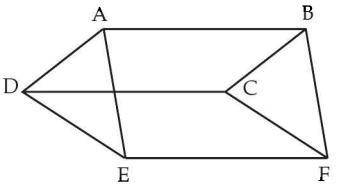
$$\Rightarrow \quad \text{ar}(\parallel \text{gm SEFR}) = \frac{1}{2}\ \text{ar}(\Delta\text{ACD}) \qquad \ldots(iv)\ \text{(using } (iii))$$

Similarly, $\text{ar}(\parallel \text{gm EPQF}) = \frac{1}{2}\ \text{ar}(\Delta\text{ABC}) \qquad \ldots(v)$

On adding (*iv*) and (*v*), we get

$$\text{ar}(\parallel \text{gm PQRS}) = \frac{1}{2}\ \text{ar}(\text{quad. } \Delta\text{ABCD}).$$

Example 26. *In the adjoining figure, ABCD, DCFE and ABFE are parallelograms. Show that ar(ΔADE) = ar(ΔBCF).* *(NCERT)*



Solution. As ABCD is a parallelogram,

$$\text{AD} = \text{BC} \qquad \text{(opp. sides of a } \parallel \text{gm)}$$

Similarly, $\text{DE} = \text{CF}$ and $\text{AE} = \text{BF}$.

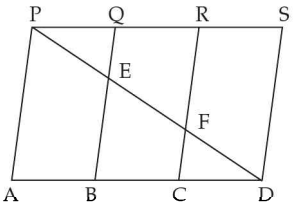
In ΔADE and ΔBCF,

$$\text{AD} = \text{BC},\ \text{DE} = \text{CF} \text{ and } \text{AE} = \text{BF}$$

$\therefore \quad \Delta\text{ADE} \cong \Delta\text{BCF}$ (by SSS rule of congruency)

$\therefore \quad \text{ar}(\Delta\text{ADE}) = \text{ar}(\Delta\text{BCF})$ (congruent figures have equal areas)

Example 27. *In the adjoining figure PSDA is a parallelogram. Points Q and R are taken on PS such that PQ = QR = RS and PA ∥ QB ∥ RC. Prove that ar (ΔPQE) = ar (ΔDCF).* *(NCERT Exemplar)*



Solution. Since PA ∥ QB ∥ RC ∥ SQ and PQ = QR = RS, therefore, by theorem of intercepts, PE = EF = FD.

$$\Rightarrow \quad \text{PE} = \text{FD}$$

Also, as PABQ is a parallelogram, PQ = AB.

Similarly, RS = CD. But PQ = RS, so PQ = CD.

In ΔPQE and ΔDCF,

$\text{PE} = \text{FD}$ (proved above)

$\text{PQ} = \text{CD}$ (proved above)

$\angle\text{QPE} = \angle\text{CDF}$ (alt. $\angle$s, PA ∥ SD and PD is a transversal)

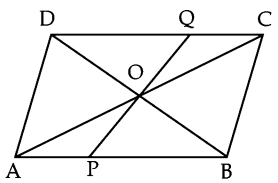
$\therefore \quad \Delta\text{PQE} \cong \Delta\text{DCF}$ (by SAS axiom of congruency)

$\therefore \quad \text{ar}(\Delta\text{PQE}) = \text{ar}(\Delta\text{DCF})$ (congruent figures have equal area)

Example 28. *The diagonals of a parallelogram ABCD intersect at O. A line through O meets AB at P and CD at Q. Prove that:*

$$\text{ar}(\text{quad. APQD}) = \frac{1}{2}\text{ar}(\parallel \text{gm ABCD}).$$ *(NCERT)*

Solution. As the diagonal AC divides the ∥ gm ABCD into two triangles of equal area,



$$\text{ar}(\Delta\text{ACD}) = \frac{1}{2}\ \text{ar}(\parallel \text{gm ABCD}) \qquad \ldots(i)$$

In ΔOAP and ΔOCQ,

$\angle\text{AOP} = \angle\text{COQ}$ (vert. opp. $\angle$s)

$\angle\text{OAP} = \angle\text{OCQ}$

(alt. $\angle$s, AB ∥ DC and AC is a transversal)

$\text{OA} = \text{OC}$ (diagonals of a ∥ gm bisect each other)

$\therefore \quad \Delta\text{OAP} \cong \Delta\text{OCQ}$ (by ASA rule of congruency)

$\therefore \quad \text{ar}(\Delta\text{OAP}) = \text{ar}(\Delta\text{OCQ})$ (congruent figures have equal areas)

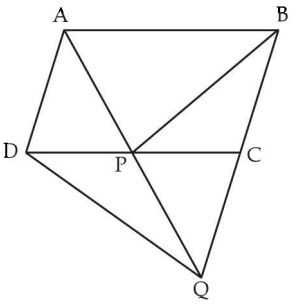
$\Rightarrow$ ar (ΔOAP) + ar (quad. AOQD) = ar (ΔOCQ) + ar (quad. AOQD)

(adding equal area to both sides)

$\Rightarrow$ ar (quad. APQD) = ar (ΔACD)

$\Rightarrow$ ar (quad. APQD) = $\frac{1}{2}$ ar (|| gm ABCD) [using (*i*)]

Example 29. *In the adjoining figure, ABCD is a parallelogram and BC is produced to a point Q such that CQ = AD. If AQ intersects DC at P, show that ar (ΔBPC) = ar (ΔDPQ).*



Solution. Join AC.

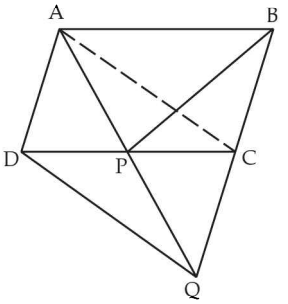
As triangles BPC and APC have same base PC and are between the same parallels (AB || DC *i.e.* AB || PC),

$\therefore$ ar (ΔBPC) = ar (ΔAPC) ...(*i*)

In quad. ADQC, AD || CQ

($\because$ AD || BC, opp. sides of || gm ABCD)

AD = CQ (given)



$\therefore$ ADQC is a parallelogram, so its diagonals AQ and DC bisect each other *i.e.* DP = PC and AP = PQ.

In ΔAPC and ΔQPD

PC = DP

AP = PQ

∠APC = ∠QPD (vert. opp. ∠s)

ΔAPC ≅ ΔQPD

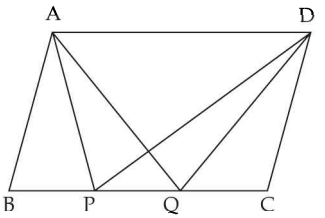
$\therefore$ ar (ΔAPC) = ar (ΔDPQ) ...(*ii*)

From (*i*) and (*ii*), we get

ar (ΔBPC) = ar (ΔDPQ).

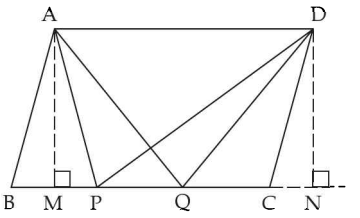
Example 30. *In the adjoining figure, ABCD is a parallelogram. Points P and Q on BC trisect BC in three equal parts. Prove that ar (ΔAPQ) = ar (ΔDPQ) = $\frac{1}{4}$ ar (|| gm ABCD).*

(NCERT Exemplar)



Solution. Given points P and Q on BC trisect BC, so BP = PQ = QC

$\Rightarrow$ PQ = $\frac{1}{3}$ BC.



From A, draw AM ⊥ BC and from D, draw DN ⊥ BC (produced).

As AM and DN are both perpendicular to BC, so AM || DN. Also, AD || BC *i.e.* AD || MN, therefore, AMND is a parallelogram.

$\therefore$ AM = DN (opp. sides of a || gm are equal)

Now ar (ΔAPQ) = $\frac{1}{2}$ PQ × AM = $\frac{1}{2}$ ($\frac{1}{3}$ BC) × AM

= $\frac{1}{6}$ BC × AM = $\frac{1}{6}$ ar (|| gm ABCD) ...(*i*)

and ar (ΔDPQ) = $\frac{1}{2}$ PQ × DN = $\frac{1}{2}$ ($\frac{1}{3}$ BC) × AM

= $\frac{1}{6}$ BC × AM = $\frac{1}{6}$ ar (|| gm ABCD) ...(*ii*)

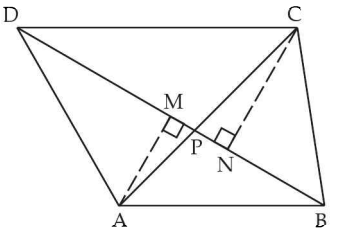
From (*i*) and (*ii*), we get

ar (ΔAPQ) = ar (ΔDPQ) = $\frac{1}{6}$ ar (|| gm ABCD).

Example 31. *Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that : ar(ΔAPB) × ar(ΔCPD) = ar(ΔAPD) × ar(ΔBPC).* *(NCERT)*

Solution. From A, draw AM ⊥ BD and from C, draw CN ⊥ BD.

Let AM = h and CN = h'.



Now, $\text{ar}(\Delta APB) = \frac{1}{2} BP \times AM = \frac{1}{2} BP \times h$ ...(*i*)

$\text{ar}(\Delta CPD) = \frac{1}{2} DP \times CN = \frac{1}{2} DP \times h'$ ...(*ii*)

$\text{ar}(\Delta APD) = \frac{1}{2} DP \times AM = \frac{1}{2} DP \times h$ ...(*iii*)

$\text{ar}(\Delta BPC) = \frac{1}{2} BP \times CN = \frac{1}{2} BP \times h'$ ...(*iv*)

From (*i*) and (*ii*), we get

$$\text{ar}(\Delta APB) \times \text{ar}(\Delta CPD) = \frac{1}{4} BP \times DP \times hh' \quad \text{...(v)}$$

From (*iii*) and (*iv*), we get

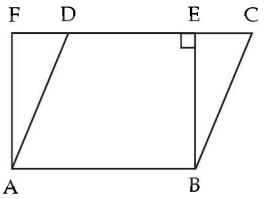
$$\text{ar}(\Delta APD) \times \text{ar}(\Delta BPC) = \frac{1}{4} BP \times DP \times hh' \quad \text{...(vi)}$$

From (*v*) and (*vi*), we get

$$\text{ar}(\Delta APB) \times \text{ar}(\Delta CPD) = \text{ar}(\Delta APD) \times \text{ar}(\Delta BPC).$$

Example 32. *Parallelogram ABCD and rectangle ABEF are on the same base and have equal areas. Show that the perimeter of the parallelogram is greater than that of the rectangle.* *(NCERT)*

Solution. As the parallelogram ABCD and the rectangle ABEF have equal areas and are on the same base, therefore, they lie between the same parallels *i.e.* DC and FE lie along the same line as shown in the adjoining figure.



In ΔEBC, ∠BEC = 90°, so ∠ECB < 90°

⇒ ∠BEC > ∠ECB ⇒ BC > BE

⇒ AB + BC > AB + BE ⇒ 2(AB + BC) > 2(AB + BE)

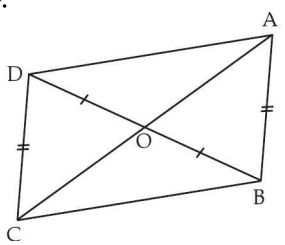
⇒ the perimeter of ‖ gm ABCD > the perimeter of rect. ABEF.

Example 33. *In the adjoining figure, diagonals AC and BD of quadrilateral ABCD intersect at O such that OB = OD. If AB = DC, show that*

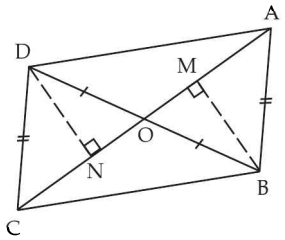
(i) ar(ΔDOC) = ar(ΔAOB)

(ii) ar(ΔDCB) = ar(ΔACB)

(iii) ABCD is a parallelogram. *(NCERT)*



Solution. (*i*) From B, draw BM ⊥ AC and from D, draw DN ⊥ AC.



In ΔOND and ΔOMB,

OD = OB (given)

∠DON = ∠BON (vert. opp. ∠s)

∠OND = ∠OMB (each = 90°)

∴ ΔOND ≅ ΔOMB (by AAS rule of congruency)

∴ DN = BM and ON = OM (c.p.c.t.)

In ΔDCN and ΔBAM,

DN = BM (proved above)

DC = AB (given)

∠CND = ∠AMB (each = 90°)

$\therefore$ $\Delta DCN \cong \Delta BAM$ (by RHS rule of congruency)

$\therefore$ $CN = AM$ (c.p.c.t.)

Now $CN = AM$ and $ON = OM$

$\Rightarrow$ $CN + ON = AM + OM \Rightarrow OC = OA.$

In ΔDOC and ΔBOA,

$OD = OB$ (given)

$DC = AB$ (given)

$OC = OA$ (proved above)

$\therefore$ $\Delta DOC \cong \Delta BOA$ (by SSS rule of congruency)

$\therefore$ $ar(\Delta DOC) = ar(\Delta AOB)$

(*ii*) Adding $ar(\Delta OCB)$ to both sides, we get

$ar(\Delta DOC) + ar(\Delta OCB) = ar(\Delta AOB) + ar(\Delta OCB)$

$\Rightarrow$ $ar(\Delta DCB) = ar(\Delta ACB)$

(*iii*) In the quad. ABCD, $OB = OD$ (given)

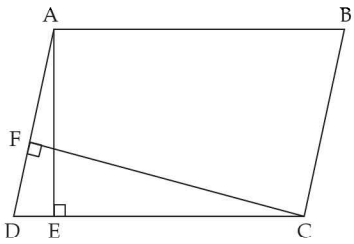
and $OC = OA$ (proved above)

$\Rightarrow$ the diagonals AC and BD of quad. ABCD bisect each other

$\Rightarrow$ ABCD is a parallelogram.

Exercise 10

1. In the adjoining figure, ABCD is a parallelogram. If AB = 16 cm, AE = 8 cm and CF = 10 cm, find AD. *(NCERT)*

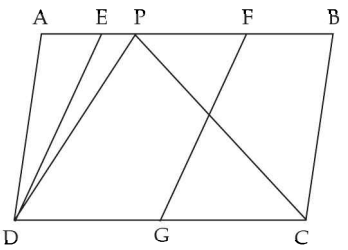


2. Write whether the following statements are true or false. Justify your answer:

(*i*) PQRS is a parallelogram whose area is 180 cm^2 and A is any point on the diagonal QS. The area of ΔASR = 90 cm^2. *(NCERT Exemplar)*

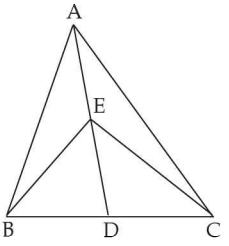
(*ii*) ABC and BDE are two equilateral triangles such that D is mid-point of BC. Then $ar(\Delta BDE) = \frac{1}{4} ar(\Delta ABC)$. *(NCERT Exemplar)*

(*iii*) In the adjoining figure, ABCD and EFGD are two parallelograms and G is mid-point of CD. Then $ar(\Delta DPC) = \frac{1}{2} ar(\parallel gm\ EFGD)$. *(NCERT Exemplar)*

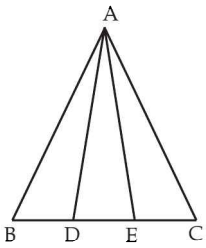


3. In the adjoining figure, E is mid-point of the median AD of a ΔABC. Show that

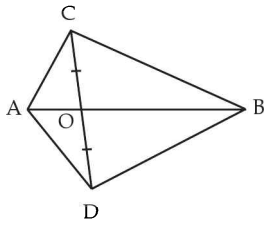
$ar(\Delta ABE) = ar(\Delta ACE)$. *(NCERT)*



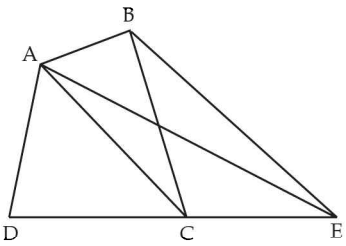
4. In the adjoining figure, D and E are points on the side BC of a triangle ABC such that BD = DE = EC. Show that ar (ΔABD) = ar (ΔADE) = ar (ΔAEC). *(NCERT)*



5. In the adjoining figure, ABC and ABD are two triangles on the same base AB. If the line segment CD is bisected by AB at O, show that ar (ΔABC) = ar (ΔABD). *(NCERT)*



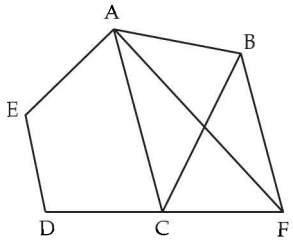
6. In the adjoining figure, ABCD is a quadrilateral, BE ∥ AC and BE meets DC produced at E. Show that the area of ΔADE is equal to the area of the quadrilateral ABCD. *NCERT)*



7. In the adjoining figure, ABCDE is a pentagon. A line through B and parallel to AC meets DC produced at F. Show that

(*i*) ar (ΔACB) = ar (ΔACF)

(*ii*) ar (quad. AEDF) = ar (pent. ABCDE). *(NCERT)*

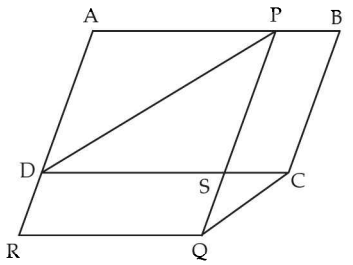


8. D, E and F are respectively the mid-points of the sides BC, CA and AB of a triangle ABC. Show that

(*i*) BDEF is a parallelogram (*ii*) $\text{ar}(\Delta DEF) = \frac{1}{4}\text{ar}(\Delta ABC)$

(*iii*) $\text{ar}(\|\text{gm BDEF}) = \frac{1}{2}\text{ar}(\Delta ABC)$. *(NCERT)*

9. In the adjoining figure, ABCD and APQR are two parallelograms of equal area. Prove that DP is parallel to QC.



Hint: Join CP and QD.

Given ar (∥gm ABCD) = ar (∥gm APQR)

(Subtract ar (∥gm APSD) from both sides)

⇒ ar (∥gm PBCS) = ar (∥gm DSQR)

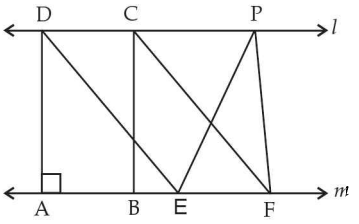
⇒ 2ar (ΔSPC) = 2ar (ΔQDS)

⇒ ar (ΔSPC) = ar (ΔQDS) (Adding ar (ΔSQC) to both sides)

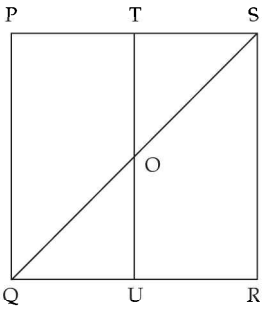
⇒ ar (ΔQPC) = ar (ΔQDC), which are two triangles on same base QC.

⇒ DP ∥ QC.

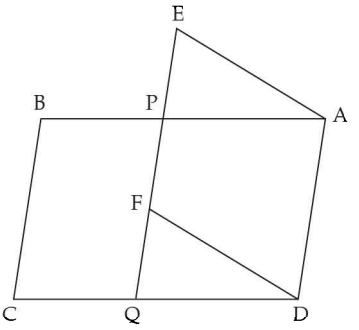
10. In the adjoining figure, $l \parallel m$, ABCD is a rectangle with sides AB = 6 cm and AD = 9 cm (not drawn to scale).
If DE || CF, find the area of
(*i*) parallelogram EFCD
(*ii*) ΔEFP.



11. In the adjoining figure, PQRS is a square. T and U are mid-points of sides PS and QR respectively. If PQ = 8 cm, find the area of ΔOTS where O is the point of intersection of TU and QS. *(NCERT Exemplar)*



12. In the adjoining figure, ABCD and AEFD are two parallelograms. Prove that ar (ΔPEA) = ar (ΔQFD). *(NCERT Exemplar)*



Hint: In ΔPEA and ΔQFD,

EA = FD (opp. sides of a || gm)

∠EPA = ∠FQD (corres. ∠s)

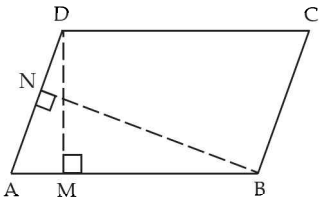
∠PEA = ∠QFD (corres. ∠s)

∴ ΔPEA ≅ ΔQFD ⇒ ar (ΔPEA) = ar(ΔQFD).

Answers

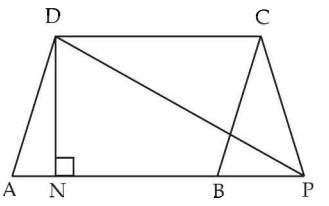
Very Short Answer Type Questions

1. In the adjoining figure, ABCD is a parallelogram, DM ⊥ AB and BN ⊥ AD. If AB = 20 cm, DM = 12 cm and BN = 16 cm, then find the length of BC.



2. ABCD is a parallelogram and P is any point on the side AD. If the area of the parallelogram ABCD is 70 cm^2, then find the area of ΔPBC.

3. In the adjoining figure, ABCD is a parallelogram, P is a point on AB (produced) and DN ⊥ AB. If AB = 8 cm and DN = 5 cm, then find the area of ΔPCD.



4. The diagonals AC and BD of a parallelogram intersect each other at the point O. If the area of the parallelogram ABCD is 140 cm^2, then find the area of ΔOBC.
5. The area of a trapezium is 30 cm^2 and the distance between its parallel sides is 5 cm. If one of the parallel side is 7 cm, then find the length of other parallel side.

Answers

Objective Type Questions

(FOR FORMATIVE/SUMMATIVE ASSESSMENT)

Mental Maths

1. Fill in the blanks:

(*i*) Two figures are called equal if they have ..... .

(*ii*) A diagonal of a parallelogram divides it into two triangles of ..... area.

(*iii*) Two parallelograms on the same base (or equal bases) and between the same parallel lines are ..... in area.

(*iv*) The area of a parallelogram is equal to the product of its any side and ..... .

(*v*) The area of a triangle is ..... the area of a parallelogram on the same base and between the same parallels.

(*vi*) Two triangles on the same base (or equal bases) and between the same parallels are ..... in area.

(*vii*) If two parallelograms on the same base (or equal bases) have equal areas, then they lie between ..... .

2. State whether the following statements are true or false.

(*i*) A parallelogram and a trapezium on the same base and between the same parallels are equal in area.

(*ii*) A diagonal of a rhombus divides it into two triangles of equal area.

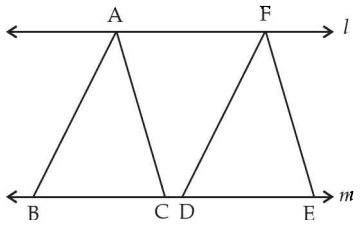
(*iii*) A diagonal of a trapezium divides it into two triangles of equal area.

(*iv*) The bisector of an angle of a triangle divides it into two triangles of equal area.

Multiple Choice Questions

Choose the correct answer from the given four options (3 to 14) :

3. In the adjoining figure, if $l \parallel m$, AB $\parallel$ FD and AC $\parallel$ FE, then which of the following two figures are on the same base and between the same parallels?



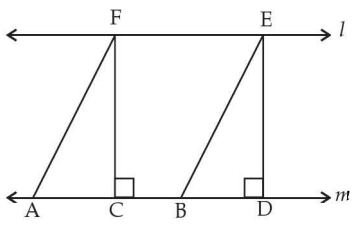
(*a*) ΔABC and quadrilateral ACDF

(*b*) $\parallel$ gm ABDF and $\parallel$ gm ACEF

(*c*) ΔFED and $\parallel$ gm CEFA

(*d*) ΔABC and ΔFDE.

4. In the adjoining figure, if $l \parallel m$, AF $\parallel$ BE, FC $\perp m$ and ED $\perp m$, then the correct statement is



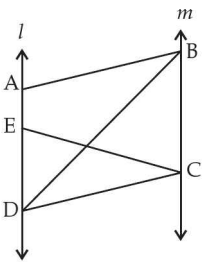
(*a*) ar ($\parallel$ABEF) = ar (rect. CDEF)

(*b*) ar ($\parallel$ABEF) = ar (quad. CBEF)

(*c*) ar ($\parallel$ABEF) = 2 ar (ΔACF)

(*d*) ar ($\parallel$ABEF) = 2 ar (ΔEBD)

5. In the adjoining figure, if $l \parallel m$ and AB $\parallel$ DC, then which of the following two figures are not on the same base and between the same parallels?



(*a*) $\parallel$ gm ABCD and ΔBCD
(*b*) $\parallel$ gm ABCD and quad. ABCE
(*c*) ΔABD and $\parallel$ gm ABCD
(*d*) ΔABD and quad. ABCE.

6. Two parallelograms are on equal bases and between the same parallels. The ratio of their areas is

(*a*) 1 : 2 (*b*) 1 : 1 (*c*) 2 : 1 (*d*) 3 : 1

(NCERT Exemplar)

7. If a triangle and a parallelogram are on the same base and between same parallels, then the ratio of area of the triangle to the area of parallelogram is

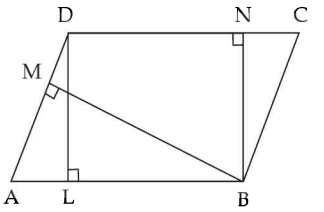
(*a*) 1 : 3 (*b*) 1 : 2 (*c*) 3 : 1 (*d*) 1 : 4

(NCERT Exemplar)

8. A median of a triangle divides it into two

(*a*) triangles of equal area (*b*) congruent triangles
(*c*) right triangles (*d*) isosceles triangles *(NCERT Exemplar)*

9. In the adjoining figure, area of parallelogram ABCD is



(*a*) AB × BM (*b*) BC × BN
(*c*) DC × DL (*d*) AD × DL *(NCERT Exemplar)*

10. The mid-points of the sides of a triangle along with any of the vertices as the fourth point make a parallelogram of area equal to

(*a*) $\frac{1}{2}$ ar (ΔABC) (*b*) $\frac{1}{3}$ ar (ΔABC) (*c*) $\frac{1}{4}$ ar (ΔABC) (*d*) ar (ΔABC)

(NCERT Exemplar)

11. The figure obtained by joining the mid-points of the adjacent sides of a rectangle of sides 8 cm and 6 cm is a

(*a*) rectangle of area 24 cm^2 (*b*) square of area 25 cm^2
(*c*) a trapezium of area 24 cm^2 (*d*) rhombus of area 24 cm^2

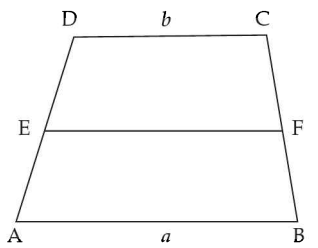
(NCERT Exemplar)

12. ABCD is a quadrilateral whose diagonal AC divides it into two parts, equal in area, then ABCD

(*a*) is a rectangle (*b*) is always a rhombus
(*c*) is a parallelogram (*d*) need not be any of (*a*), (*b*) or (*c*)

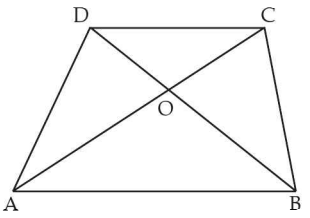
(NCERT Exemplar)

13. In the adjoining figure, ABCD is a trapezium with parallel sides AB = a cm and DC = b cm. E and F are mid-points of the non-parallel sides. The ratio of ar (ABFE) and ar (EFCD) is



(*a*) $a : b$
(*b*) $(3a + b) : (a + 3b)$
(*c*) $(a + 3b) : (3a + b)$
(*d*) $(2a + b) : (3a + b)$ *(NCERT Exemplar)*

14. In the adjoining figure, AB || DC and AB ≠ DC. If the diagonals AC and BD of the trapezium ABCD intersect at O, then which of the following statements is not true?



(*a*) ar(ΔABC) = ar(ΔABD)

(*b*) ar(ΔACD) = ar(ΔBCD)

(*c*) ar(ΔOAB) = ar(ΔOCD)

(*d*) ar (ΔOAD) = ar(ΔOBC)

Answers

Value Based Question

A farmer was having a field in the form of a parallelogram PQRS. He took any point A on RS and joined it to points P and Q. In how many parts the field is divided? What are the shapes of these parts? The farmer wants to sow wheat and pulse in equal portions of the field separately. How should he do it?

What is the value of wheat and pulses in our daily life?

Answers

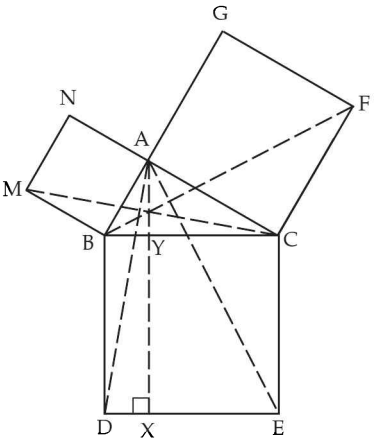
Higher Order Thinking Skills (HOTS)

1. If P and Q are respectively the mid-points of sides AB and BC of a triangle ABC and R is mid-point of AP, show that :

(*i*) $\text{ar}(\Delta PQR) = \frac{1}{2}\text{ar}(\Delta ARC)$ (*ii*) ar(ΔPBQ) = ar(ΔARC)

(*iii*) $\text{ar}(\Delta RQC) = \frac{3}{8}\text{ar}(\Delta ABC)$.

2. In the adjoining figure, ABC is a right triangle right angled at A. BCED, ACFG and ABMN are squares on the sides BC, CA and AB respectively. Line segment AX ⊥ DE meets BC at Y.



Show that :

(*i*) ΔMBC ≅ ΔABD

(*ii*) ar(rect. BYXD) = 2 ar(ΔMBC)

(*iii*) ar(rect. BYXD) = ar(square ABMN)

(*iv*) ΔFCB ≅ ΔACE

(*v*) ar(rect. CYXE) = 2 ar(ΔFCB)

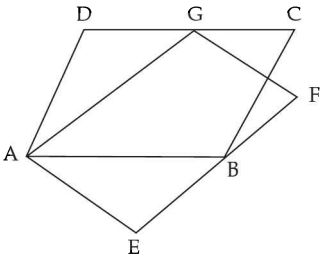
(*vi*) ar(rect. CYXE) = ar(sq. ACFG)

(*vii*) ar(sq. BCED) = ar(sq. ABMN) + ar(sq. ACFG).

Chapter Test

1. In the adjoining figure, ABCD and AEFG are two parallelograms. Prove that

ar (|| gm ABCD) = ar (|| gm AEFG).

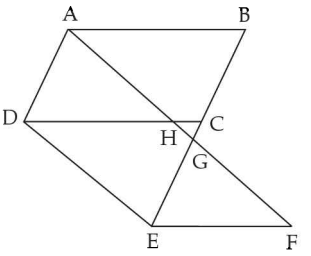


Hint: Join BG.

ar (ΔABG) = $\frac{1}{2}$ ar (|| gm ABCD);

ar (ΔABG) = $\frac{1}{2}$ ar (|| gm AEFG).

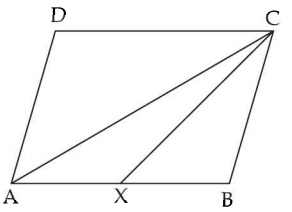
2. In the adjoining figure, AB || DC || EF, AD || BE and DE || AF. Prove that ar (|| gm DEFH) = ar (|| gm ABCD).



Hint: ar (|| gm ABCD) = ar (|| gm ADEG);

ar (|| gm ADEG) = ar (|| gm DEFH).

3. ABCD is a parallelogram and X is mid-point of AB. If ar (AXCD) = 24 cm^2, then find ar (ΔABC). *(NCERT Exemplar)*



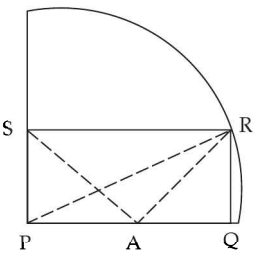
Hint: ar (AXCD) = ar (ΔACD) + ar (ΔACX)

$\Rightarrow$ 24 cm^2 = ar (ΔABC) + $\frac{1}{2}$ ar (ΔABC)

$\Rightarrow$ 24 cm^2 = $\frac{1}{2}$ ar (ΔABC)

$\Rightarrow$ ar (ΔABC) = $\left(\frac{2}{3} \times 24\right)$ cm^2 = 16 cm^2.

4. In the adjoining figure, PQRS is a rectangle inscribed in a quadrant of a circle of radius 13 cm. If PS = 5 cm and A is any point on PQ, then find ar (ΔARS). *(NCERT Exemplar)*



Hint: $SR^2 = PR^2 - SP^2 = 13^2 - 5^2 = 144$

$\Rightarrow$ SR = 12 cm.

ar (rect. PQRS) = (12 × 5) cm^2 = 60 cm^2

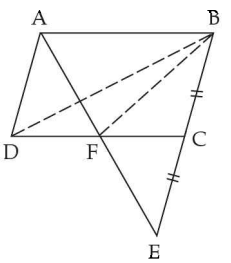
5. ABCD is a trapezium in which AB || DC, AB = 50 cm and DC = 30 cm. If X and Y are mid-points of sides AD and BC respectively, prove that ar (DCYX) = $\frac{7}{9}$ ar (XYBA).

(NCERT Exemplar)

Hint: XY = $\frac{1}{2}$ (AB + DC) = $\frac{1}{2}$ (50 + 30) cm = 40 cm.

Height of trap. DCYX = height of trap. XYBA = h cm (say).

6. In the adjoining figure, ABCD is a parallelogram in which BC is produced to E such that CE = BC. AE intersects CD at F. If ar (ΔDFB) = 3 cm^2, find the area of parallelogram ABCD. *(NCERT Exemplar)*

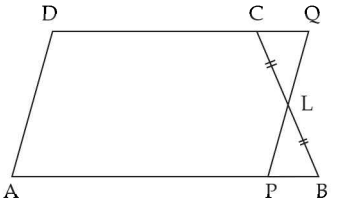


Hint: Join AC and DE. ACED is a parallelogram (why?). Diagonals AE and DC of ||gm ACED bisect each other, so F is mid-point of DC.

ar(ΔBFC) = ar(ΔDFB) = 3 cm^2 (given).

7. In the adjoining figure, ABCD is a trapezium in which AB || DC and L is mid-point of BC. Through L, a line PQ || AD has been drawn which meets AB in P and DC produced in Q. Prove that ar (ABCD) = ar (APQD)

(NCERT Exemplar)

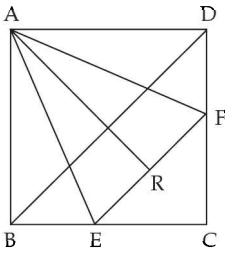


Hint: Show that $\Delta LPB \cong \Delta LQC$

$\Rightarrow$ ar (ΔLPB) = ar (ΔLQC).

Add area of pentagon APLCD to both sides.

8. In the adjoining figure, ABCD is a square. E and F are mid-points of sides BC and CD respectively. If R is mid-point of EF, prove that ar(ΔAER) = ar(ΔAFR). *(NCERT Exemplar)*



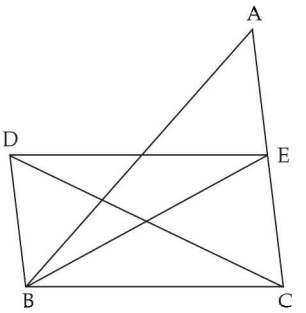
Hint: Show that $\Delta ABE \cong \Delta ADF$.

$\Rightarrow$ AE = AF.

Now, show that $\Delta AER \cong \Delta AFR$.

9. In the adjoining figure, BD || CA, E is mid-point of CA and BD = $\frac{1}{2}$ CA. Prove that ar(ΔABC) = 2 ar(ΔDBC).

(NCERT Exemplar)



Hint: BD = CE (why) and BD || CE, therefore, BCED is a parallelogram.

$\therefore$ ar(ΔDBC) = ar(ΔEBC) ...(*i*)

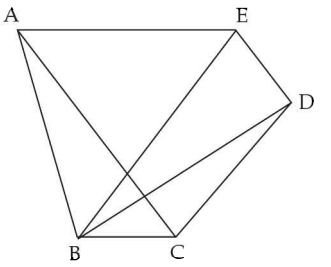
(Same base BC and between same parallels BC || DE)

In ΔABC, BE is median.

$\therefore$ ar(ΔABC) = 2 ar(ΔEBC)

$\Rightarrow$ ar(ΔABC) = 2 ar(ΔDBC) (using (*i*))

10. In the adjoining figure, BC || AE and CD || BE. Prove that ar(ΔABC) = ar(ΔEBD).



Hint: Join CE and let CE intersect BD at F.

ar(ΔABC) = ar(ΔEBC).

(Same base BC and between parallels BC and AE)

Also, ar (ΔBCD) = ar(ΔECD).

(Same base CD and between parallels CD and BE)

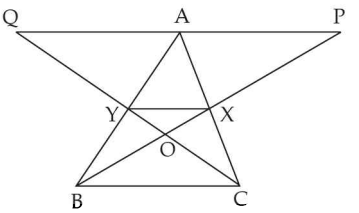
$\Rightarrow$ ar(ΔBCD) – ar(ΔFCD) = ar(ΔEBC) – ar(ΔFCD)

$\Rightarrow$ ar(ΔFBC) = ar(ΔEFD)

$\Rightarrow$ ar(ΔFBC) + ar(ΔEBF) = ar(ΔEFD) + ar(ΔEBF)

$\Rightarrow$ ar(ΔEBC) = ar(ΔEBD).

11. In the adjoining figure, X and Y are mid-points of the sides AC and AB respectively of ΔABC. QP || BC and CYQ and BXP are straight lines. Prove that ar(ΔABP) = ar(ΔCAQ). *(NCERT Exemplar)*



Hint: Since X and Y are mid-points of AC and AB, YX || BC. But QP || BC, so YX || QP. In ΔBAP, Y is mid-point of AB and YX || AP, so X is mid-point of BP and YX = $\frac{1}{2}$ AP.

Similarly, Y is mid-point of CQ and YX = $\frac{1}{2}$ QA $\Rightarrow$ AP = QA.

Thus, ABP and CAQ are two triangles on equal bases AP, QA and are between same parallels BC and QP, therefore, ar(ΔABP) = ar(ΔCAQ).

Answers

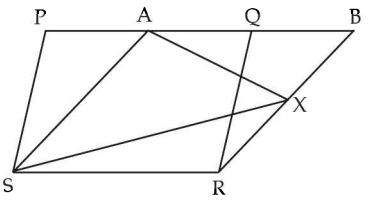
Model Question Paper 4

(FOR FORMATIVE ASSESSMENT III - TERM II)

Time: 1 Hour *Maximum Marks: 30*

Note. *Questions 1-2 carry 1 mark each, 3-4 carry 2 marks each, 5-8 carry 3 marks each and 9-11 carry 4 marks each.*

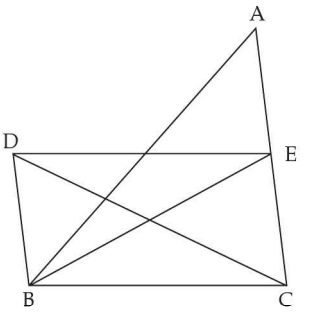
1. If (–5, 7) is a solution of the equation $3x - 4y + 5k = 0$, then find the value of k.
2. In ΔABC, AB = 5 cm, BC = 8 cm and AC = 7 cm. If D and E are mid-points of sides AB and AC respectively, find the perimeter of the quadrilateral DBCE.
3. Diagonals of a quadrilateral ABCD bisect each other. If $\angle B = 50°$, then find $\angle C$.
4. In the adjoining figure, PQRS and ABRS are parallelograms and X is any point on the side BR. Show that



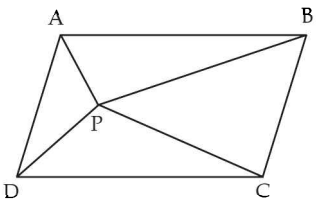
   (*i*) ar(|| gm PQRS) = ar (|| gm ABRS)

   (*ii*) $\text{ar}(\Delta AXS) = \frac{1}{2}\text{ar}$ (|| gm PQRS).

5. Solve the equation $3y - 1 = 9 - 2y$, and represent the solution (s) on:
   (*i*) the number line (*ii*) the cartesian plane.
6. E is mid-point of the median AD of any triangle ABC and BE is produced to meet AC at F. Show that $AF = \frac{1}{3}$ AC.
7. If a pair of opposite sides of a quadrilateral is equal and parallel, then prove that it is a parallelogram.



8. In the adjoining figure, BD || CA, E is mid-point of CA and CA = 2BD. Prove that:
   ar(ΔABC) = 2 ar (ΔDBC).
9. The autorickshaw fare in a city is as follows:
   For the first kilometre, the fare is ₹ 10 and for the subsequent distance, it is ₹ 4 per km. Taking the distance covered as x km and the total fare as ₹ y, write a linear equation for this information. Also draw its graph.
10. Show that the bisectors of the angles of a parallelogram form a rectangle.
11. In the adjoining figure, P is a point in the interior of a parallelogram ABCD. Show that



   (*i*) $\text{ar}(\Delta APB) + \text{ar}(\Delta PDC) = \frac{1}{2}\text{ar}$ (|| gm ABCD)

   (*ii*) ar(ΔAPD) + ar(ΔPBC) = ar (ΔAPB) + ar (ΔPDC).

Answers

Circles



INTRODUCTION

In our day-to-day life, we come across many objects which are round in shape, such as dials of many clocks, wheels of a vehicle, bangles, key rings, coins of denomination ₹ 1, ₹ 2, ₹ 5, ₹ 10, etc. In a clock, we observe that second's hand goes round the dial of the clock rapidly and its tip moves in a round path. This path traced by the tip of the second's hand is called a **circle**. In this chapter, we shall study about circles, some terms related with a circle and some properties of a circle.

11.1 CIRCLE AND SOME TERMS RELATED WITH IT

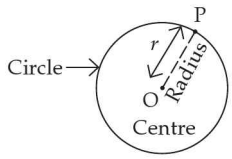
❑ **Circle**

A circle is the collection of all those points, say, P, in a plane each of which is at a constant distance from a fixed point in that plane. In other words, a circle is the path of a point which moves in a plane so that it remains at a constant distance from a fixed point in the plane.

The fixed point is called the ***centre*** *and the constant distance is called the* ***radius*** *of the circle.*

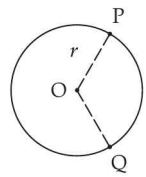
The radius of a circle is always positive.

The adjoining figure shows a circle with O as its centre and r as its radius.



Note that the centre of a circle does not lie on the circle.

Let O be the centre of a circle and r its radius. If P is a point on the circle, then the line segment $\overline{OP}$ is a radius of the circle and its length is r. If Q is another point on the circle then $\overline{OQ}$ is another radius of the circle. Note that all radii (plural of radius) have one point in common, which is the centre of the circle. Also OP = OQ = r. Thus:



All radii of a circle are equal.

Note. The line segment joining the centre and any point on the circle is also called a radius of the circle. Thus, 'radius' is used in two senses — in the sense of a line segment and also in the sense of its length.

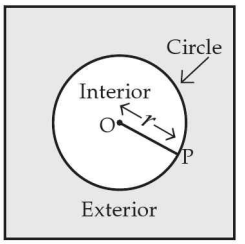
❑ **Some terms related with a circle**

1. Circle-interior and exterior

A circle is a closed curve. It divides the plane region into three parts. They are:

(*i*) **Circle** — The collection of all points P of the plane such that OP = r form a circle with ***centre*** O and radius r ($r > 0$).

(*ii*) **Interior of a circle** — The collection of all points of the plane which lie inside the circle *i.e.* the collection of all points P of the plane such that OP < r form the **interior of the circle.**

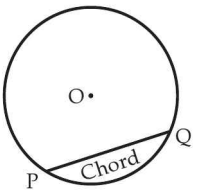


The collection of all points of the plane which either lie on the circle or are inside the circle form the **circular region**.

(*iii*) **Exterior of a circle** — The collection of all points of the plane which lie outside the circle *i.e.* the collection of all points of the plane such that OP > r form the **exterior of the circle.**

2. Chord of a circle

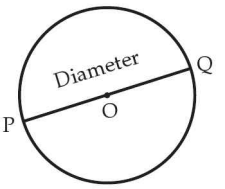
A line segment joining any two points of a circle is called a **chord of the circle.**



In the adjoining figure, PQ is a chord of the circle with centre O. The distance PQ is called the **length of the chord.**

3. Diameter of a circle

A chord of a circle which passes through its centre is called a **diameter of the circle.**



A diameter of a circle is the **longest chord** of the circle and all diameters have equal length.

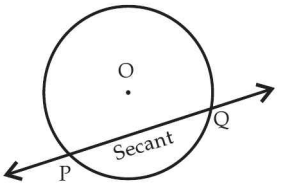
In the adjoining figure, $\overline{PQ}$ is a diameter of the circle with centre O. Note that $\overline{OP}$ and $\overline{OQ}$ are both radii of the circle, so OP = OQ = r. It follows that PQ = $2r$ = 2 × radius. Thus:

***Length of a diameter* = 2 × *radius*.**

Note that there are infinitely many lines passing through the point O (centre of circle), so a circle has infinitely many diameters.

4. Secant of a circle

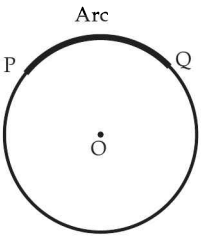
A line which meets a circle in two points is called a **secant** of the circle. In the adjoining figure, line PQ is a secant of the circle with centre O.



5. Arc of a circle

A (continuous) part of a circle is called an **arc of the circle.**

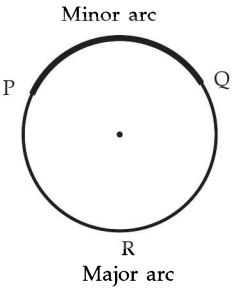
The arc of a circle is denoted by the symbol "$\frown$".



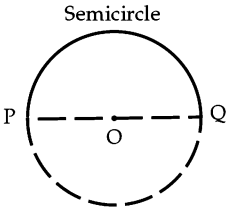
In the adjoining figure, $\overset{\frown}{PQ}$ denotes the arc PQ of the circle with centre O.

Arc of a circle is divided into following categories:

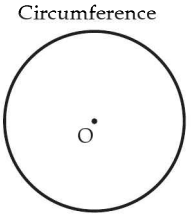
(*i*) **Minor and major arc** — An arc less than one-half of the whole arc of a circle is called a **minor arc** of the circle and an arc greater than one-half of the whole arc is called a **major arc** of the circle. Here, $\overset{\frown}{PQ}$ is the minor arc and $\overset{\frown}{PRQ}$ is the major arc.



(*ii*) **Semicircle**—When PQ is a diameter, then both arcs are equal and each is called a **semicircle**. Thus, one-half of the whole arc of a circle is called a semicircle.



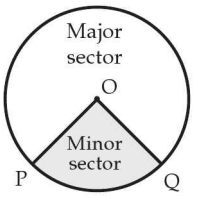
(*iii*) **Circumference**—The whole arc of a circle is called the **circumference** of the circle. The length of the circumference of a circle is the length of the whole arc. However, in general, the term **'circumference of a circle'** refers to its length.



6. Sector of a circle

The part of the plane region enclosed by an arc of a circle and its two bounding radii is called a **sector of the circle**.

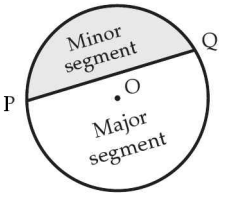
The part containing the minor arc is called **minor sector** and the part containing the major arc is called **major sector**.



7. Segment of a circle

A chord of a circle divides its circular region into two parts. Each part of the circular region is called a segment of the circle.

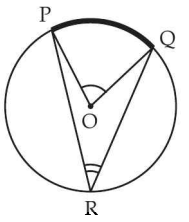
The part of the circular region containing the minor arc is called a **minor segment** and the part containing the major arc is called a **major segment**.



8. Angle subtended by an arc

The angle subtended by the two bounding radii of an arc of a circle at the centre of the circle is called the **angle subtended by the arc**.

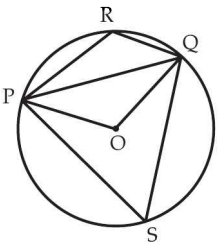
In the adjoining figure, ∠POQ is the angle subtended by the minor arc PQ of a circle with centre O and the reflex ∠POQ is the angle subtended by the major arc PQ at O. If R is any point on the major arc, then ∠PRQ is called the angle subtended by the minor arc PQ at the point R.



9. Angle subtended by a chord at a point

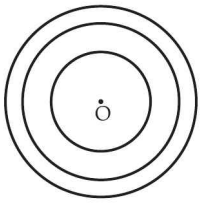
Let PQ be a chord of a circle with centre O and R, S be points on the minor and major arcs of the circle (as shown in the adjoining figure).

Join OP and OQ, then ∠POQ is the angled subtended by the chord PQ at the centre. Join PR, QR, PS and QS, then ∠PRQ and ∠PSQ are the angles subtended by the chord PQ at the points R and S respectively on the minor and major arcs. ***Note that smaller is the arc bigger is the angle subtended by the chord.***



10. Concentric circles

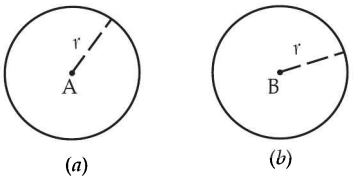
Two or more circles are called concentric circles if and only if they have same centre but different radii.



11. Equal (or congruent) circles

Two or more circles are called **equal (or congruent)** circles if and only if they have same radius.

In the adjoining figures, two circles with centres A and B have equal radius (= r), so, these are equal circles.



(a) (b)

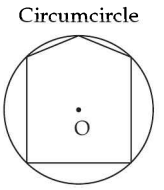
12. Circumscribed circle

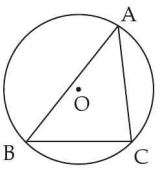
A circle passing through all the vertices of a polygon is called a ***circumscribed*** circle of the polygon, and its centre is called ***circumcentre***.

The polygon is called ***inscribed polygon***.

The points, which lie on the circumference of the same circle, are called ***concyclic points***.

In particular, the circle which passes through all the vertices of a triangle is called the ***circumcircle*** of the triangle and its centre is called ***circumcentre***. The adjoining figure shows circumcircle of ΔABC with O as its circumcentre. The triangle ABC is called ***inscribed*** in the circle.

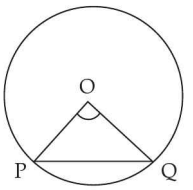




11.2 CHORD AND ARC PROPERTIES OF CIRCLES

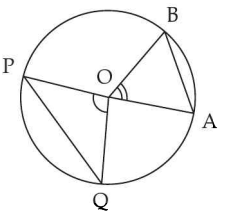
❑ Angle subtended by a chord of a circle at the centre

Let PQ be a chord of a circle with centre O. Join OP and OQ, then ∠POQ is called the *angle subtended* by the chord PQ at the centre O of the circle.



Let us examine the relationship between the length of the chord and the angle subtended by it at the centre of a circle.

Draw chords of different size (length) and measure the angle subtended by them at the centre of a circle. We shall observe that longer chord subtends bigger angle at the centre.

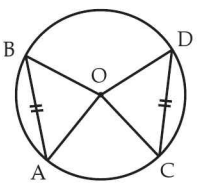


In the adjoining figure, PQ > AB, we find that ∠POQ > ∠AOB.

The question arises! What happens if the two chords are of equal length? On measuring the angles subtended by them at the centre, we find that these angles are equal. In fact, we have:

Theorem 11.1

Equal chords of a circle subtend equal angles at the centre.



Given. Two equal chords AB and CD of a circle with centre O.

To prove. ∠AOB = ∠COD.

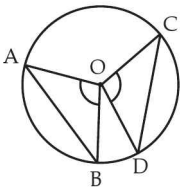
Proof.

Statements	*Reasons*
In ΔOAB and ΔOCD	
1. OA = OC	1. Radii of same circle.
2. OB = OD	2. Radii of same circle.
3. AB = CD	3. Given
4. ΔOAB ≅ ΔOCD	4. SSS rule of congruency.
5. ∠AOB = ∠COD	5. c.p.c.t.

Converse of the above theorem is also true. In fact, we have:

Theorem 11.2

If the angles subtended by the chords of a circle at the centre are equal, then the chords are equal.



Given. Two chords AB and CD of a circle with centre O such that $\angle AOB = \angle COD$.

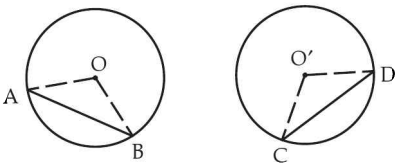
To prove. AB = CD.

Proof.

Statements	*Reasons*
In ΔOAB and ΔOCD	
1. OA = OC	1. Radii of same circle.
2. OB = OD	2. Radii of same circle.
3. $\angle AOB = \angle COD$	3. Given
4. $\Delta OAB \cong \Delta OCD$	4. SAS axiom of congruency.
5. AB = CD	5. c.p.c.t.

❑ ***Prove that the equal chords of congruent circles subtend equal angles at their centre.***

Solution. Given two chords AB and CD of two congruent circles with centres O and O′ such that AB = CD.



We need to prove that $\angle AOB = \angle CO'D$.

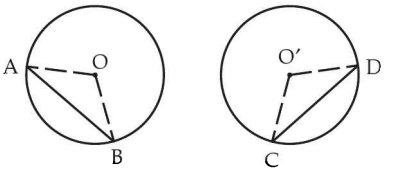
As the circles are congruent, they have equal radii.

In ΔOAB and $\Delta O'CD$,

	OA = O′C	(radii of congruent circles)
	OB = O′D	(radii of congruent circles)
	AB = CD	(given)
∴	$\Delta OAB \cong \Delta O'CD$	(by SSS rule of congruency)
∴	$\angle AOB = \angle CO'D$	(c.p.c.t.)

❑ ***Prove that if chords of congruent circles subtend equal angles at their centres, then the chords are equal.***

Solution. Given two chords AB and CD of two congruent circles with centres O and O′ such that



$\angle AOB = \angle CO'D$.

We need to prove that AB = CD.

As the circles are congruent, they have equal radii.

In ΔOAB and $\Delta O'CD$,

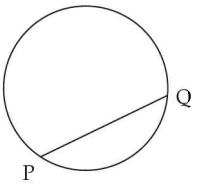
	OA = O′C	(radii of congruent circles)
	OB = O′D	(radii of congruent circles)
	$\angle AOB = \angle CO'D$	(given)
∴	$\Delta OAB \cong \Delta O'CD$	(by SAS rule of congruency)
∴	AB = CD	(c.p.c.t.)

From theorems 11.1 and 11.2 and the above results, it follows that:

✦ ***If two chords of a circle (or of congruent circles) are equal, then they subtend equal angles at the centre (or centres) and conversely, if they subtend equal angles at the centre (or centres), then the chords are equal.***

❑ Axiom of equal arcs

If $\overset{\frown}{PQ}$ is an arc of a circle, then the line segment PQ is called the corresponding chord of the arc PQ and conversely, if PQ is a chord of a circle then the arc PQ is called the corresponding arc of the chord PQ.



✦ **Congruent arcs**

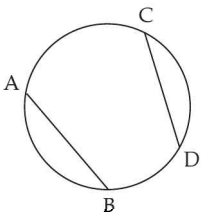
Two arcs of the same circle (or of congruent circles) are called ***congruent (or equal) arcs*** *if one arc is put on the other arc without bending or twisting then one arc superimposes on the other completely.*

Let us examine relationship between two arcs and their corresponding chords.

Draw two equal chords of a circle. What can you say the size of their corresponding arcs? Is the arc made by the first chord equal to the corresponding arc made by another chord? In fact, these arcs are congruent.

To verify this fact, we have:

In the adjoining figure, AB and CD are two equal chords of a circle. Cut the arc corresponding to the chord AB from the circle along AB and put it on the corresponding arc made by the chord CD. We find that arc AB superimposes the arc CD completely *i.e.* these arcs are congruent. This shows that the equal chords make congruent arcs and conversely congruent arcs make equal chords of a circle. We can state this result as follows:



If two chords of a circle (or of congruent circles) are equal, then their corresponding arcs are congruent and conversely, if two arcs are congruent then their corresponding chords are equal.

The above result is known as **axiom of equal (or congruent) arcs**.

In view of the above axiom and theorem 11.1 and theorem 11.2, the following result is true:

✦ ***Congruent arcs (or equal arcs) of a circle subtend equal angles at the centre and conversely, if two arcs of a circle subtend equal angles at the centre then the arcs are congruent (or equal).***

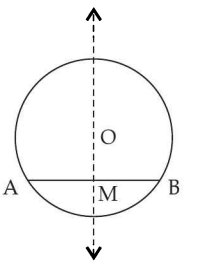
In fact, the above result is true for congruent circles also. We have:

✦ ***If two arcs of a circle (or of congruent circles) are equal, then they subtend equal angles at the centre (or centres) and conversely, if the two arcs of a circle (or of congruent circles) subtend equal angles at the centre (or centres), then the arcs are equal.***

❑ Perpendicular from the centre of a circle to a chord

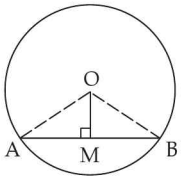
Perform the following activity:

Draw a circle on a tracing paper. Let O be its centre. Draw any chord AB of the circle. Fold the paper along a line through O so that a portion of the chord falls on the other portion and form a crease. Let the crease (shown dotted in the adjoining figure) cut the chord AB at the point M. Then $\angle OMA = \angle OMB = 90°$ *i.e.* OM is perpendicular to AB. We observe that B coincides with A, so AM = BM *i.e.* the perpendicular from the centre to the chord bisects the chord. Moreover, this result is true for all circles and every chord (not a diameter) of a circle. In fact, we have:



Theorem 11.3

The perpendicular from the centre of a circle to a chord bisects the chord.



Given. A chord AB of a circle with centre O and OM ⊥ AB.

To prove. AM = BM.

Proof.

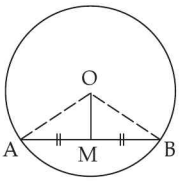
Statements	*Reasons*
In ΔOAM and ΔOBM	
1. OA = OB	1. Radii of same circle.
2. ∠OMA = ∠OMB	2. Each ∠ = 90°, since OM ⊥ AB.
3. OM = OM	3. Common
4. ΔOAM ≅ ΔOBM	4. RHS rule of congruency.
5. AM = BM	5. c.p.c.t.

The converse of the above theorem is also true. In fact, we have:

Theorem 11.4

The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.

Given. A chord AB of a circle with centre O and OM bisects the chord AB.



To prove. OM ⊥ AB.

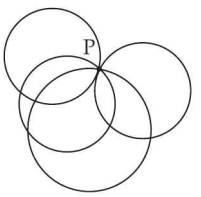
Construction. Join OA and OB.

Proof.

Statements	*Reasons*
In ΔOAM and ΔOBM	
1. OA = OB	1. Radii of same circle.
2. AM = BM	2. M is mid-point of AB.
3. OM = OM	3. Common
4. ΔOAM ≅ ΔOBM	4. SSS rule of congruency.
5. ∠OMA = ∠OMB	5. c.p.c.t.
6. ∠OMA + ∠OMB = 180°	6. Linear pair.
7. ∠OMA + ∠OMA = 180° ⇒ ∠OMA = 90° Hence, OM ⊥ AB.	7. From 5 and 6.

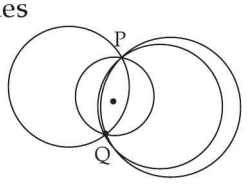
❑ Circle passing through three points

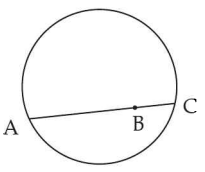
We know that two points are sufficient to determine a line *i.e.* there is one and only line passing through two points. The question arises! How many points are sufficient to determine a circle?



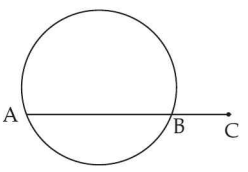
Take a point P. We see that infinitely many circles (shown in the adjoining figure) can be drawn passing through the point P.

Now take two points P and Q. Again we see that infinitely many circles (shown in the adjoining figure) can be drawn passing through two points P and Q. Now take three points A, B and C. If the points are collinear, then no circle can be drawn passing through these three points because then the third point will lie either inside or outside the circle (as shown in the figure given below).





(i)



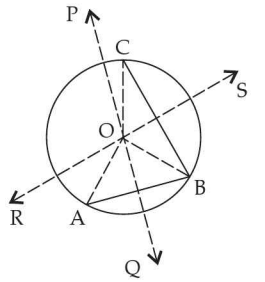
(ii)

If the points A, B and C are non-collinear, then there is one and only one circle passing through them. In fact, we have:

Theorem 11.5

There is one and only one circle passing through three given non-collinear points.

Given. Three non-collinear points A, B and C.



To prove. One and only one circle can be drawn passing through the points A, B and C.

Construction. Join AB and BC.

Draw perpendicular bisectors of line segments AB and BC, say PQ and RS respectively. Let these bisectors meet at O. Join OA, OB and OC.

Proof.

Statements	*Reasons*
1. OA = OB	1. O lies on the perpendicular bisector PQ of AB and every point on the perpendicular bisector of a line segment is equidistant from its end points.
2. OB = OC	2. O lies on the perpendicular bisector RS of BC and every point on the perpendicular bisector of a line segment is equidistant from its end points.
3. OA = OB = OC	3. From 1 and 2.
4. If a circle is drawn with O as centre and radius = OA, it passes through the points B and C.	4. From 3, O is equidistant from points A, B and C.
5. There is only one circle passing through the points A, B and C.	5. Since two lines can intersect at only one point, the perpendicular bisectors of AB and BC meet at only one point O.
Hence, one and only one circle can be drawn passing through three given non-collinear points.	

Corollary 1. ***In the plane of a circle, the perpendicular bisector of a chord of a circle passes through its centre.***

Corollary 2. ***The perpendicular bisectors of two (non-parallel) chords of a circle intersect at the centre of the circle.***

Corollary 3. ***As there is one and only one circle passing through three non-collinear points, two different circles can meet atmost in two different points.***

❑ Equal chords and their distances from the centre

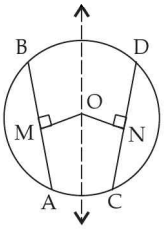
A circle has infinitely many chords. We can observe by drawing chords of a circle that longer chord is nearer to the centre than the smaller chord. It may be verified by drawing several chords of a circle of different lengths and measuring their distances from the centre. Note that the distance of the diameter from the centre is zero because the centre lies on it.

Let us examine relationship between the lengths of chords and their distances from the centre.

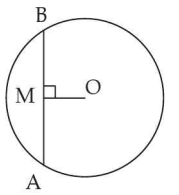
Perform the following activity:

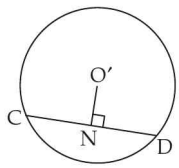
Draw a circle of any radius on a tracing paper. Draw two equal chords AB and CD and also the perpendiculars OM and ON respectively on them. Fold the figure so that

C falls on A and D falls on B (as shown in figure (*i*)). We observe that O lies on the increase and N falls on M. Thus, OM = ON.



(*i*)



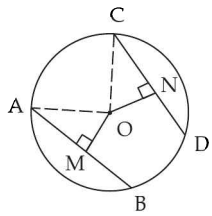


(*ii*)

Repeat the activity by drawing congruent circles with centres O and O′ and taking equal chords AB and CD one on each. Draw perpendiculars OM and O′N on AB and CD respectively (as shown in figure (*ii*)). Cut one circular disc and put on the other so that AB coincides with CD. We shall find that O coincides with O′ and M coincides with N. Thus, OM = O′N. In fact, we have:

Theorem 11.6

Equal chords of a circle are equidistant from the centre.



Given. AB and CD are chords of a circle with centre O, and AB = CD.

To prove. AB and CD are equidistant from O *i.e.* if OM ⊥ AB and ON ⊥ CD, then OM = ON.

Construction. Join OA and OC.

Proof.

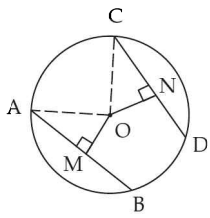
Statements	*Reasons*
1. AM = $\frac{1}{2}$ AB	1. Perpendicular from centre bisects the chord (Theorem 11.3).
2. CN = $\frac{1}{2}$ CD	2. Same as above.
3. AM = CN In Δs OAM and OCN	3. AB = CD (given).
4. AM = CN	4. From 3.
5. ∠AMO = ∠CNO	5. Each = 90°, OM ⊥ AB and ON ⊥ CD.
6. OA = OC	6. Radii of same circle.
7. ΔOAM ≅ ΔOCN	7. RHS rule of congruency.
8. OM = ON	8. 'c.p.c.t.'

In a similar way, we can prove that equal chords of congruent circles are equidistant from their centres. Thus, we have:

Equal chords of a circle (or of congruent circles) are equidistant from the centre (or centres).

The converse of the above theorem is also true. In fact, we have:

Theorem 11.7



Chords of a circle that are equidistant from the centre of a circle are equal.

Given. AB and CD are chords of a circle with centre O; OM ⊥ AB, ON ⊥ CD and OM = ON.

To prove. AB = CD.

Construction. Join OA and OC.

Statements	*Reasons*
In ΔOAM and ΔOCN	
1. $OM = ON$	1. Given.
2. $OA = OC$	2. Radii of same circle.
3. $\angle AMO = \angle CNO$	3. Each $= 90°$, $OM \perp AB$ and $ON \perp CD$.
4. $\Delta OAM \cong \Delta OCN$	4. RHS rule of congruency.
5. $AM = CN$	5. 'c.p.c.t.'
6. $AM = \frac{1}{2} AB$	6. Perpendicular from centre bisects the chord.
7. $CN = \frac{1}{2} CD$	7. Perpendicular from centre bisects the chord.
8. $\frac{1}{2} AB = \frac{1}{2} CD$ $\Rightarrow AB = CD$.	8. $AM = CN$, from 5.

In a similar way, we can prove that chords of congruent circles that are equidistant from their centres are equal. Thus, we have:

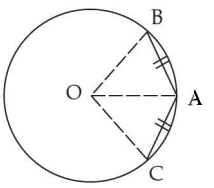
✦ ***Chords of a circle (or of congruent circles) that are equidistant from the centre (or centres) are equal in length.***

Illustrative Examples

Example 1. *AB and AC are two equal chords of a circle. Prove that the bisector of $\angle BAC$ passes through the centre of the circle.* *(NCERT Exemplar)*

Solution. AB and AC are chords of a circle with centre O such that AB = AC.

Join OB, OC and OA.



In ΔOAB and ΔOAC,

$AB = AC$ (given)

$OB = OC$ (radii of same circle)

$OA = OA$ (Common)

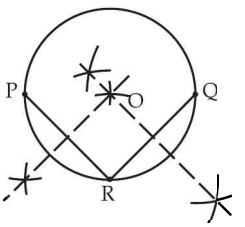
$\therefore \quad \Delta OAB \cong \Delta OAC$ (SSS rule of congruency)

$\therefore \quad \angle OAB = \angle OAC$

Hence, the centre O of the circle lies on the bisector of $\angle BAC$.

Example 2. *Given an arc of a circle, complete the circle.* *(NCERT)*

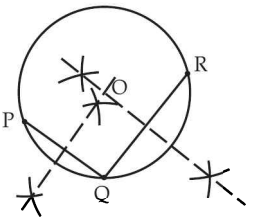
Solution. Let arc PQ of a circle be given. Take any point R on the arc PQ. Join PR and RQ.



Draw perpendicular bisectors of PR and RQ. If these bisectors meet at the point O, then O is the centre of the circle. Taking O as centre and OP as radius, complete the circle.

Example 3. *Suppose you are given a circle. Give a construction to find its centre.* *(NCERT)*

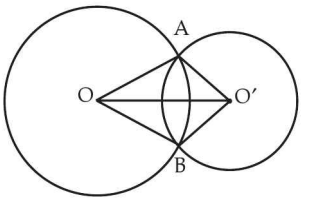
Solution. Given a circle. We are to find its centre by construction. Take any three points on the circle, say P, Q and R, as shown in the adjoining figure. Join PQ and QR. Draw perpendicular bisectors of PQ and QR. The point of intersection of these bisectors is the centre of the given circle.



Note. In fact, we can take any two non-parallel chords of the given circle. The point of intersection of the perpendicular bisectors of the chords is the centre of the given circle.

Example 4. *Prove that the line of centres of two intersecting circles subtend equal angles at the two points of intersection.* *(NCERT)*

Solution. Given two circles with centres O and O′, and intersecting at the points A and B. Join OO′.



We need to prove that $\angle OAO' = \angle OBO'$.

In $\Delta OAO'$ and $\Delta OBO'$,

$OA = OB$ (radii of same circle)

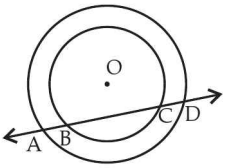
$O'A = O'B$ (radii of same circle)

$OO' = OO'$ (common)

$\therefore \quad \Delta OAO' \cong \Delta OBO'$ (by SSS rule of congruency)

$\therefore \quad \angle OAO' = \angle OBO'$ (c.p.c.t.)

Example 5. *If a line intersects two concentric circles with centre O at A, B, C and D (shown in the adjoining figure), then prove that AB = CD.* *(NCERT)*

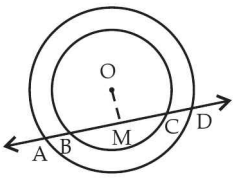


Solution. From O, draw $OM \perp AD$.

In the larger circle, as $OM \perp AD$,

so OM bisects AD

$\Rightarrow \quad AM = MD$ ...(*i*)



In the smaller circle, $OM \perp BC$,

so OM bisects BC

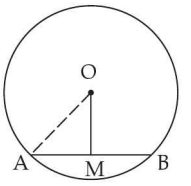
$\Rightarrow \quad BM = MC$ ...(*ii*)

From (*i*) and (*ii*), we get

$AM - BM = MD - MC \Rightarrow AB = CD$, as required.

Example 6. *A chord of length 16 cm is drawn in a circle of diameter 20 cm. Calculate its distance from the centre of the circle.*

Solution. Let AB be a chord of a circle with centre O such that AB = 16 cm and radius of the circle = OA = $\frac{1}{2}$ of diameter = $\frac{1}{2} \times 20$ cm = 10 cm.



From O, draw $OM \perp AB$.

Since $OM \perp AB$, AB is bisected at M. (Theorem 11.3)

$\therefore \quad AM = \frac{1}{2} AB = \frac{1}{2} \times 16 \text{ cm} = 8 \text{ cm}.$

From right angled ΔOAM, by Pythagoras theorem, we get

$OA^2 = OM^2 + AM^2$

$\Rightarrow \quad 10^2 = OM^2 + 8^2$

$\Rightarrow \quad OM^2 = 10^2 - 8^2 = 100 - 64 = 36$

$\Rightarrow \quad OM = 6$ cm.

Hence, the given chord is at a distance of 6 cm from the centre of the circle.

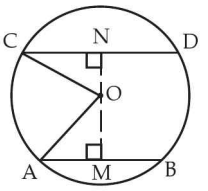
Example 7. *The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at a distance of 4 cm from the centre, find the distance of the other chord from the centre.* *(NCERT)*

Solution. Let AB = 6 cm, CD = 8 cm and O be the centre of the circle and r cm be its radius.

From O, draw $OM \perp AB$ and $ON \perp CD$.

As OM ⊥ AB, OM bisects AB

$\therefore$ AM = $\frac{1}{2}$ × 6 cm = 3 cm.



Given OM = 4 cm.

In ΔOAM, ∠M = 90°, by Pythagoras theorem,

$OA^2 = AM^2 + OM^2 \Rightarrow r^2 = 3^2 + 4^2 \Rightarrow r^2 = 25 \Rightarrow r = 5$.

As ON ⊥ CD, ON bisects CD

$\therefore$ CN = $\frac{1}{2}$ × 8 cm = 4 cm.

In ΔOCN, ∠N = 90°, by Pythagoras theorem,

$OC^2 = ON^2 + CN^2 \Rightarrow 5^2 = ON^2 + 4^2$ ($\because$ OC = radius = 5 cm)

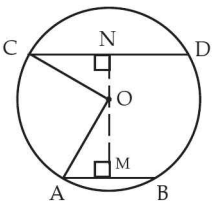
$\Rightarrow$ $ON^2 = 25 - 16 = 9 \Rightarrow ON = 3$ cm.

Note. In the above problem, if the chords AB and CD are not parallel, even then we obtain the same answer.

Example 8. *Two chords AB and CD of lengths 5 cm and 11 cm respectively of a circle are parallel to each other and are on the opposite sides of its centre. If the distance between AB and CD is 6 cm, find the radius of the circle.* *(NCERT)*

Solution. (*i*) Let O be the centre of the circle.

From O, draw OM ⊥ AB and ON ⊥ CD.



$\therefore$ AM = $\frac{1}{2}$ AB = $\frac{1}{2}$ × 5 cm = $\frac{5}{2}$ cm and

CN = $\frac{1}{2}$ CD = $\frac{1}{2}$ × 11 cm = $\frac{11}{2}$ cm.

Since AB ∥ CD, points M, O and N are collinear.

Let OM = x cm, then ON = $(6 - x)$ cm ($\because$ MN = 6 cm, given)

Let the radius of the circle be r cm.

From right angled triangles OMA and ONC (by Pythagoras theorem), we get

$OA^2 = AM^2 + OM^2$ and $OC^2 = CN^2 + ON^2$

$\Rightarrow$ $r^2 = \left(\frac{5}{2}\right)^2 + x^2$ ...(*i*) and $r^2 = \left(\frac{11}{2}\right)^2 + (6 - x)^2$ ...(*ii*)

$\Rightarrow$ $\left(\frac{5}{2}\right)^2 + x^2 = \left(\frac{11}{2}\right)^2 + (6 - x)^2 \Rightarrow \frac{25}{4} + x^2 = \frac{121}{4} + 36 - 12x + x^2$

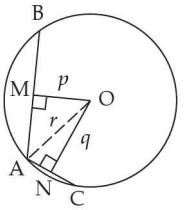
$\Rightarrow$ $12x = 60 \Rightarrow x = 5$.

Putting $x = 5$ in (*i*), we get

$$r^2 = \left(\frac{5}{2}\right)^2 + 5^2 = \frac{25}{4} + 25 = \frac{125}{4} \Rightarrow r = \frac{5\sqrt{5}}{2} \text{ cm}.$$

Example 9. *AB and AC are two chords of a circle of radius r such that AB = 2 AC. If p and q are the distances of AB and AC from the centre, prove that* $4q^2 = p^2 + 3r^2$. *(NCERT Exemplar)*

Solution. Let O be the centre of circle and AC = $2x$, then AB = 2AC = 2 × $2x$ = $4x$. From O, draw OM ⊥ AB and ON ⊥ AC. As OM bisects AB and ON bisects AC, AM = $\frac{1}{2}$ AB = $\frac{1}{2}$ × $4x$ = $2x$ and AN = $\frac{1}{2}$ AC = $\frac{1}{2}$ × $2x$ = x.



From right angled triangles OAM and OAN (by Pythagoras theorem), we get

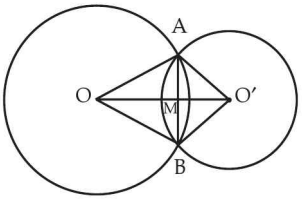
$OA^2 = AM^2 + OM^2$ and $OA^2 = AN^2 + ON^2$

$\Rightarrow \quad r^2 = (2x)^2 + p^2$ and $r^2 = x^2 + q^2$

$\Rightarrow \quad r^2 = 4x^2 + p^2$ and $x^2 = r^2 - q^2$

$\Rightarrow \quad r^2 = 4(r^2 - q^2) + p^2 \Rightarrow 4q^2 = p^2 + 3r^2$.

Example 10. *If two circles intersect at two points, prove that their centres lie on the perpendicular bisector of the common chord.*



Solution. Given two circles with centre O and O′, and intersecting at the points A and B so that AB is their common chord. Join OO′ to meet AB at M. We need to prove that OO′ is the perpendicular bisector of AB *i.e.* AM = BM and OO′ ⊥ AB.

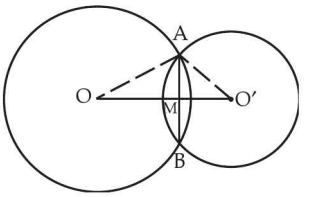
In ΔOAO′ and ΔOBO′,

	OA = OB	(radii of same circle)
	O′A = O′B	(radii of same circle)
	OO′ = OO′	(common)
∴	ΔOAO′ ≅ ΔOBO′	(by SSS rule of congruency)
∴	∠AOM = ∠BOM	(c.p.c.t.)

In ΔAOM and ΔBOM,

	OA = OB	(radii of same circle)
	OM = OM	(common)
	∠AOM = ∠BOM	(proved above)
∴	ΔAOM ≅ ΔBOM	(by SAS rule of congruency)
∴	AM = BM and ∠OMA = ∠OMB	(c.p.c.t.)
But	∠OMA + ∠OMB = 180°	(linear pair)
⇒	2 ∠OMA = 180° ⇒ ∠OMA = 90° ⇒ OO′ ⊥ AB.	

Example 11. *Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.* *(NCERT)*



Solution. Let O, O′ be the centres of circles with radii 5 cm, 3 cm respectively and AB be the common chord. As the line of centres is the perpendicular bisector of the common chord, OO′ ⊥ AB and M is mid-point of AB. Let AM = y cm and OM = x cm.

As OO′ = 4 cm (given), so MO′ = $(4 - x)$ cm.

In ΔOMA, ∠M = 90°, by Pythagoras theorem,

$OA^2 = OM^2 + AM^2 \Rightarrow 5^2 = x^2 + y^2$ ...(*i*)

In ΔO′MA, ∠M = 90°, by Pythagoras theorem,

$O'A^2 = OM'^2 + AM^2 \Rightarrow 3^2 = (4 - x)^2 + y^2$ ...(*ii*)

Subtracting (*ii*) from (*i*), we get

$25 - 9 = x^2 - (4 - x^2) \Rightarrow 16 = 8x - 16 \Rightarrow x = 4$.

Putting $x = 4$ in (*ii*), we get

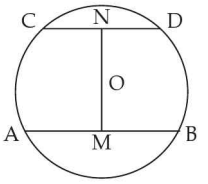
$3^2 = 0^2 + y^2 \Rightarrow y^2 = 9 \Rightarrow y = 3$ $(\because y > 0)$

∴ Length of common chord AB = 2 × AM = (2 × 3) cm = 6 cm.

Note. In the above question, we observe that $x = 4 \Rightarrow$ MO′ = 0 ⇒ the points M and O′ coincide ⇒ the centre of the smaller circle lies on the common chord ⇒ the common chord is a diameter of the smaller circle.

Example 12. *If a line segment joining mid-points of two chords of a circle passes through the centre, prove that the two chords are parallel.* *(NCERT Exemplar)*

Solution. Let AB, CD be two chords of a circle with centre O and M, N be the mid-points of AB, CD respectively. We are given that the line segment joining M and N passes through O *i.e.* M, O and N are collinear. We want to prove that AB ∥ CD.



Since M is mid-point of AB and O is the centre of circle, OM ⊥ AB.

$\therefore \quad \angle OMB = 90°$ ...(*i*)

Similarly, ON ⊥ CD.

$\therefore \quad \angle OND = 90°$ ...(*ii*)

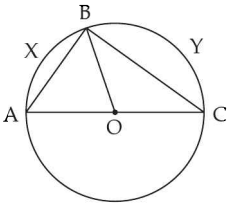
From (*i*) and (*ii*), we get

$\angle OMB + \angle OND = 90° + 90° = 180°.$

But these are interior angles on the same side of the transversal MON and line segments AB and CD,

$\therefore \quad AB \parallel CD.$

Example 13. *In the adjoining figure, AOC is a diameter of the circle and arc AXB* $= \frac{1}{2}$ *arc BYC. Find* $\angle BOC$. *(NCERT Exemplar)*



Solution. As arc AXB $= \frac{1}{2}$ arc BYC,

$\angle AOB = \frac{1}{2} \angle BOC$ ...(*i*)

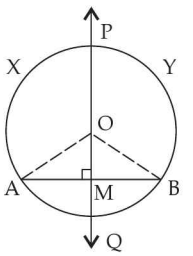
Since AOC is a diameter, $\angle AOB + \angle BOC = 180°$

$\Rightarrow \quad \frac{1}{2} \angle BOC + \angle BOC = 180°$ (using (*i*))

$\Rightarrow \quad \frac{3}{2} \angle BOC = 180° \Rightarrow \angle BOC = \frac{2}{3} \times 180° = 120°.$

Example 14. *If the perpendicular bisector of a chord AB of a circle PXAQBY intersects the circle at P and Q, prove that arc PXA = arc PYB.* *(NCERT Exemplar)*

Solution. Given the perpendicular bisector of a chord AB of a circle intersects the circle at points P and Q. Let the centre of the circle be O and PQ meet chord AB at M. Since the right bisector of a chord always passes through the centre of the circle, so PQ passes through O *i.e.* PQ is a diameter of the circle.



Join OA and OB.

In ΔOAM and ΔOBM,

OA = OB (radii of same circle)

AM = BM (∵ M is mid-point of AB)

OM = OM (common)

$\therefore \quad \Delta OAM \cong \Delta OBM$ (SSS rule of congruency)

$\therefore \quad \angle AOM = \angle BOM$ (c.p.c.t.)

$\Rightarrow \quad \angle AOQ = \angle BOQ$

$\Rightarrow \quad$ arc AQ = arc BQ ...(*i*)

(∵ If two arcs of a circle subtend equal angles at the centre then the arcs are equal). As PQ is a diameter of the circle, PQ divides the whole arc of the circle into two equal arcs, so

arc PXAQ = arc PYBQ ...(*ii*)

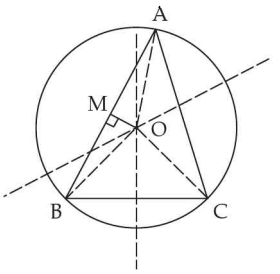
From (*i*) and (*ii*), we get

arc PXAQ – arc AQ = arc PYBQ – arc BQ

$\Rightarrow$ arc PXA = arc PYB.

Example 15. *A, B and C are three points on a circle. Prove that the perpendicular bisectors of AB, BC and CA are concurrent.* *(NCERT Exemplar)*

Solution. Let the perpendicular bisectors (shown dotted) of BC and CA meet at O. From O, draw OM $\perp$ AB. As O lies on the perpendicular bisector of BC,



OB = OC. ...(*i*)

Also, as O lies on the perpendicular bisector of CA,

OC = OA ...(*ii*)

From (*i*) and (*ii*), we get OA = OB.

In ΔOAM and ΔOBM,

OA	= OB	(proved above)
$\angle$OMA	= $\angle$OMB	(each = 90°, $\because$ OM $\perp$ AB)
OM	= OM	(common)
$\therefore$ ΔOAM	$\cong$ ΔOBM	(RHS rule of congruency)
$\therefore$ AM	= BM	(c.p.c.t.)

$\Rightarrow$ M is mid-point of AB.

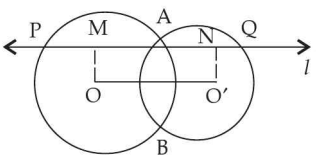
Therefore, OM is the right bisector of AB.

Hence, the right bisectors of AB, BC and CA pass through O *i.e.* they are concurrent.

Example 16. *Two circles with centres O and O′ intersect at two points A and B. A line PQ is drawn parallel to OO′ through A (or B) intersecting the circles at P and Q. Prove that PQ = 2OO′.* *(NCERT Exemplar)*

Solution. From O, draw OM $\perp$ PQ and from O′, draw O′N $\perp$ PQ.

Then M bisects PA and N bisects AQ



$\Rightarrow$ MA = $\frac{1}{2}$ PA and AN = $\frac{1}{2}$ AQ

$\Rightarrow$ MA + AN = $\frac{1}{2}$ (PA + AQ)

$\Rightarrow$ MN = $\frac{1}{2}$ PQ

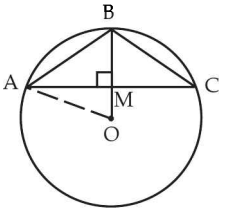
$\Rightarrow$ OO′ = $\frac{1}{2}$ PQ ($\because$ MOO′N is a rectangle, why?)

$\Rightarrow$ PQ = 2OO′.

Example 17. *Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5 m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma, and Salma and Mandip is 6 m each, what is the distance between Reshma and Mandip?*

What is the utility of playing outdoor games? ***(Value Based)***

Solution. Let the points A, B and C on the circle represent the three girls Reshma, Salma and Mandip respectively.



Then AB = 6 m and BC = 6 m.

If O is the centre of the circle, then

OA = radius of the circle = 5 m.

From B, draw BM $\perp$ AC.

As ABC is an isosceles triangle with AB = BC, M is mid-point of AC. Therefore, BM is the perpendicular bisector of AC, and hence it passes through the centre of the circle.

Let AM = y metres and OM = x metres, then BM = $(5 - x)$ metres.

In ΔOAM, $\angle M = 90°$, by Pythagoras theorem,

$$OA^2 = OM^2 + AM^2 \Rightarrow 5^2 = x^2 + y^2 \quad \ldots(i)$$

In ΔAMB, $\angle M = 90°$, by Pythagoras theorem,

$$AB^2 = BM^2 + AM^2 \Rightarrow 6^2 = (5 - x)^2 + y^2 \quad \ldots(ii)$$

Subtracting (i) from (ii), we get

$$36 - 25 = (5 - x)^2 - x^2 \Rightarrow 11 = 25 - 10x$$

$$\Rightarrow \quad 10x = 14 \Rightarrow x = \frac{7}{5}.$$

Substituting this value of x in (i), we get

$$y^2 + \frac{49}{25} = 25 \Rightarrow y^2 = 25 - \frac{49}{25} = \frac{625 - 49}{25}$$

$$\Rightarrow \quad y^2 = \frac{576}{25} \Rightarrow y = \frac{24}{5}. \qquad (\because\ y > 0)$$

$$\therefore \quad AC = 2 \times AM = (2 \times y)\ \text{m} = \left(2 \times \frac{24}{5}\right)\ \text{m} = \frac{48}{5}\ \text{m} = 9.6\ \text{m}.$$

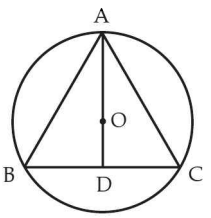
Hence, the distance between Reshma and Mandip is 9.6 m.

Playing outdoor games is good for health and keep the mind alert.

Example 18. *A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distances on its boundary each having a toy telephone in his hands to talk to each other. Find the length of the string of each phone.*

What is the value of a park in a colony? ***(Value Based)***

Solution. Let Ankur, Syed and David be represented by the points A, B and C respectively on the circle. As the three boys are at equal distances from each other, ABC is an equilateral triangle.



From A, draw AD ⊥ BC, then AD is a median and it passes through the centre O of the circle.

Also, the point O is the centroid of ΔABC,

so $\quad OA = \frac{2}{3}\ AD$

Let a side of ΔABC be a metres, then $BD = \frac{a}{2}$ metres.

In ΔABD, $\angle D = 90°$, by Pythagoras theorem, we get

$$AB^2 = BD^2 + AD^2 \Rightarrow AD^2 = a^2 - \left(\frac{a}{2}\right)^2 = \frac{3}{4}a^2 \Rightarrow AD = \frac{\sqrt{3}}{2}a.$$

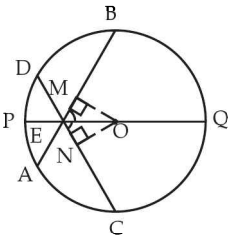
Now $OA = \frac{2}{3}\ AD \Rightarrow 20\ \text{m} = \frac{2}{3} \times \frac{\sqrt{3}}{2}a \Rightarrow a = 20\sqrt{3}\ \text{m}.$

$\therefore$ The length of the string of each phone = $20\sqrt{3}$ m.

Parks are very useful for the people. Children play, ladies and gents pass their free time and enjoy the nature.

Example 19. *If two intersecting chords of a circle make equal angles with the diameter passing through their point of intersection, then prove that the chords are equal.* *(NCERT)*

Solution. Given two chords AB and CD of a circle with centre O intersecting at the point E and PQ is the diameter passing through E such that $\angle BEQ = \angle CEQ$.



We want to prove that AB = CD.

From O, draw OM ⊥ AB and ON ⊥ CD.

In ΔOEM and ΔOEN

$\angle OME = \angle ONE$ (each = 90°, by construction)

$\angle OEM = \angle OEN$ ($\because$ $\angle BEQ = \angle CEQ$, given)

$OE = OE$ (common)

$\therefore \quad \Delta OEM \cong \Delta OEN$ (by AAS rule of congruency)

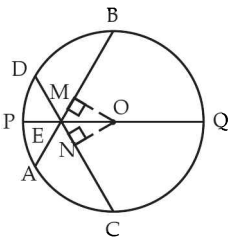
$\therefore \quad OM = ON$ (c.p.c.t.)

$\therefore \quad AB = CD$

(because chords which are equidistant from the centre of a circle are equal)

Example 20. *If two equal chords of a circle intersect within the circle, then prove that the line joining the point of intersection to the centre makes equal angles with the chords.* *(NCERT)*

Solution. Given two chords AB and CD of a circle with centre O intersecting at the point E and PQ is the diameter passing through E such that AB = CD.



We need to prove that $\angle BEQ = \angle CEQ$.

From O, draw $OM \perp AB$ and $ON \perp CD$.

Since equal chords of a circle are at equal distances from its centre,

$OM = ON$

In ΔOEM and ΔOEN,

$OM = ON$ (from above)

$OE = OE$ (common)

$\angle OME = \angle ONE$ (each = 90°, by construction)

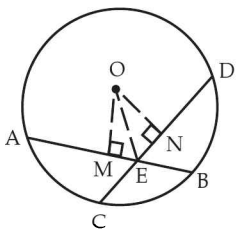
$\therefore \quad \Delta OEM \cong \Delta OEN$ (by RHS rule of congruency)

$\therefore \quad \angle MEO = \angle NEO$ (c.p.c.t.)

$\Rightarrow \quad \angle BEQ = \angle CEQ$, as required.

Example 21. *If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.* *(NCERT)*

Solution. Given AB and CD are chords of a circle with centre O intersecting at the point E such that AB = CD.



We need to prove that AE = ED and EB = CE.

From O, draw $OM \perp AB$ and $ON \perp CD$.

Join OE.

As $OM \perp AB$, OM bisects AB

$\Rightarrow \quad AM = MB = \frac{1}{2} AB.$

Also $ON \perp CD$, ON bisects CD

$\Rightarrow \quad CN = ND = \frac{1}{2} CD$ but $AB = CD$ (given)

$\therefore \quad AM = ND$ ...(*i*) and $MB = CN$ ...(*ii*)

In ΔOME and ΔONE,

$\angle OME = \angle ONE$ (each = 90°, by construction)

$OE = OE$ (common)

$OM = ON$ ($\because$ equal chords are equidistant from the centre)

$\therefore \quad \Delta OME \cong \Delta ONE$ (by RHS rule of congruency)

$\therefore \quad ME = EN$ ...(*iii*) (c.p.c.t.)

From (*i*) and (*iii*), we get

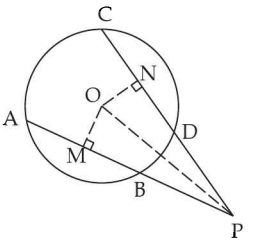
$AM + ME = ND + EN \Rightarrow AE = ED.$

From (*ii*) and (*iii*), we get

$MB - ME = CN - EN \Rightarrow EB = CE.$

Example 22. *Two equal chords AB and CD of a circle when produced meet at a point P. Prove that PB = PD.* *(NCERT Exemplar)*

Solution. Let O be the centre of the circle. From O, draw OM ⊥ AB and ON ⊥ CD. Join OP. As equal chords are equidistant from the centre, OM = ON.



In ΔOMP and ΔONP,

OM = ON

∠M = ∠N (each = 90°)

OP = OP (common)

∴ ΔOMP ≅ ΔONP (RHS rule of congruency)

∴ MP = NP ...(*i*)

Given AB = CD ⇒ $\frac{1}{2}$ AB = $\frac{1}{2}$ CD

⇒ MB = ND ...(*ii*)

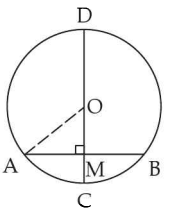
(∵ OM ⊥ AB ⇒ M is mid-point of AB, ON ⊥ CD ⇒ N is mid-point of CD)

From (*i*), we get

MB + PB = ND + PD ⇒ PB = PD. (using (*ii*))

Exercise 11.1

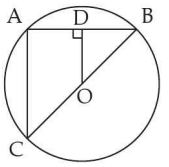
1. Write whether the following statements are true or false. Justify your answer:
   (*i*) Two congruent circles with centres O and O′ intersect at two points A and B. Then ∠AOB = ∠AO′B. *(NCERT Exemplar)*
   (*ii*) A circle of radius 3 cm can be drawn through two points A, B such that AB = 6 cm. *(NCERT Exemplar)*
   (*iii*) Two chords AB and CD of a circle are each at a distance of 4 cm from the centre. Then AB = CD. *(NCERT Exemplar)*
   (*iv*) Two chords of a circle of lengths 10 cm and 8 cm are at the distances 8.0 cm and 3.5 cm respectively from the centre. *(NCERT Exemplar)*
   (*v*) Two chords AB and AC of a circle with centre O are on opposite sides of OA. Then ∠OAB = ∠OAC. *(NCERT Exemplar)*
2. If arcs AXB and CYD of a circle are congruent, then find the ratio of AB and CD. *(NCERT Exemplar)*
3. Calculate the length of a chord which is at a distance of 6 cm from the centre of a circle of diameter 20 cm.
4. A chord of length 16 cm is at a distance 6 cm from the centre of a circle. Find the length of the chord of the same circle which is at a distance 8 cm from the centre.
5. In the adjoining figure, the diameter CD of a circle with centre O is perpendicular to the chord AB. If AB = 8 cm and CM = 2 cm, find the radius of the circle.



6. Two chords AB, CD of lengths 24 cm, 10 cm respectively of a circle are parallel. If the chords lie on the same side of the centre and the distance between them is 7 cm, then find the length of the diameter of the circle.
7. In a circle of radius 5 cm, AB and CD are two parallel chords of length 8 cm and 6 cm respectively. Calculate the distance between the chords, if they are on
   (*i*) the same side of centre (*ii*) the opposite sides of the centre.
8. An equilateral triangle of side 6 cm is inscribed in a circle. Find the radius of the circle.

9. The length of the common chord of two intersecting circles is 30 cm. If the radii of two circles are 25 cm and 17 cm, find the distance between their centres.

10. In the adjoining figure, OD is perpendicular to the chord AB of a circle with centre O. If BC is a diameter, show that CA = 2 OD.



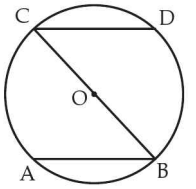
Hint: OD ⊥ AB ⇒ D is mid-point of AB.

O being the centre of circle is mid-point of BC.

In ΔABC, D is mid-point of AB and O is mid-point of BC.

∴ OD ∥ CA and OD = $\frac{1}{2}$ CA (Mid-point theorem)

11. In the adjoining figure, chords AB and CD are parallel and BC is a diameter of the circle with centre O. Show that AB = CD.



Hint: From O, draw OM ⊥ AB and ON ⊥ CD.

Prove that ΔOMB ≅ ΔONC (AAS)

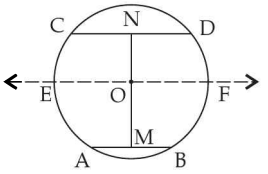
⇒ OM = ON ⇒ AB = CD

(∵ Chords which are equidistant from centre of a circle are equal.)

12. Prove that the line joining the mid-points of two parallel chords of a circle passes through the centre of circle.

Hint: Let AB, CD be chords of a circle with centre O.

AB ∥ CD, M and N are mid-points of AB and CD respectively. We want to prove that M, O, N are collinear.



Through O, draw EF ∥ AB, then EF ∥ CD.

As M is mid-point of AB, OM ⊥ AB ⇒ ∠OMA = 90°.

As EF ∥ AB and OM is transversal, ∠EOM + ∠OMA = 180°

⇒ ∠EOM + 90° = 180° ⇒ ∠EOM = 90°

Similarly, ∠EON = 90° ⇒ ∠EOM + ∠EON = 180°

⇒ MON is a straight line.

Answers

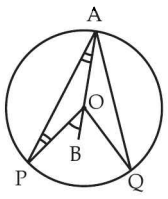
11.3 ANGLE PROPERTIES OF CIRCLES

❑ The following theorem gives a relationship between the angles subtended by an arc at the centre and at a point on the circle.

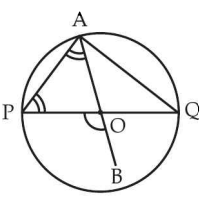
Theorem 11.8

The angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.

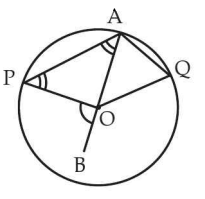
Given. A circle with centre O. An arc PQ subtends ∠POQ at the centre O and ∠PAQ at any point A on the remaining part of the circle.



(i)



(ii)



(iii)

To prove. ∠POQ = 2∠PAQ.

Construction. Join OA and produce it to a point B.

Proof. Consider three different cases as shown in the above figure. In (*i*), arc PQ is minor; in (*ii*), arc PQ is a semicircle and in (*iii*), arc PQ is major.

In all the cases, $\angle POB = \angle OAP + \angle OPA$ ...(*i*)

($\because$ an ext. $\angle$ of a Δ = sum of its two int. opp. $\angle$s)

Also in ΔOAP, $OA = OP$ (radii of same circle)

$\therefore$ $\angle OAP = \angle OPA$ ...(*ii*) ($\angle$s opp. equal sides are equal)

From (*i*) and (*ii*), we get

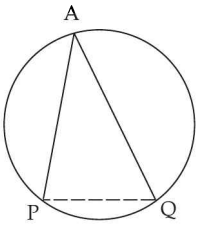
$\angle POB = 2\angle OAP$ ...(*iii*)

Similarly, $\angle BOQ = 2\angle OAQ$ ...(*iv*)

From (*iii*) and (*iv*), we get

$\angle POB + \angle BOQ = 2(\angle OAP + \angle OAQ)$

$\Rightarrow$ $\angle POQ = 2\angle PAQ$.



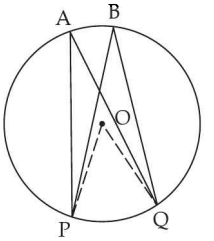
❑ **Angle in a segment**

Let PQ be a chord of a circle and A be any point on the circle, then $\angle PAQ$ is called angle formed in the segment PAQP by the chord PQ or the arc PQ.

Theorem 11.9

Angles in the same segment of a circle are equal.

Given. A circle with centre O, $\angle PAQ$ and $\angle PBQ$ are angles in the same segment.



To prove. $\angle POQ = \angle PBQ$.

Construction. Join OP and OQ.

Proof. $\angle POQ = 2\angle PAQ$ ...(*i*)

($\because$ angle subtended by an arc at the centre = double the angle subtended by it at any point on the remaining part of circle)

Similarly, $\angle POQ = 2\angle PBQ$ ...(*ii*)

From (*i*) and (*ii*), we get

$2\angle PAQ = 2\angle PBQ \Rightarrow \angle PAQ = \angle PBQ$.

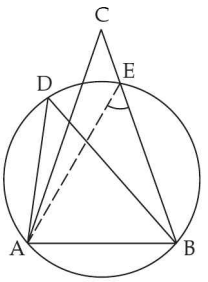
The converse of the above theorem is also true. In fact, we have:

Theorem 11.10

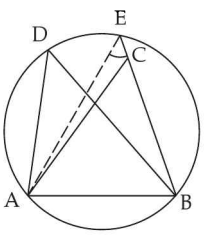
If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, then the four points lie on the same circle (i.e. they are concyclic).

Given. AB is a line segment and C, D are two points lying on the same side of the line containing the segment AB and $\angle ACB = \angle ADB$.

To prove. Points A, B, C and D are concyclic.



(*i*)



(*ii*)

Construction. If the points A, B, C and D are not concyclic, let a circle passing through three non-collinear points A, B and D meet BC at E (figure (*i*)) or BC produced at E (figure (*ii*)) so that the points A, B, E and D are concyclic.

Proof. $\angle AEB = \angle ADB$ (angles in the same segment)

But $\angle ACB = \angle ADB$ (given)

$\Rightarrow \quad \angle AEB = \angle ACB.$

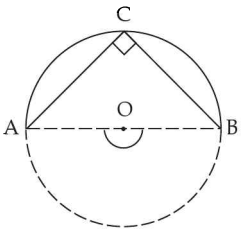
But $\angle AEB \neq \angle ACB$ because ext. angle of a triangle cannot be equal to int. opposite angle.

$\therefore$ Our supposition is wrong.

Hence, points A, B, C and D are concyclic.

Theorem 11.11

Angle in a semicircle is a right angle.



Given. A semicircle ACB of a circle with centre O.

To prove. $\angle ACB = 90°$.

Proof. $\angle AOB = 2\angle ACB$

($\because$ angle subtended by an arc at the centre = double the angle subtended by it at any point on the remaining part)

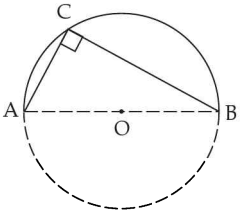
But AOB is a straight line, so $\angle AOB = 180°$.

$\Rightarrow \quad 2\angle ACB = 180° \Rightarrow \angle ACB = 90°$.

The converse of the above theorem is also true. In fact, we have:

Theorem 11.12

In an arc of a circle subtends a right angle at any point on the remaining part of the circle, then the arc is a semicircle.



Given. A circle with centre O, and arc AB (shown dotted) subtends $\angle ACB$ on the remaining part of the circle such that $\angle ACB = 90°$.

To prove. Arc AB is a semicircle.

Construction. Join OA and OB.

Proof. $\angle AOB = 2\angle ACB$

($\because$ angle subtended by an arc at the centre = double the angle subtended by it at any point on the remaining part)

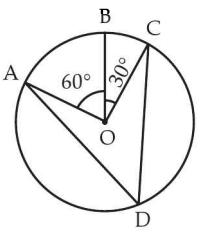
$\Rightarrow \quad \angle AOB = 2 \times 90°$ ($\because \angle ACB = 90°$, given)

$\Rightarrow \quad \angle AOB = 180° \Rightarrow$ AOB is a straight line, so it is a diameter. Hence, arc AB is a semicircle.

Remark. A circle drawn with hypotenuse of a right triangle as diameter passes through its opposite vertex.

Illustrative Examples

Example 1. *In the adjoining figure, A, B and C three points on a circle with centre O such that $\angle BOC = 30°$ and $\angle AOB = 60°$. If D is a point on the circle other than the arc ABC, find $\angle ADC$.* *(NCERT)*



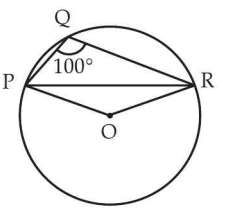
Solution. $\angle AOC = 60° + 30° = 90°$.

Now $\angle AOC = 2 \times \angle ADC$

($\because$ angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of circle)

$\Rightarrow \quad 90° = 2 \times \angle ADC \Rightarrow \angle ADC = 45°$.

Example 2. *In the adjoining figure, P, Q and R are points on a circle with centre O. If $\angle PQR = 100°$, then find $\angle OPR$.* *(NCERT)*



Solution. Reflex $\angle POR = 2 \times \angle PQR = 2 \times 100° = 200°$,

$\therefore \quad \angle POR = 360° - 200° = 160°$.

In ΔOPR, $\quad OP = OR$ (radii of same circle)

$\therefore \quad \angle OPR = \angle ORP.$

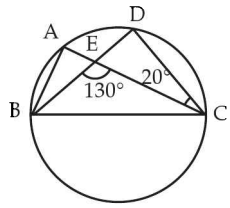
As the sum of angles in a triangle is 180°,

$\angle OPR + \angle ORP + \angle POR = 180°$

$\Rightarrow \angle OPR + \angle OPR + 160° = 180°$

$\Rightarrow 2\angle OPR = 20° \Rightarrow \angle OPR = 10°$.

Example 3. *In the adjoining figure, A, B, C and D are four points on a circle. AC and BD intersect at a point E. If $\angle BEC = 130°$ and $\angle ECD = 20°$, find $\angle BAC$.* *(NCERT)*



Solution. We note that $\angle BEC$ is an exterior angle of ΔCDE,

$\therefore \quad \angle BEC = \angle ECD + \angle EDC$

(ext. $\angle$ = sum of two int. opp. $\angle$s)

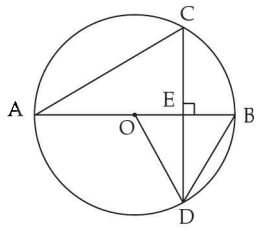
$\Rightarrow \quad 130° = 20° + \angle EDC \Rightarrow \angle EDC = 110°$

$\Rightarrow \quad \angle BDC = 110°$.

Now, $\angle BAC = \angle BDC$ (angles in the same segment of a circle)

$\Rightarrow \quad \angle BAC = 110°$.

Example 4. *In the adjoining figure, O is the centre of the circle, BD = OD and CD $\perp$ AB. Find $\angle CAB$.* *(NCERT Exemplar)*



Solution. Given BD = OD.

As OB = OD (radii of same circle)

So, BD = OD = OB

$\Rightarrow$ ΔOBD is equilateral $\Rightarrow \angle OBD = 60°$.

Also CD $\perp$ AB, so $\angle BED = 90°$.

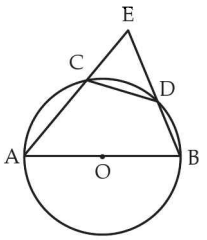
In ΔEBD, $\angle BED + \angle EBD + \angle BDE = 180°$ (sum of $\angle$s of a Δ)

$\Rightarrow 90° + 60° + \angle BDE = 180° \Rightarrow \angle BED = 30° \Rightarrow \angle BDC = 30°$

But $\angle CAB = \angle BDC$ (angles in the same segment of a circle)

$\Rightarrow \angle CAB = 30°$. ($\because \angle BDC = 30°$)

Example 5. *In the adjoining figure, AB is a diameter of a circle with centre O and CD is a chord equal to the radius of the circle. AC and BD when extended intersect at E. Prove that $\angle AEB = 60°$.* *(NCERT)*



Solution. Join OC, OD and CB.

As CD = OC = OD (radii of same circle)

ΔOCD is equilateral

$\Rightarrow \angle COD = 60°$.

$\therefore \quad \angle CBD = \frac{1}{2} \times \angle COD = \frac{1}{2} \times 60° = 30°$

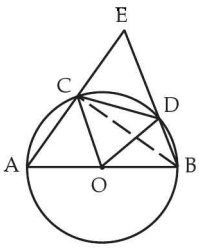
As AB is a diameter, $\angle ACB = 90°$ (angle in a semicircle)

Since $\angle ACB$ is an exterior angle of ΔCBE,

$\therefore \quad \angle ACB = \angle CEB + \angle CBE$ (ext. $\angle$ = sum of two int. opp. $\angle$s)

$\Rightarrow 90° = \angle CEB + 30°$ ($\because \angle CBE = \angle CBD = 30°$)

$\Rightarrow \angle CEB = 60°$, as required.

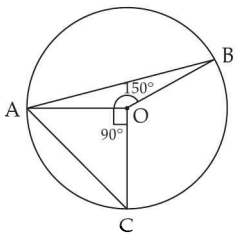


Example 6. *Two chords AB and AC of a circle subtend angles equal to 150° and 90° respectively at the centre. Find $\angle BAC$, if AB and AC lie on the opposite sides of the centre.* *(NCERT Exemplar)*

Solution. Let O be the centre of the circle, then

$\angle AOB = 150°$ and $\angle AOC = 90°$ (given)

$\therefore \quad \angle BOC = 360° - 150° - 90° = 120°$

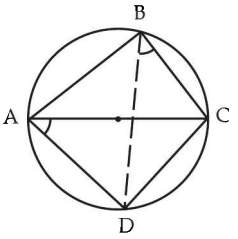


Now $\angle BAC = \frac{1}{2} \angle BOC$ (Theorem 11.8)

$\Rightarrow \quad \angle BAC = \frac{1}{2} \times 120° = 60°.$

Example 7. *If ABC and ADC are two right triangles with common hypotenuse AC, prove that* $\angle CAD = \angle CBD$. *(NCERT)*

Solution. Given two right triangles ABC and ADC with common hypotenuse AC.



We need to prove that $\angle CAD = \angle CBD$.

In ΔABC, $\angle ABC = 90°$.

Therefore, the circle drawn on AC as diameter passes through the point B.

In ΔADC, $\angle ADC = 90°$.

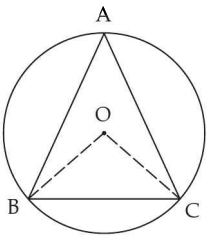
Therefore, the circle drawn on AC as diameter passes through D.

Thus, points A, B, C and D lie on a circle.

$\therefore \quad \angle CAD = \angle CBD$ (angles in the same segment of a circle are equal)

Example 8. *The circumcentre of a triangle ABC is O. Prove that* $\angle OBC + \angle BAC = 90°$. *(NCERT Exemplar)*

Solution. O is the circumcentre of ΔABC. Join OB and OC.



In ΔOBC, $OB = OC$ (radii of same circle)

$\therefore \quad \angle OBC = \angle OCB$ ($\because$ angles opposite sides are equal)

$\angle BOC + \angle OBC + \angle OCB = 180°$

$\Rightarrow \quad \angle BOC + 2\angle OBC = 180°$

$\Rightarrow \quad \frac{1}{2} \angle BOC + \angle OBC = 90°$ ...(*i*)

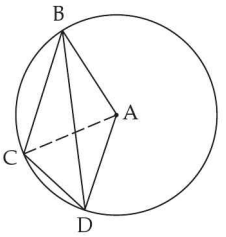
Now, $\angle BAC = \frac{1}{2} \angle BOC$ (Theorem 11.8)

$\Rightarrow \quad \angle BAC = 90° - \angle OBC$ (using (*i*))

$\Rightarrow \quad \angle OBC + \angle BAC = 90°$.

Example 9. *ABCD is such a quadrilateral that A is the centre of the circle passing through B, C and D. Prove that* $\angle CBD + \angle CDB = \frac{1}{2} \angle BAD$. *(NCERT Exemplar)*

Solution. Join AC.



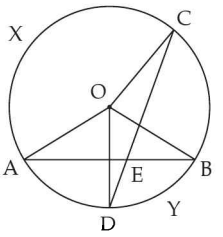
As the angle subtended by an arc at the centre = double the angle subtended by it at any point on the remaining part,

$\angle CBD = \frac{1}{2} \angle CAD$ and

$\angle CDB = \frac{1}{2} \angle CAB$

$\Rightarrow \quad \angle CBD + \angle CDB = \frac{1}{2} (\angle CAD + \angle CAB) = \frac{1}{2} \angle BAD.$

Example 10. *In the adjoining figure, AB and CD are two chords of a circle with centre O intersecting each other at E.*

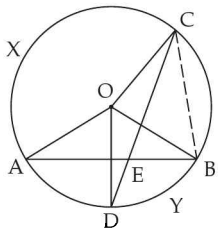


Prove that $\angle AEC = \frac{1}{2}$ *(angle subtended by arc CXA at centre + angle subtended by arc DYB at centre).* *(NCERT Exemplar)*

Solution. Join BC.

$\angle EBC = \angle ABC = \frac{1}{2}$ (angle subtended by arc CXA at centre) ...(*i*)

$\angle ECB = \angle DCB = \frac{1}{2}$ (angle subtended by arc DYB at centre) ...(*ii*)



In ΔEBC,

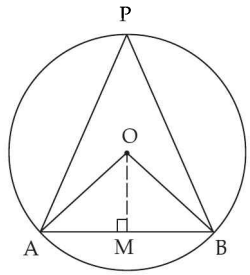
$\angle AEC = \angle EBC + \angle ECB$ (ext. $\angle$ = sum of two int. opp. $\angle$s)

$\Rightarrow$ $\angle AEC = \frac{1}{2}$ (angle subtended by arc CXA at centre) + angle subtended by arc DYB at centre) (using (*i*) and (*ii*))

Example 11. *A circle has radius $\sqrt{2}$ cm. It is divided into two segments by a chord of length 2 cm. Prove that the angle subtended by the chord at a point in major segment is 45°.*

(*NCERT Exemplar*)

Solution. Let O be the centre of a circle of radius $\sqrt{2}$ cm and AB be a chord of the circle such that AB = 2 cm.



From O, draw OM $\perp$ AB.

Then AM = $\frac{1}{2}$ AB = $\frac{1}{2}$ × 2 cm = 1 cm.

In ΔOAM, $\angle M = 90°$. By Pythagoras theorem, we get

$OA^2 = AM^2 + OM^2 \Rightarrow \left(\sqrt{2}\right)^2 = 1^2 + OM^2$

$\Rightarrow$ $OM^2 = 1 \Rightarrow OM = 1$ cm $\Rightarrow OM = AB$.

Thus, OAM is a right angled isosceles triangle

$\therefore$ $\angle AOM = 45°$

Similarly, $\angle BOM = 45°$

$\therefore$ $\angle AOB = \angle AOM + \angle BOM = 45° + 45° = 90°$.

If P is any point on the circle in the major segment, then

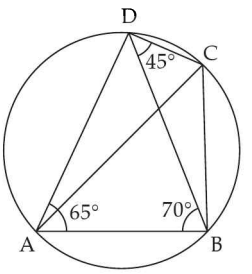
$\angle APB = \frac{1}{2}$ $\angle AOB$ (Theorem 11.8)

$\Rightarrow$ $\angle APB = \frac{1}{2} \times 90° = 45°$.

Example 12. *In the given figure, $\angle BAD = 65°$, $\angle ABD = 70°$ and $\angle BDC = 45°$.*

(i) Prove that AC is a diameter of the circle.

(ii) Find $\angle ACB$.



Solution. (*i*) In ΔABD,

$\angle ADB + \angle BAD + \angle ABD = 180°$ (sum of angles of a triangle)

$\Rightarrow$ $\angle ADB + 65° + 70° = 180°$

$\Rightarrow$ $\angle ADB = 180° - 65° - 70° = 45°$

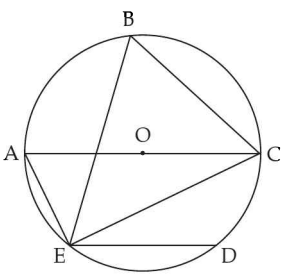
From figure, $\angle ADC = \angle ADB + \angle BDC = 45° + 45° = 90°$.

Thus, an arc of a circle subtends a right angle at any of the circumference, so the arc is a semicircle $\Rightarrow$ AC is a diameter of the circle.

(*ii*) $\angle ACB = \angle ADC$ (angles in same segment of a circle)

$\Rightarrow$ $\angle ACB = 45°$ ($\because$ $\angle ADC = 45°$, proved above)

Example 13. *In the adjoining figure, ED is a chord parallel to the diameter AC of the circle ABCDE. If $\angle CBE = 63°$, calculate $\angle DEC$.*



Solution. Given $\angle CBE = 63°$.

$\angle CAE = \angle CBE = 63°$.

(angles in same segment are equal)

$\angle AEC = 90°$ (angle in a semicircle = 90°)

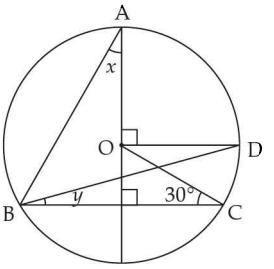
$\therefore$ $\angle ACE = 180° - (63° + 90°)$ (sum of angles of a $\Delta = 180°$)

$= 27°$

$\therefore$ $\angle DEC = \angle ACE = 27°$ (alt. $\angle$s, ED $\parallel$ AC)

Hence, $\angle DEC = 27°$.

Example 14. *In the adjoining figure, O is the centre of the circle. If $\angle BCO = 30°$, find x and y.* *(NCERT Exemplar)*



Solution. Join OB and produce DO to meet the circle at E.

In ΔOBC, OB = OC (radii of same circle)

$\Rightarrow$ $\angle OBC = \angle BCO$

$\Rightarrow$ $\angle OBC = 30°$ ($\because$ $\angle BCO = 30°$, given)

As OD and BC are both perpendicular to OA, therefore,

OD $\parallel$ BC *i.e.* ED $\parallel$ BC.

$\angle EOB = \angle OBC$ (alt. $\angle$s, OB is transversal)

$\Rightarrow$ $\angle EOB = 30°$ ($\because$ $\angle OBC = 30°$, proved above)

$\angle AOB = \angle AOE + \angle EOB = 90° + 30°$ ($\because$ OA $\perp$ ED)

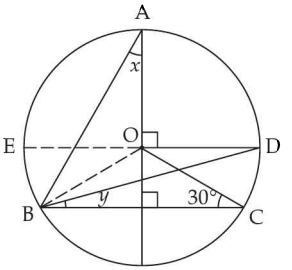
$\Rightarrow$ $\angle AOB = 120°$

In ΔAOB, OA = OB (radii of same circle)

$\angle OBA = \angle OAB \Rightarrow \angle OBA = x$ ($\because$ $\angle OAB = x$, given)

Also $\angle AOB + \angle OBA + \angle OAB = 180°$ (sum of $\angle$s of a Δ)

$\Rightarrow$ $120° + x + x = 180° \Rightarrow 2x = 60° \Rightarrow x = 30°$.



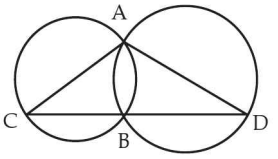
$\angle EDB = \frac{1}{2} \angle EOB$ (Theorem 11.8)

$\Rightarrow$ $\angle EDB = \frac{1}{2} \times 30° = 15°$.

$\angle DBC = \angle EBD$ (alt. $\angle$s, ED $\parallel$ BC and BD is a transversal)

$\Rightarrow$ $y = 15°$. ($\because$ $\angle EDB = 15°$)

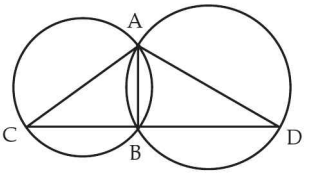
Example 15. *In the adjoining figure, two circles intersect at two points A and B. AC and AD are two diameters to the two circles. Prove that B lies on the line segment CD.* *(NCERT)*



Solution. Join AB.

As AC is a diameter, $\angle ABC = 90°$ (angle in a semicircle)

As AD is a diameter, $\angle ABD = 90°$ (angle in a semicircle)

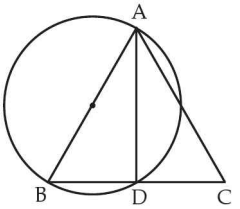


Now, $\angle ABC + \angle ABD = 90° + 90° = 180°$,

therefore, CBD is a straight line *i.e.* the point B lies on the line segment CD.

Example 16. *Prove that the circle described on any one of equal sides of an isosceles triangle as diameter bisects the base.*

Solution. Given an isosceles ΔABC in which AB = AC. The circle drawn on AB as diameter intersects the base BC of ΔABC at the point D. We need to prove that D is mid-point of BC.



As AB is a diameter, $\angle ADB = 90°$ (angle in a semicircle)

$\Rightarrow$ AD $\perp$ BC.

In ΔABD and ΔACD,

AB = AC (given)

AD = AD (common)

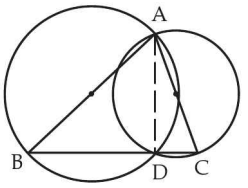
$\angle ADB = \angle ADC$ (each = 90°)

$\therefore \quad \Delta ABD \cong \Delta ACD$ (by RHS rule of congruency)

$\therefore \quad BD = DC$ (c.p.c.t.)

Example 17. *If circles are drawn taking two sides of a triangle as diameters, prove that the other point of intersection of these circles lie on the third side.* (*NCERT*)

Solution. Given a ΔABC. Circle drawn on AB as diameter intersects BC at D. We need to prove that the circle drawn on AC as diameter passes through D.



Join AD.

As AB is a diameter, $\angle ADB = 90°$ (angle in a semicircle)

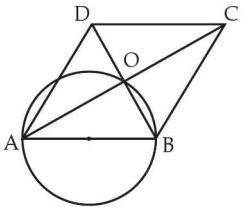
But $\angle ADB$ and $\angle ADC$ form a linear pair,

so $\angle ADC + \angle ADB = 180° \Rightarrow \angle ADC + 90° = 180° \Rightarrow \angle ADC = 90°$.

Since $\angle ADC = 90°$, therefore, the circle drawn on AC as diameter will pass through D.

Example 18. *Prove that the circle drawn with any side of a rhombus as diameter, passes through the point of intersection of its diagonals.* (*NCERT*)

Solution. Given a rhombus ABCD and its diagonals AC and BD intersect at the point O. We need to prove that the circle drawn on any side, say AB, as a diameter passes through the point O. Since the diagonals of a rhombus intersect at right angles, $\angle AOB = 90°$.

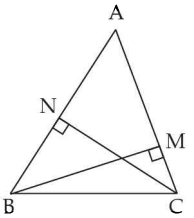


Therefore, the circle drawn on AB as a diameter passes through the point O.

Example 19. *If BM and CN are the perpendiculars drawn on the sides AC and AB of the triangle ABC, prove that the points B, C, M and N are concyclic.* (*NCERT Exemplar*)

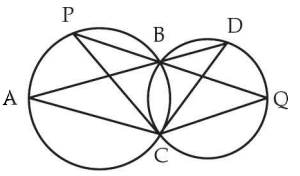
Solution. Given BM ⊥ AC and CN ⊥ AB

$\Rightarrow \quad \angle BMC = \angle BNC$ (each = 90°)



Thus, the line segment BC subtends equal angles at two other points M and N lying on the same side of the line segment BC, therefore, the points B, C, M and N are concyclic. (Theorem 11.10).

Example 20. *In the adjoining figure, two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D, P and Q respectively. Prove that* $\angle ACP = \angle QCD$. (*NCERT*)



Solution. $\angle ABP = \angle QBP$ ...(*i*) (vert. opp. ∠s)

$\angle ACP = \angle ABP$ ...(*ii*)

(angles in the same segment of a circle subtended by arc AP)

$\angle QCD = \angle QBD$ ...(*iii*)

(angles in the same segment of a circle subtended by arc QD)

From (*i*), (*ii*) and (*iii*), we get

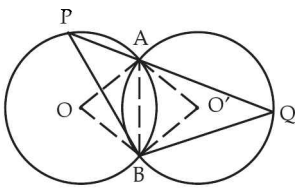
$\angle ACP = \angle QCD$, as required.

Example 21. *Two congruent circles intersect each other at points A and B. Through A, any line segment PAQ is drawn so that P, Q lie on the two circles. Prove that BP = BQ.* (*NCERT*)

Solution. Given two congruent circles with centres O, O′ and intersecting each other at points A and B.

We need to prove that BP = BQ.

As AB is a common chord,



$\angle AOB = \angle AO'B$ ...(*i*)

($\because$ equal chords subtend equal angles at the centres of two congruent circles)

$\angle AOB = 2\angle APB$ ...(*ii*)

and $\angle AO'B = 2\angle AQB$ ...(*iii*)

($\because$ angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle)

From (*i*), (*ii*) and (*iii*), we get

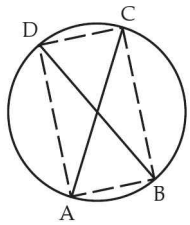
$2\angle APB = 2\angle AQB \Rightarrow \angle APB = \angle AQB$

$\Rightarrow$ BP = BQ (In ΔBPQ, sides opp. equal angles are equal)

Example 22. *AC and BD are chords of a circle which bisect each other. Prove that :*

(i) AC and BD are diameters (ii) ABCD is a rectangle. (*NCERT*)

Solution. (*i*) Given AC and BD are chords of a circle which bisect each other.



Join AB, BC, CD and DA.

ABCD is a quadrilateral such that its diagonals AC and BD bisect each other, therefore, ABCD is a parallelogram.

$\therefore$ AB = DC and BC = AD

(opp. sides of a || gm are equal)

$\Rightarrow$ arc AB = arc DC and arc BC = arc AD

($\because$ if chords of a circle are equal, then their corresponding arcs are also equal)

$\Rightarrow$ arc AB + arc BC = arc DC + arc AD

$\Rightarrow$ arc ABC = arc CDA

$\Rightarrow$ the chord AC divides the circle into two equal parts

$\Rightarrow$ the chord AC is a diameter of the circle.

Similarly, chord BD is also a diameter of the circle.

Hence, AC and BD are both diameters of the circle.

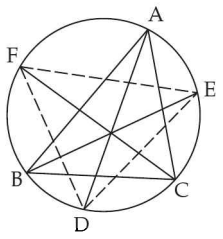
(*ii*) As AC is a diameter of the circle.

$\angle B = 90°$ (angle in a semicircle)

Thus, ABCD is a || gm in which $\angle B = 90°$. Therefore, ABCD is a rectangle.

Example 23. *Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at points D, E and F respectively. Prove that the angles of the triangle DEF are* $90° - \frac{1}{2}\angle A$, $90° - \frac{1}{2}\angle B$ *and* $90° - \frac{1}{2}\angle C$. (*NCERT*)

Solution. $\angle ADE = \angle ABE$ (angles in same segment)



and $\angle ADF = \angle ACF$ (angles in same segment)

But $\angle ABE = \frac{1}{2}\angle B$ ($\because$ BE is bisector of $\angle B$)

and $\angle ACF = \frac{1}{2}\angle C$ (CF is bisector of $\angle C$)

$\therefore$ $\angle ADE = \frac{1}{2}\angle B$ and $\angle ADF = \frac{1}{2}\angle C$

$\therefore \quad \angle EDF = \angle ADE + \angle ADF = \frac{1}{2}(\angle B + \angle C)$

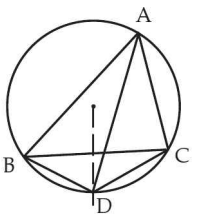
$= \frac{1}{2}(180° - \angle A)$ $(\because \angle A + \angle B + \angle C = 180°)$

$= 90° - \frac{1}{2}\angle A.$

Similarly, $\angle DEF = 90° - \frac{1}{2}\angle B$ and $\angle EFD = 90° - \frac{1}{2}\angle C$.

Example 24. *In any triangle ABC, if the angle bisector of ∠A and perpendicular bisector of BC intersect, prove that they intersect on the circumcircle of the triangle ABC.* (*NCERT*)

Solution. Let the bisector of ∠A intersect the circumcircle of ΔABC at D. Join BD and CD.



$\angle CBD = \angle CAD$ (angles in same segment)

and $\angle BCD = \angle BAD$ (angles in same segment)

As AD is bisector of ∠A,

$\angle CAD = \angle BAD = \frac{1}{2}\angle A.$

$\therefore \quad \angle CBD = \angle BCD.$

In ΔBCD, $\angle CBD = \angle BCD$

$\therefore \quad BD = CD$ (sides opp. equal angles are equal)

Thus, BCD is an isosceles triangle in which BD = CD,

so, BC is the base of an isosceles triangle with vertex D.

Since, the perpendicular bisector of the base of an isosceles triangle passes through the vertex, therefore, the perpendicular bisector of BC passes through the point D.

Exercise 11.2

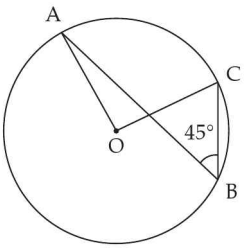
1. Write whether the following statements are true or false. Justify your answer:

(*i*) The angles subtended by a chord at any two points of a circle are equal. (*NCERT Exemplar*)

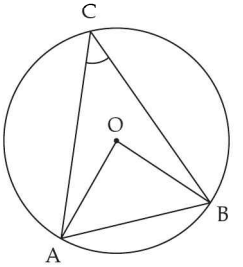
(*ii*) AOB is a diameter of a circle and C is any point on the circle, then $AC^2 + BC^2 = AB^2$. (*NCERT Exemplar*)

(*iii*) If A, B, C, D are four points such that ∠BAC = 30° and ∠BDC = 60°, then D is the centre of the circle through A, B and C. (*NCERT Exemplar*)

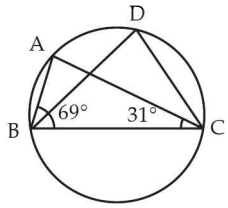
2. In the adjoining figure, O is the centre of the circle. If ∠ABC = 45°, then prove that OA ⊥ OC. (*NCERT Exemplar*)



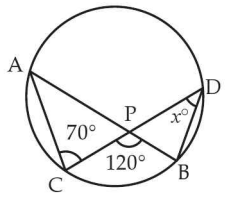
3. In the adjoining figure, O is the centre of the circle. If ∠ACB = 40°, find ∠OAB. (*NCERT Exemplar*)



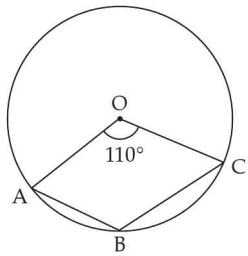
4. In the adjoining figure, points A, B, C and D lie on a circle. If $\angle ABC = 69°$ and $\angle ACB = 31°$, find $\angle BDC$.
(*NCERT*)



5. In the adjoining figure, chords AB and CD of a circle intersect at P. If $\angle CPB = 120°$ and $\angle ACP = 70°$, then find the value of x.



6. In the adjoining figure, O is the centre of the circle. If $\angle AOC = 110°$, find $\angle ABC$.



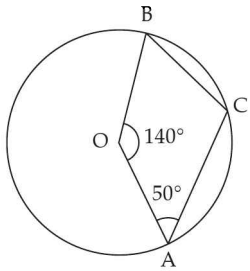
Hint: Reflex $\angle AOC = 360° - 110° = 250°$.

$$\angle ABC = \frac{1}{2} \text{ reflex } \angle AOC = \frac{1}{2} \times 250° = 125°.$$

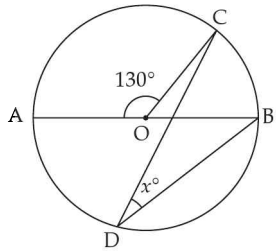
7. In the adjoining figure, O is the centre of the circle. If $\angle AOB = 140°$ and $\angle OAC = 50°$, find:

(*i*) $\angle ACB$ (*ii*) $\angle OBC$

(*iii*) $\angle OAB$ (*iv*) $\angle ABC$.



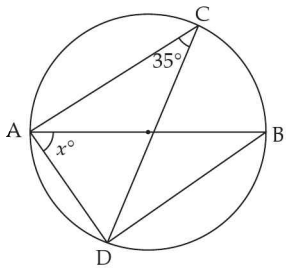
8. In the adjoining figure, O is the centre of the circle. If $\angle AOC = 130°$, find the value of x.



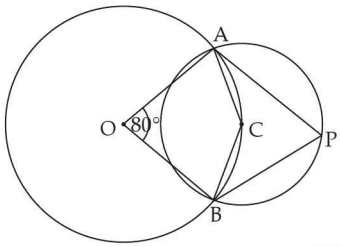
Hint: $\angle AOC + \angle BOC = 180°$ (linear pair)

$\Rightarrow \quad 130° + \angle BOC = 180° \Rightarrow \angle BOC = 50°$.

9. In the adjoining figure, AB is a diameter of the circle. If $\angle ACD = 35°$, find the value of x.



10. In the adjoining figure, two circles with centres O, C intersect at points A, B. The centre C of the smaller circle lies on the circumference of the other circle. If $\angle AOB = 80°$, find $\angle APB$.



Hint: Reflex $\angle AOB = 360° - 80° = 280°$.

$$\angle ACB = \frac{1}{2} \times 280° = 140°$$

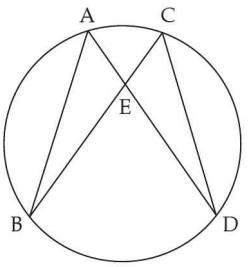
11. O is the circumcentre of the triangle ABC and D is mid-point of the base BC. Prove that $\angle BOD = \angle A$. *(NCERT Exemplar)*

Hint: $\angle BOC = 2\angle A$. $\Delta OBD \cong \Delta OCD$ (SSS rule of congruency)

$\therefore \quad \angle BOD = \angle COD \Rightarrow \angle BOC = 2\angle BOD$

$\Rightarrow \quad 2\angle A = 2\angle BOD \Rightarrow \angle A = \angle BOD.$

12. In the adjoining figure, AB and CD are equal chords. AD and BC intersect at E. Prove that AE = CE and BE = DE.



Hint: In ΔAEB and ΔCED,

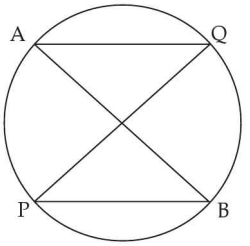
$\angle A = \angle C$ (angles in same segment of a circle)

$\angle B = \angle D$ (angles in same segment of a circle)

AB = CD (given)

$\therefore \quad \Delta AEB \cong \Delta CED \Rightarrow$ AE = CE and BE = DE.

13. In the adjoining figure, a diameter AB of a circle bisects a chord PQ. If AQ || PB, prove that chord PQ is also a diameter of the circle.



Hint: Let AB and PQ intersect at O.

In ΔOPB and ΔOQA,

OP = OQ ($\because$ AB bisects PQ)

$\angle P = \angle Q$ (alt. $\angle$s, AQ || PB, PQ is a transversal)

$\angle POB = \angle QOB$ (vert. opp. $\angle$s)

$\therefore \quad \Delta OPB \cong \Delta OQA$ (ASA axiom of congruency)

$\Rightarrow \quad$ OB = OA (c.p.c.t.)

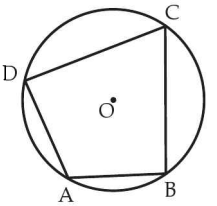
$\Rightarrow$ O is mid-point of AB but AB is a diameter, so O is centre of circle. As PQ passes through O, so PQ is also a diameter.

Answers

11.4 CYCLIC QUADRILATERALS

A quadrilateral inscribed in a circle is called a **cyclic quadrilateral**. Thus, in a cyclic quadrilateral all its vertices lie on the circumference of the circle.

cyclic quadrilateral



In the adjoining figure, ABCD is a cyclic quadrilateral. The points A, B, C and D are **concyclic points**. The cyclic quadrilateral ABCD has two pairs of

(*i*) opposite vertices—A, C; B, D

(*ii*) opposite angles— $\angle A$, $\angle C$; $\angle B$, $\angle D$.

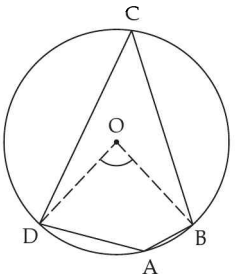
A cyclic quadrilateral has a special property. Measure all the angles of the quadrilateral. We find that:

$\angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$.

We can draw several cyclic quadrilaterals with different sides. This can be done more easily by drawing circles of different radii and taking four points on each of them. On measuring opposite angles, we find that their sum is always 180°. In fact, we have:

Theorem 11.13

The sum of either pair of opposite angles of a cyclic quadrilateral is 180°.



Given. A cyclic quadrilateral ABCD inscribed in a circle with centre O.

To prove. $\angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$.

Construction. Join OB and OD.

Proof. $\angle DOB = 2\angle C$ ...(*i*)

($\because$ angle subtended by an arc at the centre = double the angle subtended by it at any point on the remaining part of the circle)

Reflex $\angle DOB = 2\angle A$ ...(*ii*)

From (*i*) and (*ii*), we get

$2\angle C + 2\angle A = \angle DOB + \text{Reflex } \angle DOB$

$\Rightarrow$ $2(\angle A + \angle C) = 360°$ ($\because$ sum of angles at a point = 360°)

$\Rightarrow$ $\angle A + \angle C = 180°$.

Now $\angle A + \angle C + \angle B + \angle D = 360°$ ($\because$ sum of angles of a quad. = 360°)

$\Rightarrow$ $180° + \angle B + \angle D = 360°$

$\Rightarrow$ $\angle B + \angle D = 180°$.

Hence, $\angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$

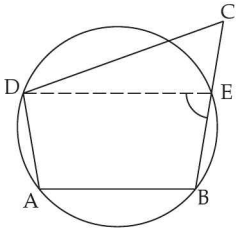
The converse of the above theorem is also true. In fact, we have:

Theorem 11.14

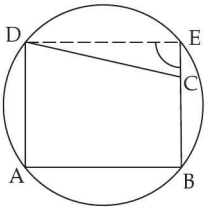
If the sum of a pair of opposite angles of a quadrilateral is 180°, then the quadrilateral is cyclic.

Given. A quadrilateral ABCD, and $\angle A + \angle C = 180°$.

To prove. ABCD is a cyclic quadrilateral.



(*i*)



(*ii*)

Construction. If ABCD is not a cyclic quadrilateral, let a circle passing through the points D, A and B meet BC at E (figure (*i*)) or BC produced at E (figure (*ii*)) so that DABE is a cyclic quadrilateral.

Proof. As ABED is a cyclic quadrilateral, sum of opposite angles is 180°.

$\therefore$ $\angle A + \angle E = 180°$

But $\angle A + \angle C = 180°$ (given)

$\Rightarrow$ $\angle A + \angle E = \angle A + \angle C \Rightarrow \angle E = \angle C$.

But $\angle E \neq \angle C$ because exterior angle of a triangle cannot be equal to interior opposite angle.

$\therefore$ Our supposition is wrong.

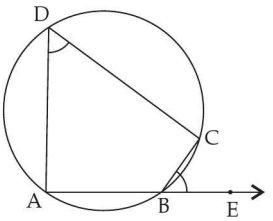
Hence, ABCD is a cyclic quadrilateral.

Theorem 11.15

An exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.

Given. A cyclic quadrilateral ABCD whose side AB is produced to a point E.

To prove. $\angle CBE = \angle ADC$.



Proof. As ABCD is a cyclic quadrilateral, sum of opposite angles is 180°

$\therefore$ $\angle ABC + \angle ADC = 180°$ ...(*i*)

Also $\angle ABC + \angle CBE = 180°$ ...(*ii*) (linear pair)

From (*i*) and (*ii*), we get

$\angle ABC + \angle CBE = \angle ABC + \angle ADC$

$\Rightarrow$ $\angle CBE = \angle ADC$.

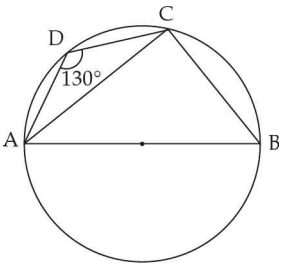
Hence, an exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.

The converse of the above theorem is also true. Thus, we have:

An exterior angle of a cyclic quadrilateral is equal to the interior opposite angle and conversely, if an exterior angle of a quadrilateral is equal to the interior opposite angle then the quadrilateral is cyclic.

Illustrative Examples

Example 1. *A quadrilateral ABCD is inscribed in a circle such that AB is a diameter and ∠ADC = 130°. Find ∠BAC.* *(NCERT Exemplar)*



Solution. As ABCD is a cyclic quadrilateral,

$\angle ABC + \angle ADC = 180°$ (sum of opposite angles = 180°)

$\Rightarrow \angle ABC + 130° = 180°$

$\Rightarrow \angle ABC = 50°$

Since AB is a diameter of the circle.

$\angle ACB = 90°$ (angle in a semicircle = 90°)

In ΔABC, $\angle BAC + \angle ABC + \angle ACB = 180°$

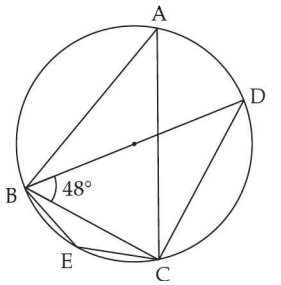
(sum of $\angle$s of a $\angle\Delta = 180°$)

$\Rightarrow \angle BAC + 50° + 90° = 180°$

$\Rightarrow \angle BAC = 40°$.

Example 2. *In the adjoining figure, BD is a diameter of the circle. If ∠DBC = 48°, find*

(i) ∠BDC (ii) ∠BEC (iii) ∠BAC.



Solution. (*i*) As BD is a diameter,

$\angle BCD = 90°$ (angle in a semicircle = 90°)

$\therefore \angle BDC = 180° - (48° + 90°)$

(sum of angles of a $\Delta = 180°$)

$\Rightarrow \angle BDC = 42°$.

(*ii*) As BECD is a cyclic quadrilateral,

$\angle BEC + \angle BDC = 180°$ (sum of opp. $\angle$s of a cyclic quad. = 180°)

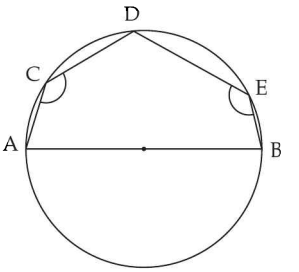
$\Rightarrow \angle BEC + 42° = 180°$

$\Rightarrow \angle BEC = 138°$

(*iii*) $\angle BAC = \angle BDC$ (angles in the same segment of a circle)

$\Rightarrow \angle BAC = 42°$. ($\because \angle BDC = 42°$)

Example 3. *In the adjoining figure, AB is a diameter of the circle and C, D, E are any three points on the semicircle. Find the value of ∠ACD + ∠BED.* *(NCERT Exemplar)*



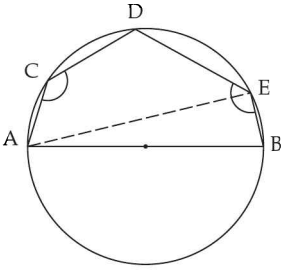
Solution. Join AE.

As ACDE is a cyclic quadrilateral,

$\angle ACD + \angle AED = 180°$ ...(*i*) (sum of opp. $\angle$s = 180°)

Since AB is a diameter of the circle,

$\angle AEB = 90°$ ...(*ii*) (angle in a semicircle = 90°)

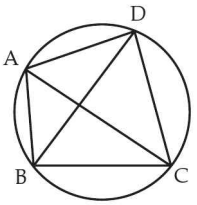


Adding (*i*) and (*ii*), we get

$\angle ACD + \angle AED + \angle AEB = 180° + 90°$

$\Rightarrow \angle ACD + \angle BED = 270°$.

Example 4. *In the adjoining figure, ABCD is a cyclic quadrilateral in which AC and BD are its diagonals. If ∠DBC = 55° and ∠BAC = 45°, find ∠BCD.* *(NCERT)*



Solution. ∠DAC = ∠DBC (angles in the same segment of a circle)

⇒ ∠DAC = 55° (∵ ∠DBC = 55°, given)

Now, ∠BAD = ∠BAC + ∠DAC = 45° + 55° = 100°.

But ∠BCD + ∠BAD = 180° (sum of opp. ∠s of a cyclic quad. = 180°)

⇒ ∠BCD + 100° = 180° ⇒ ∠BCD = 80°.

Example 5. *ABCD is a cyclic quadrilateral whose diagonals intersect at E. If ∠DBC = 70° and ∠BAC = 30°, find ∠BCD. Further, if AB = BC, find ∠ECD.* *(NCERT)*

Solution. ∠DAC = ∠DBC (angles in same segment of a circle)

⇒ ∠DAC = 70°

∴ ∠DAB = 70° + 30° = 100°.

As ABCD is a cyclic quad.,

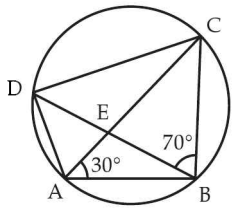
∠BCD + ∠DAB = 180° ⇒ ∠BCD + 100° = 180°

⇒ ∠BCD = 80°

In ΔABC, AB = BC ⇒ ∠BCA = ∠BAC but ∠BAC = 30°

⇒ ∠BCA = 30°.

∴ ∠ECD = ∠BCD − ∠BCA = 80° − 30° = 50°.



Example 6. *A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the major arc and also a point on the minor arc.*

Solution. Let AB be a chord of a circle with centre O and AB = radius of circle. Let P and Q be any points on major arc and minor arc respectively.

As AB = OA = OB (radii of same circle)

⇒ ΔOAB is equilateral ⇒ ∠AOB = 60°.

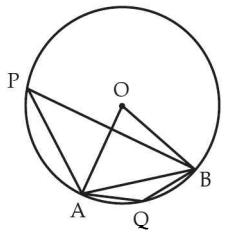
Now ∠AOB = 2 × ∠APB ⇒ 60° = 2 ∠APB

⇒ ∠APB = 30°.

As points P, A, Q and B lie on the circle, PAQB is a cyclic quad.

⇒ ∠AQB + ∠APB = 180° ⇒ ∠AQB + 30° = 180°

⇒ ∠AQB = 150°.



Example 7. *In the adjoining figure, AB is a diameter of the circle with centre O. If ∠ADC = 130° and chord BC = chord BE, find ∠CBE.* *(NCERT Exemplar)*

Solution. Join OC and OE.

As ABCD is a cyclic quadrilateral,

∠ABC + ∠ADC = 180°

(sum of opp. ∠s of a cyclic quad.)

⇒ ∠ABC + 130° = 180°

⇒ ∠ABC = 50° *i.e.* ∠OBC = 50°

In ΔOBC and ΔOBE,

OC = OE (radii of same circle)

BC = BE (given)

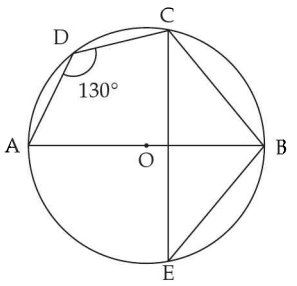
OB = OB (common)

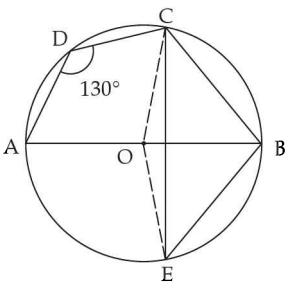
∴ ΔOBC ≅ ΔOBE (SSS rule of congruency)

∴ ∠OBC = ∠OBE (c.p.c.t.)

⇒ ∠OBE = 50° (∵ ∠OBC = 50°)

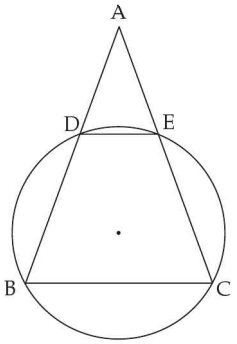
∴ ∠CBE = ∠OBC + ∠OBE = 50° + 50° = 100°.





Example 8. *ABC is an isosceles triangle with AB = AC and ∠BAC = 40°. If a circle passing through B and C intersects side AB and AC at the points D and E respectively, find ∠ADE.*

Solution. In ΔABC, AB = AC.



$\therefore$ $\angle ABC = \angle ACB$ (angle opp. equal sides are equal)

Also $\angle BAC + \angle ABC + \angle ACB = 180°$

$\Rightarrow$ $40° + 2\angle ACB = 180°$

$\Rightarrow$ $2\angle ACB = 140° \Rightarrow \angle ACB = 70°$

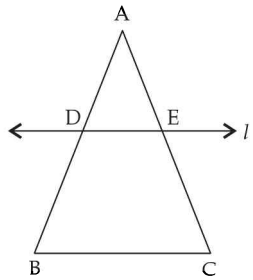
Since BCED is a cyclic quadrilateral,

$\angle ADE = \angle ECB$ (ext. ∠ = opp. int. ∠)

$\Rightarrow$ $\angle ADE = 70°$ ($\because$ $\angle ECB = \angle ACB = 70°$)

Example 9. *If a line is drawn parallel to the base of an isosceles triangle to intersect its equal sides, prove that the quadrilateral so formed is cyclic.* *(NCERT Exemplar)*

Solution. Let ABC be an isosceles triangle with AB = AC and l be a line drawn parallel to the base BC to intersect sides AB and AC at points D and E respectively. We need to show that BCED is a cyclic quadrilateral.



In ΔABC, AB = AC.

$\therefore$ $\angle ABC = \angle ACB$ ...(*i*)

Since line l is parallel to BC and AB is a transversal,

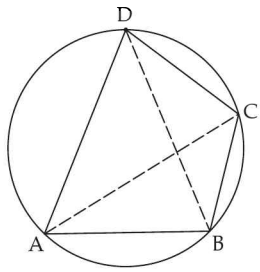
$\angle ADE = \angle ABC$ (corres. ∠s are equal)

$\Rightarrow$ $\angle ADE = \angle ACB$ (using (*i*))

Thus, in quadrilateral BCED, an exterior angle is equal to opposite interior angle. Therefore, BCED is a cyclic quadrilateral (converse of theorem 11.15).

Example 10. *If a pair of opposite sides of a cyclic quadrilateral is equal, prove that its diagonals are also equal.* *(NCERT Exemplar)*

Solution. Let ABCD be a cyclic quadrilateral such that AB = CD. We want to prove that AC = BD.



Since AB = CD (given),

arc AB = arc CD

($\because$ equal chords make equal arcs on the same circle)

$\Rightarrow$ arc AB + arc BC = arc BC + arc CD

(adding equal arc BC to both sides)

$\Rightarrow$ arc ABC = arc BCD

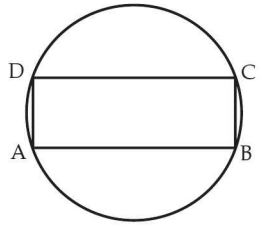
$\Rightarrow$ AC = BD

($\because$ equal arcs make equal chords on the same circle)

Example 11. *Prove that a cyclic parallelogram is a rectangle.* *(NCERT)*

Solution. Given ABCD is a cyclic parallelogram.

We need to prove that ABCD is a rectangle.



As ABCD is cyclic,

$\angle A + \angle C = 180°$

(sum of opp. ∠s of a cyclic quad. = 180°)

As ABCD is a parallelogram

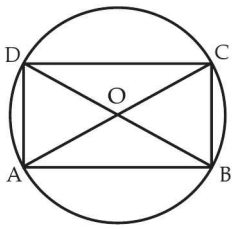
$\angle A = \angle C$ (opp. ∠s of a || gm are equal)

$\therefore$ $2\angle A = 180° \Rightarrow \angle A = 90°$

Thus, ABCD is a || gm in which $\angle A = 90°$, therefore, ABCD is a rectangle.

Example 12. *If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.* *(NCERT)*

Solution. Given ABCD is a cyclic quadrilateral such that its diagonals AC and BD are diameters of the circle. Then the point of intersection O of the diagonals *i.e.* of the diameters is the centre of the circle.



Now OA = OC (radii of same circle)

and OB = OD (radii of same circle)

So, the diagonals AC and BD of the quadrilateral bisect each other. Therefore, ABCD is a parallelogram.

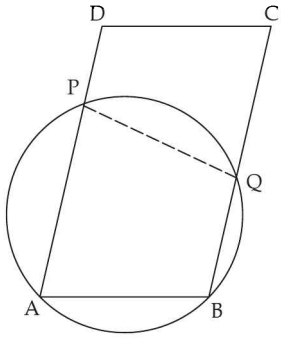
Also, as AC is diameter, $\angle ABC = 90°$ (angle in a semicircle is 90°)

Thus, ABCD is a parallelogram in which one angle is 90°, therefore, ABCD is a rectangle.

Example 13. *ABCD is a parallelogram. A circle through A and B is so drawn that it intersects AD at P and BC at Q. Prove that points P, Q, C and D are concyclic.* *(NCERT Exemplar)*

Solution. Join PQ.

As points A, B, Q and P lie on a circle, ABQP is a cyclic quadrilateral.



$\therefore$ $\angle DPQ = \angle B$ ...(*i*)

($\because$ an exterior angle of a cyclic quadrilateral = opp. interior angle)

Since ABCD is a || gm, AD || BC and AB is a transversal,

$\therefore$ $\angle A + \angle B = 180°$

$\Rightarrow$ $\angle A + \angle DPQ = 180°$ (using (*i*))

$\Rightarrow$ $\angle C + \angle DPQ = 180°$

($\because$ ABCD is a || gm, $\angle A = \angle C$, opp. angles are equal)

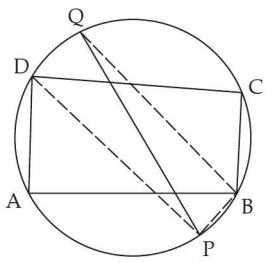
$\Rightarrow$ PQCD is a cyclic quadrilateral (theorem 11.14)

$\Rightarrow$ points P, Q, C and D are concyclic.

Example 14. *If the bisectors of opposite angles of a cyclic quadrilateral intersect the circle circumscribing it at the points P and Q, prove that PQ is a diameter of the circle.*

(NCERT Exemplar)

Solution. Let DP be bisector of $\angle D$ and BQ be bisector of $\angle B$,



then $\angle D = 2\ \angle ADP$ and $\angle B = 2\ \angle ABQ$.

As ABCD is a cyclic quadrilateral,

$\therefore$ $\angle D + \angle B = 180°$ (sum of opp. $\angle$s = 180°)

$\Rightarrow$ $2\ \angle ADP + 2\ \angle ABQ = 180°$

$\Rightarrow$ $\angle ADP + \angle ABQ = 90°$ ...(*i*)

But $\angle ADP = \angle ABP$ (angles in the same segment of a circle subtended by arc AP)

From (*i*), we get

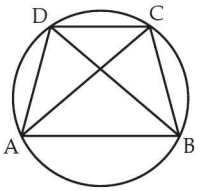
$\angle ABP + \angle ABQ = 90°$

$\Rightarrow$ $\angle PBQ = 90° \Rightarrow$ PBQ is a semicircle

$\Rightarrow$ PQ is a diameter of the circle.

Example 15. *If two sides of a cyclic quadrilateral are parallel, prove that the remaining two sides are equal and the diagonals are also equal.*

Solution. Given a cyclic quadrilateral ABCD in which AB || DC.



We need to prove that AD = BC and AC = BD.

As AB || CD,

$\angle A + \angle D = 180°$ ...(*i*) (sum of co-int. $\angle$s)

As ABCD is a cyclic quadrilateral

$\angle D + \angle B = 180°$ ...(*ii*) (sum of $\angle$s of a cyclic quad.)

From (*i*) and (*ii*), we get

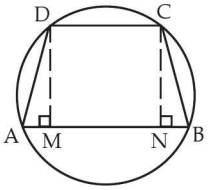
$\angle A + \angle D = \angle D + \angle B \Rightarrow \angle A = \angle B$.

In ΔABC and ΔBAD,

	$\angle B = \angle A$	(proved above)
	$\angle ACB = \angle BDA$	(angles in the same segment of a circle)
	$AB = BA$	(common)
$\therefore$	$\Delta ABC \cong \Delta BAD$	(by AAS rule of congruency)
$\therefore$	$BC = AD$	(c.p.c.t.)
and	$AC = BD$	(c.p.c.t.)

Example 16. *If the non-parallel sides of a trapezium are equal, prove that it is cyclic.* *(NCERT)*

Solution. Given ABCD is a trapezium in which AB || DC and AD = BC.



We need to prove that ABCD is cyclic.

From C, draw CN $\perp$ AB and from D, draw DM $\perp$ AB.

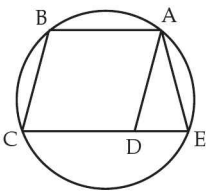
In ΔDAM and ΔCBN,

	$AD = BC$	(given)
	$\angle AMD = \angle BNC$	(each = 90°, as DM $\perp$ AB and CN $\perp$ AB)
	$DM = CN$	(distance between parallel lines)
$\therefore$	$\Delta DAM \cong \Delta CBN$	(by RHS rule of congruency)
$\therefore$	$\angle A = \angle B$	...(*i*)
As	$AB \parallel DC$	
	$\angle B + \angle C = 180°$	(sum of co-int. $\angle$s)
$\Rightarrow$	$\angle A + \angle C = 180°$	(using (*i*))

$\Rightarrow$ trapezium ABCD is cyclic (because sum of a pair of opposite angles is 180°).

Example 17. *ABCD is a parallelogram. If the circle through A, B and C intersects CD (produced) at E, prove that AE = AD.* *(NCERT)*

Solution. Given ABCD is a parallelogram and a circle passing through A, B and C intersects CD (produced) at E.



We need to prove that AE = AD.

As ABCD is a || gm

$\angle ABC = \angle ADC$ ...(*i*) (opp. $\angle$s of a || gm are equal)

As points A, B, C and E lie on the circle, ABCE is a cyclic quadrilateral,

$\therefore$ $\angle ABC + \angle AED = 180°$ ...(*ii*) (sum of opp. $\angle$s)

Since CDE is a straight line

$\angle ADC + \angle ADE = 180°$ ...(*iii*)

From (*ii*) and (*iii*), we get

$\angle ABC + \angle AED = \angle ADC + \angle ADE$

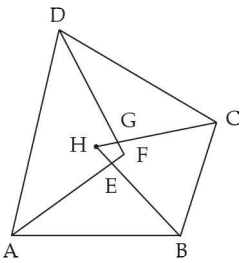
$\Rightarrow \quad \angle AED = \angle ADE$ (using (*i*))

Thus, in ΔADE, $\angle AED = \angle ADE$

$\therefore \quad AE = AD$ (sides opp. equal angles are equal)

Example 18. *Prove that the quadrilateral formed (if possible) by the internal angle bisectors of any quadrilateral is cyclic.* *(NCERT)*

Solution. Given a quadrilateral ABCD in which internal angle bisectors AF, BH, CH and DF of angles A, B, C and D respectively form a quadrilateral EFGH.



We need to prove that quad. EFGH is cyclic.

$\angle HEF = \angle AEB$ (vert. opp. $\angle$s)

In ΔAEB,

$$\angle AEB + \frac{1}{2}\angle A + \frac{1}{2}\angle B = 180°$$

($\because$ AE is bisector of $\angle A$ and BE is bisector of $\angle B$)

$$\Rightarrow \quad \angle AEB = 180° - \frac{1}{2}(\angle A + \angle B) \qquad ...(ii)$$

From (*i*) and (*ii*), we get

$$\angle HEF = 180° - \frac{1}{2}(\angle A + \angle B) \qquad ...(iii)$$

Similarly, $\angle HGF = 180° - \frac{1}{2}(\angle C + \angle D)$ ...(*iv*)

From (*iii*) and (*iv*), we get

$$\angle HEF + \angle HGF = 360° - \frac{1}{2}(\angle A + \angle B + \angle C + \angle D)$$

$$= 360° - \frac{1}{2} \times 360° \qquad \text{(sum of } \angle\text{s of a quad.)}$$

$$= 360° - 180° = 180°$$

Therefore, EFGH is a cyclic quadrilateral. ($\because$ sum of opp. pairs of angles is 180°)

Exercise 11.3

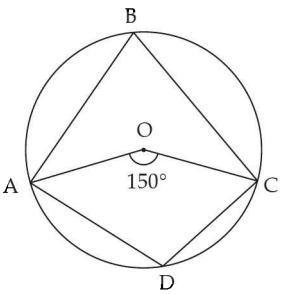
1. Write whether the following statements are true or false. Justify your answer:

(*i*) ABCD is a cyclic quadrilateral such that $\angle A = 90°$, $\angle B = 70°$, $\angle C = 95°$ and $\angle D = 105°$. *(NCERT Exemplar)*

(*ii*) If A, B, C and D are four points such that $\angle BAC = 45°$ and $\angle BDC = 45°$, then points A, B, C and D are concyclic. *(NCERT Exemplar)*

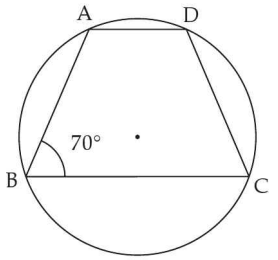
2. In the adjoining figure, O is the centre of the circle. If $\angle AOC = 150°$, find:

(*i*) $\angle ABC$ (*ii*) $\angle ADC$

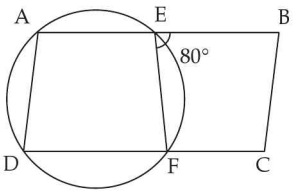


3. In the adjoining figure, ABCD is a cyclic trapezium in which AD is parallel to BC. If $\angle ABC = 70°$, find:

(*i*) $\angle BAD$ (*ii*) $\angle BCD$

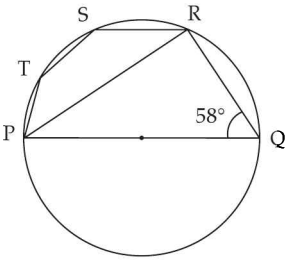


4. In the adjoining figure, ABCD is a parallelogram. A circle passing through A and D cuts AB at E and DC at F. If $\angle BEF = 80°$, find $\angle ABC$.

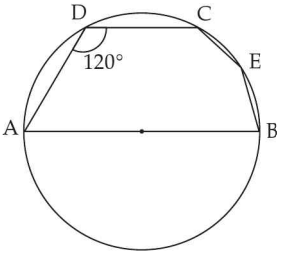


5. In the adjoining figure, PQ is a diameter of the circle. Chord SR is parallel to PQ. If $\angle PQR = 58°$, find:

(*i*) $\angle RPQ$ (*ii*) $\angle STP$



6. In the adjoining figure, AB is a diameter of the circle. If $\angle ADC = 120°$, find $\angle CAB$. *(NCERT Exemplar)*



Hint: Join BC, then ABCD is a cyclic quad.

$\therefore \quad \angle ABC + 120° = 180° \Rightarrow \angle ABC = 60°$

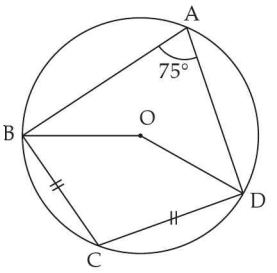
In ΔABC, $\angle ACB = 90°$

$\therefore \quad \angle CAB + 60° + 90° = 180° \Rightarrow \angle CAB = 30°$.

7. In the adjoining figure, O is the centre of the circle. If $\angle BAD = 75°$ and chord BC = chord CD. Find:

(*i*) $\angle BOD$ (*ii*) $\angle BCD$

(*iii*) $\angle BOC$ (*iv*) $\angle OBD$



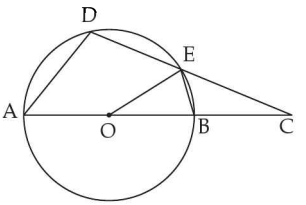
Hint:

(*iii*) As equal chords subtend equal at the centre and chord BC = chord CD, so $\angle BOC = \angle COD$

$\Rightarrow \angle BOC = \frac{1}{2} \angle BOD = \frac{1}{2} \times 150° = 75°$.

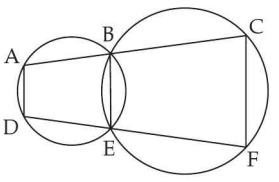
8. In the adjoining figure, O is the centre of the circle. If $\angle AOE = 150°$ and $\angle DAO = 55°$, find :

(*i*) $\angle CEB$ (*ii*) $\angle CBE$



9. Prove that a cyclic rhombus is a square.

10. In the adjoining figure, ABC and DEF are straight lines. Prove that AD is parallel to CF.



Hint: As ABED is a cyclic quad.

$\angle DAB + \angle DEB = 180°$ ...(*i*)

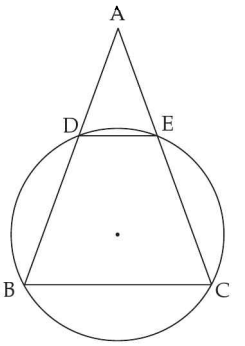
Also BCFE is a cyclic quadrilateral,

so $\angle DEB = \angle BCF$ ...(*ii*) (ext $\angle$ = opp. int. $\angle$s)

From (*i*) and (*ii*), we get $\angle DAB + \angle BCF = 180°$

$\Rightarrow \angle DAC + \angle ACF = 180° \Rightarrow AD \parallel CF$.

11. In the adjoining figure, ABC is an isoceles triangle in which AB = AC and circle passing through B and C intersects sides AB and AC at points D and E. Prove that DE $\parallel$ BC.



Hint: AB = AC $\Rightarrow \angle ABC = \angle ACB$. As BCED is a cyclic quadrilateral,

$\angle ADE = \angle BCE \Rightarrow \angle ADE = \angle ABC$

$\Rightarrow$ DE $\parallel$ BC (corres. $\angle$s are equal)

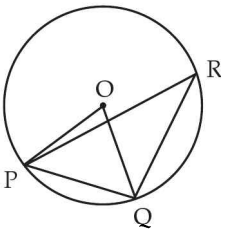
Very Short Answer Type Questions

Answers

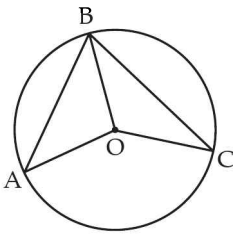
1. If PQ is a chord of a circle with radius r units and R is a point on the circle such that $\angle PRQ = 90°$, then find the length of PQ.

2. If an equilateral triangle PQR is inscribed in a circle with centre O, then find $\angle QOR$.

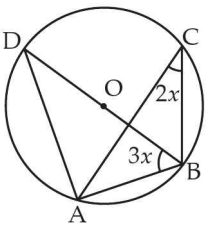
3. In the adjoining figure, O is the centre of a circle. If the length of chord PQ is equal to the radius of the circle, then find $\angle PRQ$.



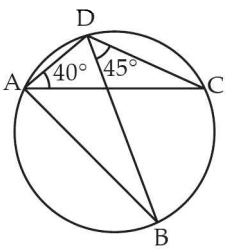
4. In the adjoining figure, O is the centre of a circle and A, B and C are points on the circle. If $\angle AOB = 100°$ and $\angle BOC = 120°$, then find $\angle ABC$.



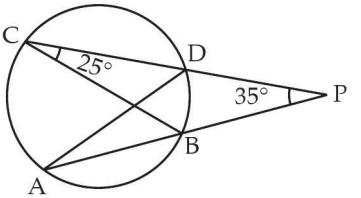
5. In the adjoining figure, O is the centre of a circle. Find the value of x.



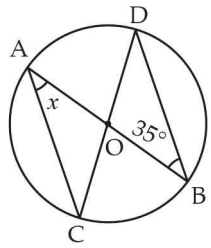
6. In the adjoining figure, ABCD is a quadrilateral inscribed in a circle and AC, BD are its diagonals. If $\angle DAC = 40°$ and $\angle BDC = 45°$, then find $\angle BCD$.



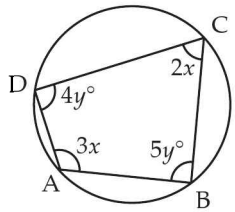
7. In the adjoining figure, chords AB and CD of a circle when produced meet at P. If $\angle APD = 35°$ and $\angle BCD = 25°$, then find $\angle ADC$.



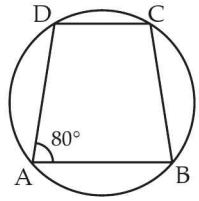
8. In the adjoining figure, AB and CD are diameters of a circle with centre O. If $\angle OBD = 35°$, then find the value of x.



9. In the adjoining figure, ABCD is a cyclic quadrilateral. If the angles are as given in the figure, then find the values of x and y.



10. In the adjoining figure, ABCD is a cyclic quadrilateral in which AB || DC. If $\angle A = 80°$, then find $\angle B$.



Answers

Objective Type Questions

(FOR FORMATIVE/SUMMATIVE ASSESSMENT)

Mental Maths

1. Fill in the blanks using the correct word given in the brackets :
   (*i*) The centre of a circle lies in ..... of the circle. (exterior, interior)
   (*ii*) A point, whose distance from the centre of a circle is greater than its radius lies in ..... of the circle. (exterior, interior)
   (*iii*) If a circle is divided into three equal parts, then each part of the circle is a ..... arc. (major, minor)
   (*iv*) The opposite angles of a cyclic quadrilateral are ..... . (complementary, supplementary)
   (*v*) In a circle, a chord which is nearer to the centre of the circle is ..... . (greater, smaller)
2. Fill in the blanks :
   (*i*) The longest chord of a circle is a ..... of the circle. (*NCERT*)
   (*ii*) The collection of all points of the plane which lie ..... the circle form its interior.
   (*iii*) A circle divides the plane, on which it lies, in ..... parts. (*NCERT*)
   (*iv*) A diameter of a circle is the ..... chord and all diameters have ..... length.
   (*v*) An arc is a ..... when its ends are the ends of a diameter. (*NCERT*)
   (*vi*) A sector of a circle is the region between an arc and its ..... .
   (*vii*) A segment of a circle is the region between an arc and ..... of the circle. (*NCERT*)
   (*viii*) Two circles are called concentric circles if and only if they have ..... but ..... radii.

(*ix*) Two circles are called equal (or congruent) if and only if they have ..... .

(*x*) Equal chords of a circle (or of congruent circles) subtend ..... angles at the centre (or centres).

(*xi*) If two chords of a circle are equal then their corresponding arcs are ..... and conversely, if two arcs of a circle are ..... then their corresponding chords are equal.

(*xii*) If two arcs of a circle are equal, then they subtend ..... angles at the centre and conversely, if two arcs of a circle subtend equal angles at the centre then they are ..... .

(*xiii*) The perpendicular from the centre of a circle to a chord ..... the chord and conversely, the line drawn through the centre of a circle to bisect a chord is ..... to the chord.

(*xiv*) Angle in a semicircle is a ..... .

3. State whether the following statements are true or false. If a statement is false, then write the corresponding correct statement.

(*i*) Line segment joining the centre of a circle to any point on the circle is a radius of the circle. (*NCERT*)

(*ii*) A chord of a circle, which is twice as long as radius, is a diameter of the circle. (*NCERT*)

(*iii*) A circle has only finite number of equal chords. (*NCERT*)

(*iv*) There is one and only one circle passing through any three points.

(*v*) Chords which are equidistant from the centre of a circle are equal in length.

(*vi*) Equal chords of a circle are equidistant from the centre of the circle.

(*vii*) Angles in the same segment of a circle are equal.

(*viii*) The angle subtended by an arc of a circle at the centre is the same as the angle subtended by it at any point on the remaining part of the circle.

(*ix*) If an arc of a circle subtends a right angle at any point on the remaining part of the circle, then the arc is a semicircle.

(*x*) An exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.

(*xi*) A radius of a circle is a chord of the circle which passes through its centre.

4. State whether the following statements are true or false. Justify your answer.

(*i*) A circle is a plane figure. (*NCERT*)

(*ii*) Circle and circumference are same.

(*iii*) The length of a chord of a circle may be longer than its diameter.

Multiple Choice Questions

Choose the correct answer from the given four options (5 to 19) :

5. If P and Q are any two points on a circle, then the line segment PQ is called a

(*a*) radius of the circle (*b*) diameter of the circle

(*c*) chord of the circle (*d*) secant of the circle

6. If P is a point in the interior of a circle with centre O and radius r, then

(*a*) $OP = r$ (*b*) $OP > r$ (*c*) $OP \geq r$ (*d*) $OP < r$

7. The circumference of a circle must be

(*a*) a positive real number (*b*) a whole number

(*c*) a natural number (*d*) an integer

8. AD is a diameter of a circle and AB is a chord. If AD = 34 cm and AB = 30 cm, then the distance of AB from the centre of circle is

(*a*) 17 cm (*b*) 15 cm (*c*) 4 cm (*d*) 8 cm

(*NCERT Exemplar*)

9. If AB = 12 cm, BC = 16 cm and AB is perpendicular to BC, then the radius of the circle passing through the points A, B and C is

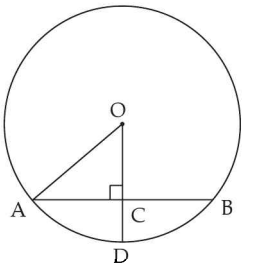
(*a*) 6 cm (*b*) 8 cm (*c*) 10 cm (*d*) 12 cm

(*NCERT Exemplar*)

10. In the adjoining figure, O is the centre of the circle. If OA = 5 cm, AB = 8 cm and OD ⊥ AB, then length of CD is equal to

(*a*) 2 cm (*b*) 3 cm

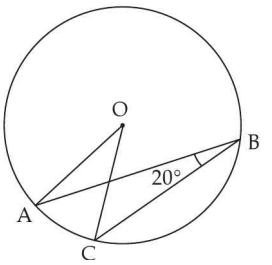
(*c*) 4 cm (*d*) 5 cm (*NCERT Exemplar*)



11. In the adjoining figure, O is the centre of the circle. If ∠ABC = 20°, then ∠AOC is equal to

(*a*) 20° (*b*) 40°

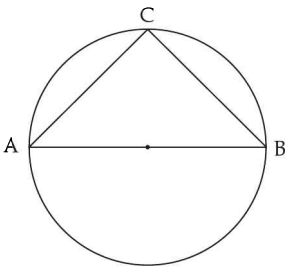
(*c*) 60° (*d*) 10° (*NCERT Exemplar*)



12. In the adjoining figure, AB is a diameter of the circle. If AC = BC, then ∠CAB is equal to

(*a*) 30° (*b*) 60°

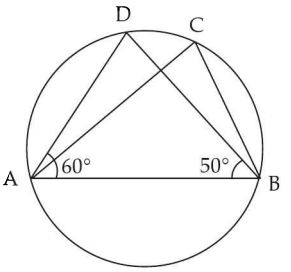
(*c*) 90° (*d*) 45° (*NCERT Exemplar*)



13. In the adjoining figure, if ∠DAB = 60° and ∠ABD = 50° then ∠ACB is equal to

(*a*) 60° (*b*) 50°

(*c*) 70° (*d*) 80° (*NCERT Exemplar*)

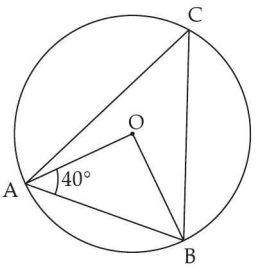


14. In the adjoining figure, O is the centre of the circle. If ∠OAB = 40°, then ∠ACB is equal to

(*a*) 50° (*b*) 40°

(*c*) 60° (*d*) 70°

(*NCERT Exemplar*)



15. ABCD is a cyclic quadrilateral such that AB is a diameter of the circle circumscribing it and $\angle ADC = 140°$, then $\angle BAC$ is equal to

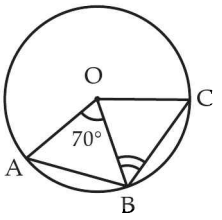
(*a*) 80° (*b*) 50° (c) 40° (*d*) 30°

(*NCERT Exemplar*)

16. In the adjoining figure, O is the centre of a circle. If AB = BC and $\angle AOB = 70°$, then $\angle OBC$ is

(*a*) 55° (*b*) 65°

(*c*) 70° (*d*) 75°

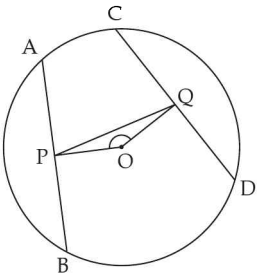


17. In the adjoining figure, AB and CD are two equal chords of a circle with centre O. If OP ⊥ AB, OQ ⊥ CD and $\angle POQ = 150°$, then $\angle APQ$ is equal to

(*a*) 30° (*b*) 75°

(*c*) 15° (*d*) 60° (*NCERT Exemplar*)

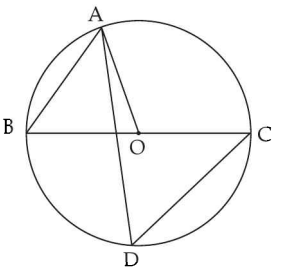
Hint: AB = CD ⇒ OP = OQ.



18. In the adjoining figure, O is the centre of the circle. If $\angle BAO = 60°$, then $\angle ADC$ is equal to

(*a*) 30° (*b*) 45°

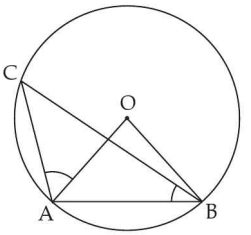
(*c*) 60° (*d*) 120° (*NCERT Exemplar*)



19. In the adjoining figure, O is the centre of the circle. If $\angle AOB = 90°$ and $\angle ABC = 30°$, then $\angle CAO$ is equal to

(*a*) 30° (*b*) 45°

(*c*) 90° (*d*) 60° (*NCERT Exemplar*)



Answers

Higher Order Thinking Skills (HOTS)

1. ABCD is a cyclic quadrilateral whose diagonals AC and BD intersect at E. If AB = CD, prove that:

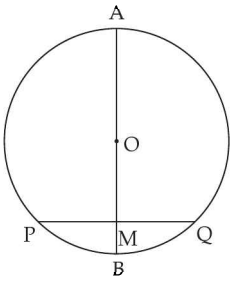
(*i*) $\Delta AEB \cong \Delta DEC$ (*ii*) AE = ED and EB = EC

(*iii*) AD || BC.

2. ABC is an isosceles triangle in which AB = AC. D and E are points on the side AB and AC respectively such that AD = AE. Prove that points B, C, E and D are concyclic.

3. If the vertex of an angle ABC be located outside a circle and the sides of the angle intersect equal chords AD and CE with the circle, then prove that $\angle ABC$ is equal to half the difference of the angles subtended by the chords AC and DE at the centre.

Chapter Test



1. In the adjoining figure, a chord PQ of a circle with centre O and radius 15 cm is bisected at M by a diameter AB. If OM = 9 cm, find the lengths of:
   (*i*) PQ (*ii*) AP (*iii*) BP

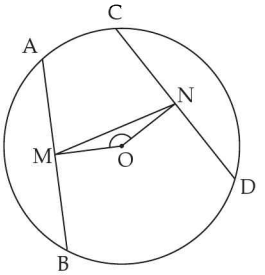
2. AB is a diameter of a circle. M is a point in AB such that AM = 18 cm and BM = 8 cm. Find the length of shortest chord through M.

   Hint: Diameter of circle = (18 + 8) cm = 26 cm, so its radius = 13 cm. Shortest chord through M is perpendicular to AB.

3. AB and CD are two parallel chords of a circle of lengths 10 cm and 4 cm respectively. If the chords lie on the same side of centre and the distance between them is 3 cm, find the diameter of the circle.

4. ABC is an isosceles triangle inscribed in a circle. If AB = AC = $12\sqrt{5}$ cm and BC = 24 cm, find the radius of the circle.

5. Prove that the line joining mid-points of two equal chords of a circle subtend equal angles with the chord.



   Hint: Let AB and CD be two equal chord of a circle with centre O, and M, N be mid-point of AB, CD respectively. Join OM and ON, then OM ⊥ AB and ON ⊥ CD.

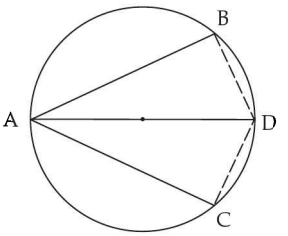
   As equal chords are at equal distances from the centre, OM = ON

   ⇒ ∠OMN = ∠ONM.

   ⇒ 90° − ∠OMN = 90° − ∠ONM and 90° + ∠OMN = 90° + ∠ONM

   ⇒ ∠AMN = ∠CNM and ∠BMN = ∠DNM.

6. In the adjoining figure, AD is a diameter of a circle. If the chords AB and AC of the circle are equidistant from its centre, prove that AD bisects ∠BAC and ∠BDC.

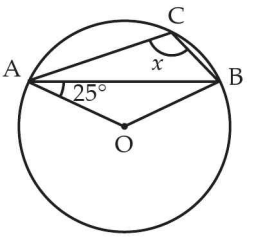


   Hint: As the chords AB and AC are equidistant from the centre, so AB = AC.

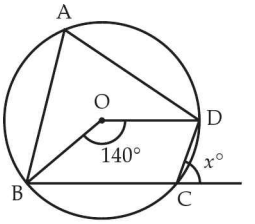
   Since angle in a semicircle is 90°. ∠B = ∠C (each is 90°)

   ΔABD ≅ ΔACD (why?).

7. In the adjoining figure, O is the centre of a circle and points A, B and C lie on the circle. If ∠OAB = 25°, then find the value of x.

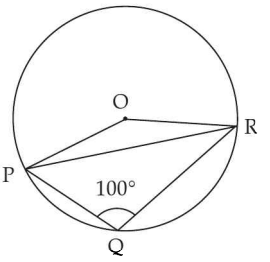


8. In the adjoining figure, O is the centre of the circle. If ∠BOD = 140°, then find the value of x.

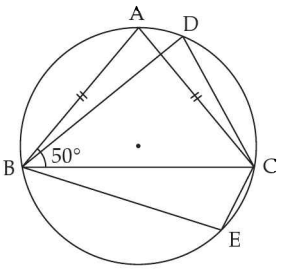


9. If in a cyclic quadrilateral opposite angles are in the ratio 5 : 4, find the angles in degrees.

10. In the adjoining figure, O is the centre of the circle. If $\angle PQR = 100°$, then find $\angle OPR$.

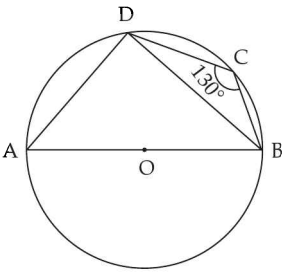


11. In the adjoining figure, ABC is an isosceles triangle with AB = AC. If $\angle ABC = 50°$, find $\angle BDC$ and $\angle BEC$.

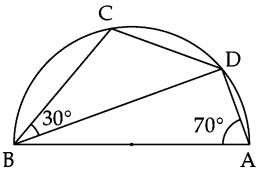


12. In the adjoining figure, O is the centre of the circle. If $\angle BCD = 130°$, then find:

(*i*) $\angle DAB$ (*ii*) $\angle DBA$



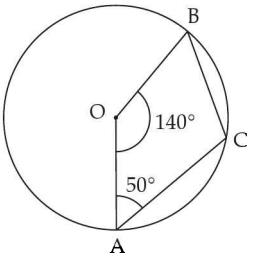
13. In the adjoining figure, C and D are points on the circumference of the semicircle described on AB as diameter. Given $\angle BAD = 70°$ and $\angle DBC = 30°$, calculate $\angle ABD$ and $\angle BDC$.



14. In the adjoining figure, O is the centre of the circle. If $\angle AOB = 140°$ and $\angle OAC = 50°$, find

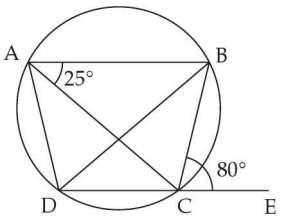
(*i*) $\angle ACB$ (*ii*) $\angle OBC$

(*iii*) $\angle OAB$ (*iv*) $\angle CBA$



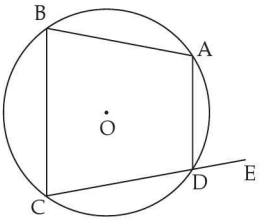
15. In the adjoining figure, AB is parallel to DC. If $\angle BCE = 80°$ and $\angle BAC = 25°$, find:

(*i*) $\angle CAD$ (*ii*) $\angle CBD$ (*iii*) $\angle ADC$

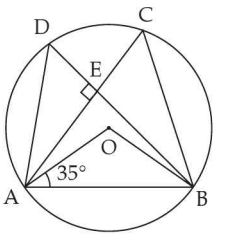


16. In the adjoining figure, ABCD is a quadrilateral inscribed in a circle with centre O. CD is produced to E. If $\angle ADE = 70°$ and $\angle OBA = 45°$, find:

(*i*) $\angle OCA$ (*ii*) $\angle BAC$



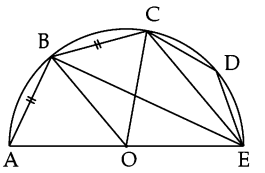
17. In the adjoining figure, O is the centre of the circle. If chords AC and BD intersect at right angles at E and $\angle OAB = 35°$, calculate $\angle EBC$.



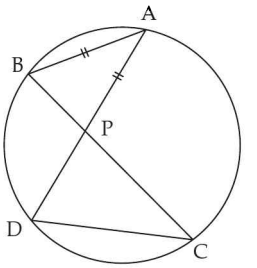
18. In the adjoining figure, O is the centre and AOE is the diameter of the semicircle. If AB = BC and $\angle OEC = 50°$, find:

(*i*) $\angle CBE$ (*ii*) $\angle CDE$ (*iii*) $\angle AOB$.

Prove that OB is parallel to EC.



19. In the adjoining figure, P is a point on the chord BC of a circle such that AB = AP. Prove that CP = CD.



Hint: In ΔABP, $AB = AP \Rightarrow \angle ABP = \angle APB$.

But $\angle APB = \angle CPD$ (vert. opp. $\angle$s)

$\therefore \quad \angle ABP = \angle CPD$ ...(*i*)

Also $\angle ABP = \angle CDP$ ...(*ii*)

(angles in the same segment of a circle on arc CA)

From (*i*) and (*ii*), $\angle CPD = \angle CDP$

Thus, in ΔCPD, $\angle CPD = \angle CDP \Rightarrow CP = CD$.

20. ABC is an isosceles triangle in which AB = AC. If D and E are the mid-points of sides AB and AC respectively, then prove that the BCED is a cyclic quadrilateral.

Hint: As D and E are the mid-points of the sides AB and AC respectively, so DE is parallel to BC. Now, see example 9 on page 296.

Answers

Constructions



INTRODUCTION

In this chapter, you will learn how to draw accurate and scaled drawings. To do this, you must have the proper geometrical instruments:

(*i*) A graduated scale or ruler (to draw or measure line segments).

(*ii*) A protractor (to draw or measure angles).

(*iii*) A pair of set-squares, one with angles 90°, 60° and 30° and other with angles 90°, 45° and 45°.

(*iv*) A divider with adjustments (to help in measuring line segments accurately).

(*v*) A compass with provision of fixing a sharp pencil at one end (to draw circles or arcs of circles).

(*vi*) An extra sharp pencil and a rubber (to rub incorrect work, if any).

Make sure that your geometrical instruments are in good conditions with accurate markings on them and your compass is reasonably tight. Your drawing must be neat and clean. Remember that small errors in the work tend to build up big error in the end.

In this chapter, we shall review some basic constructions and will learn the construction of some special type of triangles – when base, base angle and sum/difference of other sides is given or when its perimeter and base angles are given.

12.1 BASIC CONSTRUCTIONS

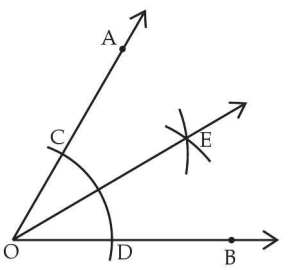
In earlier classes, you have learnt how to construct the perpendicular bisector of a line segment, angles of measure 30°, 45°, 60°, 90° and 120°, and the bisector of a given angle without giving justification for these constructions. In this section, we shall construct some of these, with reasoning behind, why these constructions are justified.

❑ **Construction 12.1**

To construct the bisector of an angle.

Given any ∠AOB, we want to construct its bisector.

Steps of construction.



1. Taking O as centre and any suitable radius, draw an arc to intersect the rays OA and OB at points C and D respectively.
2. Taking C as centre and any suitable radius (not necessarily equal to radius of step 1 but > $\frac{1}{2}$ CD), draw an arc. Also, taking D as centre and same radius draw another arc to intersect the previous arc at E.
3. Draw the ray OE. Then OE is the required bisector of ∠AOB.

Justification

Join CE and DE.

In ΔOCE and ΔODE,

	OC	$= OD$	(radii of same arc)
	CE	$= DE$	(arcs of equal radii)
	OE	$= OE$	(common)
$\therefore$	ΔOCE	$\cong \Delta ODE$	(by SSS rule of congruency)
$\therefore$	$\angle COE$	$= \angle DOE$	(c.p.c.t.)

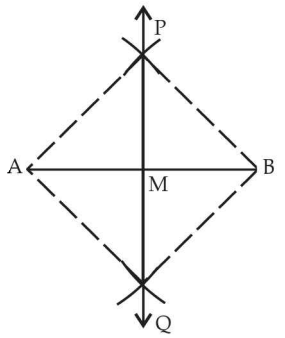
❑ **Construction 12.2**

To construct the perpendicular bisector of a given line segment.

Given any line segment AB, we want to construct its perpendicular bisector.

Steps of construction.

1. Taking A as centre and any suitable radius ($> \frac{1}{2}$ AB), draw two arcs, one on each side of AB.
2. Taking B as centre and same radius (as in step 1), draw two more arcs, one on each side of AB intersecting the previous arcs at P and Q.
3. Join PQ to intersect AB at M. Then the line PQ is the required perpendicular bisector of segment AB.



Justification

Join AP, BP, AQ and BQ.

In ΔAPQ and ΔBPQ,

	AP	$= BP$	(arcs of equal radii)
	AQ	$= BQ$	(arcs of equal radii)
	PQ	$= PQ$	(common)
$\therefore$	ΔAPQ	$\cong \Delta BPQ$	(by SSS rule of congruency)
$\therefore$	$\angle APM$	$= \angle BPM$	(c.p.c.t.)

In ΔAMP and ΔBMP,

	AP	$= BP$	(arcs of equal radii)
	MP	$= MP$	(common)
	$\angle APM$	$= \angle BPM$	(proved above)
$\therefore$	ΔAMP	$\cong \Delta BMP$	(by SAS rule of congruency)
$\therefore$	AM	$= BM$	(c.p.c.t.)
and	$\angle AMP$	$= \angle BMP$	(c.p.c.t.)
Now	$\angle AMP + \angle BMP$	$= 180°$	(linear pair)

$2\angle AMP = 180° \Rightarrow \angle AMP = 90° \Rightarrow MP \perp AM \Rightarrow PQ \perp AB.$

Therefore, the line PQ is the perpendicular bisector of the line segment AB.

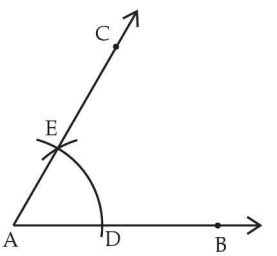
❑ **Construction 12.3**

To construct an angle of 60° at the initial point of a given ray.

Given a ray AB, we want to construct another ray AC such that $\angle CAB = 60°$.

Steps of construction.

1. Taking A as centre and any suitable radius, draw an arc to intersect the ray AB at D.
2. Taking D as centre and same radius (as in step 1), draw another arc to intersect the previous arc at E.



3. Draw ray AC passing through E. Then $\angle CAB = 60°$.

Justification

Join DE.

Then $AD = AE = DE$ (radii of same arc)

$\therefore$ ΔADE is equilateral $\Rightarrow \angle EAD = 60°$

$\therefore$ $\angle CAB = 60°$.

Illustrative Examples

Example 1. *Draw an angle of 80° with the help of a protractor. Then (using ruler and compass) construct angles of*

(i) 40° *(ii) 160°* *(iii) 120°* *(NCERT Exemplar)*

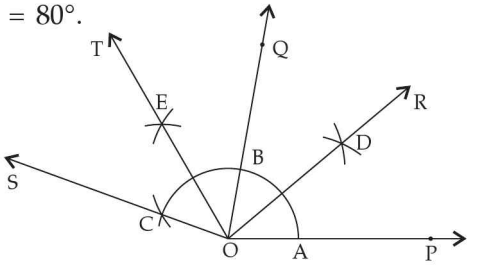
Solution. ***Steps of construction.***

1. Draw a ray OP.

2. With the help of a protractor, construct $\angle QOP = 80°$.

3. Taking O as centre and any suitable radius draw an arc to intersect rays OP and OQ at points A and B respectively.

4. Bisect $\angle QOP$ as in construction 12.1. Let ray OR be the bisector of $\angle QOP$, then $\angle ROP = \frac{1}{2} \angle QOP = \frac{1}{2} \times 80° = 40°$.



5. With B as centre and radius equal to AB, draw an arc to cut the extended arc AB at C. Join OC and produce it to form ray OS, then $\angle SOP = 2 \angle QOP = 2 \times 80° = 160°$.

6. Bisect $\angle SOQ$ as in construction 12.1. Let OT be the bisector of $\angle SOQ$, then $\angle TOP = \angle TOQ + \angle QOP = \frac{1}{2} \angle SOQ + \angle QOP = \frac{1}{2} \times 80° + 80° = 40° + 80° = 120°$.

Example 2. *Construct an angle of 30° at the initial point of a given ray and justify the construction.*

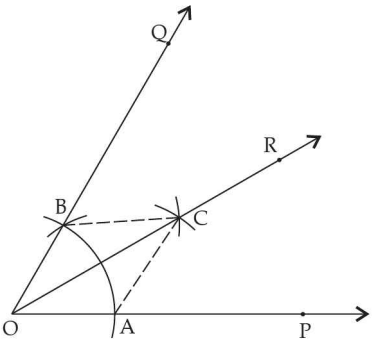
Solution. ***Steps of construction.***

1. Let OP be the given ray with O as its initial point.

2. Taking O as centre and any suitable radius, draw an arc to intersect ray OP at A.

3. Taking A as centre and same radius (as in step 2), draw another arc to cut the previous arc at B.

4. Draw ray OQ passing through B, then $\angle QOP = 60°$ (by construction 12.3).



5. Taking A as centre and any suitable radius (not necessarily equal to radius of step 2 but $> \frac{1}{2}$ AB), draw an arc. Also, taking B as centre and same radius draw another arc to cut the previous arc at C.

6. Draw ray OR passing through C, then $\angle ROP = 30°$.

Justification

Join CA and CB.

In ΔOAC and ΔOBC,

$OA = OB$ (radii of same arc)

$AC = BC$ (arcs of equal radii)

$OC = OC$ (common)

$\therefore \quad \Delta OAC \cong \Delta OBC$ (SSS rule of congruency)

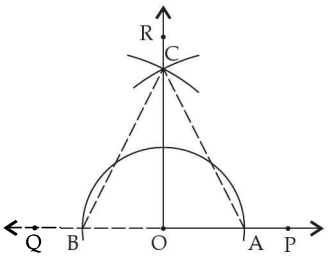
$\Rightarrow \quad \angle COA = \angle BOC$ *i.e.* $\angle ROP = \angle QOR$.

$\therefore \quad \angle ROP = \frac{1}{2} \angle QOP = \frac{1}{2} \times 60° = 30°$.

Example 3. *Construct an angle of 90° at the initial point of a given ray and justify the construction.* *(NCERT)*

Solution. ***Steps of construction.***

1. Let OP be the given ray with O as its initial point.
2. Produce OP backwards to get the opposite ray OQ so that QOP is a straight line.
3. Taking O as centre and any suitable radius, draw an arc to intersect ray OP at A and ray OQ at B.
4. Taking A as centre and any suitable radius (not equal to radius of step 3 but $> \frac{1}{2}$ AB), draw an arc. Also, taking B as centre and same radius, draw another arc to cut the previous arc at C.
5. Draw ray OR passing through C, then $\angle ROP = 90°$.



Justification

Join CA and CB.

In ΔOAC and ΔOBC

$OA = OB$ (radii of same arc)

$AC = BC$ (arc of equal radii)

$OC = OC$ (common)

$\therefore \quad \Delta OAC \cong \Delta OBC$

$\therefore \quad \angle COA = \angle COB$ (c.p.c.t.)

Now, $\angle COB + \angle COA = 180°$ (linear pair)

$\Rightarrow \quad \angle COA + \angle COA = 180° \Rightarrow 2\angle COA = 180°$

$\Rightarrow \quad \angle COA = 90°$ *i.e.* $\angle ROP = 90°$.

Exercise 12.1

1. Draw an angle of 110° with the help of a protractor and bisect it. Measure each angle. *(NCERT Exemplar)*
2. Draw an angle of 72° with the help of a protractor. Then (using ruler and compass) construct angles of 36° and 108°.
3. Construct an angle of 45° at the initial point of a given ray and justify the construction. *(NCERT)*
4. Draw a line segment of length 5.6 cm. Construct its perpendicular bisector and justify your construction. Measure the length of each part.
5. Draw a circle of radius 3.4 cm. Draw its chord PQ and draw the perpendicular bisector of PQ. Does it pass through the centre of the circle?
6. Draw a line segment AB of 4 cm in length. Draw lines perpendicular to AB through A and B respectively. Are these lines parallel? *(NCERT Exemplar)*
7. Construct the angles of the following measurements :

   (*i*) 45° (*ii*) $22\frac{1}{2}°$ (*iii*) 15° *(NCERT)*

8. Construct the following angles and verify them by measuring them with protractor:
   (*i*) 75° (*ii*) 105° (*iii*) 135° (*NCERT*)
9. State whether the following statements are true or false. Give reasons for your answer:
   (*i*) An angle of 67.5° can be constructed. (*NCERT Exemplar*)
   (*ii*) An angle of 42.5° can be constructed. (*NCERT Exemplar*)

Answers

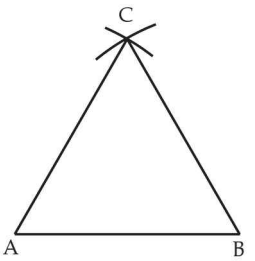
12.2 CONSTRUCTIONS OF SOME SPECIAL TRIANGLES

❑ **Construction 12.4**

To construct an equilateral triangle when its side is given.

Steps of construction.

1. Draw a line segment, say AB, whose length is equal to the length of the given side.
2. Taking A as centre and radius equal to length of side, draw an arc.
3. Taking B as centre and same radius (as in step 1), draw another arc to intersect the previous arc at C.
4. Join AC and BC. Then, ΔABC is the required equilateral triangle with given side.



Justification

AC = AB (each = length of side)

BC = AB (each = length of side)

⇒ AB = BC = AC

⇒ ΔABC is equilateral with given side.

❑ **Construction 12.5**

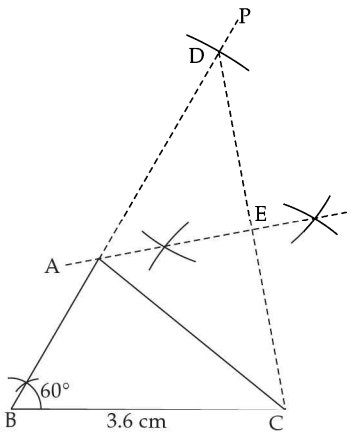
To construct a triangle given its base, a base angle and sum of other two sides.

Given. BC = 3.6 cm, AB + AC = 5.4 cm and ∠B = 60°.

Required. To construct ΔABC.

Steps of construction.

1. Draw BC = 3.6 cm.
2. At B, construct ∠CBP = 60°.
3. From BP, cut off BD = 5.4 cm.
4. Join CD and draw its perpendicular bisector to intersect BD at A.
5. Join AC. Then, ABC is the required triangle.



Justification

Here, base BC = 3.6 cm and base angle B = 60°.

As the point A lies on the perpendicular bisector of CD, AC = AD.

Now, BD = 5.4 cm ⇒ AB + AD = 5.4 cm

⇒ AB + AC = 5.4 cm (∵ AD = AC)

Hence, ABC is the required triangle with the given data.

Remark. The construction of ΔABC is not possible if AB + AC ≤ BC.

❑ **Construction 12.6**

To construct a triangle given its base, a base angle and the difference of the other two sides.

Given the base BC, a base angle, say ∠B and the difference of other two sides AB – AC or AC – AB. So, there are the following two cases :

(*i*) AB > AC *i.e.* AB – AC is given

(*ii*) AC > AB *i.e.* AC – AB is given.

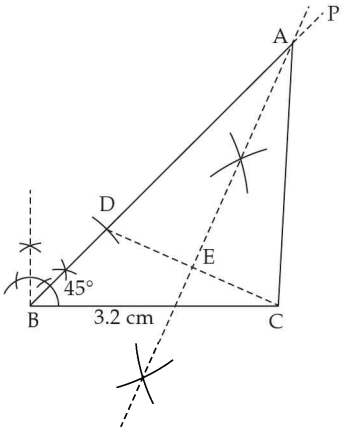
Case I

Given. BC = 3.2 cm, AB – AC = 1.4 cm and ∠B = 45°.

Required. To construct ΔABC.

Steps of construction.

1. Draw BC = 3.2 cm.
2. At B, construct ∠CBP = 45°.
3. From BP, cut off BD = 1.4 cm.
4. Join CD and draw its perpendicular bisector to intersect BP at A.
5. Join AC. Then, ABC is the required triangle.



Justification

Here, base BC = 3.2 cm and base angle B = 45°.

As the point A lies on the perpendicular bisector of CD,

AD = AC.

Now, BD = 1.4 cm ⇒ AB – AD = 1.4 cm

⇒ AB – AC = 1.4 cm (∵ AD = AC).

Hence, ABC is the required triangle with the given data.

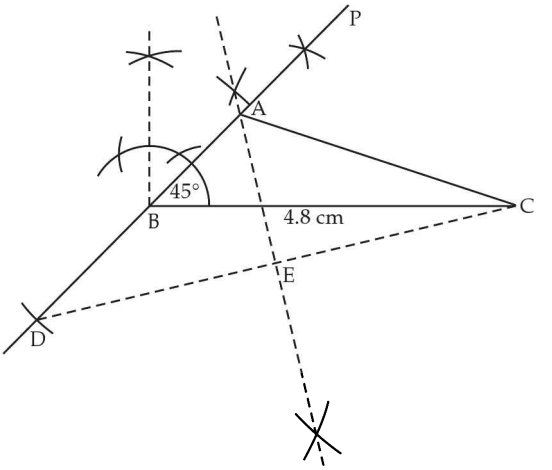
Case II

Given. BC = 4.8 cm, AC – AB = 2.1 cm and ∠B = 45°.

Required. To construct ΔABC.

Steps of construction.

1. Draw BC = 4·8 cm.
2. At B, construct ∠PBC = 45°.
3. With B as centre and radius 2.1 cm draw an arc to intersect PB produced at D.
4. Join DC and draw its perpendicular bisector to intersect BP at A.
5. Join CA. Then, ABC is the required triangle.



Justification

Here, base BC = 4.8 cm and base angle B = 45°.

As the point A lies on the perpendicular bisector of CD,

AD = AC.

Now, BD = 2.1 cm ⇒ AD – AB = 2.1 cm

⇒ AC – AB = 2.1 cm (∵ AD = AC).

Hence, ABC is the required triangle with the given data.

Remark. In case I, the construction of ΔABC is not possible if AB – AC ≥ BC and in case II, the construction of ΔABC is not possible if AC – AB ≥ BC.

❑ **Construction 12.7**

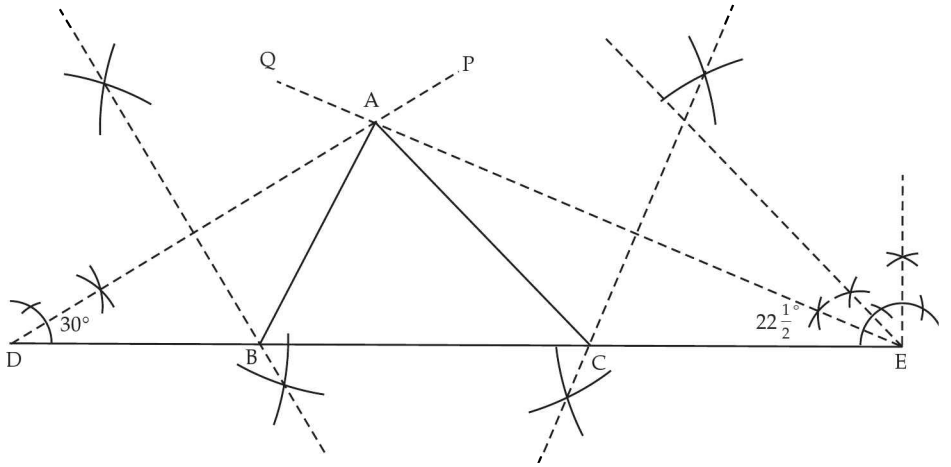
To construct a triangle given its perimeter and its two base angles.

Given. $\angle B = 60°$, $\angle C = 45°$ and perimeter = 11.8 cm.

Required. To construct ΔABC.

Steps of construction.

1. Draw DE = 11.8 cm.
2. At D, construct $\angle EDP = \frac{1}{2}$ of $60° = 30°$ and at E, construct $\angle DEQ = \frac{1}{2}$ of $45° = 22\frac{1}{2}°$.
3. Let DP and EQ meet at A.
4. Draw perpendicular bisector of AD to meet DE at B.
5. Draw perpendicular bisector of AE to meet DE at C.
6. Join AB and AC. Then, ABC is the required triangle.



Justification

As point B lies on the perpendicular bisector of AD, DB = AB.

Similarly, CE = AC.

$\therefore$ AB + BC + AC = DB + BC + CE = DE = 11.8 cm

In ΔAOB, DB = AB $\Rightarrow \angle ADB + \angle DAB$.

$\therefore$ $\angle ABC = \angle ADB + \angle DAB$ (ext. $\angle$ = sum of two int. opp. $\angle$s)

$\Rightarrow$ $\angle ABC = 2\angle ADB = 2 \times 30° = 60° \Rightarrow \angle B = 60°$

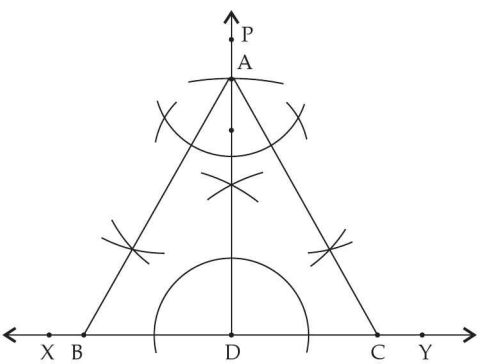
Similarly, $\angle ACB = 2\angle CEA = 2 \times 22\frac{1}{2}° = 45° \Rightarrow \angle C = 45°$.

Illustrative Examples

Example 1. *Construct an equilateral if its altitude is 3.4 cm. Give justification for your construction.* *(NCERT Exemplar)*

Solution. ***Steps of construction.***

1. Draw a line XY and take any point D on it.
2. At D, construct ray DP $\perp$ XY.
3. Taking D as centre and radius = 3.4 cm, draw an arc to cut DP at A.
4. At A, construct angles equal to 30° on both sides of AD to meet the line XY at B and C *i.e.* $\angle BAD = 30°$ and $\angle CAD = 30°$. Then ABC is the required equilateral triangle with altitude 3.4 cm.



Justification

In ΔABD and ΔACD,

$\angle BAD = \angle CAD$ (each = 30°)

$\angle ADB = \angle ADC$ (each = 90°, $\because$ PD $\perp$ XY)

$AD = AD$ (common)

$\therefore \quad \Delta ABD \cong \Delta ACD$

$\therefore \quad AB = AC$ (c.p.c.t.)

Also $\angle BAC = \angle BAD + \angle CAD = 30° + 30° = 60°$.

Thus, ABC is an isosceles triangle with vertical angle = 60°.

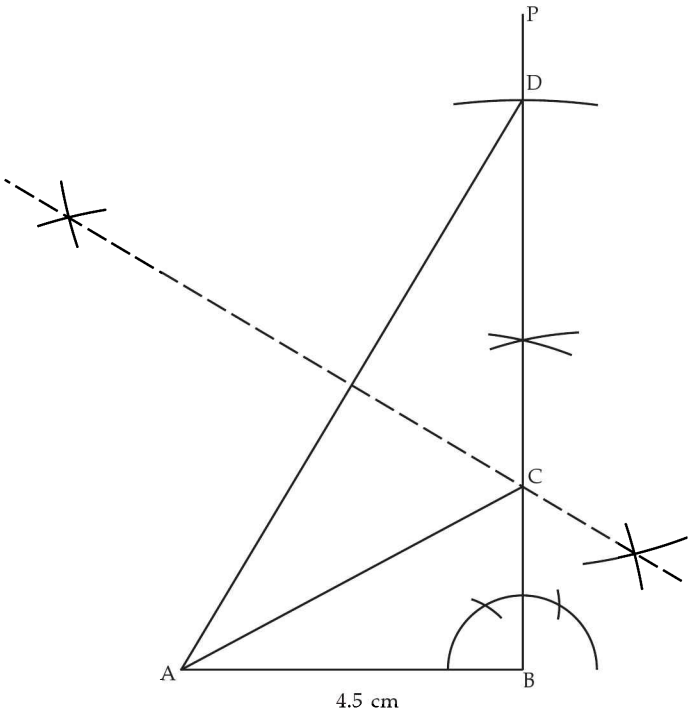
So, ΔABC is equilateral.

Hence, ABC is an equilateral triangle with altitude 3.4 cm.

Example 2. *Construct a right triangle whose base is 4.5 cm and sum of its hypotenuse and other side is 7.5 cm.*

Solution. ***Steps of construction.***

1. Draw AB = 4.5 cm.
2. At B, construct $\angle ABP = 90°$.
3. From BP, cut off DB = 7.5 cm.
4. Join AD.
5. Construct the perpendicular bisector of segment AD to intersect BD at C.
6. Join AC. Then ΔABC is the required triangle with the given data.



Exercise 12.2

1. State whether the following statements are true or false. Give reasons for your answer:
   (*i*) A triangle ABC can be constructed in which $\angle B = 105°$, $\angle C = 90°$ and AB + BC + AC = 10 cm. (*NCERT Exemplar*)
   (*ii*) A triangle ABC can be constructed in which, $\angle B = 60°$, $\angle C = 45°$ and AB + BC + AC = 12 cm. (*NCERT Exemplar*)
   (*iii*) A triangle ABC can be constructed in which AB = 5 cm, $\angle A = 45°$ and BC + AC = 5 cm. (*NCERT Exemplar*)
   (*iv*) A triangle ABC can be constructed in which BC = 6 cm, $\angle C = 30°$ and AC – AB = 4 cm. (*NCERT Exemplar*)
2. Construct a triangle whose sides are 3.6 cm, 3.0 cm and 4.8 cm. Bisect the smaller angle and measure each part. (*NCERT Exemplar*)
3. Construct an equilateral triangle with side 4.5 cm and justify the construction.

4. Construct an equilateral triangle if its altitude is 3.2 cm. *(NCERT Exemplar)*
5. Construct a triangle ABC in which BC = 5 cm, $\angle B = 60°$ and AC + AB = 7.5 cm. *(NCERT Exemplar)*
6. Construct a right triangle whose base is 6 cm and sum of its hypotenuse and other side is 9 cm. *(NCERT)*
7. Construct a triangle ABC in which BC = 8 cm, $\angle B = 45°$ and AB – AC = 3.5 cm. *(NCERT)*
8. Construct a triangle ABC, given BC = 6.5 cm, AC – AB = 2.8 cm and $\angle B = 60°$.
9. Construct a triangle PQR in which QR = 6 cm, $\angle Q = 60°$ and PR – PQ = 2 cm. *(NCERT)*
10. Construct a triangle ABC in which $\angle B = 60°$, $\angle C = 45°$ and AB + BC + CA = 11 cm. *(NCERT)*
11. Construct a triangle PQR in which $\angle Q = 30°$, $\angle R = 90°$ and perimeter is 11 cm. *(NCERT)*

Answers

Objective Type Questions

(FOR FORMATIVE/SUMMATIVE ASSESSMENT)

Mental Maths

1. Fill in the blanks using the correct word(s) given in the brackets :
   (*i*) A unique triangle can be constructed when its three ..... are known. (sides, angles)
   (*ii*) A triangle can be constructed when its two sides and ..... angle is known. (included, any one)
   (*iii*) A triangle can be constructed when the sum of lengths of any two sides is ..... than the length of the third side. (less, greater)

Multiple Choice Questions

Choose the correct answer from the given four options (2 to 7) :

2. A triangle can be constructed when the lengths of its three sides are
   (*a*) 7 cm, 3 cm, 4 cm (*b*) 3.6 cm, 11.5 cm, 6.9 cm
   (*c*) 5.2 cm, 7.6 cm, 4.7 cm (*d*) 33 mm, 8.5 cm, 49 mm
3. A unique triangle cannot be constructed if its
   (*a*) three angles are given
   (*b*) two angles and one side is given
   (*c*) three sides are given
   (*d*) two sides and the included angle is given
4. If the lengths of two sides of an isosceles triangle are 4 cm and 10 cm, then the length of the third side is
   (*a*) 4 cm (*b*) 10 cm (*c*) 7 cm (*d*) 14 cm
5. With the help of ruler and compass, it is possible to construct an angle of:
   (*a*) 35° (*b*) 40° (*c*) 37.5° (*d*) 47.5°

(NCERT Exemplar)

6. The construction of a triangle ABC, given that BC = 3 cm, $\angle C = 60°$ is possible when the difference of AB and AC is equal to:

(*a*) 3.2 cm (*b*) 3.1 cm (*c*) 3 cm (*d*) 2.8 cm

(*NCERT Exemplar*)

7. The construction of a triangle ABC, given that BC = 6 cm, $\angle B = 45°$ is not possible when the difference of AB and AC is equal to:

(*a*) 6.9 cm (*b*) 5.2 cm (*c*) 5.0 cm (*d*) 4.0 cm

(*NCERT Exemplar*)

Answers

Chapter Test

1. Draw an angle of 64° with the help of a protractor. Then using ruler and compass construct an angle of 32° and justify your construction.
2. Draw a line segment of length 6.4 cm. Construct its perpendicular bisector and justify your construction.
3. Construct a triangle ABC in which BC = 7 cm, $\angle B = 75°$ and AB + BC = 13 cm.

(*NCERT*)

4. Construct a right triangle when one side is 3.5 cm and the sum of its other side and hypotenuse is 5.5 cm. (*NCERT Exemplar*)
5. Construct a triangle PQR given that QR = 3 cm, $\angle PQR = 45°$ and QP – PR = 2 cm. Justify your construction. (*NCERT Exemplar*)
6. Construct a triangle if its perimeter is 10.4 cm and two angles are 45° and 120°. Also justify your construction. (*NCERT Exemplar*)

Surface Areas and Volumes



INTRODUCTION

So far, we were dealing with figures that can be easily drawn on our notebooks or black /white boards such as rectangles, squares, triangles and circles etc. These are called **plane figures**. We have learnt about the perimeters and areas of such figures. It is interesting to see what happens when such figures are cut from a cardboard sheet and stack them up in a pile. By this process, we obtain solids such as cuboid, cube, cylinder and prism etc. In the previous class, you have learnt about the surface areas and volumes of cuboids, cubes and right circular cylinders.

In this chapter, we shall review these and will extend our study to some other solids such as right circular cones and spheres.

13.1 SURFACE AREA AND VOLUME OF A SOLID

- **Solid**

  A figure which occupies a portion of space enclosed by plane or curved surfaces is called a ***solid.*** A solid has three dimensions—length, breadth and height.

- **Surface area of a solid**

  The sum of the areas of the plane or curved surfaces (faces) of a solid is called its **total surface area**. It is measured in square units.

  By the words 'Surface area of a solid' we shall mean the total surface area of the solid.

- **Lateral surface area of a solid**

  The surface area of the solid excluding the top and the bottom, if any, is called **lateral surface area** of the solid.

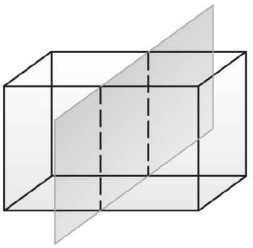
- **Volume of a solid**

  The measurement of the space enclosed by a solid is called its **volume.** It is measured in cubic units.

- **Cross-section of a solid**

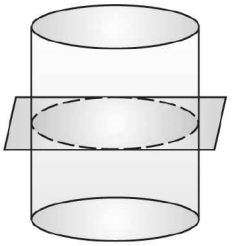
  If a cut is made through a solid perpendicular to its length (breadth or height), then the surface so obtained is called its ***cross-section.***

  If the surface made by the cut has the same shape and size at every point of its length (breadth or height), then it is called a uniform **cross-section.**



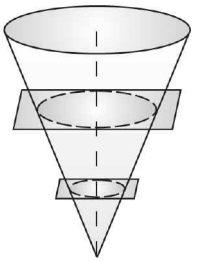
Cross-section
perpendicular to length

(i)



Cross-section
perpendicular to height

(ii)



Cross-sections
perpendicular to height

(iii)

In figures (*i*) and (*ii*) above, the solids have uniform cross-sections.

In figure (*iii*) above, the solid has non-uniform cross-sections.

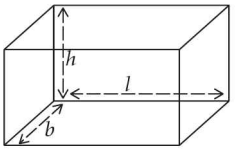
When a solid has uniform cross-section, then its

(*i*) ***volume = area of cross-section × length (breadth or height).***

(*ii*) ***lateral surface area = perimeter of cross-section × length (breadth or height).***

13.2 SURFACE AREA AND VOLUME OF A CUBOID

A solid in the shape of a box (or brick) is called a ***cuboid.*** *It has six rectangular plane surfaces, called* ***faces.***

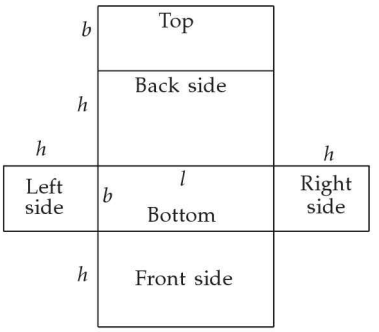


The length, breadth and height of a cuboid are usually denoted by l, b and h respectively. A cuboid has 12 edges and 8 vertices (or corners).

❑ **Surface area of a cuboid**

To find the surface area of a cuboid, perform the following activity:

Take a hollow cuboidal box of cardboard and cut it along its edges carefully to prepare a net of the cuboid as shown in the adjoining figure.



There are two identical faces (top and bottom) each with area = ($l \times b$) sq. units. There are two identical faces (front and back) each with area = ($l \times h$) sq. units.

There are two identical faces (right and left) each with area = ($h \times b$) sq. units.

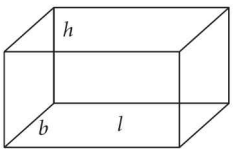
$\therefore$ The total surface area of the cuboid
= sum of areas of all its six faces
= $2(lb + lh + bh)$ sq. units.

Surface area of a cuboid = 2 (lb + lh + bh) sq. units.

❑ **Lateral surface area of a cuboid**

In a cuboid the area of side faces (*i.e.* faces excluding top and bottom) make the lateral surface area.

For example, the total area of the four walls of a cuboidal room is the lateral surface of this room.



The adjoining figure shows a rectangular room with length = l units, breadth = b units and height = h units.

There are two identical walls (front and back) each with area = ($l \times h$) sq. units.

There are two identical walls (right and left) each with area = ($b \times h$) sq. units.

$\therefore$ The area of four walls = $2(lh + bh)$ sq. units = $2(l + b)\, h$ sq. units

Thus, the area of four walls = 2 (l + b) h sq. units = perimeter of floor × height of room.

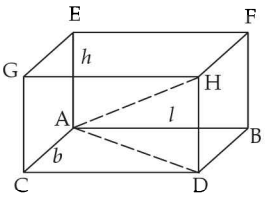
Lateral surface area of a cuboid = 2 (l + b) h sq. units.

❑ **Length of a diagonal of a cuboid**

The line joining opposite corners of a cuboid is called its ***diagonal.***

A cuboid has four diagonals. The adjoining figure shows a cuboid with length = l units, breadth = b units and height = h units. Join the opposite corners A and H, then AH is a diagonal of the cuboid. Join AD, then ADH is right angled triangle with $\angle D = 90°$. By Pythagoras theorem, we get

$AH^2 = AD^2 + HD^2$ ...(*i*)



Also ΔADB is a right angled triangle with $\angle D = 90°$.

By Pythagoras theorem, we get

$AD^2 = AB^2 + BD^2$ ...(*ii*)

From (*i*) and (*ii*), we get

$AH^2 = AB^2 + BD^2 + HD^2 = l^2 + b^2 + h^2$

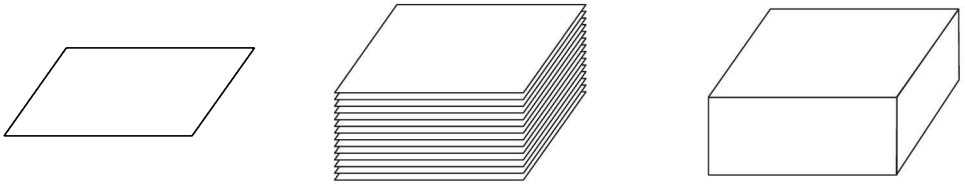
$\Rightarrow \quad AH = \sqrt{l^2 + b^2 + h^2}$

Hence, the length of a diagonal of a cuboid $= \sqrt{l^2 + b^2 + h^2}$ ***units.***

❑ **Volume of a cuboid**

Volume of a cuboid is the measurement of the space occupied by it:

To find the volume of a cuboid, take few rectangular sheets of thick paper of same size and stack them up in a vertical pile. We will obtain a cuboid as shown in the figure given below.



If A is the area of the rectangular sheet and h is the height to which the rectangles are stacked up in a vertical pile, then volume of cuboid so formed

= area of rectangular sheet × height = A × h.

Thus, volume of a cuboid = area of base × height

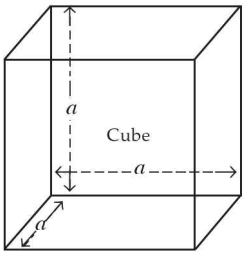
= length × breadth × height.

It follows that if a cuboid has length l units, breadth b units and height h units, then *the volume of cuboid = ($l \times b \times h$) cubic units.*

Remarks

* The capacity of a container = its internal volume.
* The volume of material in a hollow body = its external volume – its internal volume.
* If the external dimensions (length, breadth and height) of a box are l, b and h and if each side is of thickness x, then the internal dimensions of the
  (*i*) closed box are $l - 2x$, $b - 2x$ and $h - 2x$
  (*ii*) open box are $l - 2x$, $b - 2x$ and $h - x$.

13.3 SURFACE AREA AND VOLUME OF A CUBE



A rectangular solid bounded by six squares is called a ***cube****. It is a particular case of a cuboid where, $l = b = h = a$ (say).*

(*i*) ***Surface area = $6a^2$ sq. units.***

(*ii*) ***Lateral surface area = $4a^2$ sq. units.***

(*iii*) ***Volume = a^3 cubic units.***

(*iv*) ***Length of a diagonal = $\sqrt{3a^2}$ units = $\sqrt{3}\,a$ units.***

Illustrative Examples

Example 1. *The length, breadth and height of a room are 5 m, 4 m and 3 m respectively. Find the cost of white washing the walls of the room and the ceiling at the rate of ₹ 7.50 per m^2.* *(NCERT)*

Solution. Area to be white washed = area of 4 walls + area of ceiling

$= 2(l + b) \times h + l \times b = (2(5 + 4) \times 3 + 5 \times 4)\ m^2 = 74\ m^2$

$\therefore$ Cost of white washing = ₹(7.50×74) = ₹555.

Example 2. *The floor of a rectangular hall has a perimeter 250 m. If the cost of painting the four walls at the rate of ₹10 per m^2 is ₹ 15000, find the height of the hall.* *(NCERT)*

Solution. As the cost of painting the walls at the rate of ₹10 per m^2 is ₹15000,

the area painted = $\frac{15000}{10}\ m^2 = 1500\ m^2$.

Let h metres be the height of the hall.

Area of four walls = perimeter × height.

$\therefore\ 250 \times h = 1500 \Rightarrow h = 6$.

Hence, the height of the hall = 6 metres.

Example 3. *A cubical box has each edge 10 cm and a cuboidal box is 12.5 cm long, 10 cm wide and 8 cm high.*

(i) Which box has greater lateral surface area and by how much?

(ii) Which box has the smaller total surface area and by how much? *(NCERT)*

Solution. (*i*) Lateral surface area of cuboidal box = $2(l + b) \times h$

$= 2(12.5 + 10) \times 8\ cm^2 = 360\ cm^2$

Lateral surface area of cubical box = $4a^2 = (4 \times 10^2)\ cm^2 = 400\ cm^2$

$\therefore$ Lateral surface area of cubical box is greater by $(400 - 360)\ cm^2$ *i.e.* by $40\ cm^2$

(*ii*) Total surface area of cuboidal box = $2(lb + bh + hl)$

$= 2(12.5 \times 10 + 10 \times 8 + 8 \times 12.5)\ cm^2 = 610\ cm^2$

Total surface area of cubical box = $6a^2 = (6 \times 10^2)\ cm^2 = 600\ cm^2$

$\therefore$ Total surface area of cubical box is smaller by $(610 - 600)\ cm^2$ *i.e.* by $10\ cm^2$.

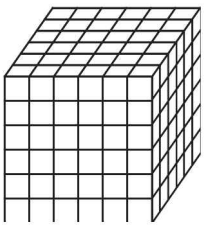
Example 4. *Mary wants to decorate her Christmas tree. She wants to place the tree on a wooden box covered with coloured papers with pictures of Santa Claus. She must know the exact quantity of paper to buy for this purpose. If the box has length 80 cm, breadth 40 cm and height 20 cm respectively, then how many square sheets of paper of side 40 cm would she require?* *(NCERT)*

Solution. Total surface area of wooden box = $2(80 \times 40 + 40 \times 20 + 80 \times 20)\ cm^2$ = $11200\ cm^2$.

Area of each square sheet of paper = $(40 \times 40)\ cm^2 = 1600\ cm^2$.

$\therefore$ The number of sheets of paper required = $\frac{11200}{1600} = 7$.

Example 5. *Hameed has built a cubical water tank with lid for his house, with each outer edge 1.5 m long. He gets the outer surface of the tank excluding the base, covered with square tiles of side 25 cm (shown in the adjoining figure). Find how much he would spend for the tiles, if the cost of the tiles is ₹ 360 per dozen.* (NCERT)



Solution. Each side (edge) of the tank = 1.5 m = 150 cm.

Outer surface area of the tank (excluding base) = $(5 \times 150 \times 150)$ cm^2.

Area of each square tile = (25×25) cm^2.

$\therefore$ No. of tiles required = $\frac{5 \times 150 \times 150}{25 \times 25} = 180$.

Cost of one dozen *i.e.* 12 tiles = ₹ 360

$\therefore$ Cost of one tile = ₹ $\frac{360}{12}$ = ₹ 30.

$\therefore$ The cost of 180 tiles = ₹ (30×180) = ₹ 5400.

Example 6. *Shanti Sweets wants to place an order for cardboard boxes for packing sweets. They need two types of boxes, one with dimensions 25 cm × 20 cm × 5 cm and another with dimensions 15 cm × 12 cm × 5 cm. For all the overlaps, 5% of the total surface is required extra. If the cost of the cardboard is ₹ 4 for 1000 cm^2, find the cost of cardboard required for supplying 250 boxes of each type.* (NCERT)

Solution. Surface area of 1 big box = $2(25 \times 20 + 20 \times 5 + 25 \times 5)$ cm^2 = 1450 cm^2

As 5% of total surface is required extra for overlaps,

$\therefore$ area of cardboard required for making one big box

$= \left(1 + \frac{5}{100}\right) \times 1450 \text{ cm}^2 = \frac{21}{20} \times 1450 \text{ cm}^2 = 1522.5 \text{ cm}^2$

$\therefore$ Area of cardboard required for making 250 big boxes

$= 1522.5 \times 250 \text{ cm}^2 = 380625 \text{ cm}^2$.

Surface area of one small box = $2(15 \times 12 + 15 \times 5 + 12 \times 5)$ cm^2 = 630 cm^2.

As 5% of total surface is required extra for overlaps,

$\therefore$ area of cardboard required for making one small box

$= \left(1 + \frac{5}{100}\right) \times 630 \text{ cm}^2 = \frac{21}{20} \times 630 \text{ cm}^2 = 661.5 \text{ cm}^2$

$\therefore$ Area of cardboard required for making 250 small boxes

$= 661.5 \times 250 \text{ cm}^2 = 165375 \text{ cm}^2$.

$\therefore$ Total area of cardboard required for supplying 250 big boxes and 250 small boxes

$= (380625 + 165375) \text{ cm}^2 = 546000 \text{ cm}^2$.

As cost of 1000 cm^2 of cardboard is ₹ 4,

$\therefore$ cost of 546000 cm^2 of cardboard = ₹ $\frac{546000 \times 4}{1000}$ = ₹ 2184.

Example 7. *A village, having a population of 4000, requires 150 litres of water per head per day. It has a tank measuring 20 m × 15 m × 6 m. For how many days will the water of this tank last?*

What value is shown by avoiding wastage of water? ***(Value Based)***

Solution. Quantity of water in the tank = $(20 \times 15 \times 6)$ m^3 = 1800 m^3.

Population of village = 4000 and quantity of water required per head per day = 150 litres.

Quantity of water required by the village per day = (4000×150) litres

$= \frac{4000 \times 150}{1000} \text{ m}^3 = 600 \text{ m}^3$.

$\therefore$ The number of days for which the water will last = $\frac{1800}{600} = 3$.

Value shown is wisdom because water is our life. Its scarcity causes many problems.

Example 8. *A river 3 m deep and 40 m wide is flowing at the rate of 2 km per hour. How much water will fall into the sea in a minute?* *(NCERT)*

Solution. As the river is flowing at the rate of 2 km per hour,

length of water delivered by the river in one minute = $\frac{2 \times 1000}{60}$ m.

$\therefore$ Quantity of water that will fall in the sea in one minute

$= \left(\frac{2 \times 1000}{60} \times 40 \times 3\right)$ $m^3 = 4000$ m^3.

Example 9. *A 10 m long wall is to be built across an open ground. The height of the wall is 4 m and the thickness of the wall is 24 cm. If this wall is to be built with bricks whose dimensions are 24 cm × 12 cm × 8 cm, how many bricks would be required?*

Solution. Length of wall = 10 m = 1000 cm, thickness = 24 cm and height = 4 m = 400 cm.

Volume of wall = (1000 × 24 × 400) cm^3

Volume of each brick = (24 × 12 × 8) cm^3.

$\therefore$ No. of bricks required to build the wall = $\frac{1000 \times 24 \times 400}{24 \times 12 \times 8} = \frac{12500}{3} = 4166\frac{2}{3}$.

But number of bricks cannot be in fraction,

$\therefore$ the number of bricks required to build the wall = 4167.

Example 10. *A storage tank is in the form of a cube. When it is full of water, the volume of the water is 15.625 m^3. If the present depth of the water is 1.3 m, find the volume of water already used from the tank.* *(NCERT Exemplar)*

Solution. Let an edge of the cubical tank be a metres, then volume of tank = a^3 m^3.

According to given, $a^3 = 15.625$

$\Rightarrow$ $a^3 = \frac{15625}{1000} = \frac{125}{8} = \left(\frac{5}{2}\right)^3 \Rightarrow a = \frac{5}{2}$.

$\therefore$ Edge of cubical tank = $\frac{5}{2}$ m.

Present depth of water in the tank = 1.3 m = $\frac{13}{10}$ m.

$\therefore$ Present volume of water in the tank = $\left(\frac{5}{2} \times \frac{5}{2} \times \frac{13}{10}\right)$ $m^3 = \frac{65}{8}$ $m^3 = 8.125$ m^3

$\therefore$ Volume of water already used = 15.625 m^3 – 8.125 m^3 = 7.5 m^3.

Example 11. *A solid cube of side 12 cm is cut into eight cubes of equal volume. What will be the side of the new cube? Also, find the ratio between their surface areas.* *(NCERT)*

Solution. Volume of given cube = (12 × 12 × 12) cm^3

As the given cube is cut into 8 cubes of equal volume,

$\therefore$ volume of each new cube = $\frac{12 \times 12 \times 12}{8}$ $cm^3 = (6 \times 6 \times 6)$ cm^3

$\Rightarrow$ side of each new cube = 6 cm.

Surface area of given cube = $6a^2 = (6 \times 12 \times 12)$ cm^2.

Surface area of each new cube = $6a^2 = (6 \times 6^2)$ cm^2

$\therefore$ $\frac{\text{Surface area of given cube}}{\text{Surface area of a new cube}} = \frac{6 \times 12 \times 12}{6 \times 6 \times 6} = 4$.

Hence, the required ratio = 4 : 1.

Example 12. *Find the length of the longest rod that can be placed in a box of length 1.2 m, breadth 90 cm and height 80 cm.*

Solution. Here, length of box = l = 1.2 m = 120 cm, breadth = b = 90 cm and height = h = 80 cm.

Length of longest rod = length of diagonal

$$= \sqrt{l^2 + b^2 + h^2} = \sqrt{(120)^2 + (90)^2 + (80)^2} \text{ cm}$$

$$= 10\sqrt{144 + 81 + 64} \text{ cm} = 10\sqrt{289} \text{ cm}$$

$$= (10 \times 17) \text{ cm} = 170 \text{ cm} = 1.7 \text{ m}.$$

Example 13. *A cuboid has length, breadth and diagonal as 4 m, 3 m and 13 m respectively. Find its volume.*

Solution. Let the height of the cuboid be h metres.

Here, l = 4 m and b = 3 m.

As the length of diagonal of a cuboid = $\sqrt{l^2 + b^2 + h^2}$,

$\therefore \quad \sqrt{4^2 + 3^2 + h^2} = 13 \Rightarrow 16 + 9 + h^2 = 169$

$\Rightarrow \quad h^2 = 169 - 25 \Rightarrow h^2 = 144 \Rightarrow h = 12 \qquad (\because h > 0)$

$\therefore$ Height of cuboid = 12 m.

$\therefore$ Volume of cuboid = $l \times b \times h$ = $(4 \times 3 \times 12)$ m^3 = 144 m^3.

Example 14. *If the areas of three adjacent faces of a cuboid are x, y and z and its volume is V, then prove that $V^2 = xyz$.*

Solution. Let l, b and h be the length, breadth and height respectively of the cuboid, then its volume $V = l \times b \times h$, ...(*i*)

According to the question, $lb = x$, $bh = y$, $lh = z$

$\Rightarrow \quad lb \times bh \times lh = xyz$

$\Rightarrow \quad (l \times b \times h)^2 = xyz$

$\Rightarrow \quad V^2 = xyz.$ (using (*i*))

Example 15. *The volume of a cuboidal block of silver is 10368 cm^3. If its dimensions are in the ratio 3 : 2 : 1, find*

(i) the dimensions of the block.

(ii) the cost of gold polishing its entire surface at ₹ 0.50 per cm^2.

Solution. (*i*) As the dimensions of the cuboidal block of silver are in the ratio 3 : 2 : 1, let its length, breadth and height be $3x$ cm, $2x$ cm and x cm respectively.

Volume of block = $(3x \times 2x \times x)$ cm^3 = 10368 cm^3

$\Rightarrow \quad 6x^3 = 10368 \Rightarrow x^3 = 1728$

$\Rightarrow \quad x^3 = 12 \times 12 \times 12 \Rightarrow x = 12.$

$\therefore$ The dimensions of the block are $l = 3x$ cm = (3×12) cm = 36 cm, $b = 2x$ cm = (2×12) cm = 24 cm and height = x cm = 12 cm.

(*ii*) Total surface area of the block of silver = $2(lb + lh + hb)$

$= 2(36 \times 24 + 36 \times 12 + 24 \times 12)$ cm^2

$= 2(864 + 432 + 288)$ cm^2 = (2×1584) cm^2 = 3168 cm^2

Since the cost of gold polishing its surface is at the rate of ₹ 0.50 per cm^2 = ₹ $\frac{1}{2}$ per cm^2

$\therefore$ the cost of gold polishing entire surface = ₹ $(\frac{1}{2} \times 3168)$ = ₹ 1584.

Example 16. *The internal dimensions of a rectangular room are 6 m, 5 m and 3.5 m. It has two doors of size 1.2 m by 2 m and three windows of size 1 m by 1.9 m. The walls of the room are to be papered with a wallpaper of width 70 cm. Find the cost of the paper at the rate of* ₹ *6.50 per metre.*

Solution. The surface area of the walls of the room $= 2(l + b) \times h$

$= 2(6 + 5) \times 3.5 \text{ m}^2$

$= (22 \times 3.5) \text{ m}^2 = 77 \text{ m}^2$.

Surface area of 2 doors $= (2 \times 1.2 \times 2) \text{ m}^2 = 4.8 \text{ m}^2$.

Surface area of 3 windows $= (3 \times 1 \times 1.9) \text{ m}^2 = 5.7 \text{ m}^2$.

$\therefore$ Surface area to be papered

= surface area of walls – surface area of doors and windows

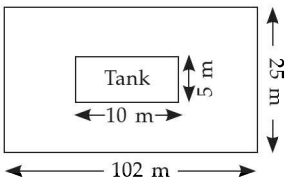
$= 77 \text{ m}^2 - (4.8 \text{ m}^2 + 5.7 \text{ m}^2) = 77 \text{ m}^2 - 10.5 \text{ m}^2 = 66.5 \text{ m}^2$

Width of the wallpaper = 70 cm = 0.7 m.

$\therefore$ The length of the wallpaper required $= \frac{66.5}{0.7}$ m = 95 m.

$\therefore$ The cost of the wallpaper = ₹(95 × 6.50) = ₹ 617.50.

Example 17. *A field is 102 m long and 25 m broad. A tank 10 m long, 5 m broad and 4 m deep is dug out from the middle of the field and the earth removed is evenly spread over the remaining part of the field. Find the rise in the level of the remaining part of the field in centimetres.*



Solution. The volume of the earth dug out

$= (10 \times 5 \times 4) \text{ m}^3 = 200 \text{ m}^3$

The upper surface area of the tank $= (10 \times 5) \text{ m}^2 = 50 \text{ m}^2$.

Total area of the field $= (102 \times 25) \text{ m}^2 = 2550 \text{ m}^2$.

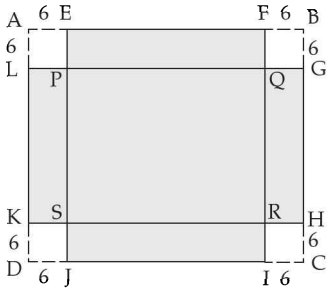
The remaining area of the field $= (2550 - 50) \text{ m}^2 = 2500 \text{ m}^2$.

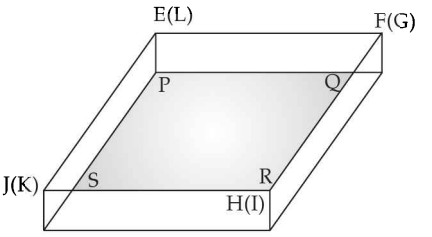
$\therefore$ The rise in the level of the remaining part of the field

$= \frac{200}{2500} \text{ m} = \frac{2}{25} \text{ m} = \left(\frac{2}{25} \times 100\right) \text{cm} = 8 \text{ cm}.$

Example 18. *Squares each of side 6 cm are cut off from the four corners of a sheet of tin measuring 42 cm by 30 cm. The remaining portion of the tin sheet is made into an open box by folding up the flaps. Find the capacity of the box.*

Solution. All measurements are in centimetres.





SR = PQ = EF = (42 – 6 – 6) cm = 30 cm,

RQ = SP = KL = (30 – 6 – 6) cm = 18 cm.

EP = LP = 6 cm.

Length of the open box = 30 cm, breadth = 18 cm and height = 6 cm.

$\therefore$ The capacity of the open box $= (30 \times 18 \times 6) \text{ cm}^3 = 3240 \text{ cm}^3$.

Example 19. *A hollow square shaped tube open at both ends, is made of iron. The internal square is of 5 cm side and the length of the tube is 8 cm. There are 192 cm^3 of iron in this tube. Find its thickness.*

Solution. Since the internal square is of 5 cm side and the length of the tube is 8 cm.

$\therefore$ Its internal volume = $(5 \times 5 \times 8)$ cm^3 = 200 cm^3.

Let the thickness of a side be x cm.

Then the external square is of $(5 + 2x)$ cm side.

As the tube is open at both ends, its length remains 8 cm.

$\therefore$ The external volume of the tube = $(5 + 2x)(5 + 2x) \times 8$ cm^3.

Since the volume of iron in the tube is 192 cm^3,

$\therefore$ $(5 + 2x)^2 \times 8 - 200 = 192$

$\Rightarrow$ $(5 + 2x)^2 \times 8 = 392$

$\Rightarrow$ $(5 + 2x)^2 = 49 \Rightarrow 5 + 2x = 7$

$\Rightarrow$ $2x = 7 - 5 \Rightarrow 2x = 2 \Rightarrow x = 1$.

$\therefore$ The thickness of the tube = 1 cm.

Example 20. *The external dimensions of an open rectangular wooden box are 98 cm by 84 cm by 77 cm. If the wood is 2 cm thick all around, find*

(i) the internal dimensions of the box.

(ii) the capacity of the box.

(iii) the volume of the wood used in making the box.

(iv) the weight of the box in kilogram correct to 1 decimal place, given that 1 cm^3 of wood weighs 0.8 g.

Solution. (*i*) The internal dimensions of the box are:

length = 98 cm – 4 cm = 94 cm,

breadth = 84 cm – 4 cm = 80 cm,

height = 77 cm – 2 cm = 75 cm.

(Since the box is open at the top, so only the thickness of bottom is to be reduced from the height).

(*ii*) The capacity of the box = internal volume of the box

= $(94 \times 80 \times 75)$ cm^3 = 564000 cm^3.

(*iii*) External volume of the box = $(98 \times 84 \times 77)$ cm^3 = 633864 cm^3

The volume of the wood used in making the box

= external volume – internal volume

= 633864 cm^3 – 564000 cm^3 = 69864 cm^3.

(*iv*) The weight of the box = the weight of the wood

= (69864×0.8) g = 55891.2 g

= $\frac{55891.2}{1000}$ kg = 55.8912 kg = 55.9 kg.

Exercise 13.1

1. State whether the following statement is true or false. Justify your answer:

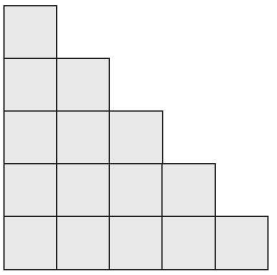
The length of the diagonal of a cube is $6\sqrt{3}$ cm, then the length of the edge of the cube is 3 cm. (*NCERT Exemplar*)

2. A plastic box 1.5 m long, 1.25 m wide and 65 cm deep is to be made. It is open at the top. Ignoring the thickness of the plastic sheet, determine :

(*i*) the area of the sheet required for making the box.

(*ii*) the cost of sheet for it, if a sheet measuring 1 m^2 costs ₹ 20. (*NCERT*)

3. A small indoor green house (herbarium) is made entirely of glass panes (including base) held together with tape. It is 30 cm long, 25 cm wide and 25 cm high.
   (*i*) What is the area of the glass?
   (*ii*) How much of tape is needed for all the 12 edges?
4. A paint in a certain container is sufficient to paint an area equal to 9.375 m^2. How many blocks of dimensions 22.5 cm × 10 cm × 7.5 cm can be painted out of this container?
5. A matchbox measures 4 cm × 2.5 cm × 1.5 cm. What will be the volume of a packet containing 12 such boxes?
6. A child playing with building blocks, which are of the shape of cubes, has built a structure as shown in the adjoining figure. If each edge of the cube is 3 cm, find the volume of the structure built by the child. (*NCERT*)



7. A cuboidal vessel is 10 m long and 8 m wide. How high must it be made to hold 380 cubic metres of a liquid? (*NCERT*)
8. Find the cost of digging a cuboidal pit 8 m long, 6 m broad and 3 m deep at the rate of ₹ 30 per m^3. (*NCERT*)
9. A cuboidal water tank is 6 m long, 5 m wide and 4.5 m deep. How many litres of water can it hold? (*NCERT*)
10. The capacity of a cuboidal tank is 50000 litres of water. Find the breadth of the tank, if its length and depth are respectively 2.5 m and 10 m. (*NCERT*)
11. A godown measures 60 m × 25 m × 10 m. Find the maximum number of wooden crates each measuring 1.5 m × 1.25 m × 0.5 m that can be stored in the godown. (*NCERT*)
12. A hall is 24 m long and 16 m wide. If the sum of the areas of the floor and ceiling is equal to the area of four walls, find the height and the volume of the hall.
13. If the length of a diagonal of a cube is $12\sqrt{3}$ cm, find its surface area and volume.
14. If the surface area of a cube is 486 cm^2, find
   (*i*) the length of a diagonal (*ii*) volume of the cube.
15. A rectangular pit 1.4 m long, 90 cm broad and 70 cm deep was dug and 1000 bricks of base 21 cm by 10.5 cm were made from the earth dug out. Find the height of each brick.
16. The length, breadth and height of a rectangular solid are in the ratio 5 : 4 : 2. If its total surface area is 1216 cm^3, find the volume of the solid.
17. The inner dimensions of a closed wooden box are 2 m, 1.2 m and 0.75 m. The thickness of the wood is 2.5 cm. Find the cost of wood required to make the box if 1 m^3 of wood costs ₹ 5400.

Answers

13.4 SURFACE AREA AND VOLUME OF A CYLINDER

Cylinder

A solid obtained by revolving a rectangular lamina about one of its sides is called a ***right circular cylinder.***

The solid shown in the adjoining figure is a right circular cylinder.

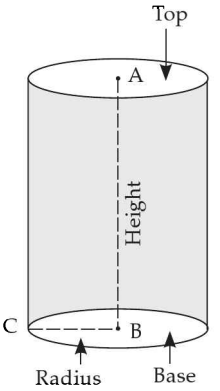
(*i*) A right circular cylinder has a circular base (bottom) and a circular top which are congruent circles. Here, A and B are the centres of the top and base.

(*ii*) The radius of the base is called the **radius** of the right circular cylinder.

(*iii*) The distance AB is called the **height** or **length** of the right circular cylinder.

(*iv*) The line joining the points A and B is called the **axis** of the cylinder.

(*v*) A right circular cylinder has three faces—a curved face and two flat faces (top and bottom).

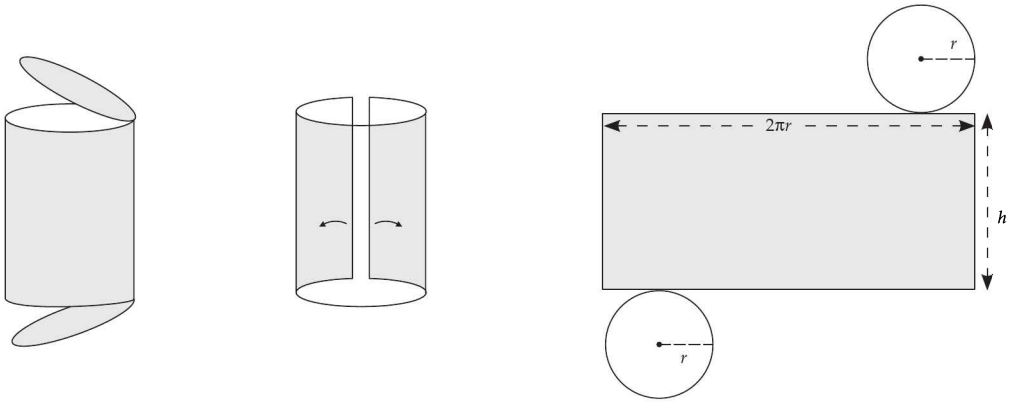


In this book, we shall be dealing only with right circular cylinders. Therefore, whenever we use the word 'cylinder', it will mean right circular cylinder.

❑ **Surface area of a right circular cylinder**

To find the surface area of a right circular cylinder, perform the following activity:

Take a hollow closed right circular cylinder made up of thick paper/cardboard and cut it as shown in the figure *i.e.* first cut its two circular ends (top and base) and then cut it along a line on its surface parallel to the axis of cylinder and prepare a net as shown in the figure.



The net of the right circular cylinder consists of:

(*i*) a rectangular piece whose length is equal to the circumference/perimeter of the circular base and its breadth is equal to the height of cylinder.

(*ii*) two circular pieces of same radius.

If r is the radius of the cylinder and h is its height, then curved surface area of the cylinder = area of rectangular piece = length × breadth = perimeter of base of cylinder × h = $2\pi r \times h$.

Hence, curved surface area of the cylinder = $2\pi rh$.

Total surface area of cylinder = curved surface area + area of two circular pieces

$$= 2\pi rh + 2 \times \pi r^2$$

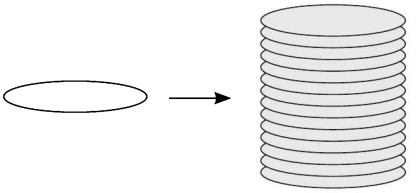
$$= 2\pi r + (h + r).$$

Hence, total surface area of cylinder = $2\pi r\ (h + r)$.

Note. We know that π is an irrational number, so it is a non-terminating, non-repeating decimal. In our calculations, we take the value of π approximately equal to $\frac{22}{7}$ or 3.14.

❑ **Volume of a right circular cylinder**

To find the volume of a right circular cylinder, take few circular sheets of thick paper of same size and stack them up in a vertical pile. We obtain a right circular cylinder as shown in the adjoining figure.



If A is the area of the circular sheet and h is the height to which the circular sheets are stacked up in a vertical pile, then

volume of the cylinder so formed = area of circular sheet × height = A × h.

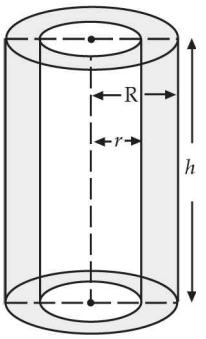
Thus, volume of a cylinder = area of base × height.

It follows that if a cylinder has radius r and height h, then

volume of cylinder $= \pi r^2 \times h = \pi r^2 h$.

Remark. When a rectangular lamina is rolled, we get a hollow right circular cylinder.

Let R and r be the external and internal radii of a hollow cylinder, and h be its height, then



(*i*) *Thickness of cylinder* $= R - r$.

(*ii*) *Area of a cross-section* $= \pi (R^2 - r^2)$.

(*iii*) *External curved surface area* $= 2\pi Rh$.

(*iv*) *Internal curved surface area* $= 2\pi rh$.

(*v*) *Total surface area = external curved area + internal curved area + area of two ends*

$= 2\pi Rh + 2\pi rh + 2\pi (R^2 - r^2)$

$= 2\pi (Rh + rh + R^2 - r^2)$.

(*vi*) *Volume of the material* $= \pi R^2 h - \pi r^2 h$

$= \pi (R^2 - r^2)h$.

Illustrative Examples

Example 1. *The diameter of a cylinder is 7 cm and its height is 16 cm. Find:*

(i) Lateral surface area of the cylinder.

(ii) Total surface area of the cylinder.

(iii) Volume of the cylinder.

Solution. Let r be the radius and h be the height of the given cylinder, then diameter $= 2r = 7$ cm (given)

$$\Rightarrow \quad r = \frac{7}{2} \text{ cm.}$$

Also height of the cylinder $= h = 16$ cm (given).

(*i*) Lateral (curved) surface area of the cylinder $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times \frac{7}{2} \times 16 \text{ cm}^2 = 352 \text{ cm}^2.$$

(*ii*) Total surface area of cylinder $= 2\pi r (h + r)$

$$= 2 \times \frac{22}{7} \times \frac{7}{2} \left(16 + \frac{7}{2}\right) \text{ cm}^2$$

$$= 22 \times \frac{39}{2} \text{ cm}^2 = 429 \text{ cm}^2.$$

(*iii*) Volume of the cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 16 \text{ cm}^3 = 616 \text{ cm}^3.$$

Example 2. *In the hot water heating system, there is a cylindrical pipe of length 28 m and diameter 5 cm. Find the total radiating surface in the system.* *(NCERT)*

Solution. Radius of cylindrical pipe $= \left(\frac{1}{2} \times 5\right)$ cm $= \frac{5}{2} \times \frac{1}{100}$ m $= 0.025$ m, length of pipe $= 28$ m.

Total radiating surface = curved surface area of the cylindrical pipe

$$= \left(2 \times \frac{22}{7} \times 0.025 \times 28\right) \text{m}^2 = 4.4 \text{ m}^2.$$

Example 3. *Curved surface area of a right circular cylinder is 4.4 m^2. If the radius of the base of the cylinder is 0.7 m, find its height.* *(NCERT)*

Solution. Let the height of the circular cylinder be h metres, then

curved surface area = $2\pi rh$

$\Rightarrow \quad 4.4 = 2 \times \frac{22}{7} \times 0.7 \times h$

$\Rightarrow \quad 4.4 = 4.4\, h \Rightarrow h = 1.$

Hence, the height of cylinder = 1 m.

Example 4. *The inner diameter of a circular well is 3.5 m. It is 10 m deep. Find:*

(i) its inner curved surface area.

(ii) the cost of plastering this curved surface area at the rate of ₹40 per m^2.

Solution. Radius of circular well = $\left(\frac{1}{2} \times 3.5\right)$ m = $\frac{7}{4}$ m.

(*i*) Inner curved surface area = $2\pi rh = \left(2 \times \frac{22}{7} \times \frac{7}{4} \times 10\right) \text{m}^2 = 110 \text{ m}^2.$

(*ii*) As the rate of plastering the surface = ₹40 per m^2,

$\therefore$ the cost of plastering the surface = ₹(110 × 40) = ₹4400.

Example 5. *30 circular plates, each of radius 14 cm and thickness 3 cm are placed one above the another to form a cylindrical solid. Find:*

(i) the total surface area.

(ii) the volume of the cylinder so formed. *(NCERT Exemplar)*

Solution. Height of the cylinder formed = number of plates × thickness of one plate

= (30 × 3) cm = 90 cm.

(*i*) Total surface area of cylinder = $2\pi r\,(h + r)$

$$= 2 \times \frac{22}{7} \times 14 \times (90 + 14) \text{ cm}^2$$

$$= (88 \times 104) \text{ cm}^2 = 9152 \text{ cm}^2.$$

(*ii*) Volume of the cylinder $= \pi r^2 h = \frac{22}{7} \times (14)^2 \times 90 \text{ cm}^3$

$$= (22 \times 28 \times 90) \text{ cm}^3 = 55440 \text{ cm}^3.$$

Example 6. *A cylindrical roller 2.5 m in length, 1.75 m in radius when rolled on a road was found to cover the area of 5500 m^2. How many revolutions did it make?* *(NCERT Exemplar)*

Solution. Length of cylindrical roller = 2.5 m = $\frac{5}{2}$, radius = 1.75 m = $\frac{7}{4}$ m.

Area covered by the roller in one revolution = curved surface area of roller

$$= 2\pi rh = \left(2 \times \frac{22}{7} \times \frac{7}{4} \times \frac{5}{2}\right) \text{m}^2 = \frac{55}{2} \text{ m}^2.$$

Let the number of revolutions made by the roller be n, then

area covered in one revolution × n = total area covered

$\Rightarrow \quad \frac{55}{2} \times n = 5500 \Rightarrow n = 200.$

Hence, the number of revolutions made by the roller = 200.

Example 7. *The circumference of the base of a cylindrical vessel is 132 cm and its height is 25 cm. How many litres of water can it hold?* *(NCERT)*

Solution. Let the radius of the cylindrical vessel be r cm, then circumference $= 2\pi r$

$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 132 \Rightarrow r = 21.$

$\therefore$ Capacity of cylindrical vessel $= \pi r^2 h = \left(\frac{22}{7} \times 21^2 \times 25\right)$ cm^3

$= 34650$ cm$^3 = \frac{34650}{1000}$ litres $= 34.65$ litres.

Example 8. *If the lateral surface area of a cylinder is 94.2 cm^2 and its height is 5 cm, then find*

(i) radius of its base *(ii) its volume* *(Use $\pi = 3.14$)* *(NCERT)*

Solution. (*i*) Lateral surface area of a cylinder $= 2\pi rh$

$\Rightarrow \quad 2 \times 3.14 \times r \times 5 = 94.2 \Rightarrow r = 3$ cm

(*ii*) Volume $= \pi r^2 h = (3.14 \times 3^2 \times 5)$ cm$^3 = 141.3$ cm^3.

Example 9. *The total surface area of a cylinder is 6512 cm^2 and the circumference of its base is 88 cm. Find the volume of the cylinder.*

Solution. Let r be the radius and h be the height of the given cylinder.

Circumference $= 2\pi r = 88$ cm (given)

$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 88 \Rightarrow r = 14$ cm.

Total surface area $= 2\pi r\,(h + r) = 6512$ cm^2 (given)

$\Rightarrow \quad 88\,(h + 14) = 6512$ $(\because\ 2\pi r = 88$ and $r = 14)$

$\Rightarrow \quad h + 14 = 74 \Rightarrow h = 60$ cm.

$\therefore$ Volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times (14)^2 \times 60$ cm$^3 = 36960$ cm^3.

Example 10. *If radii of two circular cylinders are in the ratio 3 : 4 and their heights are in the ratio 6 : 5, find the ratio of their curved surfaces.*

Solution. Since the radii of two circular cylinder are in the ratio 3 : 4, let their radii be $3r$ and $4r$ respectively.

As their heights are in the ratio 6 : 5, let their heights be $6h$ and $5h$ respectively.

Curved surface area of first cylinder $= S_1 = 2\pi \times 3r \times 6h = 36\pi rh$, and the curved surface area of second cylinder $= S_2 = 2\pi \times 4r \times 5h = 40\pi rh$.

$\therefore \quad \frac{S_1}{S_2} = \frac{36\pi rh}{40\pi rh} = \frac{9}{10}.$

Hence, the ratio of their curved surface areas $= 9 : 10$.

Example 11. *The radius and the height of a cylinder are in the ratio 2 : 7. If the volume of the cylinder is 704 cm^3, find the total surface area of the cylinder.*

Solution. Since the radius and the height of a cylinder are in the ratio 2 : 7,

let r (radius) $= 2k$ cm, then h (height) $= 7k$ cm.

$\therefore$ Volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times (2k)^2 \times 7k = 704$ (given)

$\Rightarrow \quad 88k^3 = 704 \Rightarrow k^3 = 8 \Rightarrow k = 2.$

$\therefore$ Radius of cylinder $= 2k$ cm $= 4$ cm and height of cylinder $= 7k$ cm $= 14$ cm.

$\therefore$ Total surface area of the cylinder $= 2\pi r\,(h + r)$

$$= 2 \times \frac{22}{7} \times 4 \times (14 + 4)\ \text{cm}^2$$

$$= \frac{176}{7} \times 18\ \text{cm}^2 = \frac{3168}{7}\ \text{cm}^2 = 452\frac{4}{7}\ \text{cm}^2.$$

Example 12. *The total surface area of a circular cylinder is 462 cm^2. If its curved surface area is one-third of its total surface area, find the volume of the cylinder.*

Solution. Let the radius of the circular cylinder be r cm and its height be h cm.

Total surface area $= 462\ \text{cm}^2$ (given)

$\Rightarrow \quad 2\pi rh + 2 \times \pi r^2 = 462$ ...(*i*)

Given, curved surface area $= \frac{1}{3}$ of total surface area

$\Rightarrow \quad 2\pi rh = \frac{1}{3} \times 462$

$\Rightarrow \quad 2\pi rh = 154$ ...(*ii*)

Substituting this value of $2\pi rh$ in (*i*), we get

$154 + 2\pi r^2 = 462$

$\Rightarrow \quad 2 \times \frac{22}{7} \times r^2 = 308 \Rightarrow r^2 = \frac{308 \times 7}{2 \times 22}$

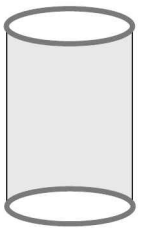
$\Rightarrow \quad r^2 = 49 \Rightarrow r = 7.$

Substituting this value of r in (*ii*), we get

$2 \times \frac{22}{7} \times 7 \times h = 154 \Rightarrow 44h = 154 \Rightarrow h = \frac{7}{2}.$

Volume of the cylinder $= \pi r^2 h = \frac{22}{7} \times 7^2 \times \frac{7}{2}\ \text{cm}^3 = 539\ \text{cm}^3.$

Example 13. *The adjoining figure shows the frame of a lampshade which is to be covered with a decorative cloth. The frame has a base diameter 20 cm and height 30 cm. A margin of 2.5 cm is to be given for folding it over the top and the bottom of the frame. Find how much cloth is required for covering the frame.* (NCERT)



Solution. Radius of frame $= \left(\frac{1}{2} \times 20\right)$ cm $= 10$ cm,

height of frame $= 30$ cm.

Actual height of cloth = height of frame + 2 margins of 2.5 cm each

$= (30 + 2 \times 2.5)$ cm $= 35$ cm.

$\therefore$ The area of cloth required to cover the frame

$$= \left(2 \times \frac{22}{7} \times 10 \times 35\right)\ \text{cm}^2 = 2200\ \text{cm}^2.$$

Example 14. *At a Ramzan Mela, a stall keeper in one of food stalls has a large cylindrical vessel of base radius 15 cm filled up to a height of 32 cm with orange juice. The juice is filled in small cylindrical glasses of radius 3 cm upto a height of 8 cm, and sold for ₹3 each. How much money does the stall keeper receive by selling the juice completely?* (NCERT)

Solution. Quantity of juice in the large cylindrical vessel $= \pi R^2 H$

$= (\pi \times 15^2 \times 32)\ \text{cm}^3.$

Quantity of juice in each small cylindrical glass $= \pi r^2 h$

$= (\pi \times 3^2 \times 8)\ \text{cm}^3.$

$\therefore$ The number of glasses of juice sold $= \frac{\text{volume of vessel}}{\text{volume of each glass}} = \frac{\pi \times 15^2 \times 32}{\pi \times 3^2 \times 8} = 100.$

$\therefore$ The money received by the stall keeper $= ₹(3 \times 100) = ₹\ 300.$

Example 15. *Rainwater which falls on a flat rectangular surface of length 6 m and breadth 4 m is transferred into a cylindrical vessel of internal radius 20 cm. What will be the height of water in the cylindrical vessel if the rain fall is 1 cm? Give your answer to the nearest integer. (Take $\pi = 3.14$)* *(NCERT Exemplar)*

Solution. Length of rectangular surface = 6 m = 600 cm, its breadth = 4 m = 400 cm.

Height of rainfall = 1 cm.

Volume of rainwater = $(600 \times 400 \times 1)$ cm^3 = 240000 cm^3.

Let h cm be the height of water level in the cylindrical vessel of radius 20 cm, then volume of water in cylindrical vessel = $(\pi \times 20^2 \times h)$ cm^3.

As the rainwater is transferred into the cylindrical vessel,

$\pi \times 400 \times h = 240000$

$\Rightarrow \quad h = \frac{600}{\pi} = \frac{600}{3.14} = 191.08.$

$\therefore$ The height of water in the cylindrical vessel nearest to the integer = 191 cm.

Example 16. *It costs ₹2200 to paint the inner curved surface of a cylindrical tank 10 m deep. If the rate of painting is ₹20 per m^2, find*

(i) the inner curved surface area of the tank

(ii) the radius of the base

(iii) capacity of the tank. *(NCERT)*

Solution. (*i*) As the cost of painting the inner curved surface of the cylindrical tank at the rate of ₹20 per m^2 is ₹2200,

inner curved surface area = $\frac{2200}{20}$ m^2 = 110 m^2.

(*ii*) Curved surface area = $2\pi rh$

$\Rightarrow \quad 2 \times \frac{22}{7} \times r \times 10 = 110 \Rightarrow r = \frac{7}{4} \Rightarrow r = 1.75$ m

(*iii*) Capacity of the tank = $\pi r^2 h$

$$= \left(\frac{22}{7} \times \left(\frac{7}{4}\right)^2 \times 10\right) \text{ m}^3 = 96.25 \text{ m}^3 = 96.25 \text{ kilolitres.}$$

($\because$ 1 m^3 = 1000 litres = 1 kilolitre)

Example 17. *The capacity of a closed cylindrical vessel of height 1 m is 15.4 litres. How many square metres of metal sheet would be needed to make it?* *(NCERT)*

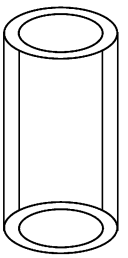
Solution. Capacity = 15.4 litres = $\frac{15.4}{1000}$ m^3 = 0.0154 m^3

$\therefore \quad \frac{22}{7} \times r^2 \times 1 = 0.0154 \Rightarrow r^2 = 0.0049 \Rightarrow r = 0.07$ m.

$\therefore$ The area of the metal sheet needed to make the vessel

$$= 2\pi(r + h)r = \left(2 \times \frac{22}{7} \times (0.07 + 1) \times 0.07\right) \text{ m}^2$$

$= 0.4708$ m^2.



Example 18. *The adjoining figure shows a metal pipe 77 cm long. The inner diameter of a cross-section is 4 cm and the outer one is 4.4 cm. Find its*

(i) inner curved surface area

(ii) outer curved surface area

(iii) total surface area. *(NCERT)*

Solution. Inner radius = $r = \left(\frac{1}{2} \times 4\right)$ cm = 2 cm,

outer radius = R = $\left(\frac{1}{2} \times 4.4\right)$ cm = 2.2 cm, height = 77 cm.

(*i*) Inner curved surface area = $\left(2 \times \frac{22}{7} \times 2 \times 77\right)$ cm^2 = 968 cm^2

(*ii*) Outer curved surface area = $\left(2 \times \frac{22}{7} \times 2.2 \times 77\right)$ cm^2 = 1064.8 cm^2

(*iii*) Area of two circular rings (at top and bottom) = $2\pi(R^2 - r^2)$

$= 2 \times \frac{22}{7} (2.2^2 - 2^2)$ cm^2 = 5.28 cm^2

$\therefore$ Total surface area = (968 + 1064.8 + 5.28) cm^2 = 2038.08 cm^2.

Example 19. *The inner diameter of a cylindrical wooden pipe is 24 cm and its outer diameter is 28 cm. The length of the pipe is 35 cm. Find the mass of the pipe, if 1 cm^3 of wood has a mass of 0.6 g.* *(NCERT)*

Solution. Outer radius = R = $\left(\frac{1}{2} \times 28\right)$ cm = 14 cm.

Inner radius = r = $\left(\frac{1}{2} \times 24\right)$ cm = 12 cm and height = h = 35 cm.

Volume of wood in the cylindrical pipe = $\pi(R^2 - r^2)h$

$= \frac{22}{7} \times (14^2 - 12^2) \times 35$ cm^3 = (52 × 110) cm^3

= 5720 cm^3.

$\therefore$ Mass of wood = (5720 × 0.6) g = $\frac{3432}{1000}$ kg = 3.342 kg.

Example 20. *Find*

(i) the lateral surface area of a closed cylindrical petrol storage tank that is 4.2 m in diameter and 4.5 m high.

(ii) how much area of steel was actually used, if $\frac{1}{12}$ of the area of steel actually used was wasted in making the tank. *(NCERT)*

Solution.

(*i*) Radius = $\left(\frac{1}{2} \times 4.2\right)$ m = 2.1 m, height = 4.5 m.

Lateral surface of the tank = $\left(2 \times \frac{22}{7} \times 2.1 \times 4.5\right)$ m^2 = 59.4 m^2

(*ii*) Total surface of the tank = $2\pi(r + h)r$

$= 2 \times \frac{22}{7} (2.1 + 4.5) \times 2.1$ m^2 = 87.12 m^2

Let x m^2 be the actual area of steel used in making the tank. As $\frac{1}{12}$ of the actual area is wasted in making, so remaining area = $\left(x - \frac{1}{12} \times x\right)$ m^2 = $\frac{11}{12}$ x m^2.

$\therefore$ $\frac{11}{12}x = 87.12 \Rightarrow x = \frac{11}{12} \times 87.12 = 95.04$.

Hence, the actual area of the steel used = 95.04 m^2.

Example 21. *A cylindrical water tank of diameter 1.4 m and height 2.1 m is being fed by a pipe of diameter 3.5 cm through which water flows at the rate of 2 m/s. Calculate the time it takes to fill the tank.*

Solution. Radius of water tank = $\frac{1}{2}$ × 1.4 m = 70 cm.

Height of water tank = 2.1 m = 210 cm.

$\therefore$ Volume (capacity) of water tank = $\pi r^2 h$

$= \frac{22}{7} \times (70)^2 \times 210$ cm^3 = 22 × 700 × 210 cm^3.

Radius of water pipe = $\frac{1}{2}$ × 3.5 cm = $\frac{7}{4}$ cm.

Length of water delivered by the pipe in one sec = 2 m = 200 cm.

$\therefore$ Volume of water delivered by the pipe in one sec = $\pi r^2 h$

$$= \frac{22}{7} \times \left(\frac{7}{4}\right)^2 \times 200 \text{ cm}^3 = 77 \times 25 \text{ cm}^3.$$

$\therefore$ Time consumed by the water pipe in filling the water tank

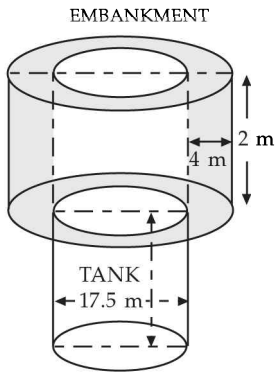
$$= \frac{22 \times 700 \times 210}{77 \times 25} \text{ sec} = 1680 \text{ sec}$$

$$= \frac{1680}{60} \text{ minutes} = 28 \text{ minutes}.$$

Example 22. *The soil taken out on digging a circular tank of diameter 17.5 m is spread all around the tank uniformly to a width of 4 m, to form an embankment of height 2 m. Calculate the depth of the circular tank correct to 2 decimal places. (The depth of the tank is uniform everywhere).*

Solution. Radius (r) of the tank = $\frac{1}{2} \times 17.5$ m = $\frac{35}{4}$ m.

Let d metres be the depth of the tank dug out,



$\therefore$ volume of the soil dug out = $\pi r^2 h$

$$= \pi \times \left(\frac{35}{4}\right)^2 \times d \text{ m}^3.$$

Internal radius of the embankment = $\frac{35}{4}$ m.

Since the width of the embankment = 4 m,

$\therefore$ the external radius of the embankment

$$= \left(\frac{35}{4} + 4\right) \text{ m} = \frac{51}{4} \text{ m}.$$

Height of the embankment = 2 m (given).

$\therefore$ The volume of the soil used in forming the embankment around the circular tank

$$= \pi(R^2 - r^2)\, h = \pi\,(R + r)\,(R - r)\, h$$

$$= \pi \left(\frac{51}{4} + \frac{35}{4}\right)\left(\frac{51}{4} - \frac{35}{4}\right) \times 2 \text{ m}^3$$

$$= \pi \times \frac{86}{4} \times 4 \times 2 \text{ m}^3 = \pi \times 86 \times 2 \text{ m}^3.$$

According to the given data,

$$\pi \times \frac{35}{4} \times \frac{35}{4} \times d = \pi \times 86 \times 2$$

$$\Rightarrow \quad d = \frac{172 \times 16}{35 \times 35} = 2.247.$$

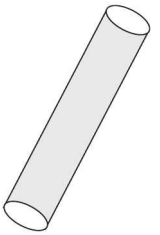
$\therefore$ The depth of the tank dug out = 2.25 m (correct to 2 decimal places).

Exercise 13.2

Take $\pi = \frac{22}{7}$, unless stated otherwise.

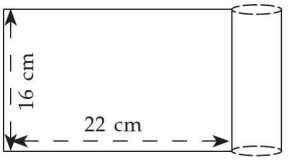
1. Write whether the following statements are true or false. Justify your answer:
   (*i*) If the radius of a cylinder is doubled and its curved surface is not changed, the height must be halved. *(NCERT Exemplar)*
   (*ii*) If the radius of a cylinder is doubled and its height is halved, the volume will be doubled. *(NCERT Exemplar)*

2. Savita had to make a model of a cylindrical kaleidoscope for her science project. She wanted to use chart paper to make the curved surface of the kaleidoscope (shown in the adjoining figure). What would be the area of the chart paper required by her, if she wanted to make a kaleidoscope of length 25 cm with a 3.5 cm radius?



(NCERT)

3. The diameter of a roller is 84 cm and its length is 120 cm. It takes 500 complete revolutions to move once over to level a playground. Find the area of the playground. *(NCERT)*
4. An electric geyser is cylindrical in shape, having a diameter of 35 cm and height 1.2 m. Neglecting the thickness of the cylinder. Calculate:
   (*i*) its outer lateral surface area.
   (*ii*) its capacity in litres.
5. (*i*) How many cubic metres of soil must be dug out to make a well 20 metres deep and 2 metres in diameter?
   (*ii*) If the inner curved surface of the well in part (*i*) above is to be plastered at the rate of ₹50 per m^2, find the cost of plastering.
6. A wooden pole is 7 m high and 20 cm in diameter. Find its weight if the wood weighs 225 kg per m^3.
7. A school provides milk to the students daily in cylindrical glasses of diameter 7 cm. If the glass is filled with milk upto a height of 12 cm, find how many litres of milk is needed to serve 1600 students. *(NCERT Exemplar)*
8. If the curved surface area of a right circular cylinder of height 14 cm is 88 cm^2, then find the diameter of the base of the cylinder. *(NCERT)*
9. A cylindrical pillar is 50 cm in diameter and 3.5 m in height. Find the cost of painting the curved surface of the pillar at the rate of ₹12.50 per m^2. *(NCERT)*
10. It is required to make a closed cylindrical tank of height 1 m and base diameter 140 cm from a metal sheet. How many square metres of the sheet are required for the same? *(NCERT)*
11. The pillars of a temple are cylindrical in shape. If each pillar has a circular base of radius 20 cm and height 10 cm, how much concrete mixture would be required to build 14 such pillars? *(NCERT)*
12. How many cylindrical glasses of diameter 8 cm and height 15 cm can be filled from a cylindrical vessel of diameter 30 cm and height 64 cm, full of orange juice?
13. In the given figure, a rectangular tin foil of size 22 cm by 16 cm is wrapped around to form a cylinder of height 16 cm. Find the volume of the cylinder.



14. The area of the curved surface of a cylinder is 4400 cm^2 and the circumference of its base is 110 cm. Find:
   (*i*) the height of the cylinder. (*ii*) the volume of the cylinder.
15. A lead pencil consists of a cylinder of wood with a solid cylinder of graphite filled in the interior. The diameter of the pencil is 7 mm and the diameter of the graphite is 1 mm. If the length of the pencil is 14 cm, find the volume of the wood and that of the graphite. *(NCERT)*
16. A cylindrical tube opened at both ends is made of iron sheet which is 2 cm thick. If the outer diameter 16 cm and its length is 1m, find how many cubic centimetres of iron has been used in making the tube? *(NCERT Exemplar)*

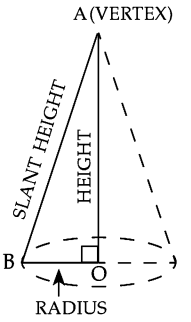
Answers

13.5 SURFACE AREA AND VOLUME OF A CONE

Cone

A solid obtained by revolving a right angled triangular lamina about any side (other than the hypotenuse) is called a ***right circular cone.***

The solid shown in the adjoining figure is a right circular cone.



(*i*) A cone has a *vertex* and a *circular* base.

(*ii*) The radius of circular base is called the *radius of the cone.*

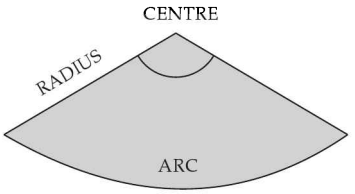
(*iii*) The distance between the centre of the circular base and the vertex is called the *height of the cone.*

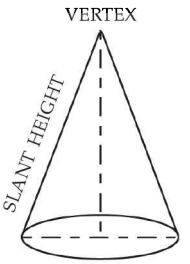
(*iv*) The distance between the vertex and any point on the circular base is called the *slant height of the cone.*

(*v*) The line joining the centre of the circular base and the vertex is called the *axis of the cone.*

Hollow Cone

If the sector of a circle is folded to make the two radii coincide, the surface so formed is a *hollow right circular cone.*





We note the following:

(*i*) *The vertex of the cone is the centre of the circle.*

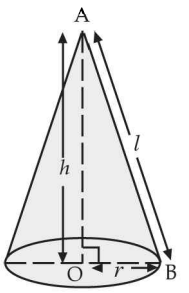
(*ii*) *The slant height of the cone is equal to the radius of the circle.*

(*iii*) *The circumference of the circular base of the cone is equal to the length of the arc of the sector.*

(*iv*) *Curved surface area of the cone is equal to the area of the sector.*

In this book, we shall be dealing only with right circular cones. Therefore, whenever we use the word '*cone*', it will mean right circular cone.

❑ **Slant height of a cone**



Let r be the radius of the cone, h be its height and l be the slant height.

In ΔAOB, $\angle AOB = 90°$. By Pythagoras theorem, we get

$$AB^2 = OB^2 + OA^2$$

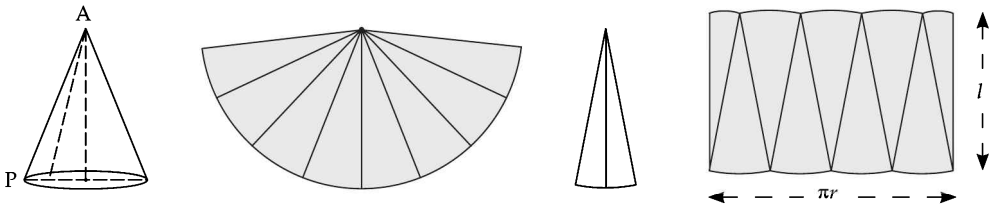
$$\Rightarrow \quad l^2 = r^2 + h^2 \Rightarrow l = \sqrt{r^2 + h^2}\ .$$

Hence, slant height of a cone = $l = \sqrt{r^2 + h^2}$.

❑ **Surface area of a right circular cone**

To find the curved (lateral) surface area of a cone, perform the following activity:

Take a hollow right circular cone made up of chart paper. Let r be the radius of the base of the cone, l be its slant height and A be the vertex (as shown in fig. (*i*)). Take a point P on its circular base and cut it along AP and open it. We will obtain a sector of a circle (as shown in fig. (*ii*)). Note that radius of circle = l and the length of arc of sector = circumference of circular base of cone = $2\pi r$.



(i) (ii) (iii) (iv)

Cut the sector into 8 equal pieces (use paper folding) and further one piece (out of these 8 pieces) into two equal pieces (as shown in fig. (*iii*)) and paste all these pieces on a page of your notebook as shown in fig. (*iv*)).

Approximately, we will obtain a rectangle.

Length of rectangle = $\frac{1}{2}$ (length of arc of sector) = $\frac{1}{2} \times 2\pi r = \pi r$ and breadth of rectangle = l.

Area of rectangle = length × breadth = $\pi r \times l = \pi rl$.

Thus, the curved surface area of a cone = area of sector = area of rectangle = πrl.

Hence, curved surface of the cone = πrl.

Total surface area of cone = curved surface area + area of circular base

$$= \pi rl + \pi r^2 = \pi r\ (l + r).$$

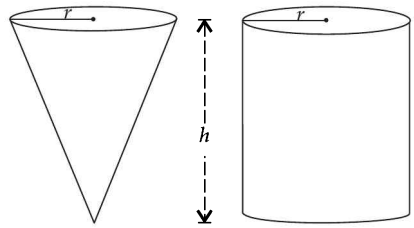
Hence, total surface area of cone = $\pi r(l + r)$.

❑ **Volume of a cone**

To find the volume of a cone, perform the following activity:

Take an open cone of radius r and height h, and a cylinder with open top of radius r and height h (as shown in fig. (*i*)).

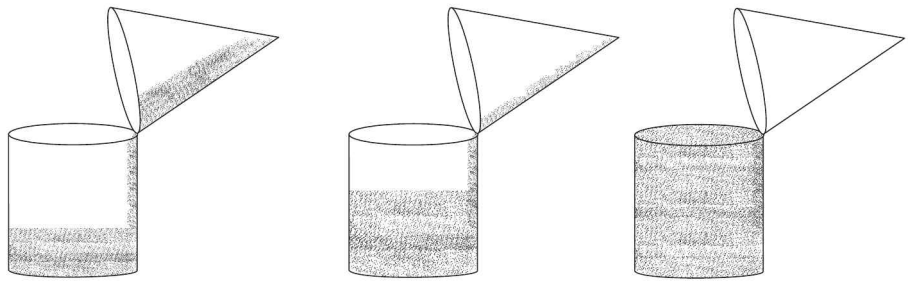
Fill the cone with sand upto brim and pour it into the cylinder. Note that the sand fills up only a part of the cylinder (as shown in fig. (*ii*)).



(i)

Fill the cone once again with sand upto the brim and pour it into the cylinder. Note that the cylinder is still not full (as shown in fig. (*iii*)).

Fill the cone once again with sand upto the brim and pour it into the cylinder. Note that the cone is completely emptied into the cylinder and the cylinder is full upto the brim (as shown in fig. (*iv*)).



(ii) (iii) (iv)

So, we observe that it needs three full pourings of sand from the cone into the cylinder to fill it completely. If follows that:

The volume of a cone = $\frac{1}{3}$ of the volume of a cylinder of same radius and same height

$$= \frac{1}{3} \times \pi r^2 h = \frac{1}{3}\ \pi r^2 h.$$

Hence, the volume of cone of radius r and height h = $\frac{1}{3}\ \pi r^2 h$.

Illustrative Examples

Example 1. *Find the total surface area of a cone, if its slant height is 21 m and diameter of its base is 24 m.* *(NCERT)*

Solution. Base radius of cone = $\left(\frac{1}{2} \times 24\right)$ m = 12 m, slant height = 21 m

Total surface area of cone $= \pi r(l + r) = \left(\frac{22}{7} \times 12 \times (21 + 12)\right)$ m^2

$= \left(\frac{22}{7} \times 12 \times 33\right)$ m^2 = $\frac{8712}{7}$ m^2 = 1244 $\frac{4}{7}$ m^2.

Example 2. *How many square metres of canvas is required for a conical tent whose height is 3.5 m and the radius of the base is 12 m.* *(NCERT Exemplar)*

Solution. Here, height of cone = h = 3.5 m = $\frac{7}{2}$ m and radius of base = r = 12 m.

Let l metres be the slant height of the cone, then

$$l = \sqrt{h^2 + r^2} = \sqrt{\left(\frac{7}{2}\right)^2 + 12^2} = \sqrt{\frac{49}{4} + 144} = \sqrt{\frac{625}{4}} = \frac{25}{2}.$$

Note that a tent has only curved surface (the floor of a tent is not covered by canvas).

Area of canvas required for a conical tent = curved surface area of the cone

$$= \pi rl = \left(\frac{22}{7} \times 12 \times \frac{25}{2}\right) \text{ m}^2 = \frac{3300}{7} \text{ m}^2 = 471.43 \text{ m}^2 \text{ (approx.)}$$

Example 3. *The height and the slant height of a cone are 21 cm and 28 cm respectively. Find the volume of the cone.* *(NCERT)*

Solution. Given height of cone = h = 21 cm and slant height = 28 cm.

As $h^2 + r^2 = l^2 \Rightarrow 21^2 + r^2 = 28^2$

$\Rightarrow \quad r^2 = 28^2 - 21^2 = 343 \Rightarrow r = 7\sqrt{7}$ cm.

Volume of cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (7\sqrt{7})^2 \times 21$ cm^3 = 7546 cm^3.

Example 4. *The height of a cone is 16 cm and its base radius is 12 cm. Find the curved surface area and the total surface area of the cone. (Use π = 3.14)* *(NCERT)*

Solution. Given height of cone = h = 16 cm and base radius = r = 12 cm.

Slant height = $\sqrt{h^2 + r^2} = \sqrt{16^2 + 12^2}$ cm = 20 cm.

Curved surface area = πrl = (3.14 × 12 × 20) cm^2 = 753.6 cm^2

Total surface area = $\pi r(l + r)$ = (3.14 × 12 × (20 + 12)) cm^2

= (3.14 × 12 × 32) cm^2 = 1205.76 cm^2.

Example 5. *A heap of wheat is in the form of a cone whose diameter is 10.5 m and height is 3 m. Find its volume. The heap is to be covered by canvas to protect it from rain. Find the area of the canvas required.*

What value is depicted by protecting wheat from rains? ***(Value Based)***

Solution. Radius of base = $\frac{10.5}{2}$ m = $\frac{21}{4}$ m.

Volume = $\frac{1}{3} \times \frac{22}{7} \times \left(\frac{21}{4}\right)^2 \times 3$ m^2 = 86.625 m^3

Slant height of cone = $\sqrt{h^2 + r^2} = \sqrt{3^2 + \left(\frac{21}{4}\right)^2}$ m = $\sqrt{9 + (5.25)^2}$ m

= $\sqrt{36.5625}$ m = 6.05 m (approximately)

Area of canvas = curved surface area of conical heap = πrl

$$= \left(\frac{22}{7} \times \frac{21}{4} \times 6.05\right) \text{ m}^2 = 99.825 \text{ m}^2 \text{ (approx.)}$$

Value depicted : It shows wisdom to save wheat from destroying. Wheat is the most used food item.

Example 6. *The volume of a right circular cone is 9856 cm^3. If the diameter of the base is 28 cm, find*

(i) height of the cone *(ii) slant height of the cone*

(iii) curved surface area of the cone.

Solution. (*i*) Let h cm be the height of the cone.

Radius of base = $\left(\frac{1}{2} \times 28\right)$ cm = 14 cm.

Volume = $\frac{1}{3} \times \frac{22}{7} \times 14^2 \times h = 9856$ (given)

$\Rightarrow$ $h = 48$

Hence, the height of the cone = 48 cm.

(*ii*) Slant height = $\sqrt{l^2 + r^2} = \sqrt{48^2 + 14^2}$ cm = $\sqrt{2500}$ cm = 50 cm.

(*iii*) Curved surface area = $\pi rl = \left(\frac{22}{7} \times 14 \times 50\right)$ cm^2 = 2200 cm^2.

Example 7. *A cloth having an area of 165 m^2 is shaped in the form of a conical tent of radius 5 m.*

(i) How many students can sit in the tent if a student, on an average, occupies $\frac{5}{7}$ m^2 area on the ground?

(ii) Find the volume of the cone. *(NCERT Exemplar)*

Solution. (*i*) Radius of the cone = r = 5 m.

$\therefore$ Floor area of the tent = $\pi r^2 = \left(\frac{22}{7} \times 5^2\right)$ m^2 = $\frac{550}{7}$ m^2.

Let n be the number of students.

As each student, on an average, occupies $\frac{5}{7}$ m^2 area,

$\therefore$ $\frac{5}{7} \times n = \frac{550}{7} \Rightarrow n = 110$.

Hence, the number of students = 110.

(*ii*) Let l metres be the slant height and h metres be the height of the cone.

The curved area of the tent = area of cloth

$\Rightarrow$ $\pi rl = 165$ (given)

$\Rightarrow$ $\frac{22}{7} \times 5 \times l = 165 \Rightarrow l = \frac{21}{2}$.

As $l^2 = h^2 + r^2$, so $\left(\frac{21}{2}\right)^2 = h^2 + 5^2$

$\Rightarrow$ $h^2 = \frac{441}{4} - 25 = \frac{341}{4} \Rightarrow h = \frac{\sqrt{341}}{2}$.

$\therefore$ Volume of cone = $\frac{1}{3} \pi r^2 h = \left(\frac{1}{3} \times \frac{22}{7} \times 5^2 \times \frac{\sqrt{341}}{2}\right)$ m^3

$= \left(\frac{275}{21} \times \sqrt{341}\right)$ m^3 = 241.8 m^3 (approx.)

Example 8. *A corn cob (shown in the adjoining figure), shaped somewhat like a cone, has the radius of its broadest end as 2.1 cm and length (height) as 20 cm. If each 1 cm² of the surface of the cob carries an average of four grains, find how many grains you would find on the entire cob.* *(NCERT)*



Solution. Slant height = $\sqrt{h^2 + r^2} = \sqrt{20^2 + 2.1^2}$ cm = $\sqrt{404.41}$ cm
= 20.11 cm (approximately).

Curved surface area of the corn cob = $\pi rl = \left(\frac{22}{7} \times 2.1 \times 20.11\right)$ cm²
= 132.726 cm²

∴ No. of grains on the whole surface = 132.726 × 4 = 531 approx.

Example 9. *What length of tarpaulin 3 m wide will be required to make conical tent of height 8 m and base radius 6 m? Assuming that the extra length of material that will be required for stitching margins and wastage in cutting is approximately 20 cm. (Use π = 3.14).* *(NCERT)*

Solution. Slant height of tent = $\sqrt{8^2 + 6^2}$ m = 10 m.

Area of tarpaulin of tent = πrl = (3.14 × 6 × 10) m²

Length of tarpaulin = $\frac{3.14 \times 6 \times 10}{3}$ m = 62.8 m.

Extra length of tarpaulin for wastage etc. = 20 cm = 0.2 m

∴ Total length of tarpaulin required = (62.8 + 0.2) m = 63 m.

Example 10. *A bus stop is barricaded from the remaining part of the road, by using 50 hollow cones made of recycled cardboard. Each cone has a base diameter of 40 cm and height 1 m. If the outside of each of the cones is to be painted and the cost of painting is ₹12 per m², what will be the cost of painting all these cones? (Use π = 3.14 and $\sqrt{1.04}$ = 1.02).* *(NCERT)*

Solution. Base radius of cone = $\left(\frac{1}{2} \times 40\right)$ cm = 20 cm = 0.2 m.

Slant height of cone = $\sqrt{1^2 + (0.2)^2}$ m = $\sqrt{1.04}$ m = 1.02 m

∴ Curved surface area of one cone = πrl = (3.14 × 0.2 × 1.02) m²

∴ Cost of painting of 50 cones = ₹(3.14 × 0.2 × 1.02 × 50 × 12)
= ₹384.34 (approximately).

Example 11. *Monica has a piece of canvas whose area is 551 m². She uses it to make a conical tent with a base radius of 7 m. Assuming that all the stitching margins and the wastage incurred while cutting, amounts to approximately 1 m², find the volume of the tent that can be made with it.* *(NCERT)*

Solution. Let l metres be the slant height and h metres be the height of the cone.

Base radius of cone = 5 m.

Total area of canvas = 551 m², wastage = 1 m².

∴ The actual area of the conical tent = (551 − 1) m² = 550 m².

Now area of tent = curved surface area of cone = πrl

$\Rightarrow \frac{22}{7} \times 7 \times l = 550 \Rightarrow l = 25.$

Also $l^2 = h^2 + r^2 \Rightarrow 25^2 = h^2 + 7^2 \Rightarrow h^2 = 625 - 49 = 576$

$\Rightarrow h = 24.$

∴ The volume of the conical tent = $\frac{1}{3} \pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 7^2 \times 24$ m³

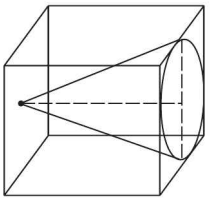
= (22 × 7 × 8) m³ = 1232 m³.

Example 12. *An edge of a cube measures r cm. If the largest possible right circular cone is cut out of this cube, then find the volume of the cone.* *(NCERT Exemplar)*

Solution. An edge of a cube = r cm.

Diameter of largest cone that can be cut out of the cube = r cm.

$\therefore$ Its radius = $\frac{1}{2}$ r cm.



Height of the largest cone that can be cut out of the cube = r cm.

$\therefore$ Volume of largest cone that can be cut out of the cube

$$= \frac{1}{3}\ \pi.\ \left(\frac{r}{2}\right)^2 .\ r\ \text{cm}^3 = \frac{1}{6}\ \pi r^3\ \text{cm}^3.$$

Example 13. *The volume of a conical tent is 1232 m^3 and the area of the base floor is 154 m^2. Calculate the:*

(i) the radius of the floor. *(ii) height of the tent.*

(iii) length of the canvas required to cover this conical tent if its width is 2 m.

Solution. (*i*) Let r metres be the radius of the base of the conical tent, then area of the base floor = πr^2 m^2

$\therefore$ $\pi r^2 = 154$ (given)

$\Rightarrow$ $\frac{22}{7} \times r^2 = 154 \Rightarrow r^2 = 49 \Rightarrow r = 7.$

Hence, the radius of the floor = 7 m.

(*ii*) Let h metres be the height of the conical tent, then the volume of the tent

$$= \frac{1}{3}\ \pi r^2 h\ \text{m}^3$$

$\therefore$ $\frac{1}{3}\pi r^2 h = 1232$ (given)

$\Rightarrow$ $\frac{1}{3} \times \frac{22}{7} \times 7^2 \times h = 1232 \Rightarrow h = \frac{1232 \times 3}{22 \times 7} = 24.$

Hence, the height of the tent = 24 m.

(*iii*) Let l metres be the slant height of the conical tent, then

$l = \sqrt{h^2 + r^2} = \sqrt{24^2 + 7^2} = \sqrt{576 + 49} = \sqrt{625} = 25.$

$\therefore$ The area of the canvas required to make the tent = curved surface area of the conical tent

$= \pi r l\ \text{m}^2 = \frac{22}{7} \times 7 \times 25\ \text{m}^2 = 550\ \text{m}^2$

The width of the canvas = 2 metres (given)

$\therefore$ The length of the canvas = $\frac{\text{area}}{\text{width}} = \frac{550}{2}$ m = 275 m.

Hence, the length of the canvas required to make the tent = 275 m.

Example 14. *(i) A right triangle ABC with sides 5 cm, 12 cm and 13 cm is revolved about the side 12 cm. Find the volume of the solid so obtained.* *(NCERT)*

(ii) If the triangle ABC in part (i) above is revolved about the side 5 cm, then find the volume of the solid so obtained. Also find the ratio of the volumes of the two solids. *(NCERT)*

Solution. (*i*) When the triangle ABC is revolved about the side 12 cm, then it generates a right circular cone of radius 5 cm and height 12 cm.

$\therefore$ Volume of solid (cone) = $\frac{1}{3}\pi \times 5^2 \times 12\ \text{cm}^3 = 100\pi\ \text{cm}^3.$

(*ii*) When the triangle ABC is revolved about the side 5 cm, then it generates a right circular cone of radius 12 cm and height 5 cm.

$\therefore$ Volume of solid (cone) $= \frac{1}{3}\pi \times (12)^2 \times 5 \text{ cm}^3$

$= 240\pi \text{ cm}^3$.

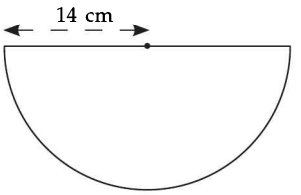
Ratio of their volumes $= \frac{100\pi}{240\pi} = \frac{5}{12} = 5 : 12$.

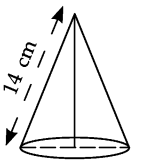
Example 15. *A semi-circular sheet of metal of diameter 28 cm is bent to form an open conical cup. Find the capacity of the cup.* *(NCERT Exemplar)*

Solution. Given diameter of semi-circular sheet = 28 cm,

$\therefore$ radius of semi-circular sheet

$= \left(\frac{1}{2} \times 28\right) \text{ cm} = 14 \text{ cm}$.





Length of the arc of the semi-circular sheet

$= \frac{1}{2} \times 2\pi r = \pi r$

$= \left(\frac{22}{7} \times 14\right) \text{ cm} = 44 \text{ cm}$

Let r cm be the radius of the cone, then

$2\pi r = 44 \Rightarrow 2 \times \frac{22}{7} \times r = 44 \Rightarrow r = 7$.

Slant height of the cone = radius of semi-circular sheet = 14 cm.

Let h cm be the height of the cone, then

$l^2 = h^2 + r^2 \Rightarrow 14^2 = h^2 + 7^2$

$\Rightarrow \quad h^2 = 196 - 49 = 147 \Rightarrow h = \sqrt{147} = 7\sqrt{3}$.

$\therefore$ The capacity of the conical cup = volume of cone $= \frac{1}{3}\pi r^2 h$

$= \left(\frac{1}{3} \times \frac{22}{7} \times 7^2 \times 7\sqrt{3}\right) \text{ cm}^3 = \frac{1078\sqrt{3}}{3} \text{ cm}^3$

$= 622.38 \text{ cm}^3$ (approx.)

Exercise 13.3

Take $= \frac{22}{7}$ unless stated otherwise.

1. Write whether the following statements are true or false. Justify your answer:
   (*i*) If the radius of a right circular cone is halved and its height is doubled, the volume will remain unchanged. *(NCERT Exemplar)*
   (*ii*) A cylinder and a right circular cone are having the same base radius and same height. The volume of the cylinder is three times the volume of the cone. *(NCERT Exemplar)*
   (*iii*) In a right circular cone, height, radius and slant height are always the sides of a right triangle. *(NCERT Exemplar)*
2. Find the curved surface area of a right circular cone whose slant height is 10 cm and base radius is 7 m. *(NCERT)*
3. Diameter of the base of a cone is 10.5 cm and slant height is 10 cm. Find its curved surface area. *(NCERT)*
4. Curved surface area of a cone is 308 cm^2 and its slant height is 14 cm. Find
   (*i*) radius of the base (*ii*) total surface area of the cone. *(NCERT)*

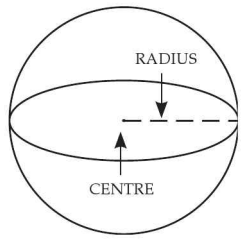
5. Find the volume of the right circular cone with
   (*i*) radius 6 cm and height 7 cm
   (*ii*) radius 3.5 cm and height 12 cm. *(NCERT)*
6. Find the capacity in litres of a conical vessel with
   (*i*) radius 7 cm, slant height 25 cm
   (*ii*) height 12 cm, slant height 13 cm *(NCERT)*
7. A conical pit of top diameter 3.5 m is 12 m deep. What is its capacity in kilolitres? *(NCERT)*
8. If the volume of a right circular cone of height 9 cm is 48π cm^3, find the diameter of its base. *(NCERT)*
9. The height of a cone is 15 cm. If its volume is 1570 cm^3, find the radius of the base. (Use $\pi = 3.14$) *(NCERT)*
10. The slant height and base diameter of a conical tomb are 25 m and 14 m respectively. Find the cost of white washing its curved surface area at the rate of ₹210 per 100 m^2. *(NCERT)*
11. A conical tent is 10 m high and the radius of its base is 24 m. Find:
    (*i*) Slant height of the tent.
    (*ii*) Cost of the canvas required to make the tent, if the cost of 1 m^2 canvas is ₹70. *(NCERT)*
12. A Joker's cap is in the form of a right circular cone of base radius 7 cm and height 24 cm. Find the area of the cloth required to make 10 such caps. *(NCERT)*
13. The curved surface area of a cone is 550 cm^2. Find the volume of the cone given that its base diameter is 14 cm.
14. The volume of a right circular cone is 9856 cm^3 and the area of its base is 616 cm^2. Find
    (*i*) the slant height of the cone.
    (*ii*) total surface area of the cone.
15. A right triangle with sides 6 cm, 8 cm and 10 cm is revolved about the side 8 cm. Find the volume and the curved surface so formed. (Take $\pi = 3.14$) *(NCERT Exemplar)*

Answers

13.6 SURFACE AREA AND VOLUME OF A SPHERE

❑ **Sphere**

If a circle is revolved about any of its diameters, a (hollow) sphere is generated.



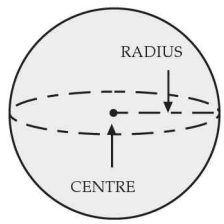
(*i*) The centre of the circle revolved is called the **centre** of the sphere.

(*ii*) The radius of the circle revolved is called the **radius** of the sphere.

*Thus, a **sphere** is a three dimensional figure, which is made up of all points in space that lie at a constant distance from a fixed point.* The constant distance is called **radius** and the fixed point is called the **centre** of the sphere.

❑ **Solid sphere**

*A solid obtained by revolving a circular lamina about any of its diameters is called a **solid sphere.***



(*i*) The centre of the circular lamina revolved is called the *centre of the sphere.*

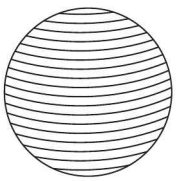
(*ii*) The radius of the circular lamina revolved is called the *radius of the sphere.*

Remark. Usually, the word 'sphere' refers to a solid sphere.

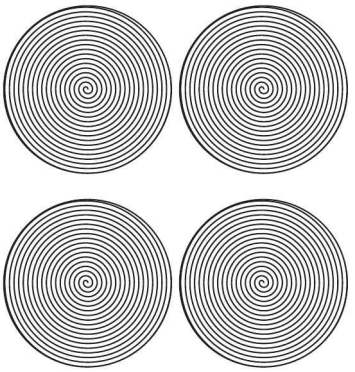
❑ **Surface area of a sphere**

To find the surface area of a sphere, perform the following activity:

Take a wooden sphere of any radius and four circles each of same radius (equal to radius of sphere). Wrap a string of uniform cross-section taking the help of a nail (fixed in the sphere) around the sphere in a spiral pattern without leaving any gap in between as shown in fig. (*i*). You may use some adhesive so that the string may stick properly.



(*i*)



(*ii*)

Take another piece of same string and of equal length. Start filling the circles one by one with the string as shown in fig. (*ii*).

We observe that the string exactly fills the region of all the four circles and no string is left. Since the same string is used in both cases and the thickness of the string is uniform, therefore, the surface area of the sphere and the region of four circles are proportional to the lengths of strings. If follows that:

The surface area of the sphere = area of regions of four circles of same radius.

Thus, if r (units) is the radius of the sphere then

surface area of sphere = 4 × area of region of a circle of radius r

$$= 4 \times \pi r^2 = 4\pi r^2$$

Hence, the surface area of a sphere of radius r = $4\pi r^2$.

❑ **Volume of a sphere**

To find the volume of a sphere, perform the following activity:

Take a hollow sphere of any radius of negligible thickness and make a hole at the top. Take an open right circular cone of same radius (as that of sphere) and same height. Fill the cone with sand upto brim and pour it into the sphere through the hole. Repeat it four times.

We observe that it need four full pourings of sand from the cone into the sphere to fill it exactly. It follows that:

Volume of a sphere of any radius = 4 × volume of cone of same radius and same height.

Thus, if r (units) is the radius of the sphere then

volume of sphere = 4 × volume of a cone of radius r and height r

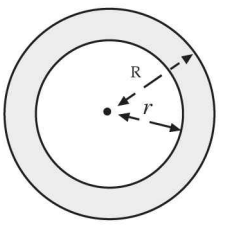
$$= 4 \times \frac{1}{3}\pi r^2 \times r = \frac{4}{3}\pi r^3.$$

Hence, the volume of a sphere of radius r = $\frac{4}{3}\pi r^3$.

❑ **Spherical shells**

The solid enclosed between two concentric spheres is called a ***spherical shell.***

Let R and r be the radii of the outer and the inner spheres, then



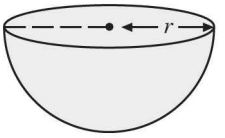
(*i*) *thickness of shell* = R – *r*.

(*ii*) *volume of the material* = $\frac{4}{3}\pi(R^3 - r^3)$.

❑ **Hemisphere**

When a sphere is cut by a plane passing through its centre, then the sphere is divided into two equal parts. Each part is called a **hemisphere**.

Let *r* be the radius of the sphere *i.e.* of the hemisphere, then



(*i*) *curved surface area* = $\frac{1}{2}$ (*surface area of sphere*)

$= \frac{1}{2} \times 4\pi r^2 = 2\pi r^2$.

(*ii*) *total surface area* = *curved surface area* + *area of circular base*

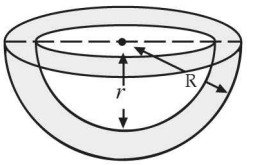
$= 2\pi r^2 + \pi r^2 = 3\pi r^2$.

(*iii*) *volume* = $\frac{1}{2}$ (*volume of sphere*) = $\frac{1}{2} \times \frac{4}{3}\pi r^3 = \frac{2}{3}\pi r^3$.

❑ **Hemispherical shell**

The solid enclosed between two concentric hemispheres is called a ***hemispherical shell***.

Let R and *r* be the radii of the outer and the inner hemispheres, then



(*i*) *thickness of shell* = R – *r*.

(*ii*) *area of base* = $\pi(R^2 - r^2)$.

(*iii*) *external curved surface area* = $2\pi R^2$.

(*iv*) *internal curved surface area* = $2\pi r^2$.

(*v*) *total surface area* = $2\pi R^2 + 2\pi r^2 + \pi (R^2 - r^2)$

$= \pi (3R^2 + r^2)$.

(*vi*) *volume of material* = $\frac{2}{3}\pi (R^3 - r^3)$.

Illustrative Examples

Example 1. *Find the surface area and the volume of a sphere of diameter 14 cm.* *(NCERT)*

Solution. Radius of sphere = $r = \left(\frac{1}{2} \times 14\right)$ cm = 7 cm.

Surface area of the sphere = $4\pi r^2 = \left(4 \times \frac{22}{7} \times 7^2\right)$ cm^2

$= (88 \times 7)$ cm^2 = 616 cm^2.

Volume of the sphere = $\frac{4}{3}\pi r^3 = \left(\frac{4}{3} \times \frac{22}{7} \times 7^3\right)$ cm^3

$= \frac{88 \times 49}{3}$ cm^3 = $\frac{4312}{3}$ cm^3 = $1437\frac{1}{3}$ cm^3.

Example 2. *The hollow sphere, in which a circus motorcyclist performs his stunts, has a diameter of 7 m. Find the surface area available to the motorcyclist for riding.* *(NCERT)*

Solution. Radius of sphere = $\frac{1}{2}$ of 7 m = $\frac{7}{2}$ m

Sphere area available for riding = inner surface area of the sphere

$= 4\pi r^2 = \left(4 \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2\right)$ m^2 = 154 m^2.

Example 3. *Find the amount of water displaced by a solid spherical ball of diameter 4.2 cm when it is completely immersed in water.* *(NCERT Exemplar)*

Solution. Radius of spherical ball = $\frac{1}{2}$ of 4.2 cm = 2.1 cm.

Amount of water displace = volume of spherical ball

$$= \left(\frac{4}{3} \times \frac{22}{7} \times (2.1)^3\right) \text{ cm}^3$$

$$= \frac{88}{21} \times \left(\frac{21}{10}\right)^3 \text{ cm}^3 = \frac{88 \times 441}{1000} \text{ cm}^3$$

$$= 38.808 \text{ cm}^3$$

Example 4. *The diameter of the moon is approximately one-fourth of the diameter of the earth. Find the ratio of their surface areas.* *(NCERT Exemplar)*

Solution. Let the radius of the moon be r, then its diameter = $2r$.

As the diameter of the moon is one-fourth of the diameter of the earth,

the diameter of the earth = $4 \times 2r = 8r$

$\Rightarrow$ radius of the earth = $\frac{1}{2} \times 8r = 4r$.

Surface area of moon = $4\pi r^2$ and surface area of earth = $4\pi \times (4r)^2 = 64\pi r^2$,

$$\therefore \quad \frac{\text{surface area of moon}}{\text{surface area of earth}} = \frac{4\pi r^2}{64\pi r^2} = \frac{1}{16}.$$

Hence, the required ratio is 1 : 16.

Example 5. *If a sphere is inscribed in a cube, then find the ratio of the volume of the cube to the volume of the sphere.* *(NCERT Exemplar)*

Solution. Let an edge of a cube be $2a$ then its volume = $(2a)^3 = 8a^3$.

As a sphere is inscribed in the cube, diameter of sphere = $2a$.

$\therefore$ Radius of sphere = $\frac{1}{2}$ of $2a = a$.

Volume of sphere = $\frac{4}{3}\pi a^3$.

$$\therefore \quad \frac{\text{Volume of cube}}{\text{Volume of sphere}} = \frac{8a^3}{\frac{4}{3}\pi a^3} = \frac{6}{\pi}.$$

Hence, the required ratio is 6 : π.

Example 6. *Metal spheres, each of radius 2 cm, are packed into a rectangular box of internal dimensions 16 cm × 8 cm × 8 cm. When 16 sphere are packed the box is filled with preservative liquid. Find the volume of this liquid. Give your answer to the nearest integer. Use* $\pi = 3.14$.

(NCERT Exemplar)

Solution. Capacity of box = internal volume of box

$$= (16 \times 8 \times 8) \text{ cm}^3 = 1024 \text{ cm}^3.$$

Radius of each sphere = 2 cm.

Volume of 16 spheres = $\left(16 \times \frac{4}{3} \times 3.14 \times 2^3\right)$ cm^3

$$= \frac{512 \times 3.14}{3} \text{ cm}^3 = 535.89 \text{ cm}^3 \text{ (approx.)}$$

$\therefore$ Volume of preservative liquid = capacity of box – volume of spheres

$$= (1024 - 535.89) \text{ cm}^3 = 488.11 \text{ cm}^3$$

$$= 488 \text{ cm}^3 \text{ (to the nearest integer).}$$

Example 7. *Find the total surface area of a hemisphere of radius 10 cm. (Use $\pi = 3.14$)* *(NCERT)*

Solution. Given radius of hemisphere = r = 10 cm.

Total surface area = curved surface + area of base

$= 2\pi r^2 + \pi r^2 = 3\pi r^2$

$= (3 \times 3.14 \times 10^2)\ \text{cm}^2 = 942\ \text{cm}^2$.

Example 8. *A hemispherical bowl is made of steel, 0.25 cm thick. The inner radius of the bowl is 5 cm. Find the outer curved surface area of the bowl.* *(NCERT)*

Solution. Radius of outer hemispherical bowl = inner radius + thickness

$= 5\ \text{cm} + 0.25\ \text{cm} = 5.25\ \text{cm} = \frac{21}{4}\ \text{cm}.$

$$\text{Outer surface area} = 2\pi r^2 = \left(2 \times \frac{22}{7} \times \left(\frac{21}{4}\right)^2\right)\ \text{cm}^2 = \frac{693}{4}\ \text{cm}^2 = 173.25\ \text{cm}^2.$$

Example 9. *A hemispherical dome of a building needs to be painted. If the circumference of the base of the dome is 17.6 m, find the cost of painting it at the rate of ₹5 per 100 cm^2.* *(NCERT)*

Solution. Circumference of base = $2\pi r$ = 17.6 m (given)

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r = \frac{176}{10} \Rightarrow r = \frac{14}{5}\ \text{m}$$

$$\text{Curved surface area of dome} = 2\pi r^2 = \left(2 \times \frac{22}{7} \times \left(\frac{14}{5}\right)^2\right)\ \text{m}^2$$

$$= \frac{1232}{25}\ \text{m}^2$$

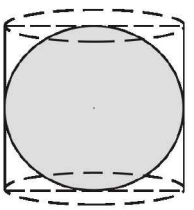
$$\therefore \quad \text{The cost of painting} = ₹\left(\frac{1232}{25} \times 500\right) = ₹\ 24640.$$

Example 10. *A right circular cylinder just encloses a sphere of radius r (shown in the adjoining figure). Find:*

(i) surface area of the sphere

(ii) curved surface area of the cylinder

(iii) ratios of the areas obtained in (i) and (ii). *(NCERT)*



Solution. (*i*) Surface area of sphere = $4\pi r^2$

(*ii*) Radius of cylinder = r and its height = $2r$.

Curved surface area of cylinder = $2\pi rh = 2\pi r \times 2r = 4\pi r^2$.

(*iii*) Ratio of the areas = $4\pi r^2 : 4\pi r^2 = 1 : 1$.

Example 11. *Two solid spheres made of the same metal have weights 5920 g and 740 g respectively. Determine the radius of the larger sphere, if the diameter of the smaller one is 5 cm.* *(NCERT Exemplar)*

Solution. Let the radius of the bigger sphere be r cm, then

its volume = $\frac{4}{3}\pi r^3\ \text{cm}^3$.

Radius of smaller sphere = $\frac{1}{2}$ of 5 cm = $\frac{5}{2}$ cm.

$\therefore$ Volume of smaller sphere = $\frac{4}{3}\pi\left(\frac{5}{2}\right)^3\ \text{cm}^3$.

As the two spheres are made up of same metal, their volumes are proportional to their weights.

$$\therefore \quad \frac{\text{volume of bigger sphere}}{\text{volume of smaller sphere}} = \frac{\text{weight of bigger sphere}}{\text{weight of smaller sphere}}$$

$$\Rightarrow \quad \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi\left(\frac{5}{2}\right)^3} = \frac{5920}{740} \Rightarrow \frac{r^3}{\frac{125}{8}} = 8$$

$$\Rightarrow \quad r^3 = 125 \Rightarrow r = 5.$$

Hence, the radius of the bigger sphere = 5 cm.

Example 12. *The volumes of two spheres are in the ratio 64 : 27. Find the ratio of their surface areas.* *(NCERT Exemplar)*

Solution. Let the radii of two spheres be r_1 and r_2.

As the ratio of their volumes is 64 : 27,

$$\therefore \quad \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{64}{27} \Rightarrow \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{4}{3}\right)^3 \Rightarrow \frac{r_1}{r_2} = \frac{4}{3}.$$

Ratio of their surface areas $= \frac{4\pi r_1^2}{4\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$.

Hence, the ratio of their surface area = 16 : 9.

Example 13. *Twenty seven solid iron spheres each of radius r and surface area S are melted to form a sphere with surface area S'. Find*

(i) the radius r' of the new sphere

(ii) ratio of S and S'. *(NCERT)*

Solution. (*i*) As the new sphere is formed by melting 27 spheres of radius r,

volume of new sphere $= \frac{4}{3}\pi r'^3 = 27 \times \frac{4}{3}\pi r^3$

$\Rightarrow \quad (r')^3 = 27r^3 = (3r)^3 \Rightarrow r' = 3r.$

(*ii*) $S = 4\pi r^2$ and $S' = 4\pi(3r)^2 = 36\pi r^2$

$$\therefore \quad \frac{S}{S'} = \frac{4\pi r^2}{36\pi r^2} = \frac{1}{9}.$$

Hence, S : S' = 1 : 9.

Example 14. *How many litres of milk can a hemispherical bowl of diameter 10.5 cm hold?* *(NCERT)*

Solution. Radius of hemispherical bowl $= \left(\frac{1}{2} \times 10.5\right)$ cm $= \frac{21}{4}$ cm.

Quantity of milk $= \frac{2}{3} \times \frac{22}{7} \times \left(\frac{21}{4}\right)^3$ cm^3 $= \frac{4851}{16}$ cm^3 $= \frac{4851}{16 \times 1000}$ litres

$= 0.303$ litres (approx.)

Example 15. *A dome of a building is in the form of a hemisphere. From inside, it was white-washed at a cost of ₹498.96. If the cost of white washing is ₹2 per square metre, find:*

(i) the inside surface of the dome (ii) volume of air inside the dome. *(NCERT)*

Solution. (*i*) As the cost of white washing at the rate of ₹2 per m^2 is ₹498.96,

$\therefore$ the area white-washed $= \frac{498.96}{2}$ m^2 $= 249.48$ m^2.

$\therefore$ The inside surface of the dome $= 249.48$ m^3.

(*ii*) Surface area of dome $= 2\pi r^2$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r^2 = 249.8 \Rightarrow r^2 = 39.69 \Rightarrow r = 6.3 \text{ m}.$$

$\therefore$ Volume of air inside the dome $= \frac{2}{3}\pi r^3$

$= \left(\frac{2}{3} \times \frac{22}{7} \times (6.3)^3\right)$ m^3 = 523.908 m^3.

Example 16. *A hemispherical tank is made up of an iron sheet 1 cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank.* *(NCERT)*

Solution. Outer radius of hemispherical tank = R = 1 m + 1 cm

= (1 + 0.01) m = 1.01 m

Inner radius = r = 1 m .

Volume of iron $= \frac{2}{3}\pi(R^3 - r^3) = \frac{2}{3} \times \frac{22}{7}((1.01)^3 - 1^3)$ m^3

$= \left(\frac{44}{21} \times 0.030301\right)$ m^3 = 0.06349 m^3 (approx.)

Example 17. *The radius of a sphere is increased by 10%. Prove that the volume will be increased by 33.1%.* *(NCERT Exemplar)*

Solution. Let r (units) be the radius of the given sphere, then

volume of sphere $= \frac{4}{3}\pi r^3$.

New radius of sphere = r + 10% of $r = r + \frac{10}{100}r = \frac{11}{10}r$.

Volume of new sphere $= \frac{4}{3}\pi\left(\frac{11}{10}r\right)^3 = \frac{4}{3}\pi \times \frac{1331}{1000}r^3$.

$\therefore$ Increase in volume $= \frac{4}{3}\pi \times \frac{1331}{1000}r^3 - \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi r^3 (1.331 - 1) = \frac{4}{3}\pi r^3 \times 0.331$.

$\therefore$ Percentage increase in volume $= \left(\frac{\frac{4}{3}\pi r^3 \times 0.331}{\frac{4}{3}\pi r^3} \times 100\right)\%$

= 33.1%.

Example 18. *The diameter of a sphere is decreased by 25%. By what percent does its curved surface area decrease?* *(NCERT)*

Solution. Let the radius of the sphere be r, then its diameter = $d = 2r$.

As the diameter decrease by 25%,

new diameter = d – 25% of $d = d - \frac{25}{100}d = \frac{3}{4}d = \frac{3}{4}(2r) = \frac{3}{2}r$

$\therefore$ New radius = $\frac{1}{2}$ of new diameter = $\frac{1}{2} \times \frac{3}{2}r = \frac{3}{4}r$.

Original surface area = $4\pi r^2$,

new surface area = $4\pi \times \left(\frac{3}{4}r\right)^2 = \frac{9}{4}\pi r^2$.

$\therefore$ Decrease in surface area = $4\pi r^2 - \frac{9}{4}\pi r^2 = \frac{7}{4}\pi r^2$.

$\therefore$ Percentage decrease in surface area $= \left(\frac{\frac{7}{4}\pi r^2}{4\pi r^2} \times 100\right)\%$

$= \left(\frac{7}{16} \times 100\right)\% = \frac{175}{4}\% = 43.75\%$

Example 19. *A sphere and a right circular cylinder of same radius have equal volumes. By what percent does the diameter of the cylinder exceed its height?* *(NCERT Exemplar)*

Solution. Let the radius of the sphere be r.

As the cylinder has same radius, so its radius = r.

Let h be the height of the cylinder.

Since the sphere and the cylinder have equal volumes,

$$\frac{4}{3}\pi r^3 = \pi r^2 h \Rightarrow h = \frac{4}{3}r.$$

$\therefore$ Excess of diameter of cylinder over its height = $2r - h$

$$= 2r - \frac{4}{3}r = \frac{2}{3}r = \frac{1}{2}\left(\frac{4}{3}r\right) = \frac{1}{2}h.$$

$\therefore$ Percentage of excess of diameter over its height = $\left(\frac{\frac{1}{2}h}{h} \times 100\right)\% = 50\%$.

Example 20. *The surface area of a sphere of radius 5 cm is five times the area of the curved surface of a cone of radius 4 cm. Find the height and the volume of cone (take* $\pi = \frac{22}{7}$*).* *(NCERT Exemplar)*

Solution. Radius of sphere = 5 cm and radius of cone = 4 cm.

Let h cm be the height and l cm be the slant height of the cone.

As the surface area of sphere = 5 × curved surface area of cone,

$$4\pi \times 5^2 = 5 \times (\pi \times 4 \times l) \Rightarrow l = 5.$$

Since $l^2 = h^2 + r^2$,

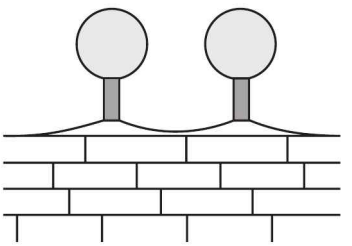
$\therefore$ $5^2 = h^2 + 4^2 \Rightarrow h^2 = 25 - 16 = 9 \Rightarrow h = 3$.

Hence, the height of the cone = 3 cm.

Volume of cone = $\frac{1}{3}\pi r^2 h = \left(\frac{1}{3} \times \frac{22}{7} \times 4^2 \times 3\right)$ cm^3

$$= \frac{352}{7} \text{ cm}^3 = 50.29 \text{ cm}^3 \text{ (approximately)}$$

Example 21. *The front compound wall of a house is decorated by wooden spheres of diameter 21 cm, placed on small support (as shown in the adjoining figure). Eight such spheres are used for this purpose, and are to be painted silver. Each support is a cylinder of radius 1.5 cm and height 7 cm and is to be painted black. Find the cost of paint required if silver paint costs 25 paise per cm^2 and black paint costs 5 paise per cm^2.* *(NCERT)*



Solution. Radius of sphere = $\frac{21}{2}$ cm, radius of cylinder = $\frac{3}{2}$ cm

Area to be painted silver

= 8(surface area of a sphere – part of area at which the sphere is resting)

$$= 8\left[4\pi \times \left(\frac{21}{2}\right)^2 - \pi\left(\frac{3}{2}\right)^2\right] \text{ cm}^2 = 8 \times \frac{22}{7}\left(441 - \frac{9}{4}\right) \text{ cm}^2$$

$$= \left(8 \times \frac{22}{7} \times 438.75\right) \text{ cm}^2 = \frac{77220}{7} \text{ cm}^2.$$

Cost of silver paint = ₹$\left(\frac{77220}{7} \times \frac{25}{100}\right)$ = ₹2757.86 (approx.)

Area to be painted black = 8 × curved surface area of a cylinder

$$= \left(8 \times 2 \times \frac{22}{7} \times \frac{3}{2} \times 7\right) \text{ cm}^2 = 528 \text{ cm}^2$$

Cost of black paint = ₹ $\frac{528 \times 5}{100}$ = ₹26.40

∴ Total cost (of both paints) = ₹(2757.86 + 26.40)

= ₹2784.26 (approx.)

Exercise 13.4

1. Find the surface area of a sphere of radius:

(*i*) 14 cm (*ii*) 10.5 cm (*iii*) 5.6 cm *(NCERT)*

2. Find the volume of a sphere of radius:

(*i*) 0.63 m (*ii*) 11.2 cm *(NCERT)*

3. Find the surface area of a sphere of diameter:

(*i*) 21 cm (*ii*) 3.5 m *(NCERT)*

4. Find the amount of water displaced by a solid spherical ball of diameter

(*i*) 28 cm (*ii*) 0.21 m *(NCERT)*

5. A shot-put is a metallic sphere of radius 4.9 cm. If the density of the metal is 7.8 g per cm^3, find the mass of the shot-put. *(NCERT)*

6. The diameter of a metallic ball is 4.2 cm. What is the mass of the ball, if the density of the metal is 8.9 g per cm^3? *(NCERT)*

7. Find the diameter of a sphere whose surface area is 154 cm^2. *(NCERT)*

8. Find

(*i*) the curved surface area.

(*ii*) the total surface area of a hemisphere of radius 21 cm. *(NCERT)*

9. A hemispherical brass bowl has inner-diameter 10.5 cm. Find the cost of tin-plating it on the inside at the rate of ₹16 per 100 cm^2. *(NCERT)*

10. The radius of a spherical balloon increases from 7 cm to 14 cm as air is pumped into it. Find the ratio of the surface areas of the balloon in two cases. *(NCERT)*

11. The diameter of the moon is approximately one-fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon? *(NCERT)*

12. A cube of side 4 cm contains a sphere touching its sides. Find the volume of the gap in between. *(NCERT Exemplar)*

13. Find the volume of a sphere whose surface area is 154 cm^2. *(NCERT)*

14. A capsule of medicine is in the shape of a sphere of diameter 3.5 mm. How much medicine (in mm^3) is needed to fill this capsule? *(NCERT)*

15. A shopkeeper has one spherical *ladoo* of radius 5 cm. With the same amount of material, how many *ladoos* of radius 2.5 cm can be made? *(NCERT Exemplar)*

16. A hemispherical bowl has a radius of 3.5 cm. What would be the volume of water it would contain? *(NCERT)*

17. The water for a factory is stored in a hemispherical tank whose internal diameter is 14 m. The tank contains 50 kilolitres of water. Water is pumped into the tank to fill to its capacity. Find the volume of water pumped into the tank. *(NCERT Exemplar)*

18. The surface area of a solid sphere is 1256 cm^2. It is cut into two hemispheres. Find the total surface area and the volume of a hemisphere. Take $\pi = 3.14$.

19. Write whether the following statements are true or false. Justify your answer:

(*i*) The volume of a sphere is equal to two-third of the volume of a cylinder whose height and diameter are equal to the diameter of the sphere. *(NCERT Exemplar)*

(*ii*) The volume of the largest right circular cone that can be fitted in a cube whose edge is $2r$ equals the volume of a hemisphere of radius r. *(NCERT Exemplar)*

(*iii*) A cone, a hemisphere and a cylinder stand on equal bases and have the same height. The ratio of their volumes is 1 : 2 : 3. *(NCERT Exemplar)*

Answers

Very Short Answer Type Questions

1. If the volume of a cube is 125 cm^3, then find its surface area.
2. If the perimeter of one face of a cube is 20 cm, then find its volume.
3. If an edge of a cube is 20 cm, then find the number of cubes of 5 cm edge that can be formed from this cube.
4. If the perimeter of a room is 18 m and its lateral surface area is 63 m^2, then find the height of the room.
5. If the radius of a solid cylinder is 3 cm and height is 4 cm, then find its total surface area.
6. If the volume of a right circular cylinder with diameter 10 cm is 200 π cm^3, then find its height.
7. If the volume of a right circular cylinder with height 14 cm is 1584 cm^3, then find its diameter.
8. If the base area of a cone is 154 cm^2 and its vertical height is 15 cm, then find its volume.
9. If the diameter of a sphere is 12 cm, then find its volume.
10. If the diameter of a solid hemisphere is 14 cm, then find its whole surface area.

Answers

Objective Type Questions

(FOR FORMATIVE/SUMMATIVE ASSESSMENT)

Mental Maths

1. Fill in the blanks :

(*i*) The sum of the areas of the plane or curved surfaces (faces) of a solid is called its ..... surface area. It is measured in ..... units.

(*ii*) The measurement of the space enclosed by a solid is called its ..... . It is measured in ..... units.

(*iii*) If a side of a cube is 4 cm, then its surface area is ..... and its volume is ..... .

(*iv*) If a side of an open cube is 75 cm, then the length of the longest rod which can be placed in it is ..... .

(*v*) If the surface area of a cube is 54 cm^2, then its volume is ..... .

(*vi*) If the surface area and the volume of a cube are numerically equal, then a side of the cube is ..... .

(*vii*) If r and h are the radius and height respectively of a right circular cylinder,

then its volume is ..... and the total surface area is ..... .

(*viii*) If r, h and l are the radius, height and slant height respectively of a right circular cone, then its volume is ..... and the total surface area is ..... .

(*ix*) The surface area of a sphere of diameter d is ..... .

(*x*) The volume of a cylinder of base diameter 10 cm and height 8 cm is ..... .

(*xi*) The volume of a hemisphere of radius 6 cm is ..... .

(*xii*) The curved surface area of a right circular cone of base radius 7 cm and slant height 10 cm is ..... .

(*xiii*) The surface area of sphere of radius 7 cm is ..... .

(*xiv*) The total surface area of hemisphere of radius 7 cm is ..... .

(*xv*) The volume of a cone of base diameter 6 cm and height 7 cm is ..... .

(*xvi*) If a rectangular piece of paper of dimensions 30 cm × 14 cm is rolled to form a hollow circular cylinder, then the curved surface area of the cylinder is ..... .

(*xvii*) If a rectangular piece of paper of dimension 44 cm × 20 cm is rolled to form a hollow cylinder of height 20 cm, then the radius of the cylinder is ..... .

(*xviii*) If the base diameter and the height of a cone are 10 cm and 12 cm respectively, then the slant height of the cone is ..... .

Multiple Choice Questions

Choose the correct answer from the given four options (2 to 22):

2. If the total surface area of a cube is 96 cm^2, then the volume of the cube is

(*a*) 8 cm^3 (*b*) 512 cm^3 (*c*) 64 cm^3 (*d*) 27 cm^3

(NCERT Exemplar)

3. The length of the longest pole that can be put in a room of dimensions (10 m × 10 m × 5 m) is

(*a*) 15 m (*b*) 16 m (*c*) 10 m (*d*) 12 m

(NCERT Exemplar)

4. The lateral surface area of a cube is 256 m^2. The volume of the cube is

(*a*) 512 m^3 (*b*) 64 m^3 (*c*) 216 m^3 (*d*) 256 m^3

(NCERT Exemplar)

5. If the perimeter of one face of a cube is 40 cm, then the sum of lengths of its edge is

(*a*) 80 cm (*b*) 120 cm (*c*) 160 cm (*d*) 240 cm

6. A cuboid container has the capacity to hold 50 small boxes. If all the dimensions of the container are doubled, then it can hold (small boxes of same size)

(*a*) 100 boxes (*b*) 200 boxes (*c*) 400 boxes (*d*) 800 boxes

7. The number of planks of dimensions (4 m × 50 cm × 20 cm) that can be stored in a pit which is 16 m long, 12 m wide and 4 cm deep is

(*a*) 1900 (*b*) 1920 (*c*) 1800 (*d*) 1840

(NCERT Exemplar)

8. A right circular cylinder has

(*a*) only one face (*b*) two faces (*c*) 3 faces (*d*) 6 faces

9. In a cylinder, if radius is halved and height is doubled then the volume will be

(*a*) same (*b*) doubled (*c*) halved (*d*) four times

(NCERT Exemplar)

10. In a cylinder, if radius is doubled and height is halved then its curved surface area will be

(*a*) halved (*b*) doubled (*c*) same (*d*) four times

(NCERT Exemplar)

11. If a well of diameter 8 m has been dug to the depth of 14 m, then the volume of the earth dug out is

(*a*) 352 m^3 (*b*) 704 m^3 (*c*) 1408 m^3 (*d*) 2816 m^3

12. If two cylinders of same lateral surface have their radii in the ratio 4 : 9, then the ratio of their heights is

(*a*) 2 : 3 (*b*) 3 : 2 (*c*) 4 : 9 (*d*) 9 : 4

13. The radii of two cylinders are in the ratio 2 : 3 and their heights are in the ratio 5 : 3. The ratio of their volumes is

(*a*) 10 : 17 (*b*) 20 : 27 (*c*) 17 : 27 (*d*) 20 : 37

(NCERT Exemplar)

14. The total surface area of a cone whose radius is $\frac{r}{2}$ and slant height $2l$ is

(*a*) $2\pi r\ (l + r)$ (*b*) $\pi r\left(l + \frac{r}{4}\right)$ (*c*) $\pi r\ (l + r)$ (*d*) $2\pi rl$

(NCERT Exemplar)

15. If the diameter of the base of a cone is 10 cm and its height is 12 cm, then its curved surface area is

(*a*) 60π cm^2 (*b*) 65π cm^2 (*c*) 90π cm^2 (*d*) 120π cm^2

16. If the diameter of the base of a cone is 12 cm and height is 20 cm, then its volume is

(*a*) 240π cm^3 (*b*) 480π cm^3 (*c*) 720π cm^3 (*d*) 960π cm^3

17. If a diameter of a sphere is increased by 20%, then its radius increases by

(*a*) 5% (*b*) 10% (*c*) 20% (*d*) 40%

18. If the radius of a sphere is $2r$, then its volume will be

(*a*) $\frac{4}{3}\pi r^3$ (*b*) $4\pi r^3$ (*c*) $\frac{8\pi r^3}{3}$ (*d*) $\frac{32\pi r^3}{3}$

(NCERT Exemplar)

19. If the diameter of a sphere is 16 cm, then its surface area is

(*a*) 64π cm^2 (*b*) 256π cm^2 (*c*) 192π cm^2 (*d*) 256 cm^2

20. If the radius of a hemisphere is 5 cm, then its volume is

(*a*) $\frac{250}{3}\pi$ cm^3 (*b*) $\frac{500}{3}\pi$ cm^3 (c) 75π cm^3 (*d*) $\frac{125}{3}\pi$ cm^3

21. If the ratio of the diameters of the two spheres is 3 : 5, then the ratio of their surface areas is

(*a*) 3 : 5 (*b*) 5 : 3 (*c*) 27 : 125 (*d*) 9 : 25

22. The radius of a hemispherical balloon increases from 6 cm to 12 cm as air is being pumped into it. The ratio of the surface areas of the balloon in the two cases is

(*a*) 1 : 4 (*b*) 1 : 3 (*c*) 2 : 3 (*d*) 2 : 1

(NCERT Exemplar)

Answers

Value Based Questions

1. Sheela donated a temporary shelter for disabled people, by making a box like structure with tarpaulin that covers all the four sides and the top (with front face as a flap which can be rolled up). Assuming that the stitching margins are very small and therefore negligible, how much tarpaulin would be required to make the shelter of height 2.5 m with base dimensions 4 m × 3 m?

   What value is shown by Sheela?
2. A soft drink is available in two packs—
   (*i*) a tin can with a rectangular base of length 5 cm and width 4 cm, having a height of 15 cm and
   (*ii*) a plastic cylinder with circular base of diameter 7 cm and height 10 cm.

   Which container has greater capacity and by how much?

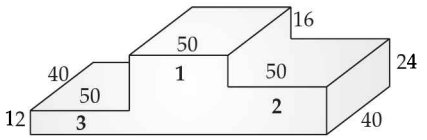
   If a company sells soft drinks in cylindrical cans, what value is depicted by it?
3. Monica has a piece of canvas whose area is 553 m^2. She uses it to make a conical tent with a base radius of 7 m. Assuming that all the stitching margins and the wastage incurred while cutting, amounts to approximately 3 m^2, find the volume of the tent that can be made with it.

   What value is depicted by Monica if she donates the tent to the poor?

Answers

Higher Order Thinking Skills (HOTS)

1. The adjoining figure shows a victory stand, each face is rectangular. All measurements are in centimetres. Find its volume and surface area (the bottom of the stand is open).

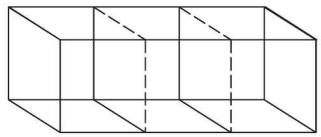


2. The length of a room is 50% more than its breadth. The cost of carpeting the room at the rate of ₹ 38.50 per m^2 is ₹ 924 and the cost of papering the wall at ₹ 3.30 per m^2 is ₹ 214.50. If the room has one door of dimensions 1 m × 2 m and two windows each of dimensions 1 m × 1.5 m, find the dimensions of the room.
3. The difference between the outer and inner curved surfaces of a cylinder 14 cm long is 88 cm^2. Find the outer and the inner radii of the cylinder, given that the volume of the metal is 176 cm^3.
4. A solid cylinder of silver 9 cm high and 4 cm in diameter is melted and recast into a right circular cone of diameter 6 cm. Find the height and the total surface area of the cone. Give your answer correct to one decimal place. Take $\pi = 3.14$.
5. The internal and external diameters of a hollow hemispherical vessel are 14 cm and 21 cm respectively. The cost of silver plating of 1 cm^2 surface are is ₹ 0.40. Find the total cost of silver plating the vessel all over.

Answers

Chapter Test

1. Find the surface area and the volume of a cuboid of dimensions 5 m × 4 m × 3 m. Also find the length of a diagonal.
2. The volume of a cuboid is 3600 cm^3 and its height is 12 cm. The cross-section is a rectangle whose length and breadth are in the ratio 4 : 3. Find the perimeter of the cross-section.
3. The length, breadth and height of a cuboid are in the ratio 5 : 4 : 2. If the total surface area is 1216 cm^2, find the volume of the solid. Also find the length of a diagonal.
4. The length of a hall is 24 m and its breadth is 16 m. If the lateral surface area of the hall is two-thirds of the sum of the areas of the roof and the floor, find its height.
5. Three cubes each of side 6 cm are joined side-by-side (as shown in the adjoining figure) to form a cuboid. Find the volume and the surface area of the cuboid.



6. A cylindrical container is to be made of tin sheet. The height of the container is 1 m and its diameter is 70 cm. If the container is open at the top and the tin sheet costs ₹ 300 per m^2, find the cost of the tin for making the container.
7. A cylinder of maximum volume is cut out from a wooden cuboid of length 30 cm and cross-section a square of side 14 cm. Find the volume of the cylinder and the volume of wood wasted.
8. The area of the curved surface of a cylinder is 4400 cm^2 and the circumference of its base is 110 cm. Find
   (*i*) the height of cylinder. (*ii*) the volume of the cylinder.
9. Find the ratio between the total surface area of a cylinder to its curved surface area given that its height and radius are 7.5 cm and 3.5 cm respectively.
10. The sum of the radius and height of a cylinder is 37 cm and the total surface area of the cylinder is 1628 cm^2. Find the height and the volume of the cylinder.
11. Find the volume and the total surface area of a cone having slant height 17 cm and base diameter 30 cm. Take $\pi = 3.14$.
12. Find the volume of a cone given that its height is 8 cm and the area of base is 156 cm^2.
13. The volume of a right circular cone is 660 cm^3. If the diameter of the base is 12 cm, find
   (*i*) the height of the cone (*ii*) the slant height of the cone.
14. Find what length of canvas 2 m in width is required to make a conical tent 20 m in diameter and 42 m in slant height allowing 10% for folds and stitching. Also find the cost of canvas at the rate of ₹ 80 per metre.
15. If the volume of a sphere is $179\frac{2}{3}$ cm^3, find its radius and surface area.
16. The circumference of the edge of a hemispherical bowl is 132 cm. Find the capacity of the bowl.
17. If the ratio of the diameters of two spheres is 3 : 7, find:
   (*i*) the ratio of their volumes. (*ii*) the ratio of their surface areas.
18. A cone is 8.4 cm high and the radius of its base is 2.1 cm. It is melted and recast into a sphere. Find the radius of the sphere. *(NCERT Exemplar)*
19. A solid sphere of radius 15 cm is melted and recast into solid right circular cones of radius 2.5 cm and height 8 cm. Find the number of cones recast.
20. The surface area of a solid metallic sphere is 616 cm^2. It is melted and recast into smaller spheres of diameter 3.5 cm. How many such spheres can be obtained?

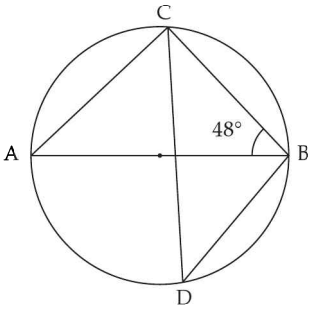
Model Question Paper 5

(FOR FORMATIVE ASSESSMENT IV - TERM II)

Time: 1 Hour *Maximum Marks: 30*

Note. *Questions 1-2 carry 1 mark each, 3-4 carry 2 marks each, 5-8 carry 3 marks each and 9-11 carry 4 marks each.*

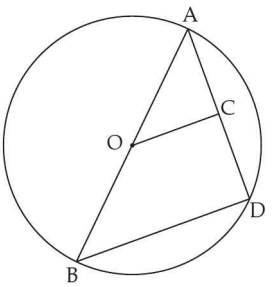
1. In the adjoining figure, AB is a diameter of the circle. If C, D are points on the circle and $\angle ABC = 48°$, then find $\angle CDB$.



2. If the total surface area of a hemisphere is 192π cm^2, then find the diameter of the hemisphere.

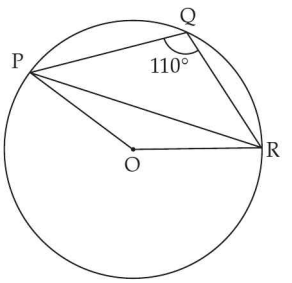
(*a*) 8 cm (*b*) 4 cm (*c*) 16 m (*d*) 16 cm

3. In the adjoining figure, AB is a diameter of the circle with centre O. If OC $\perp$ AD, then show that BD = 2OC.



4. If the surface area of a cube is 486 cm^2, then find the length of the longest rod that can be placed in it.

5. In the adjoining figure, P, Q and R are points on a circle with centre O. If $\angle PQR = 110°$, find $\angle ORP$.



6. Prove that a cyclic parallelogram is a rectangle.

7. The sum of the radius and height of a cylinder is 37 cm and the total surface area of the cylinder is 1628 cm^2. Find the height and the volume of the cylinder.

8. Find the volume and the total surface area of a cone having slant height 17 cm and base diameter 30 cm. Use $\pi = 3.14$.

9. Prove that the angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.

10. The surface area of a solid metallic sphere is 616 cm^2. It is melted and recast into smaller spheres of diameter 3.5 cm. How many such spheres can be obtained?

11. Using ruler and compass, construct a triangle ABC given BC = 6 cm, AC – AB = 2.8 cm and $\angle B = 60°$. Justify your construction.

Answers

Statistics



INTRODUCTION

The word 'Statistics' seems to have derived from the Latin word 'status' which means a 'political state'. Originally, statistics was simply the collection of numerical data on some aspects of life of the people useful to the government. However, with the passage of time, its scope broadened. Today, statistics means collection of facts or information concerning almost every aspect of life of the people with a definite purpose in the form of numerical data, organisation, summarisation and presentation of data by tables and graphs (charts), analysing the data and drawing inferences (meaningful predictions) from the data.

14.1 SOME TERMS RELATED TO STATISTICS

☆ **Primary data.** The information collected by the investigator himself or herself with a definite purpose in his or her mind is called ***primary data***.

☆ **Secondary data.** The information gathered from a source which already had the information stored is called ***secondary data.***

☆ **Raw data.** The numerical data recorded in its original form as it is collected by the investigator or received from some source is called ***raw data***.

☆ **Variable.** A quantity which is being measured in an experiment (or survey) is called a ***variable.*** Height, age and weight of people, income and expenditure of people, number of members in a family, number of workers in a factory, marks obtained by students in a test, the number of runs scored in a cricket match etc., are examples of variables.

Variables are of two types:

(*i*) **Continuous variable.** A variable which can take any value between two given values is called a ***continuous variable.***

For example, height, age and weight of people are continuous variables.

(*ii*) **Discontinuous (discrete) variable.** A variable which cannot take all possible values between two given values is called a ***discontinuous or discrete variable.***

For example, the number of members in a family and the number of workers in a factory are discrete variables (since the variable cannot take any value between 1 and 2, 2 and 3 etc.)

☆ **Range.** The difference between the maximum and minimum values of a variable is called its ***range.***

☆ **Variate.** A particular value of a variable is called ***variate (observation)***.

☆ **Frequency.** The number of times a variate (observation) occurs in a given data is called ***frequency*** of that variate.

☆ **Frequency distribution.** A tabular arrangement of given numerical data showing the frequency of different variates is called ***frequency distribution,*** and the table itself is called ***frequency distribution table.***

14.2 PRESENTATION OF DATA

Suppose there are 32 students in class IX in a school and in an examination, out of 50 marks, the marks scored by them are as follows:

39, 44, 25, 11, 21, 25, 44, 25, 7, 40, 43, 44, 49, 14, 11, 14, 25, 28, 28, 39, 44, 37, 21, 40, 43, 3, 37, 25, 25, 21, 37, 28.

The data in this form is the **raw** (or **ungrouped** or **unclassified) data**. Here, the number of marks obtained is the **variable** and each entry in the above list is an **observation** (or **variate**).

Suppose we wish to analyse the achievement of the students in the examination, the data in the above form does not give much information.

Let us arrange the above data in ascending or descending order. The above data in the ascending order is:

3, 7, 11, 11, 14, 14, 21, 21, 21, 25, 25, 25, 25, 25, 25, 28, 28, 28, 37, 37, 37, 39, 39, 40, 40, 43, 43, 44, 44, 44, 44, 49.

The presentation of data in this form gives better information. The data arranged in this form is called **arrayed data**. However, the presentation of data in this form is quite tedious and time-consuming, particularly when the number of observations is large.

To make it easily understandable, we present the above data in the form of a table called ***frequency distribution table*** (***for raw data***). To prepare table, we take each observation from the data, one at a time, and mark a stroke (|) called **tally mark** in the next column opposite to the variate. For convenience, we write tally marks in bunches of five, the fifth one crossing the four diagonally. The number of *tally marks* opposite a variate is its **frequency** and it is written in the next column opposite the tally marks of the variate. Note that the sum of all the frequencies is equal to the total number of observations in the given data.

The frequency distribution table for the above raw (ungrouped) data is given below:

Marks obtained	*Tally marks*	*Frequency*
3	|	1
7	|	1
11	||	2
14	||	2
21	|||	3
25	𝍸 |	6
28	|||	3
37	|||	3
39	||	2
40	||	2
43	||	2
44	||||	4
49	|	1
Total		32

The above table is called ***simple (or ungrouped) frequency distribution table.***

❑ **Presentation of grouped data**

Consider the following examples of grouped frequency distribution:

Example 1. *Using class intervals 1 – 5, 6 – 10, 11 – 15, … construct the frequency distribution for the following data:*

13, 6, 12, 9, 11, 14, 2, 8, 18, 16, 9, 13, 17, 11, 19, 6, 7, 12, 22, 21, 18, 1, 8, 12, 18.

(Class 1-5 means 1 to 5 including 1 and 5 both, class 6-10 means 6 to 10 including 6 and 10 both).

Solution. The grouped frequency distribution table for the given data is:

Class intervals	*Tally marks*	*Frequency*
1 – 5	||	2
6 – 10	𝍸 ||	7
11 – 15	𝍸 |||	8
16 – 20	𝍸 |	6
21 – 25	||	2
Total		25

Example 2. *The following is the survey on pocket money of 50 students in a school (pocket money in rupees per month):*

49, 55, 22, 27, 30, 27, 25, 27, 30, 42, 40, 13, 24, 38, 10, 24, 30, 33, 17, 29, 10, 50, 18, 34, 15, 40, 13, 32, 36, 32, 27, 35, 17, 41, 18, 36, 20, 41, 35, 51, 29, 27, 44, 43, 15, 32, 29, 54, 14, 45.

Form a frequency table with a grouping of 10 – 20, 20 – 30, 30 – 40 and so on (class 10 – 20 means including 10 but excluding 20, class 20 – 30 means including 20 but excluding 30).

Solution. The grouped frequency distribution table for the given data is:

Classes	*Tally marks*	*Frequency*
10 – 20	𝍸 𝍸 |	11
20 – 30	𝍸 𝍸 |||	13
30 – 40	𝍸 𝍸 |||	13
40 – 50	𝍸 ||||	9
50 – 60	||||	4
Total		50

Class limits and types of frequency distribution

In example 1, for the class interval 1 – 5, 1 is the **lower limit** and 5 is the **upper limit**.

If x is a member of this class, then $1 \leq x \leq 5$. Similarly, 6 is the ***lower limit*** and 10 is the ***upper limit*** of the class 6 – 10. In this example, the classes are non-overlapping but discontinuous. Such a frequency distribution is called **discrete** or **inclusive distribution**. In this distribution, the upper limit of one class does not coincide with the lower limit of the next class.

In example 2, for the class 1 – 10, 1 is the **lower limit** and 10 is the **upper limit**. If x is a member of this class, then $1 \leq x < 10$. Similarly, 10 is the **lower limit** and 20 is the **upper limit** of the class 10 – 20. In this example, the classes are non-overlapping but continuous. Such a frequency distribution is called **continuous** or **exclusive distribution**. In this distribution, the upper limit of one class coincides with the lower limit of the next class.

Converting discrete distribution to continuous distribution

If we measure height, weight and time, there may be fractions of a metre, kilogram and hour respectively, therefore, we need continuous distribution.

To convert discrete classes into continuous classes, we require some *adjustment*.

$$\text{Adjustment factor} = \frac{\text{lower limit of one class} - \text{upper limit of previous class}}{2}.$$

Subtract the adjustment factor from all the lower limits and add the adjustment factor to all the upper limits.

In example 1, adjustment factor $= \frac{6-5}{2} = \frac{1}{2} = 0.5$.

Continuous frequency distribution table for example 1 is:

Classes before adjustment	*Classes after adjustment*	*Tally marks*	*Frequency*
1 – 5	0.5 – 5.5	||	2
6 – 10	5.5 – 10.5	𝍸 ||	7
11 – 15	10.5 – 15.5	𝍸 |||	8
16 – 20	15.5 – 20.5	𝍸 |	6
21 – 25	20.5 – 25.5	||	2
Total			25

True class limits

In a continuous distribution, the class limits are called **true** or **actual class limits**. In a discrete distribution, the class limits obtained after adjustment are the *true* or *actual class limits*. The actual class limits are also called ***class boundaries***.

In discrete distribution, the original (given) class limits are called the ***stated class limits***.

Class size. *The difference between the actual upper limit and the actual lower limit of a class is called its* ***class size*** *or* ***class width.***

In example 2, class size of the class 10 – 20 = 20 – 10 = 10.

In example 1, class size of the class 1 – 5 = 5.5 – 0.5 = 5.

Class mark. *The* ***class mark*** *of a class is the value midway between its actual lower limit and actual upper limit.*

In example 2, class mark of the class $10 - 20 = \frac{10 + 20}{2} = 15$.

In example 1, class mark of the class $1 - 5 = \frac{0.5 + 5.5}{2} = 3$.

Note. In discrete distribution, the class mark of a class is also the value mid-way between its stated class limits. Thus in example 1, the class mark of the class

$$1 - 5 = \frac{1 + 5}{2} = 3.$$

Remark. If the classes are of equal size, then class size = difference between two successive class marks.

☆ **Formation of classes from a given raw data**

We condense the given raw data into classes (or groups) as follows:

1. Find the *range i.e.* the difference between the maximum and minimum observations. Decide about the number of classes (usually between 5 to 10). In general, the size (width) is a convenient natural number immediately greater than the quotient obtained by dividing the range by the number of classes.
2. Classes should be non-overlapping and continuous.
3. There should be no gaps between classes.
4. As far as possible, classes should be of the same size.
5. Open ended classes such as less than 5 or greater than 9 should be avoided.
6. Limits of each class should be so chosen that there is no ambiguity as to which class a particular observation of the given data belongs to.

Illustrative Examples

Example 1. *The blood groups of 30 students of class IX are recorded as follows:*

A, B, O, O, AB, O, A, O, B, A, O, B, A, O, O,

A, AB, O, A, A, O, O, AB, B, A, O, B, A, B, O.

Represent this data in the form of a frequency distribution table. Which is the most common, and which is the rarest, blood group among these students? (*NCERT*)

Solution. The frequency distribution table for the given data is:

Blood group	*Tally marks*	*Frequency*
A	𝍸 IIII	9
B	𝍸 I	6
O	𝍸 𝍸 II	12
AB	III	3
Total		30

As the blood group O has maximum frequency (12), so most common blood group is O; and the blood group AB has minimum frequency (3), so rarest blood group is AB.

Example 2. *The value of π upto 50 decimal places is given below:*

3.14159265358979323846264338327950288419716939937510

(i) Make a frequency distribution of the digits from 0 to 9 after the decimal point.

(ii) What are the most and the least frequency occurring digits? (*NCERT*)

Solution. (*i*) The required frequency distribution table for the given data after decimal point is:

Digits	*Tally marks*	*Frequency*
0	||	2
1	𝍸	5
2	𝍸	5
3	𝍸 |||	8
4	||||	4
5	𝍸	5
6	||||	4
7	||||	4
8	𝍸	5
9	𝍸 |||	8
Total		50

(*ii*) As the digits 3 and 9 each has maximum frequency (8), so the most occurring digits are 3 and 9.

The digit 0 has minimum frequency 2, so the least occurring digit is 0.

Example 3. *If m is the mid-point and l is the upper limit of a class in a continuous frequency distribution, then what is the lower limit of the class?* *(NCERT Exemplar)*

Solution. As l is the upper limit and m is the mid-point of a class in a continuous frequency distribution,

so half of the width of the class = upper limit – mid-point = $l - m$.

Let x be the lower limit of this class, then

$$x = \text{mid-point} - \frac{1}{2} \text{ of width of this class}$$

$$= m - (l - m) = 2m - l.$$

Example 4. *The distance (in km) of 40 engineers from their residence to their place of work were found as follows:*

5	*3*	*10*	*20*	*25*	*11*	*13*	*7*	*12*	*31*
19	*10*	*12*	*17*	*18*	*11*	*32*	*17*	*16*	*2*
7	*9*	*7*	*8*	*3*	*5*	*12*	*15*	*18*	*3*
12	*14*	*2*	*9*	*6*	*15*	*15*	*7*	*6*	*12*

Construct a grouped frequency distribution table with class size 5 for the data given above taking the first interval as 0–5 (5 not included). What main features do you observe from this tabular representation?

Solution. Minimum distance = 2 km and maximum distance = 32 km

Taking class size as 5 and the first interval as 0 – 5 (5 not included), the class intervals will be 0 – 5, 5 – 10, 10 – 15, 15 – 20, 20 – 25, 25 – 30 and 30 – 35.

The grouped continuous frequency distribution table for the given data is:

Distances (in km)	*Tally marks*	*No. of engineers (Frequency)*
0 – 5	𝍸	5
5 – 10	𝍸 𝍸 |	11
10 – 15	𝍸 𝍸 |	11
15 – 20	𝍸 ||||	9
20 – 25	|	1
25 – 30	|	1
30 – 35	||	2
Total		40

The distance of most of the engineers from their residence to their workplace is between 5 km to 20 km.

Example 5. *A company manufactures car batteries of a particular type. The lives (in years) of 40 such batteries were recorded as follows:*

2.6	*3.0*	*3.7*	*3.2*	*2.2*	*4.1*	*3.5*	*4.5*
3.5	*2.3*	*3.2*	*3.4*	*3.8*	*3.2*	*4.6*	*3.7*
2.5	*4.4*	*3.4*	*3.3*	*2.9*	*3.0*	*4.3*	*2.8*
3.5	*3.2*	*3.9*	*3.2*	*3.2*	*3.1*	*3.7*	*3.4*
4.6	*3.8*	*3.2*	*2.6*	*3.5*	*4.2*	*2.9*	*3.6*

Construct a grouped frequency distribution table for this data, using class intervals of size 0.5 starting from the interval 2 – 2.5.

Solution. Minimum lifetime of battery = 2 years and maximum lifetime = 4.6 years.

Taking class size as 0.5, the class intervals will be

2.0 – 2.5, 2.5 – 3.0, 3.0 – 3.5, 3.5 – 4.0,4.0 – 4.5, 4.5 – 5.0

The grouped continuous frequency distribution table for the given data is:

Life of batteries (in years)	*Tally marks*	*No. of batteries (Frequency)*
2 – 2.5	||	2
2.5 – 3	𝍸 |	6
3 – 3.5	𝍸 𝍸 ||||	14
3.5 – 4	𝍸 𝍸 |	11
4 – 4.5	||||	4
4.5 – 5	|||	3
Total		40

Example 6. *Two sections of class IX having 30 students each appeared for mathematics olympiad. The marks obtained by them are as shown below:*

46	31	74	68	42	54	14	61	83	48	37	26	8	64	57
93	72	53	59	38	16	88	75	56	46	60	45	61	54	27
27	44	63	58	43	81	64	67	36	49	50	76	38	47	55
77	62	53	40	71	60	58	45	42	34	46	40	59	42	29

Construct a group frequency of the above data using the classes 0–9, 10–19 etc. and hence find the number of students who secured more than 49 marks. *(NCERT Exemplar)*

Solution. Taking classes 0 – 9, 10 – 19, 20 – 29, ..., 90 – 99, the grouped discrete frequency distribution table of the given data is:

Classes (Marks obtained)	*Tally marks*	*No. of students (Frequency)*
0 – 9	|	1
10 – 19	||	2
20 – 29	||||	4
30 – 39	𝍸 |	6
40 – 49	𝍸 𝍸 𝍸	15
50 – 59	𝍸 𝍸 ||	12
60 – 69	𝍸 𝍸	10
70 – 79	𝍸 |	6
80 – 89	|||	3
90 – 99	|	1
Total		60

From the above table, we find that the number of students who obtained more than 49 marks = 12 + 10 + 6 + 3 + 1 = 32.

Example 7. *Convert the given frequency distribution into a continuous grouped frequency distribution:*

Class interval	*150 – 153*	*154 – 157*	*158 – 161*	*162 – 165*	*166 – 169*	*170 – 173*
Frequency	*7*	*7*	*15*	*10*	*5*	*6*

In which interval 153.5 and 157.5 be included? *(NCERT Exemplar)*

Solution. We observe that the class intervals are of uniform width 4 and the frequency distribution is discrete.

To convert discrete classes into continuous classes, we need some adjustment.

$$\text{Adjustment factor} = \frac{\text{lower limit of one class} - \text{upper limit of previous class}}{2}$$

$$= \frac{154 - 153}{2} = \frac{1}{2} = 0.5.$$

Subtract the adjustment factor 0.5 from all the lower limits and add the adjustment factor to all the upper limits.

The continuous grouped frequency distribution for the given grouped discrete frequency distribution is

Class intervals (after adjustment)	*Frequency*
149.5 – 153.5	7
153.5 – 157.5	7
157.5 – 161.5	15
161.5 – 165.5	10
165.5 – 169.5	5
169.5 – 173.5	6

153.5 is included in the class 153.5 – 157.5 and 157.5 is included in class 157.5 – 161.5.

Example 8. *The electricity bills (in rupees) of 40 houses in a locality are given below. Construct a grouped frequency distribution table:*

78, 87, 81, 52, 59, 65, 101, 108, 115, 95, 98, 65, 62, 121, 128, 63, 76, 84, 89, 91, 65, 101, 95, 81, 87, 105, 129, 92, 75, 105, 78, 72, 107, 116, 127, 100, 80, 82, 61, 118

Solution. Here maximum = 129 and minimum = 52,

$\therefore$ range = 129 – 52 = 77.

Let us form 8 classes each of size 10.

Since we want to include 129 in the last class, 130 is the upper limit of the last class, so the lower limit of the first class is 50.

The grouped frequency distribution table of the given data is:

Classes	*Tally marks*	*No. of houses (Frequency)*
50 – 60	||	2
60 – 70	𝍸 |	6
70 – 80	𝍸	5
80 – 90	𝍸 |||	8
90 – 100	𝍸	5
100 – 110	𝍸 ||	7
110 – 120	|||	3
120 – 130	||||	4
Total		40

Example 9. *The class marks of a continuous distribution are 1.04, 1.14, 1.24, 1.34, 1.44, 1.54 and 1.64. Is it correct to say that the last interval will be 1.55 – 1.73? Justify your answer.*

(NCERT Exemplar)

Solution. It is not correct; because the class marks of the continuous frequency distribution are 1.04, 1.14, 1.24, 1.34, 1.44, 1.54 and 1.64 which are at equal gaps, so the classes are of equal size and class size = difference between two consecutive class marks = 1.14 – 1.04 = 0.1

But here class size = 1.73 – 1.55 = 0.18, which is not equal to 0.1.

Example 10. *Prepare a continuous grouped frequency distribution from the following data:*

Mid-points	5	15	25	35	45
Frequency	4	8	13	12	6

Also find the size of class intervals. *(NCERT Exemplar)*

Solution. As the mid-points (class marks) of classes are 5, 15, 25, 35 and 45 which are at equal gaps, so the classes are of equal size.

Therefore, size of class = difference between two consecutive mid-points

$= 15 - 5 = 10.$

Half of class size $= \frac{10}{2} = 5.$

Therefore, the class limits of the first class are lower limit = mid-point – half of class size and upper limit = mid-point + half of class size *i.e.* 5 – 5 and 5 + 5 *i.e.* 0 and 10. Similarly, we find the class limits for all other classes. Thus, the classes are 0 – 10, 10 – 20, 20 – 30, 30 – 40 and 40 – 50. The continuous distribution of the given data is:

Classes	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	4	8	13	12	6

Exercise 14.1

1. Three coins were tossed 30 times simultaneously. Each time the number of heads occurring was noted as follows:

0 1 2 2 1 2 3 1 3 0
1 3 1 1 2 2 0 1 2 1
3 0 0 1 1 2 3 2 2 0

Prepare a frequency distribution table for the data given above.

2. Heights (in cm) of 30 girls of class IX are given below:

140, 140, 160, 139, 153, 153, 146, 150, 148, 150,
152, 146, 154, 150, 160, 148, 150, 148, 140, 148,
153, 138, 152, 150, 148, 138, 152, 140, 146, 148

Prepare a frequency distribution table for this data. *(NCERT Exemplar)*

3. The scores (out of 100) obtained by 33 students in a mathematics test are as follows:

69, 48, 84, 58, 48, 73, 83, 48, 66, 58, 84,
66, 64, 71, 64, 66, 69, 66, 83, 66, 69, 71,
81, 71, 73, 69, 66, 64, 58, 64, 66, 69, 69

Represent this data in the form of a frequency distribution. *(NCERT Exemplar)*

4. Thirty children were asked about the number of hours they watched TV programmes in the previous week. The results were found as follows:

1 6 2 3 5 12 5 8 4 8
10 3 4 12 2 8 15 1 17 6
3 2 8 5 9 6 8 7 14 12

(*i*) Make a grouped frequency distribution table for this data, taking class width 5 and one of the class intervals as 5 – 10.

(*ii*) How many children watched television for 15 or more hours a week?

(NCERT)

5. The relative humidity (in %) of a certain city for a month of 30 days was as follows:

98.1	98.6	99.2	90.3	86.5	95.3	92.9	96.3	94.2	95.1
89.1	92.3	97.1	93.5	92.7	95.1	97.2	93.3	95.2	97.3
96.2	92.1	84.9	90.2	95.7	98.3	97.3	96.1	92.1	89

(*i*) Construct a grouped frequency distribution table with classes 84 – 86, 86 – 88, etc.

(*ii*) Which month or season do you think this data is about?

(*iii*) What is the range of this data? *(NCERT)*

6. The heights of 50 students, measured to the nearest centimetres, have been found to be as follows:

161	150	154	165	168	161	154	162	150	151
162	164	171	165	158	154	156	172	160	170
153	159	161	170	162	165	166	168	165	164
154	152	153	156	158	162	160	161	173	166
161	159	162	167	168	159	158	153	154	159

(*i*) Represent the data given above by a grouped frequency distribution table, taking the class intervals as 160 – 165, 165 – 170, etc.

(*ii*) What can you conclude about their heights from the table?

7. The maximum temperatures (in degrees celsius) for Delhi for the month of April, 2014 as reported by the Meteorological Department, are given below:

27.4, 28.3, 23.9, 23.6, 25.4, 27.5, 28.1, 30.5, 29.7, 30.6,
28.4, 31.7, 32.2, 32.6, 33.4, 35.7, 36.1, 37.2, 38.4, 40.1,
40.2, 40.5, 41.1, 42.0, 42.1, 42.3, 42.4, 42.9, 43.1, 43.2

Construct a continuous grouped frequency distribution table.

8. 30 children were asked about the number of hours they watched TV programmes last week. The results were recorded as under:

Number of hours	0 – 5	5 – 10	10 – 15	15 – 20
Frequency	8	16	4	2

Can we say that the number of children who watched TV for 10 or more hours a week is 22? Justify your answer. *(NCERT Exemplar)*

Answers

14.3 GRAPHICAL REPRESENTATION OF DATA

Since pictures (or graphs) are good visual aids and leave more lasting effect on the mind of an observer, the information contained in a numerical data (or frequency distribution) can be easily understood if we represent it in the form of diagrams (graphs). It is well said that one picture is better than a thousand words. Usually, comparisons among the individual items are best shown by means of graphs.

There are various ways of representing numerical data (or frequency distribution) graphically. Here, we shall study the following graphical representations:

(*i*) Bar graphs

(*ii*) Histograms of uniform width, and of varying width

(*iii*) Frequency polygons.

14.4 BAR GRAPHS

In previous classes, you have already studied and constructed bar graphs. Bar graphs can be drawn vertically or horizontally.

In bar graphs;

(*i*) take the width of all bars (rectangles) equal.

(*ii*) space between consecutive bars should be equal. The bars can touch each other.

(*iii*) the height (or length) of a bar is equal (or proportional) to the frequency of the corresponding variate (observation).

(*iv*) the width of bars has no significance. In fact, the width of the bars is shown simply to make the representation more eye-catching.

Consider the following example.

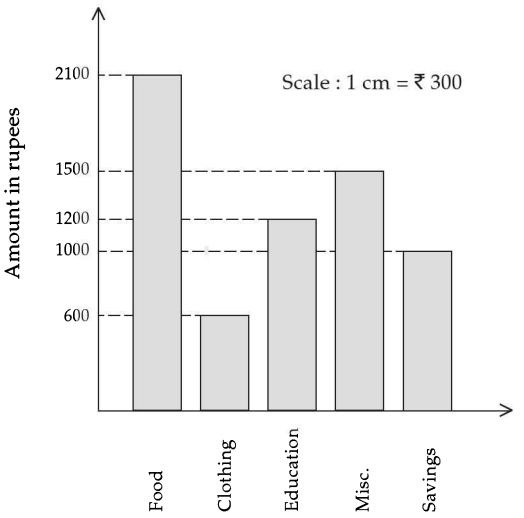
Example. *A man with a monthly salary of ₹6400 plans his budget for a month as given below:*

Item	*Food*	*Clothing*	*Education*	*Miscellaneous*	*Savings*
Amount (in ₹)	*2100*	*600*	*1200*	*1500*	*1000*

Represent the above data by a bar graph.

Solution. The required bar graph is shown alongside:

Here, we can easily visualise the relative characteristics of the data at a glance. For example, the expenditure on food is more than the expenditure on any other item and the expenditure on clothing is least. Therefore, in some ways it gives a better representation of data than the tabular form.



Illustrative Examples

Example 1. *Given below are the seats won by different political parties in the polling outcome of a state assembly elections:*

Political parties	*A*	*B*	*C*	*D*	*E*	*F*
Seats won	*75*	*55*	*37*	*29*	*10*	*37*

(i) Draw a bar graph to represent the polling results.

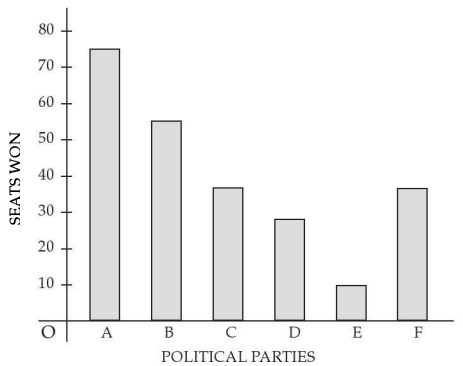
(ii) Which political party won the maximum number of seats?

(iii) What should be the criteria of selecting candidates in elections? ***(Value Based)***

Solution. (*i*) The required bar graph for the given data is shown alongside:

(*ii*) A

(*iii*) Candidates must be educated, of good character, honest and well wishers of society.



Example 2. *A survey conducted by an organisation for the cause of illness and death among the women between the ages 15 – 44 (in years), found the following figures (in %)*

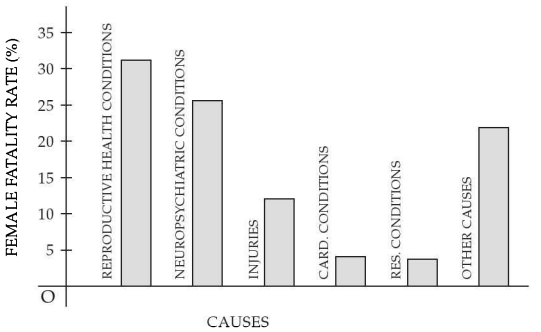
S.No.	Causes	Female fatality rate (%)
1.	Reproductive health conditions	31.8
2.	Neuropsychiatric conditions	25.4
3.	Injuries	12.4
4.	Cardiovascular conditions	4.3
5.	Respiratory conditions	4.1
6.	Other causes	22.0

(i) Represent the given information by a bar graph.

(ii) Which condition is the major cause of women's ill health and death worldwide? (NCERT)

Solution. (*i*) The required bar graph for the given data is shown alongside:

(*ii*) Reproductive health conditions.



Example 3. *The following data on the number of girls (to the nearest ten) per thousand boys in different sections of Indian society is given below:*

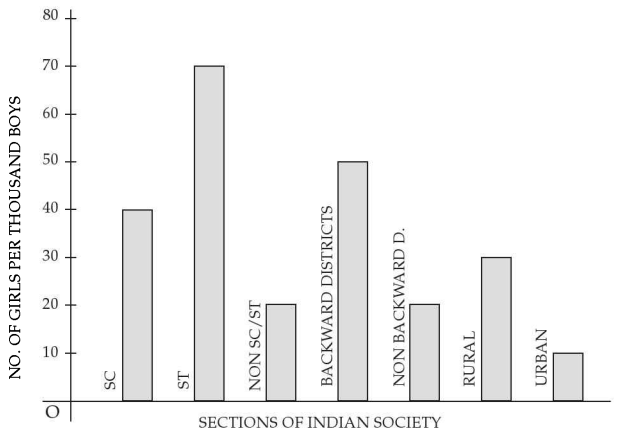
Section	Number of girls per thousand boys
Scheduled Caste (SC)	940
Scheduled Tribe (ST)	970
Non SC/ST	920
Backward districts	950
Non-backward districts	920
Rural	930
Urban	910

(i) Represent the above information by a bar graph.

(ii) What conclusions can be arrived at from the graph?

Solution. (*i*) Note that 900 is common in the whole data, so it is not shown in the graph. Only the numbers above 900 are shown in the bar graph, so that the real effects of the graph may be more eye-catching.

(*ii*) The ratio of girls to boys is the least in Urban Indian society and is best in Scheduled tribes.



Exercise 14.2

1. The expenditure of a family on different heads in a month is given below:

Head	Food	Education	Clothing	House Rent	Others	Savings
Expenditure (in ₹)	4000	2500	1000	3500	2500	1500

Draw a bar graph to represent the above data. *(NCERT Exemplar)*

2. Expenditure on Education of a country during a five year period (2002–2006), in crores of rupees, is given below:

Elementary Education	240
Secondary Education	120
University Education	190
Teachers' Training	20
Social Education	10
Other Educational Programmes	115
Cultural Programmes	25
Technical Education	125

Represent the above information by a bar graph. *(NCERT Exemplar)*

Answers

14.5 HISTOGRAMS

A histogram is used to represent continuous grouped data. It consists of adjacent rectangles.

Procedure to draw a histogram of uniform width:

(*i*) Take the breadth of a rectangle equal to a class size and mark it along x-axis, the end points of which correspond to the class limits.

(*ii*) Take the length of a rectangle equal to the frequency of that class and mark it along y-axis.

(*iii*) Construct rectangles corresponding to each class with the help of steps (*i*) and (*ii*).

Remarks

☆ A histogram (of uniform width) consists of a set of adjacent rectangles whose bases are equal to class sizes and heights are equal to class frequencies.

☆ In a bar graph, the breadth of a rectangle has no significance; whereas in a histogram, the breadth of a rectangle is meaningful and it represents the class size.

☆ In a bar graph, there may be gaps between different bars (rectangles); whereas in a histogram, there is no gap between different rectangles.

☆ A bar graph represents discrete data, while a histogram represents a continuous grouped data.

☆ If the frequency distribution is discontinuous (inclusive), change it to continuous (exclusive) and then construct histogram.

Example 1. *Draw a histogram for the following data:*

Height (in cm)	*150 – 160*	*160 – 170*	*170 – 180*	*180 – 190*	*190 – 200*
No. of students	*8*	*3*	*4*	*10*	*2*

Solution. *Steps.*

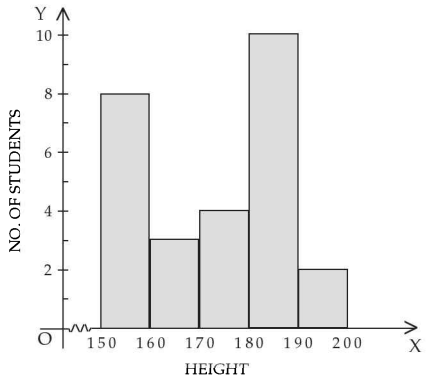
(*i*) Since the scale on x-axis starts at 150, a break (kink or zig-zag curve) is shown near the origin along x-axis to indicate that the graph is drawn to scale beginning at 150 and not at the origin itself.

(*ii*) Take 1 cm on x-axis = 10 cm (height).

(*iii*) Take 1 cm on y-axis = 2 (no. of students).

(*iv*) Construct rectangles corresponding to the given data.

The required histogram is shown in the adjoining diagram.



Example 2. *Draw a histogram for the following data:*

Wt. (in kg)	*40 – 44*	*45 – 49*	*50 – 54*	*55 – 59*	*60 – 64*	*65 – 69*
No. of students	*2*	*8*	*12*	*10*	*6*	*4*

Solution. The given frequency distribution is discontinuous. To convert it into continuous frequency distribution,

$$\text{adjustment factor} = \frac{\text{lower limit of one class} - \text{upper limit of previous class}}{2}$$

$$= \frac{45 - 44}{2} = \frac{1}{2} = 0.5.$$

Subtract the adjustment factor (0.5) from all the lower limits and add the adjustment factor (0.5) to all the upper limits.

Continuous frequency distribution for the given data is:

Classes before adjustment	*Classes after adjustment*	*Frequency*
40 – 44	39.5 – 44.5	2
45 – 49	44.5 – 49.5	8
50 – 54	49.5 – 54.5	12
55 – 59	54.5 – 59.5	10
60 – 64	59.5 – 64.5	6
65 – 69	64.5 – 69.5	4

To construct histogram:

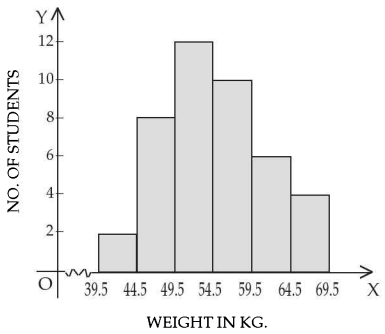
(*i*) Since the scale on x-axis starts at 39.5, a break (kink) is shown near the origin on x-axis to indicate that the graph is drawn to scale beginning at 39.5.

(*ii*) Take 1 cm along x-axis = 5 kg (weight).

(*iii*) Take 1 cm along y-axis = 2 (no. of students).

(*iv*) Construct rectangles corresponding to the continuous frequency distribution given in the above table.

The required histogram is shown in the adjoining figure.



❑ **Histograms of varying width**

In a histogram, in fact, it is the area of a rectangle which represents the frequency of the class. Therefore, if the classes are of unequal size (width) then the lengths of rectangles are not equal to the frequencies of the corresponding classes. However, in the above examples, the classes were of equal size, therefore, the widths of all rectangles were equal and hence the lengths of rectangles were proportional to the frequency and we have drawn their lengths according to their frequencies.

Now, consider the following example:

A teacher wanted to analyse the performance of students in a test of 100 marks. Looking at their performance, she found that a few students got less than 20 marks and a few got 70 marks or above. So, she decided to group them into intervals of varying sizes as follows: 0 – 20, 20 – 30, 30 – 40, 40 – 50, 50 – 60, 60 – 70, 70 – 100, and prepared the following table:

Marks	*No. of students*
0 – 20	7
20 – 30	10
30 – 40	10
40 – 50	20
50 – 60	20
60 – 70	15
70 – above	8

We note that the size of the classes is unequal, so the widths of the rectangles are unequal (varying). In a histogram, the area of a rectangle represents the frequency of the class, therefore, the frequencies of the classes are to be adjusted so that these are proportional to the lengths of rectangles of the corresponding class intervals.

Procedure to draw a histogram of varying width

(*i*) Select a class interval with minimum class size and mark all class intervals along x-axis, the end points of which correspond to the class limits.

(*ii*) Find adjusted frequencies to the minimum class size so that these are proportional to the lengths of rectangles of the corresponding class intervals.

$$\text{Adjusted frequency of a class} = \frac{\text{minimum class size}}{\text{class size}} \times \text{frequency of this class.}$$

Mark adjusted frequencies of classes along y-axis.

The adjusted frequencies are the heights (lengths) of each rectangle of histogram but widths are according to class limits.

(*iii*) Construct rectangles corresponding to each class with the help of steps (*i*) and (*ii*) above.

Here, minimum class size = 10. We construct a table as follows:

Marks	*Frequency*	*Width of class*	*Adjusted frequency*
0 – 20	7	20	$\frac{10}{20} \times 7 = 3.5$
20 – 30	10	10	$\frac{10}{10} \times 10 = 10$

30 – 40	10	10	$\frac{10}{10} \times 10 = 10$
40 – 50	20	10	$\frac{10}{10} \times 20 = 20$
50 – 60	20	10	$\frac{10}{10} \times 20 = 20$
60 – 70	15	10	$\frac{10}{10} \times 15 = 15$
70 – 100	8	30	$\frac{10}{30} \times 8 = 2.67$

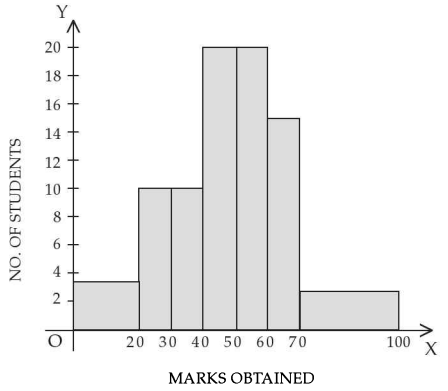
Steps

(*i*) Take 1 cm on x-axis = 10 marks.

(*ii*) Take 1 cm on y-axis = 2 students.

(*iii*) Construct rectangles corresponding to the above table.

(*iv*) The required histogram is shown in the adjoining figure.



Illustrative Examples

Example 1. *Is it correct to say that in a histogram, the area of each rectangle is proportional to the class size of the corresponding class interval? If not, correct the statement.* *(NCERT Exemplar)*

Solution. It is not correct. The correct statement is:

In a histogram, the area of each rectangle is proportional to the frequency of its class.

Example 2. *In a histogram, the areas of the rectangles are proportional to the frequencies. Can we say that the lengths of the rectangles are also proportional to the frequencies?* *(NCERT Exemplar)*

Solution. No. It is true only when all the classes are of same width (size).

Example 3. *The following table gives the weights (in kg) of 36 students:*

Weight (in kg)	*Number of students*
30.5 – 35.5	*9*
35.5 – 40.5	*6*
40.5 – 45.5	*15*
45.5 – 50.5	*3*
50.5 – 55.5	*1*
55.5 – 60.5	*2*

Represent the above continuous grouped frequency by a histogram. *(NCERT)*

Solution. *Steps*

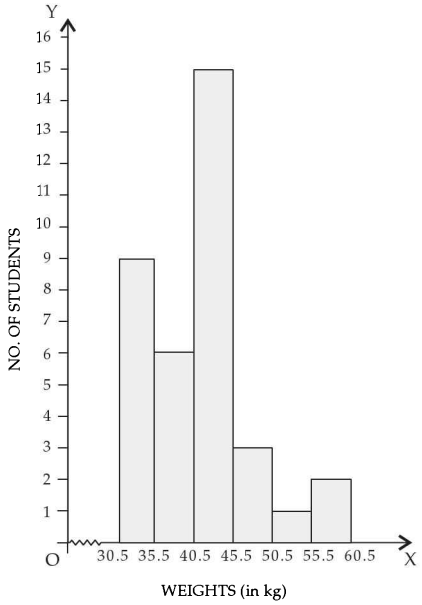
(*i*) Since the scale on x-axis starts at 30.5, a break (kink or zig-zag curve) is shown near the origin to indicate that the graph is drawn to scale beginning at 30.5 and not as the origin itself.

(*ii*) Take 1 cm on x-axis = 5 kg (weight).

(*iii*) Take 1 cm on y-axis = 2 students.

(*iv*) Construct the rectangles corresponding to the given data.

(*v*) The required histogram is shown in the adjoining diagram.



Example 4. *The following table gives the lifetimes of 400 neon lamps:*

Lifetime (in hours)	*Number of lamps*
300 – 400	*14*
400 – 500	*56*
500 – 600	*60*
600 – 700	*86*
700 – 800	*74*
800 – 900	*62*
900 – 1000	*48*

(*i*) *Represent the given information with the help of a histogram.*

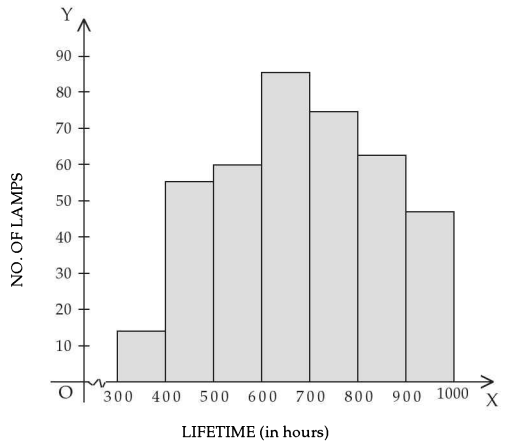
(*ii*) *How many lamps have a lifetime of more than or equal to 700 hours?*

Solution. (*i*) *Steps*

1. Since the scale on x-axis starts at 300, a break is shown near the origin.
2. Take 1 cm on x-axis = 100 hours.
3. Take 1 cm on y-axis = 10 lamps.
4. Construct rectangles corresponding to the given data. The required histogram is shown in the adjoining figure.

(*ii*) The number of lamps which have a lifetime more than (or equal to) 700 hours

= 74 + 62 + 48 = 184.



Example 5. *The lengths of 62 leaves of a plant are measured in millimetres and the data is represented in the following table:*

Length (in mm)	*118-126*	*127-135*	*136-144*	*145-153*	*154-162*	*163-171*	*172-180*
Number of leaves	*8*	*10*	*12*	*17*	*7*	*5*	*3*

Draw a histogram to represent the above data. *(NCERT Exemplar)*

Solution. The given frequency distribution is discontinuous. To convert it into continuous frequency distribution,

$$\text{adjustment factor} = \frac{\text{lower limit of one class} - \text{upper limit of previous class}}{2}$$

$$= \frac{127 - 126}{2} = \frac{1}{2} = 0.5.$$

Subtract the adjust factor (0.5) from all the lower limits and add the adjustment factor (0.5) to all the upper limits.

Continuous frequency distribution for the given data is:

Classes before adjustment	*Classes after adjustment*	*Frequency*
118 – 126	117.5 – 126.5	8
127 – 135	126.5 – 135.5	10
136 – 144	135.5 – 144.5	12
145 – 153	144.5 – 153.5	17
154 – 162	153.5 – 162.5	7
163 – 171	162.5 – 171.5	5
172 – 180	171.5 – 180.5	3

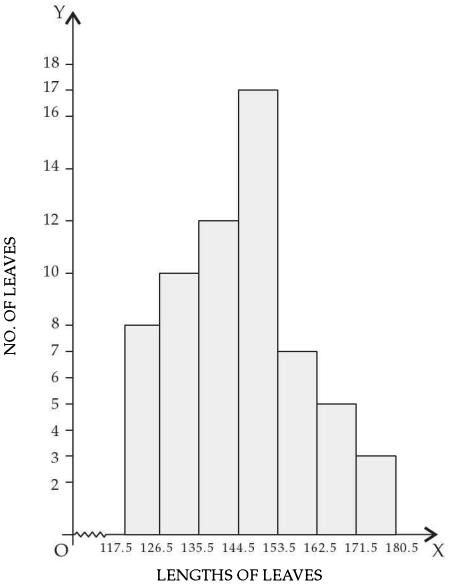
Steps

(*i*) Since the scale starts at 117.5, a break is shown near the origin.

(*ii*) Take 1 cm on x-axis = 9 mm.

(*iii*) Take 1 cm on y-axis = 2 leaves.

(*iv*) Construct rectangles corresponding to the given data. The required histogram is shown in the adjoining diagram.



Example 6. *The marks obtained (out of 100) by a class of 80 students are given below:*

Marks	*10 – 20*	*20 – 30*	*30 – 50*	*50 – 70*	*70 – 100*
Number of students	*6*	*17*	*15*	*16*	*26*

Construct a histogram to represent the above data. *(NCERT Exemplar)*

Solution. Here the classes are of unequal size, so it is case of histogram with varying width. Minimum class size = 10. Therefore, we are to find the adjusted frequencies to the minimum class size so that these are proportional to the lengths of rectangles of the corresponding class intervals.

$$\text{Adjusted frequency} = \frac{\text{minimum class size}}{\text{size of class}} \times \text{frequency of this class.}$$

We prepare the table as under:

Marks	*Frequency*	*Width of class*	*Adjusted frequency*
10 – 20	6	10	$\frac{10}{10} \times 6 = 6$
20 – 30	17	10	$\frac{10}{10} \times 17 = 17$
30 – 50	15	20	$\frac{10}{20} \times 15 = 7.5$
50 – 70	16	20	$\frac{10}{20} \times 16 = 8$
70 – 100	26	30	$\frac{10}{30} \times 26 = 8.66$

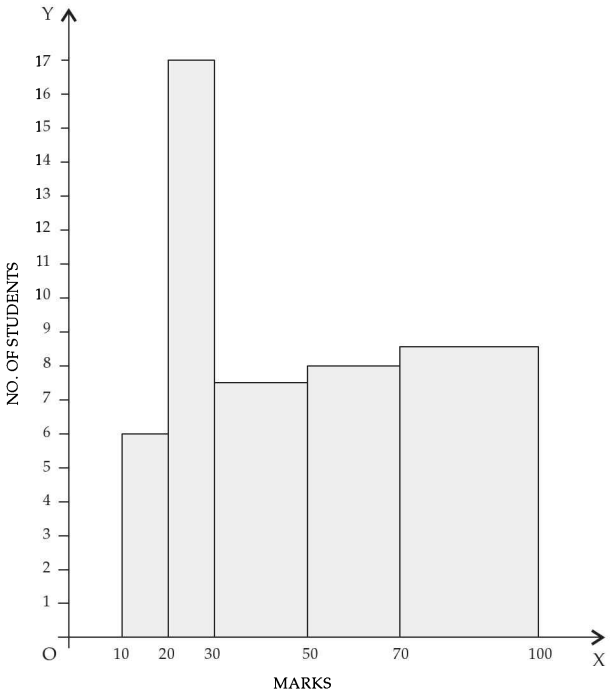
Steps

(*i*) Take 1 cm on x-axis = 10 marks and mark the class intervals along x-axis, the end points of which correspond to the class limits.

(*ii*) Take 1 cm on y-axis = 2 students and mark the lengths of rectangles along the y-axis.

(*iii*) Construct the rectangles corresponding to the above table.

The required histogram is shown in the adjoining figure.



Example 7. *A random survey of the number of children of various age groups playing in a park was found as follows:*

Age (in years)	*1 – 2*	*2 – 3*	*3 – 5*	*5 – 7*	*7 – 10*	*10 – 15*	*15 – 17*
Number of children	*5*	*3*	*6*	*12*	*9*	*10*	*4*

Draw a histogram to represent the above data. (*NCERT*)

Solution. Here the classes are of unequal size, so it is a case of histogram with varying width. Minimum class size is 1.

Therefore, we are to find the adjusted frequencies to the minimum class size so that these are proportional to the lengths of rectangles of the corresponding class intervals.

Adjusted frequency = $\frac{\text{minimum class size}}{\text{size of class}}$ × frequency of this class.

We prepare the table as under:

Age (in years)	*No. of children*	*Width of class*	*Adjusted Frequency*
1 – 2	5	1	$\frac{1}{1} \times 5 = 5$
2 – 3	3	1	$\frac{1}{1} \times 3 = 3$
3 – 5	6	2	$\frac{1}{2} \times 6 = 3$
5 – 7	12	2	$\frac{1}{2} \times 12 = 6$
7 – 10	9	3	$\frac{1}{3} \times 9 = 3$
10 – 15	10	5	$\frac{1}{5} \times 10 = 2$
15 – 17	4	2	$\frac{1}{2} \times 4 = 2$

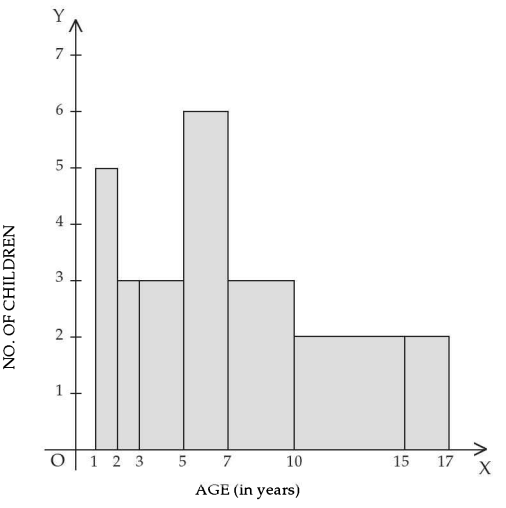
Steps

(*i*) Take 1 cm on x-axis = 2 years and mark the class intervals along x-axis, the end points of which correspond to the class limits.

(*ii*) Take 1 cm on y-axis = 1 child and mark the length of rectangles along the y-axis.

(*iii*) Construct the rectangles corresponding to the above table.

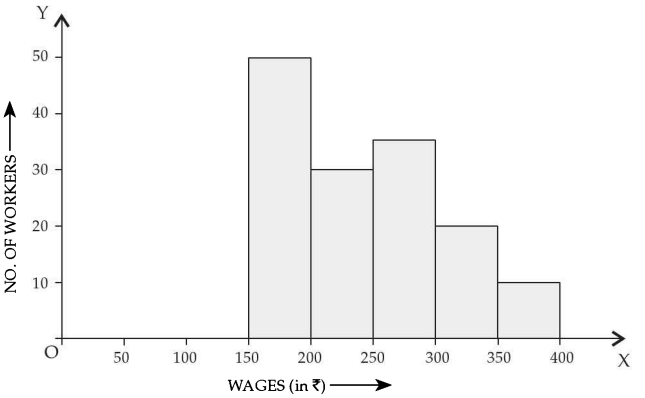
The required histogram is shown in the adjoining figure:



Exercise 14.3

1. In the adjoining figure, there is a histogram depicting daily wages of workers in a factory. Construct the frequency distribution table.

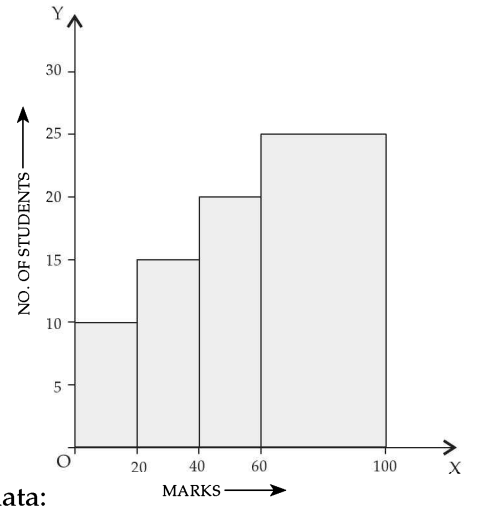
(*NCERT Exemplar*)



2. The frequency distribution has been represented graphically (shown in the adjoining figure). Do you think this representation is correct? Why?

Marks	*Number of students*
0 – 20	10
20 – 40	15
40 – 60	20
60 – 100	25

(*NCERT Exemplar*)



3. Draw a histogram to represent the following data:

Marks obtained	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
No. of students	4	10	6	8	5	9

4. Draw a histogram to represent the following grouped frequency distribution:

Age (in years)	20 – 24	25 – 29	30 – 34	35 – 39	40 – 44	45 – 49	50 – 54
No. of teachers	10	28	32	48	50	35	12

(*NCERT Exemplar*)

5. 100 surnames were randomly picked up from a local telephone directory and a frequency distribution of the number of letters in the English alphabet in the surnames was found as follows:

Number of letters	1 – 4	4 – 6	6 – 8	8 – 12	12 – 20
No. of surnames	6	30	44	16	4

(*i*) Draw a histogram to depict the given information.

(*ii*) Write the class interval in which the maximum number of surnames lie. (*NCERT*)

Answers

14.6 FREQUENCY POLYGON

Usually, in continuous frequency distributions, the frequencies are represented along y-axis and class intervals along x-axis. Find the mid-points (or class marks) of class intervals. Plot the points on the graph paper representing frequencies of different classes against the corresponding mid-points of class intervals. On joining consecutive points by line segments, we get *frequency graph (curve)*. In other words, on joining the mid-points of the upper bases of

the adjacent rectangles of a histogram we get frequency curve. If the end points of *frequency graph* are joined to the mid-points of immediate lower and upper class intervals outside the range with zero frequency, we get a closed figure. The figure so obtained is called a ***frequency polygon.***

Thus, a ***frequency polygon*** *is a closed curve representing continuous grouped frequency distribution.*

Histogram and frequency polygon can be constructed simultaneously.

Illustrative Examples

Example 1. *Draw a frequency polygon to represent the following data:*

Class interval	*20 – 30*	*30 – 40*	*40 – 50*	*50 – 60*	*60 – 70*	*70 – 80*
Frequency	*5*	*10*	*19*	*24*	*18*	*6*

Solution. We find the class marks and prepare a new table as follows:

Classes	*Classes marks*	*Frequency*
20 – 30	25	5
30 – 40	35	10
40 – 50	45	19
50 – 60	55	24
60 – 70	65	18
70 – 80	75	6

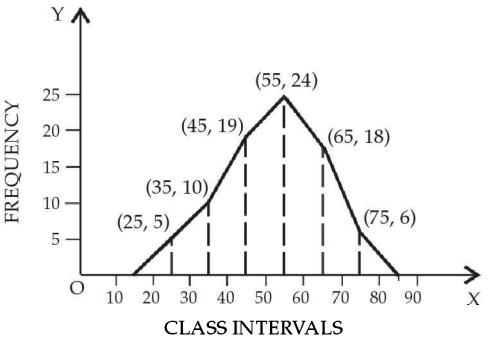
Steps to draw frequency polygon:

(*i*) Take 1 cm on x-axis = 10 units and 1 cm on y-axis = 5 frequencies.

(*ii*) Plot the points (25, 5), (35, 10), (45, 19), (55, 24), (65, 18) and (75, 6)

(*iii*) Join the consecutive points by line segments.

(*iv*) Join the first end-point with mid-point of class 10 – 20 with zero frequency, and join the other end point with the mid-point of class 80 – 90 with zero frequency.



The required frequency polygon (thick line segments) has been shown alongside.

Example 2. *The marks obtained (out of 100) by 51 students of a class are as under:*

Marks	*Number of students*
0 – 10	*5*
10 – 20	*10*
20 – 30	*4*
30 – 40	*6*
40 – 50	*7*
50 – 60	*3*
60 – 70	*2*
70 – 80	*2*
80 – 90	*3*
90 – 100	*9*

Draw a frequency polygon corresponding to the above frequency distribution. *(NCERT)*

Solution. We find the class marks and prepare a new table as follows:

Classes	*Class marks*	*Frequency*
0 – 10	5	5
10 – 20	15	10
20 – 30	25	4
30 – 40	35	6
40 – 50	45	7
50 – 60	55	3
60 – 70	65	2
70 – 80	75	2
80 – 90	85	3
90 – 100	95	9

Steps to draw frequency polygon:

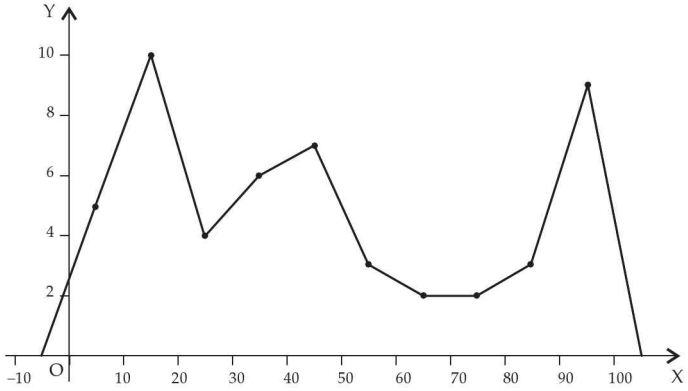
(*i*) Take 1 cm on x-axis = 10 marks and 1 cm on y-axis = 2 students.

(*ii*) Plot the points (5, 5), (15, 10), (25, 4), (35, 6), (45, 7), (55, 3), (65, 2), (75, 2), (85, 3) and (95, 9).

(*iii*) Join the consecutive points by line segments

(*iv*) Join the first end point with mid-point of class (–10) – 0 with zero frequency and the last end point with mid-point of class 100 – 110 with zero frequency.

The required frequency polygon is shown alongside.



Example 3. *100 students in a school have heights as tabulated below:*

Height (in cm)	*121–130*	*131–140*	*141–150*	*151–160*	*161–170*	*171–180*
No. of students	*12*	*16*	*30*	*20*	*14*	*8*

Draw frequency polygon of the above data.

Solution. The given frequency distribution is discontinuous. To convert it into a continuous distribution,

$$\text{adjustment factor} = \frac{\text{lower limit of one class} - \text{upper limit of previous class}}{2}$$

$$= \frac{131 - 130}{2} = \frac{1}{2} = 0.5.$$

Also, find the class marks and prepare a table as under:

Classes before adjustment	*Classes after adjustment*	*Class marks*	*Frequency*
121 – 130	120.5 – 130.5	125.5	12
131 – 140	130.5 – 140.5	135.5	16
141 – 150	140.5 – 150.5	145.5	30
151 – 160	150.5 – 160.5	155.5	20
161 – 170	160.5 – 170.5	165.5	14
171 – 180	170.5 – 180.5	175.5	8

Steps to draw frequency polygon:

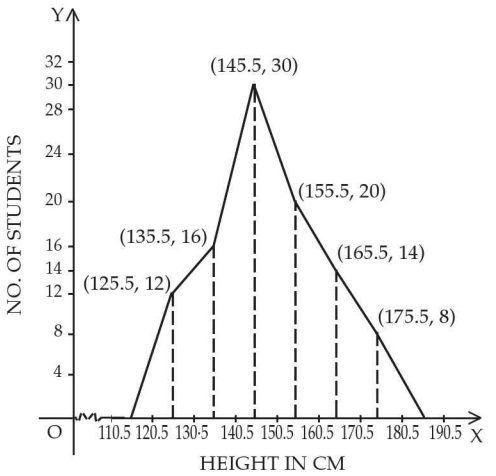
(*i*) Since the scale on x-axis starts at 110.5, a kink is shown near the origin on x-axis to indicate that the graph is drawn to scale beginning at 110.5.

(*ii*) Take 1 cm along x-axis = 10 cm (height).

(*iii*) Take 1 cm along y-axis = 4 (no. of students).

(*iv*) Plot the points (125.5, 12), (135.5, 16), (145.5, 30), (155.5, 20), (165.5, 14) and (175.5, 8).

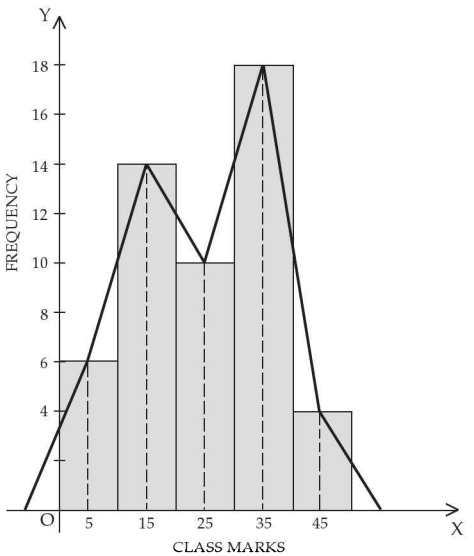
(*v*) Join consecutive points by line segments.



(*vi*) Join first end point with mid-point of class 110.5 – 120.5 with zero frequency and join the other end with mid-point of class 180.5 – 190.5 with zero frequency.

The required frequency polygon is shown by thick line segment alongside.

Example 4. *The histogram and the frequency polygon of a frequency distribution is shown alongside.*



Answer the following questions about the frequency distribution:

(i) What is the class size of each class?

(ii) What is the frequency of the class whose class mark is 25?

(iii) What is the class whose class mark is 15?

(iv) What is the class whose frequency is 18?

(v) Construct the frequency table for the given distribution.

Solution. (*i*) As the class marks are 5, 15, 25, 35 and 45 which are at equal gaps, so the classes are of equal size.

$\therefore$ Class size of each class = difference between two successive class marks
= 15 – 5 = 10.

(*ii*) 10

(*iii*) As the size of each class is 10 and the class mark is 15. So the class limits are 15 – 5 and 15 + 5 *i.e.* 10 and 20.

Hence, the class is 10 – 20.

(*iv*) The class whose frequency is 18 is 30 – 40.

(*v*) The frequency distribution table is given below:

Classes	0–10	10–20	20–30	30–40	40–50
Frequency	6	14	10	18	4

Example 5. *The following table gives the distribution of students of two sections according to the marks obtained by them:*

Section A		*Section B*	
Marks	*Frequency*	*Marks*	*Frequency*
0 – 10	*3*	*0 – 10*	*5*
10 – 20	*9*	*10 – 20*	*19*
20 – 30	*17*	*20 – 30*	*15*
30 – 40	*12*	*30 – 40*	*10*
40 – 50	*9*	*40 – 50*	*1*

Represent the marks of the students of both the sections on the same graph by two frequency polygons. From the two polygons compare the performance of the two sections. (*NCERT*)

Solution. We find the class marks and prepare a new table as under:

Class	*Class marks*	*Section A (No. of students)*	*Section B (No. of students)*
0 – 10	5	3	5
10 – 20	15	9	19
20 – 30	25	17	15
30 – 40	35	12	10
40 – 50	45	9	1

Step to draw frequency polygon:

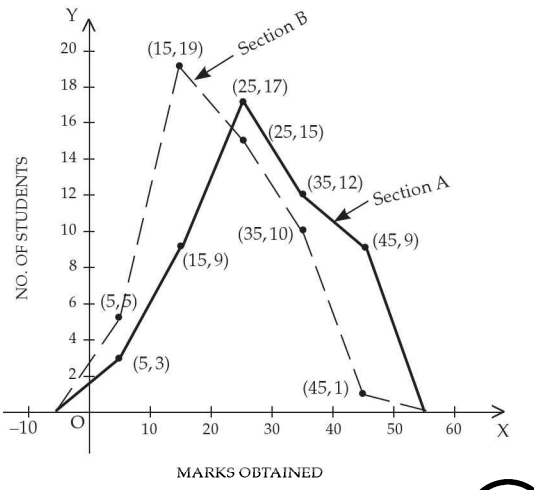
Take 1 cm on x-axis = 5 marks and 1 cm on y-axis = 2 students

For Section A

Plot the points (5, 3), (15, 9), (25, 17), (35, 12) and (45, 9).

Join the consecutive points by thick line segments.

Join the first end point with the mid-point of class (–10) – 0 with zero frequency, and join the other end point with the mid-point of class 50 – 60 with zero frequency. The required frequency polygon is shown by thick line segments in the adjoining figure.



For Section B

Plot the points (5, 5), (15, 19), (25, 15), (35, 10) and (45, 1).

Join the consecutive points with dotted line segments.

Join the first end point with the mid-point of class (–10) – 0 with zero frequency, and join the other end point with the mid-point of class 50 – 60 with zero frequency. The required frequency polygon is shown by dotted line segments in the same figure.

Performance of section A is better than performance of section B.

Exercise 14.4

1. In a city, the weekly observations made in a study on the cost of living index are given in the following table:

Cost of living index	*Number of weeks*
140 – 150	5
150 – 160	10
160 – 170	20
170 – 180	9
180 – 190	6
190 – 200	2
Total	52

Draw a frequency polygon for the data given above.

2. Following table shows a frequency distribution for the speed of cars passing through at a particular spot on a highway:

Class intervals (km/h)	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100
Frequency	3	6	25	65	50	28	14

Draw a histogram and frequency polygon representing the above data.

(*NCERT Exemplar*)

3. Following table gives the distribution of students of sections A and B of a class according to the marks obtained by them:

Section A		*Section B*	
Marks	*Frequency*	*Marks*	*Frequency*
0 – 15	5	0 – 15	3
15 – 30	12	15 – 30	16
30 – 45	28	30 – 45	25
45 – 60	30	45 – 60	27
60 – 75	35	60 – 75	40
75 – 90	13	75 – 90	10

Represent the marks of the students of both the sections on the same graph by two frequency polygons. What do you observe? (*NCERT Exemplar*)

Answers

14.7 MEASURES OF CENTRAL TENDENCY

❑ **Central tendency**

Often, we observe that every statistical data has a tendency to cluster around a certain value, possibly at the centre of observations. This tendency of the statistical data is known as its *central tendency.*

❑ **Measures of central tendency**

A numerical value which represents (approximately) the entire statistical data is called a *measure of central tendency* of that data.

The numerical value which represents the entire statistical data is neither the lowest nor the highest value in the data, rather it lies in between the two extreme values of the data.

The different ways of measuring central tendency of a statistical data are:

(*i*) ***Mean*** (*ii*) ***Median*** (*iii*) ***Mode.***

❑ **Mean of ungrouped data**

Mean (or ***arithmetic average***) *of a number of observations is the sum of the values of all the observations divided by the total number of observations.*

The ***mean*** *of n observations (variates)* $x_1, x_2, x_3, \ldots, x_n$ *is given by the formula:*

$$\textbf{Mean} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n} = \frac{\Sigma x_i}{n}$$

where $\Sigma x_i = x_1 + x_2 + x_3 + \ldots + x_n$.

Thus, $\text{mean} = \dfrac{\text{sum of all observations}}{\text{total number of observations}}$.

If the variates $x_1, x_2, x_3, \ldots, x_n$ have frequencies $f_1, f_2, f_3, \ldots, f_n$ respectively, then the mean of ungrouped frequency distribution is given by

$$\textbf{Mean} = \frac{f_1x_1 + f_2x_2 + \ldots + f_nx_n}{f_1 + f_2 + \ldots + f_n} = \frac{\Sigma f_i x_i}{\Sigma f_i}.$$

Note. The Greek letter Σ (read as sigma) represents the sum.

❑ **Median of ungrouped data**

Median *is the central value (or middle observation) of a statistical data if it is arranged in ascending or descending order.*

Thus, if there are n observations (variates) $x_1, x_2, x_3, \ldots, x_n$ arranged in ascending or descending order, then

$$\textbf{Median} = \begin{cases} \dfrac{n+1}{2}\textbf{th observation, if } n \textbf{ is odd} \\ \dfrac{\frac{n}{2}\textbf{th observation} + \left(\frac{n}{2}+1\right)\textbf{th observation}}{2}, \textbf{if } n \textbf{ is even.} \end{cases}$$

❑ **Mode of ungrouped data**

Mode (*or* ***modal value)*** *of an ungrouped data is the variate (observation) which occurs most frequently. In other words,* ***mode*** *is the variate (observation) which has maximum frequency.*

Illustrative Examples

Example 1. *The marks obtained by 12 students in a monthly test are 11, 19, 07, 13, 18, 21, 09, 05, 20, 17, 16, 21. Find:*

(i) the mean of their marks.

(ii) the mean of their marks when the marks of each student are increased by 3.

Solution. (*i*) The sum of the marks of all students

$= 11 + 19 + 07 + 13 + 18 + 21 + 09 + 05 + 20 + 17 + 16 + 21 = 177.$

$\therefore$ The mean of marks $= \frac{\Sigma x_i}{n} = \frac{\text{sum of marks of all students}}{\text{number of students}}$

$= \frac{177}{12} = \frac{59}{4} = 14.75.$

(*ii*) When the marks of each student are increased by 3, then the sum of their marks increases by 12×3 *i.e.* by 36.

$\therefore$ The new sum of marks of all students $= 177 + 36 = 213.$

$\therefore$ The new mean of marks $= \frac{\text{New sum of marks}}{\text{Number of students}} = \frac{213}{12} = \frac{71}{4} = 17.75.$

Note that the new mean of marks also increases by 3.

Example 2. *The mean of 7 variates is 12. If six of them are 5, 13, 9, 17, 14 and 10, then find the seventh variate.*

Solution. Let the seventh variate be x. By definition,

$$\text{mean} = \frac{\text{sum of variates}}{\text{number of variates}}$$

$$\Rightarrow \quad 12 = \frac{5 + 13 + 9 + 17 + 14 + 10 + x}{7}$$

$$\Rightarrow \quad 12 = \frac{68 + x}{7} \Rightarrow 68 + x = 12 \times 7$$

$$\Rightarrow \quad x = 84 - 68 = 16.$$

Hence, the seventh variate is 16.

Example 3. *There are 50 numbers. If each number is subtracted from 53, then the mean of the numbers so obtained is – 3.5. Find the mean of the given numbers.* *(NCERT Exemplar)*

Solution. Let the given 50 numbers be $x_1, x_2, x_3, \ldots, x_{50}$.

When each number is subtracted from 53, the numbers so obtained are $53 - x_1$, $53 - x_2$, $53 - x_3$, …, $53 - x_5$.

Mean of these numbers $= \frac{\sum_{i=1}^{50} (53 - x_i)}{50} = -3.5$ (given)

$$\Rightarrow \quad 53 \times 50 - \sum_{i=1}^{50} x_i = -3.5 \times 50$$

$$\Rightarrow \quad 53 \times 50 + 3.5 \times 50 = \sum_{i=1}^{50} x_i$$

$$\Rightarrow \quad 56.5 \times 50 = \sum_{i=1}^{50} x_i$$

$$\Rightarrow \quad \frac{\sum_{i=1}^{50} x_i}{50} = 56.5 \Rightarrow \text{mean of given numbers} = 56.5.$$

Example 4. *The mean height of 8 students is 152 cm. Two more students of heights 143 cm and 156 cm join the group. What is the new mean height?*

Solution. Mean height of 8 students $= \frac{\text{sum of heights of 8 students}}{8}$,

$\therefore$ 152 cm $= \frac{\text{sum of heights of 8 students}}{8}$

$\Rightarrow$ sum of heights of 8 students $= (152 \times 8)$ cm $= 1216$ cm.

Now two more students of heights 143 cm and 156 cm join the group.

$\therefore$ Sum of heights of 10 students $= (1216 + 143 + 156)$ cm $= 1515$ cm.

$\therefore$ New mean height $= \frac{\text{sum of heights of 10 students}}{10}$

$= \frac{1515}{10}$ cm $= 151.5$ cm.

Example 5. *A class consists of 50 students out of which 30 are girls. The mean of marks scored by girls in a test is 73 (out of 100) and that of boys is 71. Determine the mean score of the whole class.* *(NCERT Exemplar)*

Solution. Total number of students in the class = 50 and number of girls = 30, so the number of boys in the class = 50 – 30 = 20.

As the mean of marks scored by 30 girls is 73,

$\therefore$ sum of marks scored by all 30 girls = 73 × 30 = 2190.

Also the mean of marks scored by 20 boys is 71,

$\therefore$ the sum of marks scored by 20 boys = 71 × 20 = 1420.

$\therefore$ Sum of marks scored by all 50 students of the class = 2190 + 1420 = 3610.

$\therefore$ Mean score of the whole class $= \frac{\text{sum of marks}}{\text{number of students}} = \frac{3610}{50} = 72.2$

Example 6. *Mean of 50 observations was found to be 80.4. But later on, it was discovered that 96 was misread as 69 at one place. Find the correct mean.* *(NCERT Exemplar)*

Solution. Mean of observations $= \frac{\text{Incorrect sum of 50 observations}}{50} = 80.4$ (given)

$\Rightarrow$ Incorrect sum of observations = 80.4 × 50 = 4020.

As one observation was misread as 69 instead of 96,

correct sum of marks = 4020 – 69 + 96 = 4047.

$\therefore$ Correct mean $= \frac{4047}{50} = 80.94$

Example 7. *The mean marks (out of 100) of boys and girls in an examination are 70 and 73 respectively. If the mean marks of all the students in that examination are 71, find the ratio of number of boys to the number of girls.* *(NCERT Exemplar)*

Solution. Let the number of boys be x and that of girls be y.

As the mean of marks scored by boys is 70,

$\therefore$ the sum of marks scored by x boys = $70x$.

Also the mean of marks scored by girls is 73,

$\therefore$ the sum of marks scored by y girls = $73y$.

$\therefore$ the sum of marks scored by all $(x + y)$ students = $70x + 73y$.

Since the mean of marks scored by all students is 71,

$\therefore \quad \frac{70x + 73y}{x + y} = 71 \Rightarrow 70x + 73y = 71x + 71y$

$\Rightarrow \quad 2y = x \Rightarrow \frac{x}{y} = \frac{2}{1}$.

Hence, the ratio of number of boys to that of girls = 2 : 1.

Example 8. *The mean of 25 observations is 36. If the mean of first 13 observations is 32 and that of the last 13 observations is 40, then find the 13th observation.* *(NCERT Exemplar)*

Solution. Let the given 25 observations be $x_1, x_2, \ldots, x_{13}, \ldots, x_{25}$

As the mean of these 25 observations is 36,

$\therefore$ sum of these observations = 36 × 25 = 900

i.e. $\sum_{i=1}^{25} x_i = 900$ ...(*i*)

The mean of first 13 observations is 32,

$\therefore \quad \sum_{i=1}^{13} x_i = 32 \times 13 = 416$ ...(*ii*)

Also the mean of last 13 observations is 40,

$\therefore \quad \sum_{i=13}^{25} x_i = 40 \times 13 = 520$ ...(*iii*)

From (*ii*) and (*iii*), we get

$$\sum_{i=1}^{13} x_i + \sum_{i=13}^{25} x_i = 416 + 520$$

$\Rightarrow \quad \sum_{i=1}^{25} x_i + x_{13} = 936$

$\Rightarrow \quad 900 + x_3 = 936$ (using (*i*))

$\Rightarrow \quad x_{13} = 36$

Hence, the 13th observation = 36.

Example 9. *Find the mean salary of 60 workers of a factory from the following table:*

Salary (in ₹)	*3000*	*4000*	*5000*	*6000*	*7000*	*8000*	*9000*	*10000*
No. of workers	*16*	*12*	*10*	*8*	*6*	*4*	*3*	*1*

Solution. To find the mean salary, we construct the following table: *(NCERT)*

x_i	f_i	$f_i x_i$
3000	16	48000
4000	12	48000
5000	10	50000
6000	8	48000
7000	6	42000
8000	4	32000
9000	3	27000
10000	1	10000
Total	60	305000

Mean $= \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{305000}{60} = 5083.33.$

Hence, the mean salary = ₹ 5083.33.

Example 10. *The mean of the following distribution is 50.*

x	*10*	*30*	*50*	*70*	*90*
f	*17*	*5a + 3*	*32*	*7a – 11*	*19*

Find the value of a and hence the frequencies of 30 and 70. *(NCERT Exemplar)*

Solution. We construct the following table:

x_i	f_i	$f_i x_i$
10	17	170
30	$5a + 3$	$150a + 90$
50	32	1600
70	$7a - 11$	$490a - 770$
90	19	1710
Total	$12a + 60$	$640a + 2800$

$\therefore$ Mean $= \dfrac{\Sigma f_i x_i}{\Sigma f_i} = \dfrac{640a + 2800}{12a + 60} = 50$ (given)

$\Rightarrow$ $640a + 2800 = 600a + 3000$

$\Rightarrow$ $40a = 200 \Rightarrow a = 5$.

Hence, the value of $a = 5$ and the frequency of $30 = 5a + 3 = 5 \times 5 + 3 = 28$ and the frequency of $70 = 7a - 11 = 7 \times 5 - 11 = 24$.

Example 11. *If $\bar{x}$ is the mean of $x_1, x_2, x_3, \ldots, x_n$, then find the mean of $ax_1, ax_2, \ldots, ax_n$, $\frac{x_1}{a}, \frac{x_2}{a}, \ldots\ldots, \frac{x_n}{a}$, $a \neq 0$.* *(NCERT Exemplar)*

Solution. As $\bar{x}$ is the mean of $x_1, x_2, x_3, \ldots, x_n$,

$$\bar{x} = \frac{\Sigma x_i}{n} \Rightarrow \Sigma x_i = n\bar{x} \qquad \ldots(i)$$

Now, mean of $ax_1, ax_2, \ldots, ax_n, \frac{x_1}{a}, \frac{x_2}{a}, \ldots\ldots, \frac{x_n}{a}$

$$= \frac{(ax_1 + ax_2 + \ldots\ldots + ax_n) + \left(\frac{x_1}{a} + \frac{x_2}{a} + \frac{x_3}{a} + \ldots. + \frac{x_n}{a}\right)}{2n}$$

$$= \frac{\left(a + \frac{1}{a}\right)(x_1 + x_2 + x_3 + \ldots\ldots + x_n)}{2n} = \frac{\left(a + \frac{1}{a}\right) \times n\bar{x}}{2n} \qquad \text{(using } (i)\text{)}$$

$$= \left(a + \frac{1}{a}\right)\frac{\bar{x}}{2}.$$

Example 12. *Find the median of the following data:*

5, 3, 12, 0, 7, 11, 4, 3, 8.

Solution. Arranging the given data in ascending order, we get

0, 3, 3, 4, 5, 7, 8, 11, 12.

Total number of observations $= n = 9$, which is odd.

$\therefore$ Median $= \dfrac{n+1}{2}$ th observation $= \dfrac{9+1}{2}$ th observation

$=$ 5th observation, which is 5.

Hence, median = 5.

Example 13. *The number of goals scored by a football team in a series of matches are:*

3, 1, 0, 7, 5, 3, 3, 4, 1, 2, 0, 2.

Find the median of this data.

Solution. Arranging the number of goals scored by the team in ascending order, we get

0, 0, 1, 1, 2, 2, 3, 3, 3, 4, 5, 7.

Total number of observations (matches played) $= n = 12$, which is even.

$$\therefore \text{ Median } = \frac{\frac{n}{2}\text{th observation} + \left(\frac{n}{2} + 1\right)\text{th observation}}{2}$$

$$= \frac{6\text{th observation} + 7\text{th observation}}{2} = \frac{2+3}{2} = \frac{5}{2} = 2.5.$$

Example 14. *The following observations have been arranged in ascending order. If the median of the data is 63, find the value of x:*

29, 32, 48, 50, x, x + 2, 72, 78, 84, 95 *(NCERT)*

Solution. The numbers 29, 32, 48, 50, x, $x + 2$, 72, 78, 84, 95 are in ascending order.

Total number of observations $= n = 10$, which is even.

$$\therefore \quad \text{Median} = \frac{\frac{n}{2}\text{th observation} + \left(\frac{n}{2}+1\right)\text{th observation}}{2}$$

$$= \frac{5\text{th observation} + 6\text{th observation}}{2} = \frac{x + (x+2)}{2} = \frac{2x+2}{2} = x + 1.$$

According to given, median = 63

$\Rightarrow \quad x + 1 = 63 \Rightarrow x = 62.$

Hence, the value of x is 62.

Example 15. *Find the mode of the data: 3, 5, 1, 2, 4, 6, 0, 2, 1, 2, 3.*

Solution. In the given data, 2 is repeated more number of times than any other number.

$\therefore$ mode = 2.

Example 16. *Find the mode for the following frequency distribution:*

Marks obtained (out of 10)	*0*	*2*	*3*	*4*	*6*	*7*	*9*	*10*
Number of students	*3*	*5*	*12*	*18*	*21*	*8*	*2*	*6*

Solution. In the given distribution, the variate 6 has the maximum frequency.

$\therefore$ mode = 6.

Example 17. *The points scored by a basket ball team in a series of matches are as follows:*

17, 2, 7, 27, 25, 5, 14, 18, 10, 24, 48, 10, 8, 7, 10, 28

Find the median and mode for the data. *(NCERT Exemplar)*

Solution. The given numbers in ascending order are:

2, 5, 7, 7, 8, 10, 10, 10, 14, 17, 18, 24, 25, 27, 28, 48

Total number of observations = n = 16, which is even.

$$\therefore \quad \text{Median} = \frac{\frac{n}{2}\text{th observation} + \left(\frac{n}{2}+1\right)\text{th observation}}{2}$$

$$= \frac{8\text{th observation} + 9\text{th observation}}{2} = \frac{10+14}{2} = 12.$$

In the given data, 10 is repeated more number of times than any other number,

$\therefore$ mode = 10.

Exercise 14.5

1. 5 people were asked about the time in a week they spend in doing social work in their community. They said 10, 7, 13, 20 and 15 hours respectively. Find the mean time in a week they devoted for social work. *(NCERT)*
2. The enrolment of a school during six consecutive years was as follows:

   1620, 2060, 2540, 3250, 3500, 3710

   Find the mean enrolment.
3. Find the mean of the first twelve natural numbers.
4. Find the mean of the first six prime numbers.
5. The mean of 5 numbers is 20. If one number is excluded, mean of the remaining numbers becomes 23. Find the excluded number.
6. The mean of 25 observations is 27. If one observation is included, the mean still remains 27. Find the included observation.
7. The mean of 8 variates is 10.5. If seven of them are 3, 15, 7, 19, 2, 17 and 8, then find the 8th variate.

8. The mean weight of 8 students is 45.5 kg. Two more students having weights 41.7 kg and 53.3 kg join the group. What is the new mean weight?
9. Mean of 9 observations was found to be 35. Later on, it was detected that an observation 81 was misread as 18. Find the correct mean of the observations.
10. Obtain the mean of the following distribution:

Variable	4	6	8	10	12
Frequency	4	8	14	11	3

(NCERT Exemplar)

11. Six coins were tossed 1000 times, and at each toss the number of heads were counted and the results were recorded as under:

No. of heads	6	5	4	3	2	1	0
No. of tosses	20	25	160	283	338	140	34

Calculate the mean for this distribution.

12. If the mean of the following data is 20.2, find the value of p:

x	10	15	20	25	30
f	6	8	p	10	6

(NCERT Exemplar)

13. The points scored by kabaddi team in a series of matches are as follows:
17, 2, 7, 27, 15, 5, 14, 8, 10, 24, 48, 10, 8, 7, 18, 28
Find the median of the points scored by the team. *(NCERT)*
14. If the numbers 3, 6, 7, 10, x, 15, 19, 20, 25, 28 are in ascending order and their median is 13, then calculate the value of x.
15. Ten observations 6, 14, 15, 17, $x + 1$, $2x - 13$, 30, 32, 34, 43 are written in an ascending order. If the median of the data is 24, find the value of x. *(NCERT Exemplar)*
16. Find the mode of 14, 25, 14, 28, 18, 17, 18, 14, 23, 22, 14, 18. *(NCERT)*
17. The following number of goals were scored by a team in a series of 10 matches:
2, 3, 4, 5, 0, 1, 3, 3, 4, 3
Find the mean, median and mode of these scores. *(NCERT)*
18. In a mathematics test given to 15 students, the following marks (out of 100) are recorded:
41, 39, 48, 52, 46, 62, 54, 40, 96, 52, 98, 40, 42, 52, 60
Find the mean, median and mode of this data. *(NCERT)*
19. A child says that the median of 3, 14, 18, 20, 5 is 18. What doesn't the child understand about finding the median? *(NCERT Exemplar)*
20. A football player scored the following number of goals in the 10 matches:
1, 3, 2, 5, 8, 6, 1, 4, 7, 9
Since the number of matches is 10 (even number), therefore,

$$\text{median} = \frac{5\text{th observation} + 6\text{th observation}}{2} = \frac{8 + 6}{2} = 7.$$

Is it the correct answer and why? *(NCERT Exemplar)*

Answers

Very Short Answer Type Questions

1. If the class marks of a grouped frequency distribution are 24, 32, 40, 48, 56, 64, 72, then find the class size of the data.
2. Noori scored 14, 16, 13 and 19 marks (out of 20) in Mathematics in weekly unit tests of a month. Find the mean of marks scored by her.
3. Find the arithmetic mean of first six natural numbers.
4. Find the mean of first seven multiples of 4.
5. If the mean of 8 given numbers is 35, then find their sum.
6. Find the mean of the prime factors of 24.
7. Find the mean of the (positive) factors of 12.
8. If the mean of 3, 5, 0, 9, x, 7 and 13 is 7, then find the value of x.
9. The mean of 8 observations is 25. If an observation 18 is removed, then find the mean of the remaining observations.
10. If the mean of six observations is 10.5 and five of them are 13, 5, 7, 9 and 14, then find the sixth observation.

Answers

Objective Type Questions

(FOR FORMATIVE/SUMMATIVE ASSESSMENT)

Mental Maths

1. Fill in the blanks:
   (*i*) The information collected by the investigator herself or himself is called ..... data.
   (*ii*) The information gathered from a source which already had the information stored is called ..... data.
   (*iii*) A quantity which is being measured in an experiment or survey is called a ..... .
   (*iv*) A variable which can take any value between two given values is called a ..... variable.
   (*v*) The ..... between the maximum value and minimum value of a variable is called its range.
   (*vi*) The number of times an observation occurs in a given data is called the ..... of that observation.
   (*vii*) The difference between the actual upper limit and the actual lower limit of a class is called its ..... .
   (*viii*) The class mark of the class 16 – 24 is ..... .
   (*ix*) Mean of a number of observations is the sum of the values of all the observations divided by the ..... of observations.
   (*x*) If there are n observations arranged in an ascending order, then median = ..., if n is odd.
   (*xi*) If there are n observations arranged in descending order, then median = ..., if n is even.
   (*xii*) If the classes are of equal size, then class size = difference between two ..... class marks.

Multiple Choice Questions

Choose the correct answer from the given four options (2 to 24):

2. The marks obtained by 17 students in a mathematics test (out of 100) are given below:

91, 82, 100, 100, 96, 65, 82, 76, 79, 90, 46, 64, 72, 66, 68, 48, 49

The range of the data is

(*a*) 46 (*b*) 54 (*c*) 90 (*d*) 100

(NCERT Exemplar)

3. The class mark of the class 90–120 is

(*a*) 90 (*b*) 105 (*c*) 115 (*d*) 120

(NCERT Exemplar)

4. In a frequency distribution, the mid-value of a class is 10 and the width of the class is 6. The lower limit of the class is

(*a*) 6 (*b*) 7 (*c*) 8 (*d*) 12

(NCERT Exemplar)

5. The width of each of 5 continuous classes in a frequency distribution is 5 and the lower limit of the lowest class is 10. The upper limit of the highest class is

(*a*) 15 (*b*) 25 (*c*) 35 (*d*) 40

(NCERT Exemplar)

6. The class marks of a frequency distribution are given as follows:

15, 20, 25, ...

The class corresponding to the class mark 20 is

(*a*) 12.5 – 17.5 (*b*) 17.5 – 22.5 (*c*) 18.5 – 21.5 (*d*) 19.5 – 20.5

(NCERT Exemplar)

7. In the class intervals 10 – 20, 20 – 30, the number 20 is included in

(*a*) 10 – 20 (*b*) 20 – 30

(*c*) both the intervals (*d*) none of these intervals

(NCERT Exemplar)

8. A grouped frequency distribution table with class intervals of equal size using 250 – 270 (270 not included in this interval) as one of the class intervals is constructed for the following data:

268, 220, 368, 258, 242, 310, 272, 342, 310, 290, 300, 320, 319, 304, 402, 318, 406, 292, 354, 278, 210, 240, 330, 316, 406, 215, 258, 236.

The frequency of the class 310 – 330 is

(*a*) 4 (*b*) 5 (*c*) 6 (*d*) 7

(NCERT Exemplar)

9. A grouped frequency distribution table with classes of equal size using 63 – 72 (72 included) as one of the class is constructed for the following data:

30, 32, 45, 54, 74, 78, 108, 112, 66, 76, 88, 40, 14, 20, 15, 35, 44, 66, 75, 84, 95, 96, 102, 110, 88, 74, 112, 14, 34, 44

The number of classes in the distribution will be

(*a*) 9 (*b*) 10 (*c*) 11 (*d*) 12

(NCERT Exemplar)

10. The mean of $x - 1$, $x + 1$, $x + 3$ and $x + 5$ is

(*a*) $x + 1$ (*b*) $x + 2$ (*c*) $x + 3$ (*d*) $x + 4$

11. The mean of five numbers is 30. If one number is excluded, their mean becomes 28. The excluded number is

(*a*) 28 (*b*) 30 (*c*) 35 (*d*) 38

(NCERT Exemplar)

12. If the mean of x_1, x_2 is 7.5, and the mean of x_1, x_2, x_3 is 8, then the value of x_3 is

(*a*) 9 (*b*) 8 (*c*) 7.5 (*d*) 6

13. If the mean of the observations x, $x + 3$, $x + 5$, $x + 7$, $x + 10$ is 9, then the mean of the last three observations is

(*a*) $10\frac{1}{3}$ (*b*) $10\frac{2}{3}$ (*c*) $11\frac{1}{3}$ (*d*) $11\frac{2}{3}$

(NCERT Exemplar)

14. If each observation of the data is increased by 5, then their mean

(*a*) remains the same (*b*) becomes 5 times the original mean

(*c*) is decreased by 5 (*d*) is increased by 5 *(NCERT Exemplar)*

15. If 6, 4 and 9 occur with frequencies 5, 3 and 2 respectively, then the mean is

(*a*) 4 (*b*) 5 (*c*) 6 (*d*) 7.5

16. The mean of 100 observations is 50. If one of the observation which was 50 is replaced by 150, the resulting mean will be

(*a*) 50.5 (*b*) 51 (*c*) 51.5 (*d*) 52

(NCERT Exemplar)

17. If $\bar{x}$ is the mean of n observations x_1, x_2, x_3, ..., x_n then the value of $\sum_{i=1}^{n}(x_i - \bar{x})$ is

(*a*) –1 (*b*) 0 (*c*) 1 (*d*) $n - 1$

(NCERT Exemplar)

18. If $\bar{x}$ is the mean of x_1, x_2, ..., x_n, $\bar{y}$ is the mean of y_1, y_2, ..., y_n and $\bar{z}$ is the mean of x_1, x_2, ..., x_n, y_1, y_2, ..., y_n, then $\bar{z}$ is equal to

(*a*) $\bar{x} + \bar{y}$ (*b*) $\frac{\bar{x} + \bar{y}}{2}$ (*c*) $\frac{\bar{x} + \bar{y}}{n}$ (*d*) $\frac{\bar{x} + \bar{y}}{2n}$

(NCERT Exemplar)

19. For drawing a frequency polygon of a continuous frequency distribution, we plot the points whose ordinates the frequencies of the respective classes and abscissae are respectively:

(*a*) upper limits of the classes (*b*) lower limits of the classes

(*c*) class marks of the classes (*d*) upper limits of preceding classes

(NCERT Exemplar)

20. Median of the numbers 4, 4, 5, 7, 6, 7, 7, 3, 12 is

(*a*) 4 (*b*) 5 (*c*) 6 (*d*) 7

(NCERT Exemplar)

21. The median of the data

78, 56, 22, 34, 45, 54, 39, 68, 54, 84 is

(*a*) 45 (*b*) 49.5 (*c*) 54 (*d*) 56

(NCERT Exemplar)

22. Mode of the data

15, 14, 19, 20, 14, 15, 16, 14, 15, 18, 14, 19, 15, 17, 15 is

(*a*) 14 (*b*) 15 (*c*) 16 (*d*) 17

(NCERT Exemplar)

23. In the following frequency distribution:

x	0	2	3	5	6	9	10
f	3	7	6	21	18	15	3

The mode is

(*a*) 21 (*b*) 5 (*c*) 10 (*d*) 15

24. In a data, 10 numbers are arranged in ascending order. If the 8th entry is increased by 6, then the median increases by

(*a*) 0 (*b*) 2 (*c*) 3 (*d*) 6

Answers

Value Based Questions

1. A family with a monthly income of ₹20000 had planned the following expenditure per month under various heads:

Heads	*Expenditure (in thousand rupees)*
Grocery	4
Rent	5
Education of children	5
Medicine	2
Fuel	2
Entertainment	1
Miscellaneous	1

Draw a bar graph for the above data.

What value will be added by increasing the expenditure on education?

2. Ashok gives a scholarship of ₹4000 per month to a poor intelligent student from December to March and of ₹2500 per month from April to November in a year. Find the mean value of scholarship.

What value is depicted by Ashok?

Answers

Higher Order Thinking Skills (HOTS)

1. Draw a histogram for the following data:

Class marks	12.5	17.5	22.5	27.5	32.5	37.5
Frequency	7	12	20	28	8	11

2. If $\bar{x}_1, \bar{x}_2, \bar{x}_3, ..., \bar{x}_n$ are the means of n groups with n_1, n_2, n_3, ... n_n, number of observations, then find the mean $\bar{x}$ of all the groups taken together.

Answers

Chapter Test

1. The following are the marks (out of 100) of 60 students in mathematics:

   16, 13, 5, 80, 86, 7, 51, 48, 24, 56, 70, 16, 61, 17, 16, 36, 34, 42,
   34, 35, 72, 55, 75, 31, 52, 28, 72, 97, 74, 45, 62, 68, 86, 35, 85, 36,
   81, 75, 55, 26, 95, 31, 7, 78, 92, 62, 56, 15, 52, 63, 25, 36, 54, 44,
   47, 27, 72, 17, 4, 30.

   Construct a grouped frequency distribution table with width 10 of each class starting from 0–9. *(NCERT Exemplar)*

2. Refer to Q.1 above. Construct a grouped frequency distribution table with width 10 of each class in such a way that one of the class is 10–20 (20 not included). *(NCERT Exemplar)*

3. The following table gives the frequencies of most commonly used letters *a, e, i, o, r, t, u* from a page of a book:

Letters	*a*	*e*	*i*	*o*	*r*	*t*	*u*
Frequency	75	125	80	70	80	95	75

   Represent the above information by a bar graph. *(NCERT Exemplar)*

4. The length of 40 leaves of a plant are measured correct to one millimetre, and the data obtained is represented in the following table:

Length (in mm)	*Number of leaves*
118 – 126	3
127 – 135	5
136 – 144	9
145 – 153	12
154 – 162	5
163 – 171	4
172 – 180	2

   (*i*) Draw a histogram to represent the given data.
   (*ii*) Is there any other suitable graphical representation for the same data?
   (*iii*) Is it correct to conclude that the maximum number of leaves are 153 mm long? Why? *(NCERT)*

5. Draw a histogram and frequency polygon of the following distribution:

Heights (in cm)	150 – 153	153 – 156	156 – 159	159 – 162	162 – 165	165 – 168
No. of students	7	8	14	10	6	5

   (NCERT Exemplar)

6. The following is the frequency distribution of total marks obtained by the students of different sections of class VIII.

Marks	100 – 150	150 – 200	200 – 300	300 – 500	500 – 800
No. of students	60	100	100	80	180

   Draw a histogram for the above distribution. *(NCERT Exemplar)*

7. The following are the ages (in years) of 10 teachers in a school:

   32, 28, 54, 41, 38, 40, 23, 26, 35, 33.

   Find the mean age of these teachers.

8. Find the mean of first seven prime numbers.

9. The mean height of 10 students is 151.8 cm. Two more students of heights 157.6 cm and 154.4 cm join the group. What is the new mean height?

10. In a science test given to a group of students, the marks scored by them (out of 100) are:

    41, 39, 52, 48, 54, 62, 46, 52, 40, 96, 42, 40, 98, 60, 52.

    Find the mean and the median of this data.

11. The following observations are arranged in ascending order:

    26, 29, 42, 53, x, $x + 2$, 70, 75, 82, 93.

    If the median is 65, find the value of x. *(NCERT Exemplar)*

12. A total of 25 patients admitted to a hospital are tested for levels of blood sugar (mg/dl) and the results obtained were as follows:

    87, 71, 83, 67, 85, 77, 69, 76, 65, 85, 85, 54, 70, 68, 80, 73, 78, 68, 85, 73, 81, 78, 81, 77, 75

    Find the mean, median and mode (mg/dl) of the above data. *(NCERT Exemplar)*

Answers

Probability



INTRODUCTION

In everyday life, we come across statements such as:

(*i*) It **may** rain today.

(*ii*) **Probably** Arvind will top the board exam this year.

(*iii*) I **doubt** she will pass the test.

(*iv*) It is **unlikely** that India will win the cricket world cup.

(*v*) **Chances** are high that the prices of petrol will go up.

The words, 'may', 'probably', 'doubt', 'unlikely', 'chances' used in the above statements indicate *uncertainties*. For example, in the statement (*i*), 'may rain today' means it may rain or may not rain today. We are predicting rain today based on our past experience when it rained under similar conditions. Similarly, predictions are made in other cases listed in (*ii*) to (*v*).

Probability is a measure of uncertainty.

The concept of probability developed as a result of studies of game of chance or gambling. Suppose you pay ₹ 5 to draw a card out of a pack of 52 cards – if you get an ace, you get ₹ 25, otherwise you get nothing. Should you play such a game? Does it give a *fair chance* of winning, or are you going to lose in the long run? Search for mathematical answers to these types of questions led to the development of the modern theory of probability. Italian mathematician Jerome Cardan, French mathematicians Blaise Pascal and Pierre de Fermat, Swiss mathematician James Bernoulli, and Russian mathematician Kolmogorov were pioneers in these studies. Pierre Simon Laplace (1749-1827) wrote "*It is remarkable that a science, which began with the consideration of games of chance, should be elevated to the rank of most important subject of human knowledge.*"

Today, probability is used extensively in the fields of Physical Sciences, Commerce, Biological Sciences, Medical Sciences, Weather Forecasting etc.

15.1 SOME TERMS RELATED TO PROBABILITY

- **Trial**

A **trial** *is an action which results in one or several outcomes.*

For example:

(*i*) If we toss a coin, it is a trial. The possible outcomes are head or tail.
If we toss a coin 10 times, then these are 10 trials.

(*ii*) If we throw a die, it is a trial. The possible outcomes are the numbers 1, 2, 3, 4, 5 and 6.
If we throw a die 15 times, then these are 15 trials.

(*iii*) If we toss two coins simultaneously, it is a trial.

The possible outcomes are two heads, one head, no head; or the possible outcomes may be taken as two tails, one tail, no tail; or the possible outcomes may be taken as two heads, one head and one tail, two tails.

If we toss two coins simultaneously 20 times, these are 20 trials.

- **Experiment**

An action which results in some (well-defined) outcomes is called an **experiment**.

For example:

(*i*) tossing a coin once

(*ii*) tossing a coin 10 times

(*iii*) throwing a die once

(*iv*) throwing a die 15 times

(*v*) tossing two coins simultaneously once

(*vi*) tossing two coins simultaneously 20 times.

All these are experiments.

- **Event**

The collection of some outcomes of an experiment is called an **event** *of the experiment.*

For example:

When a die is thrown, the six outcomes are the numbers 1, 2, 3, 4, 5, 6. Then 'getting a six' and 'getting an even number' are events of the experiment.

- **Occurrence of an event**

When the outcome of an experiment satisfies the condition mentioned in the event, then we say that event has **occurred** *(or* **happened***).*

For example:

(*i*) In the experiment of tossing a coin, an event E may be getting a head. If the coin comes up with head, we say that event E has occurred; otherwise, if the coin comes up with tail, we say that event E has not occurred.

(*ii*) In the experiment of tossing a pair of coins simultaneously, an event E may be getting two heads. If the pair of coins comes up with two heads, we say that event E has occurred; otherwise, we say that event E has not occurred.

(*iii*) In the experiment of throwing a die, an event E may be 'getting an even number'. If the die comes up with any of the numbers 2, 4 or 6, we say that event E has occurred; otherwise, if the die comes up any of the numbers 1, 3 or 5, we say that event E has not occurred.

For convenience, by the word 'event E' we will mean that event E has occurred (happened).

15.2 PROBABILITY – AN EXPERIMENTAL APPROACH

To learn the concept of experimental (or empirical) probability, perform the following activities:

☆ **Activity 1**

Take a coin, toss it ten times and note down the number of times a head and a tail come up. Record your observations in the form of following table:

Number of times the coin is tossed	*Number of times head comes up*	*Number of times tail comes up*
10	...	...

Write down the values of the following fractions:

$\frac{\text{Number of times a head comes up}}{\text{Total number of times the coin is tossed}}$ and $\frac{\text{Number of times a tail comes up}}{\text{Total number of times the coin is tossed}}$.

Toss the coin twenty times and in the same way record your observations as above and find the values of the fractions given above for this collection of observations.

Repeat the same experiment by increasing the number of tosses and record the number of heads and tails and then find the values of the corresponding fractions.

You will observe that as the number of tosses gets larger and larger, the values of the fractions come closer to 0.5.

☆ **Activity 2**

Divide the class into small groups of 2 or 3 students. Let a student in each group toss a coin 20 times. Another student in each group should record the observations regarding heads and tails. Note that the coins of same denomination should be used by all groups, then it will be treated as if the same coin has been tossed by all the groups.

On the blackboard make a table as shown below. First, group 1 can write its observations and calculate the resulting fraction. Then group 2 can write its observations and calculate the fractions for the combined data of group 1 and 2, and so on. These fractions are called cumulative fractions. The table given below shows the observations and calculation of fractions made by 5 groups of a class of students.

Group	*Number of tosses*	*Number of heads*	*Number of tails*	*Cumulative number of heads* / *Total number of times the coin is tossed*	*Cumulative number of tails* / *Total number of times the coin is tossed*
(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*
1	20	8	12	$\frac{8}{20}$	$\frac{12}{20}$
2	20	11	9	$\frac{11+8}{20+20}=\frac{19}{40}$	$\frac{9+12}{20+20}=\frac{21}{40}$
3	20	13	7	$\frac{13+19}{20+40}=\frac{32}{60}$	$\frac{7+21}{20+40}=\frac{28}{60}$
4	20	9	11	$\frac{9+32}{20+60}=\frac{41}{80}$	$\frac{11+28}{20+60}=\frac{39}{80}$
5	20	6	14	$\frac{6+41}{20+80}=\frac{47}{100}$	$\frac{14+39}{20+80}=\frac{53}{100}$

From the above table we observe that as the total number of tosses of the coin increases, the values of the fractions in columns (5) and (6) come closer and closer to 0.5.

☆ **Activity 3**

Divide the class into small groups of 2 or 3 students. Let a student in each group toss two coins simultaneously 15 times. Another student in each group should record the observations regarding two heads, one head and one tail, two tails. Note that the coins of same denomination should be used by all groups so that it may be treated as if the same coins has been tossed by all the groups.

Record the observations regarding heads and tails as shown in the table given below:

Group	*Number of trials*	*Number of two heads*	*Number of one head and one tail*	*Number of two tails*
1	15	4	8	3
2	15	5	8	2
3	15	3	7	5
4	15	4	8	3
5	15	4	7	4
6	15	4	6	5
Total	90	24	44	22

Calculate the following fractions:

(*i*) $\frac{\text{Cumulative number of two heads}}{\text{Total number of trials}} = \frac{24}{90}$

(*ii*) $\frac{\text{Cumulative number of one head and one tail}}{\text{Total number of trials}} = \frac{44}{90}$

(*iii*) $\frac{\text{Cumulative number of two tails}}{\text{Total number of trials}} = \frac{22}{90}$.

From the above table and the calculation of fractions we find that as the total number of trials increases the values of the fractions come closer and closer to 0.25, 0.5 and 0.25 respectively.

From the above activities, we find that **empirical** (or **experimental**) **probability** is based on what we directly observe or on known (given) outcomes of an experiment. Thus, empirical probability can be defined as:

Empirical probability of an event E is written as P(E) and is given by

$$P(E) = \frac{\textit{Number of trials in which event E happened}}{\textit{Total number of trials}}.$$

Hence, the empirical probability depends upon the number of trials undertaken and the number of times the outcomes you are looking for coming up in these trials.

Remarks

- If E is an event, then $0 \leq P(E) \leq 1$.
- If the events E_1 and E_2 cover all the outcomes of an experiment and if no outcome is common in these events, then
  $P(E_1) + P(E_2) = 1$.
- In activity 2, if E be the event of getting heads and F be the event of getting tails, then the empirical or experimental probability in the experiment are
  $$P(E) = \frac{47}{100} \text{ and } P(F) = \frac{53}{100}.$$
  Note that $P(E) + P(F) = \frac{47}{100} + \frac{53}{100} = 1$.
- In activity 3, if E_1, E_2 and E_3 be the events of getting two heads, one head and one tail, two tails respectively then the experimental probability in the experiment are

$$P(E_1) = \frac{24}{90},\ P(E_2) = \frac{44}{90} \text{ and } P(E_3) = \frac{22}{90}.$$

Note that $P(E_1) + P(E_2) + P(E_3) = \frac{24}{90} + \frac{44}{90} + \frac{22}{90} = 1.$

Illustrative Examples

Example 1. *A coin is tossed 500 times and we get*

Head : 262 times *Tail : 238 times*

Find the probability of occurrence of these events.

Solution. Total number of trials = 500

If H and T are the events of getting a head and of getting a tail respectively, then the number of times H happened = 262 and the number of times T happened = 238.

$\therefore$ $P(H) = \frac{262}{500} = \frac{131}{250}$ and $P(T) = \frac{238}{500} = \frac{119}{250}$.

Note. In the above example, $P(H) + P(T) = \frac{131}{500} + \frac{119}{250} = \frac{250}{250} = 1$, and H and T are the only two possible outcomes of each trial.

Example 2. *Two coins are tossed simultaneously 400 times and we get*

Two heads : 96 times

One head : 210 times

No head : 94 times

Find the probability of happening of each of these events.

Solution. Total number of trials = 400.

If E_1, E_2 and E_3 are the events of getting two heads, one head and no head respectively, then

$$P(E_1) = \frac{96}{400} = 0.24,$$

$$P(E_2) = \frac{210}{400} = 0.525 \text{ and}$$

$$P(E_3) = \frac{94}{400} = 0.235.$$

Note that $P(E_1) + P(E_2) + P(E_3) = 0.24 + 0.525 + 0.235 = 1$, and E_1, E_2 and E_3 cover all the outcomes of a trial.

Example 3. *A die is thrown 1000 times with the frequencies for the outcomes 1, 2, 3, 4, 5 and 6 as given in the following table:*

Outcome	*1*	*2*	*3*	*4*	*5*	*6*
Frequency	*157*	*175*	*160*	*179*	*145*	*184*

Find the possibility of getting each outcome.

Solution. Total number of trials = 1000.

If E_1, E_2, ..., E_6 denote the events of getting the numbers 1, 2, ..., 6 respectively, then

$$P(E_1) = \frac{157}{1000}, \quad P(E_2) = \frac{175}{1000} = \frac{7}{40}, \quad P(E_3) = \frac{160}{1000} = \frac{4}{25},$$

$$P(E_4) = \frac{179}{1000}, \quad P(E_5) = \frac{145}{1000} = \frac{29}{200} \text{ and } \quad P(E_6) = \frac{184}{1000} = \frac{23}{125}.$$

Example 4. *The record of a weather station shows that out of the past 250 consecutive days, its weather forecast was correct 175 times.*

(i) What is the probability that on a given day it was correct?

(ii) What is the probability that on a given day it was not correct? *(NCERT)*

Solution. The total number of days on which the weather forecast was made = 250.

Let E and F be the events that the weather forecast was correct and not correct respectively.

(*i*) The number of days when the weather forecast was correct = 175.

$\therefore \quad P(E) = \frac{175}{250} = \frac{7}{10} = 0.7$

(*ii*) The number of days on which the weather forecast was not correct = 250 – 175 = 75

$\therefore \quad P(F) = \frac{75}{250} = \frac{3}{10} = 0.3$

Example 5. *The percentage of marks obtained by a student in the monthly unit tests are given below:*

Unit test	*I*	*II*	*III*	*IV*	*V*
Percentage of marks obtained	*69*	*71*	*73*	*68*	*74*

Based on this data, find the probability that the student gets more than 70% marks in a unit test. *(NCERT)*

Solution. Total number of unit test held = 5.

The number of unit tests in which the student obtained more than 70% marks = 3.

$\therefore \quad P(\text{getting more than 70\% marks}) = \frac{3}{5} = 0.6$

Example 6. *On a page of a telephone directory, there were 200 telephone numbers. The frequency distribution of their unit place digit is given in the following table:*

Digit	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
Frequency	*22*	*26*	*22*	*22*	*20*	*10*	*14*	*28*	*16*	*20*

Find the probability of a number, chosen at random, having unit place digit as

(i) 6 *(ii) a prime number.* *(NCERT)*

Solution. Total number of telephone numbers = 200.

(*i*) The number of telephone numbers which have the digit 6 at unit place = 14.

$\therefore \quad P(\text{having digit 6 at unit place}) = \frac{14}{200} = 0.07$

(*ii*) The number of telephone numbers which have a prime number *i.e.* 2, 3, 5 or 7 at their unit place = 22 + 22 + 10 + 28 = 82.

$\therefore \quad P(\text{having a prime number at unit place}) = \frac{82}{200} = 0.41$

Example 7. *Fifty seeds were selected at random from each of 5 bags of seeds, and were kept under standardised conditions for germination. After 20 days, the number of seeds which had germinated in each collection were collected and recorded as follows:*

Bag	*1*	*2*	*3*	*4*	*5*
No. of seeds germinated	*40*	*48*	*42*	*39*	*41*

What is the probability of germination of

(i) more than 40 seeds in a bag? *(ii) 49 seeds in a bag?*

(iii) more than 35 seeds in a bag? *(NCERT)*

Solution. Total no. of bags = 5

(*i*) No. of bags in which more than 40 seeds germinated = 3,

$\therefore$ required probability = $\frac{3}{5}$ = 0.6

(*ii*) No. of bags in which 49 seeds germinated = 0,

$\therefore$ required probability = $\frac{0}{5}$ = 0.

(*iii*) No. of bags in which more than 35 seeds germinated = 5,

$\therefore$ required probability = $\frac{5}{5}$ = 1.

Example 8. *Two dice are thrown simultaneously 500 times. Each time the sum of numbers appearing on their top is noted and recorded as given in the following table:*

Sum	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
Frequency	*14*	*30*	*42*	*55*	*72*	*75*	*70*	*53*	*46*	*28*	*15*

If the dice are thrown once more, what is the probability of getting a sum

(i) 3 *(ii) more than 10*

(iii) less than 5 or equal to 5 *(iv) between 8 and 12* *(NCERT Exemplar)*

Solution. Total number of times the two dice are thrown = 200.

When two dice are thrown once more, we find the probability on the basis of given data.

(*i*) The number of times the sum is 3 = 30.

$\therefore$ P(having sum 3) = $\frac{30}{500} = \frac{3}{50}$ = 0.06

(*ii*) The number of times the sum is more than 10

i.e. the sum is 11 or 12 = 28 + 15 = 43

$\therefore$ P(having sum more than 10) = $\frac{43}{500}$ = 0.086

(*iii*) The number of times the sum is less than 5 or equal 5

i.e. the sum is 2, 3, 4 or 5 = 14 + 30 + 42 + 55 = 141

$\therefore$ P(having sum less than 5 or equal to 5) = $\frac{141}{500}$ = 0.282

(*iv*) The number of times the sum is between 8 and 12

i.e. the sum is 9, 10 or 11 = 53 + 46 + 28 = 127.

$\therefore$ P(having sum between 8 and 12) = $\frac{127}{500}$ = 0.254

Example 9. *Here is an extract from a mortality table:*

Age (in years)	*60*	*61*	*62*	*63*	*64*	*65*
Number of persons surviving out of sample of one million	*16090*	*11490*	*8012*	*5448*	*3607*	*2320*

(i) Based on this information, what is the probability of a person 'aged 60' of dying within a year?

(ii) What is the probability that a person 'aged 61' will live for 4 years? *(NCERT Exemplar)*

Solution. (*i*) No. of persons 'aged 60' = 16090.

No. of persons 'aged 60' dying within a year *i.e.* the no. of persons 'aged 60' dying before reaching their 61st birthday = 16090 – 11490 = 4600.

$\therefore$ P(a person aged 60 dying within a year) = $\frac{4600}{16090} = \frac{460}{1609}$.

(*ii*) No. of persons 'aged 61' = 11490.

No. of persons 'aged 61' surviving for 4 years = 2320.

$\therefore$ P(a person aged 61 will live for 4 years) = $\frac{2320}{11490} = \frac{232}{1149}$.

Example 10. *A tyre manufacturing company kept a record of the distance covered before a tyre needed to be replaced. The table shows the results of 1000 cases.*

Distance (in km)	*less than 4000*	*4000 to 9000*	*9001 to 14000*	*more than 14000*
Frequency	*20*	*210*	*325*	*445*

If you buy a tyre of this company, what is the probability that:

(i) it will need to be replaced before it has covered 4000 km?

(ii) it will last more than 9000 km?

(iii) it will need to be replaced after it has covered somewhere between 4000 km and 14000 km? *(NCERT)*

Solution. Total no. of trials = 1000.

(*i*) Frequency of a tyre that needs to be replaced before it covers 4000 km = 20,

$\therefore$ required probability = $\frac{20}{1000}$ = 0.02

(*ii*) Frequency of a tyre that will last more than 9000 km = 325 + 445 = 770,

$\therefore$ required probability = $\frac{770}{1000}$ = 0.77

(*iii*) Frequency of a tyre that requires replacement between 4000 km and 14000 km = 210 + 325 = 535,

$\therefore$ required probability = $\frac{535}{1000}$ = 0.535

Example 11. *In a mathematics test, 90 students of a class obtained (out of 100 marks) the marks as given in the following table:*

Marks	*0 – 20*	*20 – 30*	*30 – 40*	*40 – 50*	*50 – 60*	*60 – 70*	*70 – above*
No. of students	*7*	*10*	*10*	*20*	*20*	*15*	*8*

(i) Find the probability that a student obtained less than 20% marks.

(ii) Find the probability that a student obtained 60 marks or above.

Solution. Total no. of students = 90.

(*i*) No. of students who obtained less than 20% marks *i.e.* 20 marks = 7,

$\therefore$ required probability = $\frac{7}{90}$.

(*ii*) No. of students who obtained 60 marks or more = 15 + 8 = 23,

$\therefore$ required probability = $\frac{23}{90}$.

Example 12. *An insurance company selected 2000 drivers at random in a particular city to find a relationship between age and accidents. The data obtained is given in the following table:*

Age of driver (in years)	*Accidents in one year*				
	0	*1*	*2*	*3*	*over 3*
18 – 29	*440*	*160*	*110*	*61*	*35*
30 – 50	*505*	*125*	*60*	*22*	*18*
above 50	*360*	*45*	*35*	*15*	*9*

Find the probabilities of the following events for a driver, chosen at random, from the city:

(i) being 18 – 29 years of age and having exactly 3 accidents in a year.

(ii) being 30 – 50 years of age and having one or more accidents in a year.

(iii) having no accidents in one year.

(iv) What is the value of experience while driving? ***(Value Based)***

Solution. Total no. of drivers = 2000.

(*i*) The no. of drivers of age 18 – 29 years and having exactly 3 accidents in a year = 61,

$\therefore$ required probability = $\frac{61}{2000}$.

(*ii*) The no. of drivers of age 30 – 50 years and having one or more accidents in a year = 125 + 60 + 22 + 18 = 225,

$\therefore$ required probability = $\frac{225}{2000} = \frac{9}{80}$.

(*iii*) The no. of drivers having no accident in one year = 440 + 505 + 360 = 1305,

$\therefore$ required probability = $\frac{1305}{2000} = \frac{261}{400}$.

(*iv*) Experienced drivers are more alert and there are less accidents.

Exercise 15

1. In a cricket match, a batswoman hits a boundary 6 times out of 30 balls she plays. Find the probability that she did not hit a boundary. (*NCERT*)

2. To know the opinion of students about the subject *statistics*, a survey of 200 students was conducted. The data is recorded in the following table:

Opinion	like	dislike
Number of students	135	65

Find the probability that a student chosen at random

(*i*) likes statistics

(*ii*) does not like it. (*NCERT*)

3. A coin is tossed 1000 times with the following frequencies:
Head : 455, Tail : 545
Find the probability for each event. (*NCERT*)

4. Two coins are tossed simultaneously 500 times, and we get
Two heads : 105 times, One head : 275 times, No head : 120 times
Find the probability of occurrence of each of these events. (*NCERT*)

5. A die is thrown 1000 times with the frequencies of the outcomes 1, 2, 3, 4, 5 and 6 as given in the following table:

Outcomes	1	2	3	4	5	6
Frequency	179	150	157	149	175	190

Find the probability of getting each outcome. (*NCERT*)

6. 1500 families with 2 children were selected randomly, and the following data were recorded:

No. of girls in a family	0	1	2
No. of families	211	814	475

Find the probability of a family, chosen at random, having

(*i*) 2 girls (*ii*) one girl (*iii*) no girl (*NCERT*)

7. Eleven bags of wheat flour, each marked 5 kg, actually contained the following weights of flour (in kg)

4.97, 5.05, 5.08, 5.03, 5.00, 5.06, 5.08, 4.98, 5.04, 5.07, 5.00

Find the probability that any of these bags, chosen at random, contains more than 5 kg of flour.

8. In Example 1, page 364 you were asked to prepare a frequency distribution table regarding the blood groups of 30 students of class IX. Use this table to determine the probability that a student of this class, chosen at random has blood group AB.

9. In Example 4, page 365, what is the empirical probability that an engineer lives:

(*i*) less than 7 km from his/her place of work?

(*ii*) more than or equal to 7 km from his/her place of work?

(*iii*) within $\frac{1}{2}$ km from his/her place of work?

10. A company selected 4000 households at random and surveyed them to find out a relationship between income level and the number of television sets in a home. The information so obtained is listed in the following table:

Monthly income (in ₹)	*Number of Televisions/Households*			
	0	*1*	*2*	*Above 2*
Less than 10000	20	80	10	0
10000 – 14999	10	240	60	0
15000 – 19999	0	380	120	30
20000 – 24999	0	520	370	80
25000 and above	0	1100	760	220

Find the probability:

(*i*) of a household earning ₹ 10000 – ₹ 14999 per month and having exactly one television.

(*ii*) of a household earning ₹ 25000 and more per month and owning 2 televisions.

(*iii*) of a household having no television. (*NCERT Exemplar*)

11. A recent survey found that the ages of workers in a factory are distributed as follows:

Age (in years)	20–29	30–39	40–49	50–59	60 and above
Number of workers	38	27	86	46	3

If a person is selected at random, find the probability that the person is:

(*i*) 40 years or more

(*ii*) under 40 years

(*iii*) having age from 30 to 39 years

(*iv*) under 60 but over 39 years. (*NCERT Exemplar*)

Answers

Very Short Answer Type Questions

1. In a one-day cricket match Virat faced 30 balls and hit 9 boundaries. Find the probability that he did not hit a boundary.

2. A bag contains 10 balls out of which 4 balls are white and the others are non-white. Find the probability of getting a non-white ball.

3. A coin is tossed 30 times and it showed head 16 times. Find the probability of getting a tail.

4. A die is thrown 35 times and it showed an even number 15 times. Find the probability of getting an odd number.

5. A pair of coins is tossed 100 times and the results are:
Two tails: 27 times and one tail : 51 times:
Find the probability of getting no tail.

Answers

Objective Type Questions
(FOR FORMATIVE/SUMMATIVE ASSESSMENT)

Mental Maths

1. Fill in the blanks:

(*i*) Probability is a measure of ..... .

(*ii*) A trial is an action which results in one or several ..... .

(*iii*) When the outcome of an experiment satisfies the condition mentioned in the event, then we say that event has ..... .

2. Fill in the blanks using the correct word given in the brackets:

(*i*) Tossing a coin 50 times is called an ..... (event, experiment).

(*ii*) The possible outcomes of an experiment are called .....(events, trials).

Multiple Choice Questions

Choose the correct answer from the given four options (3 to 10) :

3. A coin is tossed 40 times and it showed tail 24 times. The probability of getting a head is

(*a*) $\frac{2}{5}$ (*b*) $\frac{3}{5}$ (*c*) $\frac{1}{2}$ (*d*) $\frac{17}{40}$

4. In a medical examination of students of a class, the following blood groups are recorded:

Blood Group	A	AB	B	O
Number of students	10	13	12	5

A student is selected at random from the class. The probability that he/she has blood group B is

(*a*) $\frac{1}{4}$ (*b*) $\frac{13}{40}$ (*c*) $\frac{3}{10}$ (*d*) $\frac{1}{8}$

(NCERT Exemplar)

5. In a survey of 364 children aged 19–36 months, it was found that 91 liked to eat potato chips. If a child is selected at random, the probability that he/she does not like to eat potato chips is

(*a*) 0.25 (*b*) 0.50 (*c*) 0.75 (*d*) 0.80

(NCERT Exemplar)

6. A die is thrown 1000 times and the outcomes were recorded as follows:

Outcomes	1	2	3	4	5	6
Frequency	180	150	160	170	150	190

If the die is thrown once more, then the probability that it shows 5 is

(*a*) $\frac{9}{50}$ (*b*) $\frac{3}{20}$ (*c*) $\frac{4}{25}$ (*d*) $\frac{7}{25}$

(NCERT Exemplar)

7. Two coins are tossed 1000 times and the outcomes are recorded as below:

Number of heads	2	1	0
Frequency	200	550	250

Based on this information, the probability for atmost one head is

(*a*) $\frac{1}{5}$ (*b*) $\frac{1}{4}$ (*c*) $\frac{4}{5}$ (*d*) $\frac{3}{4}$

(NCERT Exemplar)

8. The probability of any event of an experiment

(*a*) is 1 (*b*) is greater than 1

(*c*) is 0 (*d*) lies between 0 and 1 (both inclusive)

9. 80 bulbs are selected at random from a lot and their lifetime (in hours) is recorded in the form of a frequency table given below:

Lifetime (in hours)	300	500	700	900	1100
Frequency	10	12	23	25	10

One bulb is selected at random from the lot. The probability that its lifetime is 1150 hours is

(*a*) $\frac{1}{80}$ (*b*) $\frac{7}{16}$ (*c*) 0 (*d*) 1

(NCERT Exemplar)

10. Refer to Q. 9 above:

The probability that bulb selected randomly from the lot has lifetime less than 900 hours is

(*a*) $\frac{11}{40}$ (*b*) $\frac{5}{16}$ (*c*) $\frac{7}{16}$ (*d*) $\frac{9}{16}$

(*NCERT Exemplar*)

Answers

Value Based Questions

1. The weights of 38 students of a class are given in the following table:

Weight (in kg)	30 – 35	35 – 40	40 – 45	45 – 50	50 – 55	55 – 60	60 – 65	65 – 70	70 – 75
No. of students	9	5	14	3	1	2	2	1	1

(*i*) Find the probability that the weight of a student in the class lies in the interval 45 – 50 kg.

(*ii*) Give two events in this context, one having probability 0 and the other having probability 1.

(*iii*) Does over weight cause physical problems?

2. The data pertaining to different modes of transport to go to school by students is given below:

Mode of transport	Cycle	Bike	Private car	School bus
Number of students	410	450	360	780

If a student is chosen at random, find the probability that he/she goes to school by:

(*i*) Private car (*ii*) Cycle (*iii*) Bike or school bus.

Which mode of transport should be promoted?

Answers

Chapter Test

1. Can the experimental probability of an event be a negative number? If not, why? *(NCERT Exemplar)*
2. Can the experimental probability of an event be greater than 1? Justify your answer. *(NCERT Exemplar)*
3. Three coins are tossed simultaneously 200 times with the following frequencies of different outcomes:

Outcome	3 heads	2 heads	1 head	no head
Frequency	23	72	77	28

If the three coins are simultaneously tossed again, then find the probability of 2 heads coming up. *(NCERT)*

4. Bulbs are packed in cartons each containing 40 bulbs. Seven hundred cartons were examined for defective bulbs and the results are given in the following table:

Number of defective bulbs	0	1	2	3	4	5	6	more than 6
Frequency	400	180	48	41	18	8	3	2

One carton was selected at random. What is the probability that it has

(*i*) no defective bulb?

(*ii*) defective bulbs from 2 to 6?

(*iii*) defective bulbs less than 4? *(NCERT Exemplar)*

5. Over the past 200 working days, the number of defective parts produced by a machine is given in the following table:

Number of defective parts	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Days	50	32	22	18	12	12	10	10	10	8	6	6	2	2

Determine the probability that tomorrow's output will have

(*i*) no defective part

(*ii*) atleast one defective part

(*iii*) not more than 5 defective parts

(*iv*) more than 13 defective parts. *(NCERT Exemplar)*

6. The marks obtained (out of 80 marks) by 40 students in a test of mathematics are given in the following table:

Marks obtained	0 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
No. of students	3	2	5	8	6	9	7

Find the probability of a student, chosen at random, getting

(*i*) less than 40 marks (*ii*) 50 or more marks

7. An organisation selected 2400 families at random and surveyed them to determine a relationship between income level and the number of vehicles in a family. The information gathered is listed in the table below:

Monthly income (in ₹)	*Vehicles per family*			
	0	*1*	*2*	*Above 2*
Less than 7000	10	160	25	0
7000 – 10000	0	305	27	2
10000 – 13000	1	535	29	1
13000 – 16000	2	469	59	25
16000 or more	1	579	82	88

Find the probability that a family, chosen at random, is

(*i*) earning ₹10000 – ₹13000 per month and owning exactly 2 vehicles.

(*ii*) earning ₹16000 or more per month and owning exactly 1 vehicle.

(*iii*) earning less than ₹7000 per month and does not own any vehicle.

(*iv*) earning ₹13000 – ₹16000 per month and owning more than 2 vehicles.

(*v*) Owning not more than 1 vehicle. (*NCERT*)

Answers

Model Question Paper 6

(FOR SUMMATIVE ASSESSMENT – SECOND TERM)

Time: 3 Hours *Maximum Marks: 90*

General Instructions:

(*i*) All questions are compulsory.

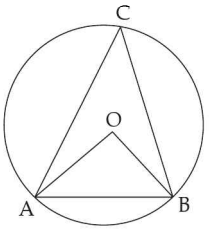
(*ii*) The question paper consists of 31 questions divided into four sections A, B, C and D. Section A comprises of 4 questions of 1 mark each, Section B comprises of 6 questions of 2 marks each, Section C comprises of 10 questions of 3 marks each and Section D comprises of 11 questions of 4 marks each.

(*iii*) Use of calculators is not permitted.

SECTION A

Question numbers 1 to 4 are of one mark each.

1. In the adjoining figure, O is the centre of the circle. If $\angle OAB = 40°$, then find $\angle ACB$.



2. If the base area of a cone is 120 cm^2 and its vertical height is 15 cm then find its volume.

3. If the volume of a sphere is numerically equal to its surface area, then find the radius of the sphere.

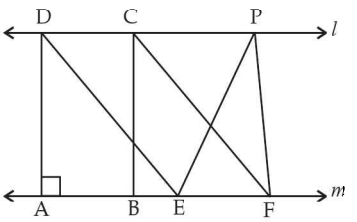
4. A pair of coins is tossed 80 times and the results are:
Two heads : 21 times and one head : 41 times.
Find the probability of getting no head.

SECTION B

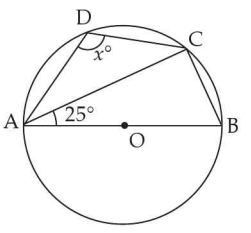
Question numbers 5 to 10 carry 2 marks each.

5. Determine the point on the graph of the linear equation $2x + 5y = 19$, whose ordinate is $1\frac{1}{2}$ times its abscissa.

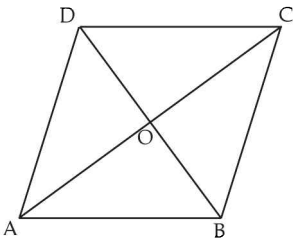
6. In the adjoining figure, $l \parallel m$, ABCD is a rectangle with sides AB = 6 cm and AD = 9 cm (not drawn to scale).
If DE || CF, find the area of
(*i*) parallelogram EFCD
(*ii*) triangle EFP.



7. In the adjoining figure, points A, B, C and D lie on a circle with centre O. Of $\angle CAB = 25°$, then find the value of x.



8. In the adjoining figure, the diagonals AC and BD of a rhombus ABCD intersect at O. If OA = 4 cm and OB = 3 cm, then find the perimeter of the rhombus ABCD.

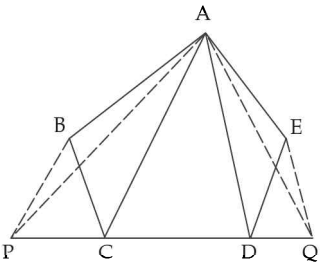


9. Curved surface area of a cone is 308 cm^2 and its slant height is 14 cm. Find
   (*i*) radius of the base (*ii*) total surface area of the cone.
10. If the median of the observations 12, 14, 15, 27, $x + 2$, $x + 3$, 35, 38, 41, 45 arranged in ascending order is 32.5, then find the value of x.

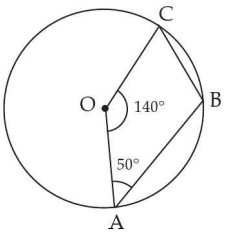
SECTION C

Question numbers 11 to 20 carry 3 marks each.

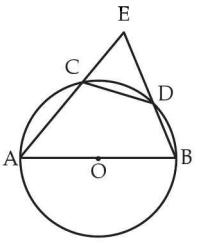
11. Draw the graph of the linear equation $2x + 3y = 12$. At what point, the graph of the equation cuts the y-axis?
12. Solve the equation $2y - 1 = y + 2$ and represent the solution (s) on:
    (*i*) the number line (*ii*) the cartesian plane.
13. If the diagonals of a parallelogram are equal, then prove that it is a rectangle.
14. In the adjoining figure, ABCDE is any pentagon. BP drawn parallel to AC meets DC produced at P and EQ drawn parallel to AD meets CD produced at Q. Prove that
    ar(pentagon ABCDE) = ar(ΔAPQ)



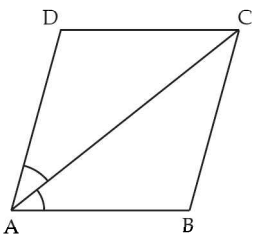
15. In the adjoining figure, points A, B and C lie on a circle with centre O. If $\angle AOC = 140°$ and $\angle OAB = 50°$, then find $\angle OCB$.



16. In the adjoining figure, AB is a diameter of a circle with centre O and CD is a chord equal to the radius of the circle. AC and BD when extended intersect at E. Prove that $\angle AEB = 60°$.



17. In the adjoining figure, the diagonal AC of a parallelogram ABCD bisects $\angle A$. Show that
    (*i*) AC bisects $\angle C$ also
    (*ii*) ABCD is a rhombus.



18. A river 3 m deep and 40 m wide is flowing at the rate of 2 km per hour. How much water will fall into the sea in a minute?

19. A conical tent is 10 m high and the diameter of its base is 48 m. Find:
    (*i*) Slant height of the tent.
    (*ii*) Cost of the canvas required to make the tent, if the cost of 1 m^2 canvas is ₹70.

20. Find the mean, median and mode of the following data:
    10, 13, 7, 17, 2, 5, 27, 15, 8, 14, 10, 48, 10, 7, 24, 8, 28, 18, 3, 37

SECTION D

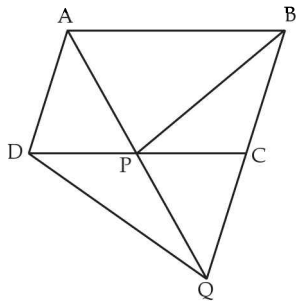
Question numbers 21 to 31 carry 4 marks each.

21. The taxi fare in a city is as follows:
    For the first kilometre, the fare is ₹8 and for the subsequent distance, it is ₹5 per km. Taking the distance covered as x km and the total fare as ₹y, write a linear equation for this information, and draw its graph.

22. If the points (2, 3) and (4, 0) lie on the graph of the equation $ax + by = 1$, then find the values of a and b. Draw the graph of the equation obtained.

23. ABCD is a rectangle and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.

24. In the adjoining figure, ABCD is a parallelogram and BC is produced to a point Q such that CQ = AD. If AQ intersects DC at P, show that
    ar (ΔBPC) = ar (ΔDPQ).



25. If two intersecting chords of a circle make equal angles with the diameter passing through their point of intersection, then prove that the chords are equal.

26. Construct a triangle PQR in which ∠Q = 30°, ∠R = 90° and perimeter is 13 cm.

27. Find
    (*i*) the lateral surface area of a closed cylindrical petrol storage tank which is 4.2 m in diameter and 4.5 m high.
    (*ii*) how much area of steel was actually used, if $\frac{1}{12}$ of the area of steel actually used was wasted in making the tank.

28. Monica has a piece of canvas whose area is 553 m^2. She uses it to make a conical tent with a base radius of 7 m. Assuming that all the stitching margins and the wastage incurred while cutting, amounts to approximately 3 m^2, find the volume of the tent that can be made with it.
    Monica donates the tent to a shelterless family. What value is depicted by her?

29. Given below are the seats won by different political parties in the polling outcome of a state assembly elections:

Political parties	A	B	C	D	E	F
Seats won	75	55	37	29	10	37

    (*i*) Draw a bar graph to represent the polling results.
    (*ii*) Which political party won the maximum number of seats?

30. A random survey of the number of children of various age groups playing in a park was found as follows:

Age (in years)	1 – 2	2 – 3	3 – 5	5 – 7	7 – 10	10 – 15	15 – 17
Number of children	5	3	6	12	9	10	4

Draw a histogram to represent the above data.

31. Two dice are thrown 500 times. Each time the sum of numbers appearing on them is noted and recorded in the following table:

Sum	2	3	4	5	6	7	8	9	10	11	12
Frequency	14	30	42	55	72	75	70	53	46	28	15

From the above data, find the probability of getting a sum

(*i*) more than 10

(*ii*) between 8 and 12 (both exclusive)

(*iii*) an odd prime number.

Answers

Answers

Term - I

EXERCISE 1.1

1. $\frac{13}{42}$ **2.** $\frac{43}{144}; \frac{3}{8}, \frac{43}{144}, \frac{2}{9}$ **3.** $\frac{7}{24}, \frac{13}{48}$

4. (*i*) 0.101, 0.102, 0.103 (*ii*) $\frac{3}{4}, \frac{11}{14}, \frac{23}{28}$ **5.** $\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}, \frac{27}{7}$

6. $\frac{19}{30}, \frac{2}{3}, \frac{7}{10}, \frac{11}{15}, \frac{23}{30}$ **7.** $-\frac{13}{35}, -\frac{12}{35}, -\frac{11}{35}, -\frac{2}{7}, -\frac{9}{35}, -\frac{8}{35}, -\frac{1}{5}, 0, \frac{1}{35}, \frac{2}{35}$

8. $\frac{11}{21}, \frac{23}{42}, \frac{4}{7}, \frac{25}{42}, \frac{13}{21}, \frac{9}{14}$

EXERCISE 1.2

2. (*i*) 0.36 ; terminating (*ii*) 4.125 ; terminating

(*iii*) $0.\bar{2}$; non-terminating repeating (*iv*) $0.\overline{18}$; non-terminating repeating

(*v*) $0.\overline{230769}$; non-terminating repeating (*vi*) 0.8225 ; terminating

3. (*i*), (*ii*), (*iii*), (*v*) and (*viii*) **4.** $\frac{15}{48}$; 0.3125

5. $\frac{1}{7} = 0.142857, \frac{2}{7} = 2 \times \frac{1}{7} = 0.\overline{285714}, \frac{3}{7} = 3 \times \frac{1}{7} = 0.\overline{428571}$,

$\frac{4}{7} = 4 \times \frac{1}{7} = 0.\overline{571428}, \frac{5}{7} = 5 \times \frac{1}{7} = 0.\overline{714285}, \frac{6}{7} = 6 \times \frac{1}{7} = 0.\overline{857142}$

6. 16 ; $0.\overline{0588235294117647}$

7. (*i*) $\frac{1}{3}$ (*ii*) $\frac{47}{9}$ (*iii*) $\frac{40}{99}$ (*iv*) $\frac{43}{90}$ (*v*) $\frac{133}{990}$ (*vi*) $\frac{1}{999}$

8. 1; yes

9. (*i*) and (*v*) are irrational ; (*ii*), (*iii*) and (*iv*) are rational

10. 1.1010010001..., 2.2020020002..., 3.50101101110...

11. (*i*) 2.5; 2.5050050005....., (*ii*) 0.05 ; 0.07007000700007...

(*iii*) $\frac{5}{12}$; 0.4141141114... (*iv*) 0; 0.1515515551... (*v*) 0.152 ; 0.15202202220....

12. 0.5050050005...., 0.6060060006.... **13.** 1.5 **14.** 3.5, 3.6

15. $\sqrt{6}$ **16.** $\sqrt{5}, \sqrt{6}$

EXERCISE 1.4

1. (*i*) irrational (*ii*) rational (*iii*) irrational (*iv*) rational

(*v*) rational (*vi*) irrational

2. $3\sqrt{2} + 2\sqrt{3}$

3. (*i*) $\sqrt{5}$ (*ii*) $\frac{34\sqrt{3}}{3}$ (*iii*) 60 (*iv*) $4\sqrt{5}$ (*v*) $\frac{7\sqrt{6}}{12}$ (*vi*) $\frac{5\sqrt{2}}{4}$

4. (*i*) $10 + 5\sqrt{5} + 2\sqrt{7} + \sqrt{35}$ (*ii*) 20 (*iii*) $7 + 2\sqrt{10}$

(*iv*) $10 - 2\sqrt{21}$ (*v*) $\sqrt{10} + \sqrt{14} + \sqrt{15} + \sqrt{21}$

(*vi*) $4\sqrt{3} - 4\sqrt{7} + \sqrt{15} - \sqrt{35}$

6. (*i*) 28.28 (*ii*) –36.764 **7.** (*i*) 8.66 (*ii*) 121.24

8. (*i*) $2-\sqrt{3}$ (*ii*) $\frac{\sqrt{7}+2}{3}$ (*iii*) $\sqrt{41}+5$ (*iv*) $\sqrt{7}+\sqrt{6}$ (*v*) $\frac{\sqrt{5}-\sqrt{2}}{3}$ (*vi*) $9+2\sqrt{15}$

9. (*i*) $a=\frac{9}{11}$ (*ii*) $a=2, b=-\frac{5}{6}$ (*iii*) $a=0, b=1$

10. 0 **11.** (*i*) 0.414 (*ii*) 0.318 **12.** $2\sqrt{3}$ **13.** 16

14. 98 **15.** (*i*) –18 (*ii*) $8\sqrt{5}$ (*iii*) 322 (*iv*) $-144\sqrt{5}$

EXERCISE 1.5

1. (*i*) 1 (*ii*) 4 (*iii*) $(221)^{\frac{1}{5}}$ (*iv*) 27 (*v*) $\frac{1}{5}$ (*vi*) $\frac{8}{27}$ **2.** (*i*) $\frac{16}{25}$ (*ii*) $\frac{27}{16}$ (*iii*) $\frac{1}{8}$

3. (*i*) $\frac{2025}{64}$ (*ii*) 16 (*iii*) –3 **4.** (*i*) 12 (*ii*) 25

5. (*i*) $\sqrt[6]{294}$ (*ii*) $\sqrt[6]{500}$ (*iii*) $\sqrt[12]{2000}$ (*iv*) $\sqrt[6]{\frac{49}{2}}$ (*v*) $\sqrt[6]{\frac{8}{9}}$

6. (*i*) $2\sqrt{3}, \sqrt{15}, 4, 3\sqrt{2}$ (*ii*) $4, 3\sqrt{2}, 2\sqrt{8}, 4\sqrt{3}, \sqrt{50}$

7. (*i*) $\sqrt[3]{2}, \sqrt[6]{5}, \sqrt{3}$ (*ii*) $\sqrt[4]{5}, \sqrt[3]{4}, \sqrt{3}$ **8.** $\sqrt[3]{4}, \sqrt[4]{3}, \sqrt[3]{2}$

VERY SHORT ANSWER TYPE QUESTIONS

1. $\frac{19}{30}$ **2.** 0.62101101110 ... **3.** 1.1 **4.** 1.5010010001 ...

5. Infinitely many **6.** Infinitely many **7.** non-terminating recurring

8. $8\sqrt{3}$ **9.** $\frac{8}{9}$ **10.** –29 **11.** $3+\sqrt{11}$ **12.** 14

13. 6 **14.** $\frac{3}{2}$ **15.** $1\frac{7}{9}$

OBJECTIVE TYPE QUESTIONS

1. (*i*) rational (*ii*) an irrational (*iii*) an irrational

(*iv*) terminating; recurring (*v*) non-terminating non-recurring

(*vi*) an irrational (*vii*) infinitely many (*viii*) infinitely many

(*ix*) infinitely many (*x*) unique (*xi*) other than 2 or 5

2. (*i*) True (*ii*) False; for example, –3 is an integer but it is not a whole number.

(*iii*) True; consider any integer m, then it can be written as $m=\frac{m}{1}$.

(*iv*) False; for example, $\frac{2}{3}$ is a rational number but it is not an integer.

(*v*) False; for example, –5 is a real number but it is not irrational.

(*vi*) True

(*vii*) False; there is a point on the number line which represents the number –2 and it is not of the form $\sqrt{n}$, where n is a natural number.

(*viii*) False; 0 is rational and $\sqrt{3}$ is irrational but their product $= 0 \times \sqrt{3} = 0$, which is rational.

(*ix*) False; since division by zero is not allowed, $\frac{1}{0}$ is meaningless *i.e.* $\frac{1}{0}$ is not defined, so reciprocal of 0 does not exist and 0 is a rational number.

(*x*) False; $\sqrt{4} = 2$, which is rational.

(*xi*) False; for example, between 1 and 2, there is no integer.

(*xii*) False; for example, $1+\sqrt{2}$ is an irrational number and $(1+\sqrt{2})^2 = 1+2+2\sqrt{2} = 3+2\sqrt{2}$, which is an irrational number.

3. (*c*) **4.** (*b*) **5.** (*d*) **6.** (*c*) **7.** (*c*) **8.** (*c*)
9. (*d*) **10.** (*d*) **11.** (*d*) **12.** (*c*) **13.** (*a*) **14.** (*c*)
15. (*c*) **16.** (*d*) **17.** (*b*) **18.** (*b*) **19.** (*d*) **20.** (*a*)
21. (*c*) **22.** (*c*) **23.** (*d*) **24.** (*b*) **25.** (*c*) **26.** (*c*)
27. (*d*) **28.** (*b*) **29.** (*a*) **30.** (*d*) **31.** (*b*) **32.** (*c*)
33. (*c*) **34.** (*b*) **35.** (*a*) **36.** (*a*) **37.** (*c*)

HIGHER ORDER THINKING SKILLS (HOTS)

1. (*i*) $\frac{2+\sqrt{6}-\sqrt{2}}{4}$ (*ii*) $\frac{5\sqrt{6}+6\sqrt{5}+\sqrt{330}}{60}$ **2.** (*i*) -4 (*ii*) $\frac{4}{7}$ (*iii*) $\frac{7}{2}$

CHAPTER TEST

1. (*b*) **2.** (*c*) **3.** (*b*) **4.** (*b*) **5.** (*c*) **6.** (*d*)
7. (*d*) **8.** $\frac{3}{80}$; 0.0375
9. Six ; the repeating string of remainders is 4, 2, 8, 5, 7, 1
10. $\frac{121}{150}, \frac{61}{75}, \frac{41}{50}, \frac{62}{75}$ **11.** $\frac{27}{110}$ **12.** $\sqrt{17}, \sqrt{15}, \sqrt{14}, \sqrt{13}$ **15.** $\frac{167}{90}$
16. $a = 3, b = -2$ **17.** $a = -\frac{2}{5}, b = \frac{3}{5}$ **18.** 2.063 **19.** 7 **20.** 98
21. $\sqrt[4]{3}, \sqrt{2}, \sqrt[4]{5}, \sqrt[3]{4}$

EXERCISE 2.1

1. (*i*) polynomial, degree 4 (*ii*) not a polynomial (*iii*) not a polynomial
(*iv*) polynomial ; degree 5 (*v*) polynomial; degree 2 (*vi*) polynomial ; degree 8
(*vii*) not a polynomial in one variable (*viii*) not a polynomial in one variable
(*ix*) polynomial; degree 2

2. (*i*) $\frac{\pi}{2}$ (*ii*) 1 (*iii*) 3

3. (*i*) Coefficient of $x^2 = -7$; leading term = $\sqrt{11}x^5$
(*ii*) Coefficient of $y^3 = 3$; leading coefficient = -3

4. (*i*) 6 (*ii*) $\frac{1}{5}$ (*iii*) -1 (*iv*) $\frac{1}{5}$

5. (*i*) binomial (*ii*) trinomial (*iii*) monomial (*iv*) multinomial
(*v*) trinomial (*vi*) binomial

6. $5x^{35} - 7x$; $3y^{100}$ (we can write many more polynomials satisfying given conditions)

7. (*i*) cubic; 3 (*ii*) quadratic; 3 (*iii*) linear ; 2 (*iv*) biquadratic; 4
(*v*) constant ; 1 (*vi*) quadratic ; 1

8. (*i*) -3 ; 3 ; -39 (*ii*) -4 ; -3; 0 (*iii*) 2; 4; 16 **9.** (*i*) 4 (*ii*) $16 + \sqrt{11}$ (*iii*) 0

EXERCISE 2.2

1. 2 is not a zero, -2 is a zero

2. 0 and -3 both are zeroes

4. (*i*) $-\frac{1}{3}$ is a zero (*ii*) $\frac{4}{5}$ is not a zero (*iii*) 1 and -1 both are zeroes
(*iv*) -1 and 2 both are zeroes (*v*) 0 is a zero, 7 is not a zero

(*vi*) $-\frac{1}{\sqrt{3}}$ is a zero, $\frac{2}{\sqrt{3}}$ is not a zero

6. (*i*) $-\frac{5}{2}$ (*ii*) $-\frac{b}{a}$ (*iii*) $-1, -\frac{3}{2}$

7. (*i*) $\frac{3}{5}$ (*ii*) $\frac{2}{3}, -\frac{2}{3}$ (*iii*) $\frac{2}{\sqrt{5}}, \frac{3}{\sqrt{5}}, -\frac{3}{\sqrt{5}}$ (*iv*) $1, -\frac{2}{3}$ (*v*) $-2, 3, \frac{7}{2}$

8. -5

EXERCISE 2.3

1. Quotient = $x^2 - 2x + 4$, remainder = -9

2. Quotient = $x^3 + x^2 + x + 1$, remainder = 2

3. 7 **4.** (*i*) 31 (*ii*) 0 (*iii*) $-\frac{27}{8}$ (*iv*) $-\frac{136}{27}$ (*v*) 2 **5.** No

6. Yes **7.** (*i*) No (*ii*) No

8. $5a$ **9.** 3 **10.** $-\frac{13}{2}$ **11.** 4

EXERCISE 2.4

1. No **4.** (*i*) Yes (*ii*) Yes (*iii*) No (*iv*) Yes

5. (*i*) $\frac{3}{2}$ (*ii*) $\sqrt{2} - 1$ (*iii*) $-2 - \sqrt{2}$ (*iv*) $-\frac{7}{6}$ **6.** (*i*) 2 (*ii*) $\frac{1}{3}$ (*iii*) $\frac{3}{2}$

7. (*i*) 1 (*ii*) -1 (*iii*) 25 **8.** $a = 2, b = -1$ **10.** $a = 5, b = 8$

EXERCISE 2.5

1. (*i*) $(x + 5)(x + 17)$ (*ii*) $(y - 2)(y - 21)$ **2.** (*i*) $(m + 3)(m - 10)$ (*ii*) $(t + 27)(t - 4)$

3. (*i*) $(x + 3)(2x + 1)$ (*ii*) $(2x + 3)(3x - 2)$ **4.** (*i*) $(x + 1)(3x - 4)$ (*ii*) $(2x + 3)(3x - 1)$

5. (*i*) $(1 - 21y)(1 + 3y)$ (*ii*) $(4x + 5)(3x - 2)$

6. (*i*) $(x - 1)(x - \sqrt{2})$ (*ii*) $(x + \sqrt{2})(\sqrt{2}x + 5)$

7. (*i*) $(x - 1)(x + 5)$ (*ii*) $(y + 3)(y - 5)$

EXERCISE 2.6

1. $(x - 1)(x - 10)(x - 12)$ **2.** (*i*) $(x + 1)(x + 1)(x - 5)$ (*ii*) $(x + 1)(x - 2)(x + 2)$

3. (*i*) $(x - 1)(x - 2)(x - 3)$ (*ii*) $(x + 1)(x + 2)(x + 10)$

4. (*i*) $(y - 1)(y + 1)(2y + 1)$ (*ii*) $(x - 1)(x + 1)(3x - 1)$

EXERCISE 2.7

1. (*i*) $x^2 + 6x + 9$ (*ii*) $x^2 + 2x - 15$ (*iii*) $\frac{4}{9}y^2 - \frac{4}{3}y + 1$ (*iv*) $y^4 - \frac{9}{4}$

2. (*i*) $(2x + 5)(2x + 5)$ (*ii*) $(2y - 1)(2y - 1)$ (*iii*) $\left(x + \frac{y}{10}\right)\left(x - \frac{y}{10}\right)$ (*iv*) $\left(3x - \frac{1}{6}\right)\left(x + \frac{5}{6}\right)$

3. (*i*) $4x^2 - 4xy + y^2 - 9$ (*ii*) $9x^2 - 30x + 25 - y^2$

4. (*i*) $9a^2 + 16b^2 + 25c^2 + 24ab + 40bc + 30ca$ (*ii*) $9a^2 + 25b^2 + c^2 - 30ab + 10bc - 6ca$

(*iii*) $4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8zx$ (*iv*) $4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12zx$

5. (*i*) $8x^3 + 12x^2 + 6x + 1$ (*ii*) $125p^3 - 27q^3 - 225p^2q + 135pq^2$

(*iii*) $\frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1$ (*iv*) $64 - \frac{16}{x} + \frac{4}{3x^2} - \frac{1}{27x^3}$

6. (*i*) $8p^3 + 27q^3$ (*ii*) $27p^3 - 64q^3$ (*iii*) $\frac{x^3}{8} + 8y^3$

7. (*i*) $8x^3 + 27y^3 + 64z^3 - 72xyz$ (*ii*) $8x^3 - y^3 + 27z^3 + 18xyz$

8. (*i*) $(a + b + 1)(a - b - 1)$ (*ii*) $(x + y - 3)(x - y + 3)$

9. (*i*) $(2x + 3y - 4z)(2x + 3y - 4z)$ (*ii*) $(-5x + 4y + 2z)(-5x + 4y + 2z)$
(*iii*) $(x + y + 4)(x + y + 4)$

10. (*i*) $(2x + 3y)(2x + 3y)(2x + 3y)$ (*ii*) $(4a - 3b)(4a - 3b)(4a - 3b)$
(*iii*) $(3 - 5a)(3 - 5a)(3 - 5a)$ (*iv*) $(1 - 4a)(1 - 4a)(1 - 4a)$

11. (*i*) $(4m - 7n)(16m^2 + 28mn + 49n^2)$ (*ii*) $(1 + 4x)(1 - 4x + 16x^2)$

12. (*i*) $(3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3zx)$
(*ii*) $(a - 2b - 4c)(a^2 + 4b^2 + 16c^2 + 2ab - 8bc + 4ca)$

13. $3(x - 2y)(2y - 3z)(3z - x)$

15. (*i*) one possible answer is : length = $5x - 3$, breadth = $5x - 4$
(*ii*) one possible answer is : length = $7x - 3$, breadth = $5x + 4$

16. (*i*) one possible answer is : 3, x and $x - 4$
(*ii*) one possible answer is : $4k$, $3y + 5$ and $y - 1$

EXERCISE 2.8

1. (*i*) 11130 (*ii*) 9120 (*iii*) 9604

2. (*i*) 1061208 (*ii*) 912673 (*iii*) 994011992

3. (*i*) –1260 (*ii*) – 0.018 **4.** 68 **5.** (*i*) 26 (*ii*) 5 **6.** 100

7. 22 **8.** 63 **9.** 135 **10.** 370 **11.** – 25 **12.** 0

VERY SHORT ANSWER TYPE QUESTIONS

1. 11 **2.** 35 **3.** 5 **4.** $7x^{24} - 5x^2 + 9$

5. –1 **6.** – 3 **7.** $\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}$ **8.** –1 **9.** – 5 **10.** 4

11. $-\frac{5}{3}$ **12.** 9409 **13.** 45 **14.** 35 **15.** $(x - 2y)(x^2 + 2xy + 4y^2)$

OBJECTIVE TYPE QUESTIONS

1. (*i*) whole number, variable x (*ii*) not defined (*iii*) zero (*iv*) four
(*v*) 9 (*vi*) three (*vii*) one, one (*viii*) no (*ix*) zero (*x*) zero (*xi*) 0

2. (*i*) False; for example, $2x^2 - 3xy + 4y^2 - 5x + 7y + 6$ is a polynomial of degree 2 *i.e.* a quadratic polynomial in two variables x, y and it has 6 terms.

(*ii*) False ; because a binomial has exactly two terms.

(*iii*) False ; because $2x^3 + 2x - 5$ is a polynomial but not a binomial.

(*iv*) False; because zero polynomial is a constant polynomial and the degree of zero polynomial is not defined.

(*v*) True ; for example, $7x^5 + 3x$ is a binomial whose degree is 5.

(*vi*) False ; because, $x - 3$ is a polynomial whose zero is 3.

(*vii*) True ; for example, $x^2 - 2x$ is a polynomial whose one zero is 0.

(*viii*) False ; for example, $x^2 - 4$ is a polynomial which has two zeroes 2 and –2.

(*ix*) False; for example, consider the polynomial $f(x) = x^2 - 4x + 9$, then $f(x) = (x - 2)^2 + 5$ but $(x - 2)^2 \geq 0$ for all real values of $x \Rightarrow f(x) \geq 5$ for all real values of $x \Rightarrow f(x)$ has no real zeroes.

(*x*) True; as 3 is a zero of $f(x)$, so $f(3) = 0$. Now $3f(0) = 3 \times 0 = 0 \Rightarrow 3$ is a zero of the polynomial $3f(x)$.

(*xi*) True; for example, $3x^5 - 4x + 7$ is a polynomial of degree 5 in the variable x and it has three terms.

(*xii*) False ; for example, $f(x) = 2x^5 - 3x$ and $g(x) = -2x^5 + 2x^2 + 1$ are polynomials of degree 5 but $f(x) + g(x) = 2x^2 - 3x + 1$, which is a polynomial of degree 2.

(*xiii*) False; for example, 2 is one of the zeroes of the polynomial $f(x) = x^2 - 4$ and 3 is one of the zeroes of the polynomial $g(x) = x^2 - 9$ then $f(x) + g(x) = 2x^2 - 13$ and we note that the value of $f(x) + g(x)$ at $x = 5$ is $2 \times 5^2 - 13$ *i.e.* 37, therefore, 5 is not a zero of $f(x) + g(x)$.

3. (*c*) **4.** (*b*) **5.** (*c*) **6.** (*b*) **7.** (*a*) **8.** (*c*)
9. (*a*) **10.** (*d*) **11.** (*b*) **12.** (*c*) **13.** (*b*) **14.** (*d*)
15. (*b*) **16.** (*a*) **17.** (*d*) **18.** (*c*) **19.** (*c*) **20.** (*b*)
21. (*d*) **22.** (*b*) **23.** (*c*) **24.** (*b*) **25.** (*d*) **26.** (*d*)
27. (*b*) **28.** (*d*) **29.** (*d*) **30.** (*c*) **31.** (*d*)

HIGHER ORDER THINKING SKILLS (HOTS)

1. 13. (**Hint:** It will be found that $a = 5$, $b = 3$) **2.** 17 **3.** $2x + 3$
4. 29 **5.** 3 **6.** $(a + b)(b + c)(c + a)$.

CHAPTER TEST

1. (*i*) ↔ (*c*) (*ii*) ↔ (*d*) (*iii*) ↔ (*b*) (*iv*) ↔ (*a*)
2. (*i*) –1 (*ii*) –2 **3.** True **4.** (*c*) **5.** (*a*) **6.** (*b*) **7.** (*a*)
8. (*c*) **11.** 2 **12.** –2 **13.** 5 **14.** $a = 3$, $b = -15$
15. (*i*) $(3x + 2)(5x - 3)$ (*ii*) $\left(\sqrt{2}x - 1\right)\left(3x - \sqrt{2}\right)$
16. (*i*) $-3, -\frac{1}{2}$ (*ii*) $-1, \frac{4}{3}$ **17.** $(x + 2)(x - 3)(3x + 2)$ **18.** $(x + 1)(x + 3)(2x + 1)$
19. (*i*) $4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8zx$ (*ii*) $8a^3 - 27b^3 - 36a^2b + 54ab^2$
(*iii*) $\frac{1}{x^3} + \frac{y^3}{27} + \frac{y}{x^2} + \frac{y^2}{3x}$
20. (*i*) $8x^3 - 27y^3$ (*ii*) $27x^3 - 8y^3 + 125z^3 + 90xyz$
21. (*i*) $(3x - 2)(3x - 2)$ (*ii*) $(2a - b)(2a - b)(2a - b)$ (*iii*) $\left(2p + \frac{1}{5}\right)\left(2p + \frac{1}{5}\right)\left(2p + \frac{1}{5}\right)$
(*iv*) $(3x + 2y)(9x^2 - 6xy + 4y^2)$ (*v*) $-10y(12x^2 + 25y^2)$
(*vi*) $(2x - 3y + 4z)(4x^2 + 9y^2 + 16z^2 + 6xy + 12yz - 8zx)$
22. (*i*) 77760 (*ii*) $-\frac{7}{48}$ **23.** $3(2x - y - z)(2y - z - x)(2z - x - y)$
25. (*i*) 26 (*ii*) 16, – 16 **26.** (*i*) – 10 (*ii*) 29

MODEL QUESTION PAPER 1

1. $\frac{1}{9}$ **2.** –2 **3.** $p = 7$, $q = -4$ **4.** Yes
5. Infinitely many; 1.8010010001..., 2.020020002....., 2.131131113....
6. $\frac{47}{198}$ **8.** 0 **9.** 101 **11.** $(x - 2)(x + 1)(2x + 1)$

EXERCISE 3

4. Since it is true for everything at any place of the world.

5. Yes, if a straight line l falls on two straight lines m and n such that the sum of the interior angles on one side of l is 180°, then by Euclid's fifth postulate the lines m and n will not meet on this sides of l. Also, we find that the sum of the interior angles on the other side of line l is again 180°. Therefore, the lines m and n will not meet on the other sides of l. Hence, the lines m and n do not meet, therefore, these lines are parallel.

OBJECTIVE TYPE QUESTIONS

1. (*i*) equal (*ii*) greater than (*iii*) axioms, postulates (*iv*) postulates (*v*) theorems (*vi*) equal (*vii*) non-Euclidean

2. (*i*) True, because if we define these terms then we have to define many terms and we get a long chain of definitions. So, mathematician accept these terms as undefined.

(*ii*) True ; because it fails on curved surface. For example, on curved surfaces, the sum of angles of a triangle may be greater than 180°.

(*iii*) False ; because the whole is greater than the part (Euclid's 5th axiom).

(*iv*) False ; because boundaries of solids are surfaces.

(*v*) False; because the edges of surfaces are lines.

(*vi*) True; because it is due to Euclid's 5th axiom.

(*vii*) True; because it is equivalent version of Euclid's fifth postulate.

3. (*b*) **4.** (*b*) **5.** (*a*) **6.** (*d*) **7.** (*a*) **8.** (*c*)
9. (*a*) **10.** (*c*) **11.** (*b*) **12.** (*a*) **13.** (*a*) **14.** (*b*)
15. (*b*) **16.** (*a*) **17.** (*a*)

EXERCISE 4.1

1. 180° **4.** (*i*) $x = 55°, y = 125°, z = 125°$ (*ii*) $x = 115°, y = 140°, z = 40°$

5. $x = 32$, $\angle AOC = 40°$, $\angle AOD = 140°$, $\angle COE = 103°$

EXERCISE 4.2

1. (*i*) Parallel (*ii*) not parallel **3.** (*a*) $\left(32\frac{1}{2}\right)°$ (*b*) 120°

4. (*a*) $x = 75°, y = 42°, z = 105°$ (*b*) $x = 39°$

5. (*a*) $\angle AED = 54°$, $\angle DEF = 58°$, $\angle PDQ = 58°$

EXERCISE 4.3

1. (*i*) No ; because the sum of angles cannot be less than 180°.

(*ii*) No ; because the sum of angles cannot be greater than 180°.

(*iii*) No ; because the sum of angles cannot be greater than 180°.

3. None ; because the sum of angles cannot be 181°.

4. Infinitely many; because sum of angles of every such triangle will be 180°.

5. 40°, 60°, 80° **6.** (*i*) 55 (*ii*) 26 **7.** (*i*) $x = 70, y = 40$ (*ii*) $x = 60, y = 80$

8. (*a*) 98° (*b*) 35 **9.** (*a*) 50° (*b*) 110 **10.** (*a*) 130° (*b*) $x = 30°, y = 70°, z = 110°$

VERY SHORT ANSWER TYPE QUESTIONS

1. 115° **2.** 24° **3.** $x = 56°$; $y = 84°$ **4.** 25°.

5. 15 **6.** 180° **7.** 108° **8.** 56° **9.** 27° **11.** 40°

12. 35

OBJECTIVE TYPE QUESTIONS

1. (*i*) different sides (*ii*) 180° (*iii*) a line (*iv*) equal to 360° (*v*) two (*vi*) equal (*vii*) do not intersect each other (*viii*) remains the same (*ix*) same (*x*) opposite sides (*xi*) (1) equal (2) equal (3) 180° (*xii*) parallel (*xiii*) equal (*xiv*) supplementary (*xv*) parallel (*xvi*) two (*xvii*) interior opposite (*xviii*) greater, two (*xix*) linear pair of angles

2. (*i*) False; two angles with a common vertex, common arm and their non-common arms lie on different sides of common arm are called adjacent angles.

(*ii*) False; if a transversal intersects two parallel lines, then each pair of corresponding angles is equal.

(*iii*) False; if a transversal intersects two parallel lines, then each pair of alternate interior angles is equal.

(*iv*) False; if a transversal intersects two parallel lines, then the sum of each pair of consecutive interior angles is two right angles.

3. (*b*) **4.** (*b*) **5.** (*b*) **6.** (*b*) **7.** (*b*) **8.** (*d*)

9. (*b*) **10.** (*c*) **11.** (*c*) **12.** (*c*) **13.** (*a*) **14.** (*b*)

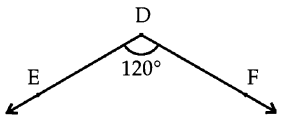
15. (*c*) **16.** (*a*) **17.** (*b*)

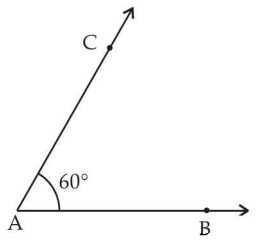
HIGHER ORDER THINKING SKILLS (HOTS)

1. 68° **2.** 135° **3.** $x = 30°$, $y = 40°$

CHAPTER TEST

1. (*a*) True; all linear pairs are certainly supplementary angles but all supplementary angles may not be linear pairs because supplementary angles need not be adjacent. Look at the figure given below, $\angle CAB + \angle EDF = 60° + 120° = 180°$, so these angles are supplementary but these are not adjacent angles and hence they do not form a linear pair.





(*b*) No ; each of these will be a right angle only when they form a linear pair.

3. (*a*) $\angle BOC = 70°$, $\angle AOC = 110°$ (*b*) $x = 35$, $y = 45$, $z = 100$

5. (*a*) 50° (*b*) $x = 72°$, $y = 108°$, $z = 115°$

6. (*a*) $p = 85°$, $q = 62°$, $r = 118°$, $s = 118°$ (*b*) $x = 132°$, $y = 42°$, $z = 48°$

7. (*a*) 30 (*b*) 130 **9.** (*i*) $x = 65$, $y = 50$ (*ii*) $x = 60$, $y = 80$ (*iii*) $x = 34°$, $y = 124°$

10. (*a*) $x = 55°$, $y = 75°$, $z = 105°$ (*b*) $x = 39°$, $y = 96°$, $z = 39°$

EXERCISE 5.1

1. No ; because BC and QR are not corresponding sides. In fact, BC = PQ.

2. No ; angles must be included angles.

EXERCISE 5.2

1. AB = QR, then $\Delta ABC \cong \Delta QRP$ by ASA congruence rule.

2. BC = RP, then $\Delta ABC \cong \Delta QRP$ by AAS congruence rule.

3. No ; sides must be corresponding sides.

EXERCISE 5.3

1. $\angle B = 45°$, $\angle C = 45°$ **2.** 50°

EXERCISE 5.5

1. PR; because $\angle Q = 80°$ and the side opposite the greatest angle is longest.

5. (*i*) No ; sum of the lengths of two sides = length of third side.

(*ii*) No ; sum of the lengths of two sides is less than the length of third side

(*iii*) Yes ; because in each case the sum of lengths of two sides is greater than the length of third side.

6. Yes ; in ΔABD, AB + BD > AD and in ΔACD, CA + CD > AD

$\Rightarrow$ AB + BC + CA > 2 AD

7. Yes ; in ΔABM, AM + BM > AM and in ΔACM, CA + CM > AM

$\Rightarrow$ AB + BC + CA > 2 AM $\Rightarrow$ perimeter of ΔABC > 2 AM.

VERY SHORT ANSWER TYPE QUESTIONS

1. No; BC and QR are not corresponding sides. In fact, BC = RP.

2. AC = PQ, then $\Delta ABC \cong \Delta PRQ$ by AAS congruence rule.

3. AAS **4.** RHS **5.** 80° **6.** 50° **7.** $\angle B = \angle C = 50°$

8. 57.5° **9.** BC **10.** BC, AC

OBJECTIVE TYPE QUESTIONS

1. (*i*) same, size (*ii*) equal (*iii*) equal (*iv*) equal

(*v*) equal (*vi*) equal (*vii*) greater than (*viii*) less than

2. (*i*) True (*ii*) True (*iii*) True

(*iv*) False; because the sides may not be equal. (*v*) True

(*vi*) True (*vii*) True (*viii*) True

(*ix*) False ; in a right angled triangle, the hypotenuse is the longest side

3. (*b*)	**4.** (*b*)	**5.** (*b*)	**6.** (*b*)	**7.** (*a*)	**8.** (*b*)
9. (*c*)	**10.** (*c*)	**11.** (*d*)	**12.** (*c*)	**13.** (*b*)	**14.** (*b*)
15. (*c*)	**16.** (*a*)	**17.** (*a*)	**18.** (*a*)	**19.** (*d*)	**20.** (*d*)
21. (*c*)	**22.** (*b*)	**23.** (*d*)	**24.** (*a*)	**25.** (*d*)	**26.** (*b*)

HIGHER ORDER THINKING SKILLS (HOTS)

1. BD, AB, DC **3.** 36°

CHAPTER TEST

1. Two triangles need not be congruent, because AB and EF are not corresponding sides in two triangles.

MODEL QUESTION PAPER 2

1. 20°, 60° **2.** PQ = 5 cm and $\angle R = 60$ **4.** 100° **9.** 40°

EXERCISE 6.1

1. (*i*) True; because a point on y-axis is of the form $(0, y)$.

(*ii*) False; because the perpendicular distance of a point from the x-axis is the absolute value of its ordinate. Hence, it is 3 not 4.

2. (*i*) (0, 0) (*ii*) (0, –4) (*iii*) (5, 0) **3.** II, IV, x-axis, I, III respectively

4. C, D, E and G

5. (*i*) 4; 3; (4, 3) (*ii*) –3; 4; (–3, 4) (*iii*) –5; –4; (–5, –4) (*iv*) 3; –4; (3, –4)

6. (*i*) B(– 5, 2), C(– 2, – 3), E(3, – 1) (*ii*) F (*iii*) 1 (*iv*) A (0, 3), D (4, 0)

7. (*i*) P(3, 2), R(3, 0), Q(3, – 1) (*ii*) 0 **8.** (0, 0), (6, 0), (6, 4), (0, 4)

9. (0, 0), (– 6, 0), (– 6, –4), (0, – 4) **10.** A(– 3, 0), B(3, 0), C(3, 3), D(– 3, 3)

EXERCISE 6.2

2. The point (–2, 4) lies in second quadrant; (3, –1) lies in fourth quadrant; (–1, 0) lies on x-axis; (1, 2) lies in first quadrant and (–3, –5) lies in the third quadrant.

5. (*i*) Collinear (*ii*) Not collinear **6.** Trapezium (in which PS parallel to QR)

7. (4, 3) **8.** (*i*) (2, 1) (*ii*) (5, 7)

VERY SHORT ANSWER TYPE QUESTIONS

1. (*i*) False; the coordinates of any point on the x-axis are of the form $(k, 0)$ where k is a real number.

(*ii*) False; the coordinates of any point on the y-axis are of the form $(0, b)$ where b is a real number.

(*iii*) False; a point lies in the IInd quadrant if its abscissa is negative and ordinate is positive.

2. $\left(5, -\frac{3}{2}\right)$ **3.** (0, –3) **4.** (–4, 0) **5.** IInd **6.** (3, 0), (–3, 0)

OBJECTIVE TYPE QUESTIONS

1. (*i*) origin (*ii*) quadrants (*iii*) (0, 0) (*iv*) 7; –3

(*v*) fourth (*vi*) zero (*vii*) zero (*viii*) x-coordinate or abscissa

(*ix*) y-coordinate or ordinate (*x*) (–5, 3) (*xi*) different

2. (*i*) True (*ii*) True (*iii*) True (*iv*) True (*v*) False

(*vi*) True (*vii*) True

3. (*b*) **4.** (*c*) **5.** (*a*) **6.** (*c*) **7.** (*b*) **8.** (*c*)

9. (*a*) **10.** (*c*) **11.** (*d*) **12.** (*d*) **13.** (*c*) **14.** (*d*)

15. (*b*) **16.** (*b*) **17.** (*d*) **18.** (*b*) **19.** (*c*)

HIGHER ORDER THINKING SKILLS (HOTS)

1. $\left(3, 2\sqrt{3}\right)$ **2.** D (–4, –3) ; (0, –3) ; 40 sq. units.

CHAPTER TEST

1. (*i*) II (*ii*) III (*iii*) II (*iv*) I

2. P(1, 1), Q(–3, 0) R(–2, –3), S(2, 1), T(4, –2), O(0, 0)

3. (*i*) (3, 2) (*ii*) (–3, 2) (*iii*) (–3, –2) (*iv*) (3, –2)

4. A(–1, 3), C(2, –2) **5.** M(–3, 0), N(0, 4) **6.** Non-collinear

8. Rectangle ; 15 sq. units **9.** Rhombus ; 12 sq. units **10.** (2, –2) ; 25 sq. units

EXERCISE 7.1

1. (*i*) False ; because area of triangle = $\frac{1}{2} \times 4 \times 6$ cm^2 = 12 cm^2

(*ii*) False ; because area of parallelogram = 10×3.5 cm^2 = 35 cm^2

(*iii*) True ; because it will be found that other diagonal of rhombus = 12 cm, so its area = $\frac{1}{2} \times 16 \times 12$ cm^2 = 96 cm^2

(*iv*) False ; because area of a regular hexagon with side '*a*' is the sum of the areas of the six equilateral triangles each with side *a*.

2. 114 m^2 **3.** 23 cm, 27 cm **4.** $256\sqrt{3}$ cm^2

5. (*i*) 60 cm^2 (*ii*) $\frac{120}{17}$ cm

EXERCISE 7.2

1. $75\sqrt{15}$ cm^2 **2.** ₹ 10500 **3.** $8\sqrt{30}$ cm^2 **4.** $9\sqrt{15}$ cm^2 **5.** $\frac{60}{13}$ cm

6. $1500\sqrt{3}$ m^2 **7.** $20\sqrt{2}$ m^2 **8.** ₹ 84000 **9.** $375\sqrt{15}$ m^2 ; ₹ 4940.

EXERCISE 7.3

1. $600\sqrt{15}$ m^2 **2.** 9600 m^2 **3.** 15.2 cm^2 (approx.) **4.** $24(\sqrt{6} + 1)$ cm^2

5. 3 cm **6.** ₹ 705.60 (approx.) **7.** $1000\sqrt{6}$ cm^2 ; $1000\sqrt{6}$ cm^2.

VERY SHORT ANSWER TYPE QUESTIONS

1. 22 cm^2 **2.** 48 cm^2 **3.** 40 cm^2 **4.** 30 cm **5.** 96 cm^2 **6.** $5\sqrt{2}$ cm

7. $2\sqrt{3}$ cm^2 **8.** 16.5 cm^2 **9.** 120 cm^2 **10.** $21\sqrt{15}$ cm^2

OBJECTIVE TYPE QUESTIONS

1. (*i*) $\frac{\sqrt{3}}{4}a^2$ (*ii*) $\frac{\sqrt{3}}{2}a$ (*iii*) $\frac{1}{4}b\sqrt{4a^2 - b^2}$ (*iv*) $\sqrt{s(s-a)(s-b)(s-c)}$

(*v*) diagonals (*vi*) height (*vii*) height (*viii*) right angle

2. (*i*) True (*ii*) True (*iii*) True

3. (*a*) **4.** (*d*) **5.** (*d*) **6.** (*b*) **7.** (*d*) **8.** (*c*)

9. (*c*) **10.** (*a*) **11.** (*b*) **12.** (*b*) **13.** (*c*)

VALUE BASED QUESTIONS

1. ₹ 364 ; value : keeping ecological balance. Tiger is our national animal.
2. 9000 cm^2 ; values promoted are : Honesty and Truth.
3. ₹ 750 ; values promoted are : no corruption, work within time, power to Gram Panchayats.
4. First group cleaned more area by 54 m^2; total area cleaned = 306 m^2
   Value: social service and well being of people.

CHAPTER TEST

1. $2100\sqrt{15}$ m^2 **2.** 50 cm **3.** (*a*) 54 cm^2 (*b*) 1074 m^2
4. Yellow : 484 m^2 ; Red : 242 m^2 ; Green : 373.04 m^2 (approx.)
5. Area of design : 1632 cm^2 ; Remaining area : 1868 cm^2.

MODEL QUESTION PAPER 3

1. $\frac{4}{9}$ **2.** 72° **3.** (0, 4), (0, −4) **4.** 24 m **5.** Irrational

6. $(x - \sqrt{2})\ (7\sqrt{2}\,x + 4)$ **7.** 54432 **8.** square **9.** 3°

11. $\frac{389}{165}$ **12.** $a = 0,\ b = 1$

13. $(\sqrt{2}\,a + 2b - 3c)\ (2a^2 + 4b^2 + 9c^2 - 2\sqrt{2}\,ab + 6bc + 3\sqrt{2}\,ca)$

14. 0 **16.** $x = 110°,\ y = 110°$ **17.** 115°

19. CX, AX, BX **21.** $2(\sqrt{2} - 1)$

22. 93 **23.** $140\sqrt{2}$ **24.** $a = 4$

25. $(x + 1)\ (x + 3)\ (2x + 1)$

31. First group cleaned more area by 54 m^2; total area cleaned = 306 m^2
Value: social service and well being of people.

Term - II

EXERCISE 8.1

1. (*i*) $5x - 3y - 4 = 0$; $a = 5,\ b = -3,\ c = -4$

(*ii*) $2x + 3y - 9.3\bar{5} = 0$; $a = 2,\ b = 3,\ c = -9.3\bar{5}$

(*iii*) $-2x + 3y - 6 = 0$; $a = -2,\ b = 3,\ c = -6$

(*iv*) $2x - y + 0 = 0$; $a = 2,\ b = -1,\ c = 0$

(*v*) $x - \frac{y}{5} - 10 = 0$; $a = 1,\ b = -\frac{1}{5},\ c = -10$

(*vi*) $2x + 5y + 0 = 0$; $a = 2,\ b = 5,\ c = 0$

2. (*i*) $0.x + 1.y - 2 = 0$ (*ii*) $2x + 0.y - 3 = 0$ (*iii*) $0.x + 5y + \sqrt{7} = 0$

3. (*i*) $7x + 0.y + 3 = 0$; $a = 7,\ b = 0,\ c = 3$

(*ii*) $0.x + 1.y - 2 = 0$; $a = 0,\ b = 1,\ c = -2$

(*iii*) $2x + 0.y - 5 = 0$; $a = 2,\ b = 0,\ c = -5$

4. $y = \frac{x}{2} - 5$, where ₹ x is the cost of a notebook and ₹ y is that of a pen

5. (*i*) No (*ii*) Yes (*iii*) No (*iv*) No (*v*) Yes **6.** 8

7. (*i*) 0 (*ii*) 5 (*iii*) 3 **8.** (*i*) Yes (*ii*) No (*iii*) Yes

9. (*i*) (0, 0), $(1, -\frac{3}{5})$ (*ii*) $(\frac{11}{2}, 0)$, (0, 11) (*iii*) $(-\frac{5}{3}, 0)$, $(-\frac{5}{3}, 1)$

10. (*i*) (0, 7), (1, 5), (2, 3), (3, 1) (*ii*) (0, 0), (4, 1), (8, 2), (–4, –1)
(*iii*) (4, 0), (0, – 6), (2, – 3), (6, 3)

EXERCISE 8.2

2. The x-axis at (2, 0) and the y-axis at $(0, \frac{3}{2})$ **3.** (*i*) (3, 0) (*ii*) (0, 2)

5. $\frac{9}{2}$ **6.** (2, 3) **7.** $x + y = 3$; infinitely many.

8. (*i*) (8, 0) (*ii*) (0, 4)

10. Graph of each equation is a line passing through the point (0, 0).

11. $y = 4x + 6$, where x km is the distance covered and ₹ y is the total fare.

12. $y = mx$, where y denotes force, x denotes acceleration and m is constant (mass); (*i*) 30 Newton (*ii*) 36 Newton

13. (*a*) For figure (1) (*iii*) (*b*) For figure (2) (*iv*)

EXERCISE 8.3

2. (*i*) It represents a point on the number line (draw the graph).
(*ii*) It represents a line parallel to x-axis at a distance of 3 units from it above the x-axis (draw the graph).

3. (*i*) It represents a point on the number line (draw the graph).
(*ii*) It represents a line parallel to y-axis at a distance of $\frac{9}{2}$ units from it towards the left of y-axis (draw the graph).

VERY SHORT ANSWER TYPE QUESTIONS

1. 21 **2.** –1 **3.** –5 **4.** 3 **5.** (–4, 0), (0, –3)

6. (–2, 0) **7.** $-\frac{8}{3}$ **8.** $2x - 5 = 0$, $2x + 5 = 0$

OBJECTIVE TYPE QUESTIONS

1. (*i*) $y = 0$ (*ii*) $x = 0$ (*iii*) y (*iv*) x (*v*) $y = mx$, where m is any real number
(*vi*) straight line (*vii*) a point on the number line

2. (*i*) True; because geometrically every polynomial equation of degree one in two variables represents a straight line.
(*ii*) False ; because a linear equation in two variables has infinitely many solutions.
(*iii*) True ; because (0, 3) is a solution of the given equation and solution of the equation lies on its graph.
(*iv*) False ; because (0, 7) is not a solution of $x + 2y = 7$.
(*v*) False ; because the points (4, 2) and (0, 5) do not lie on x-axis.
(*vi*) False ; because $x = 0$, $y = 0$ does not satisfy the given equation unless $c = 0$.
(*vii*) False ; because (3, –5) does not satisfy the given equation.
(*viii*) False ; because every point on the graph of a linear equation is a solution.
(*ix*) False ; because graph of every linear equation in two variables is always a straight line.
(*x*) True ; because the equation of a line parallel to y-axis at a distance of a units to the left of y-axis is given by the equation $x = -a$.

(*xi*) True; because when $ax + b = 0$ is considered as an equation in one variable x, then it has unique solution $x = -\frac{b}{a}$ but when this equation is taken as $ax + 0y + b = 0$ *i.e.* when it is considered as an equation in two variables then it has infinitely many solutions $\left(-\frac{b}{a}, k\right)$, where k is any real number.

3. (*c*) **4.** (*c*) **5.** (*a*) **6.** (*a*) **7.** (*d*) **8.** (*c*)
9. (*c*) **10.** (*a*) **11.** (*c*) **12.** (*c*) **13.** (*b*) **14.** (*d*)
15. (*c*) **16.** (*b*) **17.** (*c*) **18.** (*d*) **19.** (*b*) **20.** (*c*)
21. (*b*)

VALUE BASED QUESTIONS

If the cost of a pen is ₹x and the cost of a notebook is ₹y, then the linear equation corresponding to the given statement is $3x - y = 0$.

For education, pens and notebooks are equally important for writing as thread and needle for sewing clothes.

CHAPTER TEST

1. (*i*) $3x - \sqrt{2}y + 5 = 0$; $a = 3$, $b = -\sqrt{2}$, $c = 5$

(*ii*) $2x + 5y = 0$; $a = 2$, $b = 5$, $c = 0$

(*iii*) $0x + 2y - \frac{5}{7} = 0$; $a = 0$, $b = 2$, $c = -\frac{5}{7}$

2. $(0, 5)$, $(1, 5 - \sqrt{2})$, $(-1, 5 + \sqrt{2})$, $(2, 5 - 2\sqrt{2})$

3. (*i*) unique solution (*ii*) infinitely many solutions

4. (3, 1) is a solution and (– 1, 2) is not a solution

5. Four solutions are (6, 0), (0, 3), (2, 2) and (4, 1)

6. (*i*) 8 (*ii*) – 6 **7.** $h = \frac{3}{2}$, $k = 2$

8. (*ii*) 86°F (*iii*) 35°C (*iv*) 32°F; $\left(-17\frac{7}{9}\right)$°C (*v*) Yes; –40° (in both F and C).

EXERCISE 9.1

1. (*i*) 84° each (*ii*) 36°, 60°, 108°, 156°

2. (*i*) No ; because the sum of angles ≠ 360°

(*ii*) No ; because the sum of angles will be greater than 360°

(*iii*) No ; because the sum of angles will be less than 360°

(*iv*) Yes ; because the sum of angles = 360°

3. (*i*) Trapezium (*ii*) Rectangle (*iii*) Rhombus

4. (*i*) No ; because the diagonals of a parallelogram bisect each other but need not be perpendicular to each other.

(*ii*) No ; because the diagonals of a rectangle are equal but need not be perpendicular to each other.

(*iii*) No ; because the diagonals of a rhombus are perpendicular to each other but need not be equal.

(*iv*) Not a parallelogram ; because diagonals of a parallelogram bisect each other and here OA ≠ OC.

5. (*i*) 90° (*ii*) 4 cm (*iii*) $\angle C = \angle D = 135°$

6. (*i*) 55° (*ii*) 70° (*iii*) 110° **7.** 90° **14.** 40 cm

EXERCISE 9.2

1. 3.5 cm **2.** (*i*) 13 cm (*ii*) 8.2 cm

VERY SHORT ANSWER TYPE QUESTIONS

1. AC = 7 cm, BD = 10 cm **2.** 86° each **3.** 120° **4.** 100° **5.** 100°

6. 38° **7.** 97° **8.** $x = 78°$, $y = 35°$ **9.** 4.5 cm **10.** 5 cm

OBJECTIVE TYPE QUESTIONS

1. (*i*) four (*ii*) trapezium (*iii*) parallelogram (*iv*) rectangle (*v*) rhombus (*vi*) kite (*vii*) two, triangles (*viii*) equal (*ix*) parallelogram (*x*) equal (*xi*) parallelogram (*xii*) bisect (*xiii*) parallelogram (*xiv*) parallel, half (*xv*) bisects

2. (*i*) False (*ii*) True (*iii*) False (*iv*) True (*v*) False (*vi*) False (*vii*) False (*viii*) True (*ix*) False (*x*) False (*xi*) False (*xii*) True (*xiii*) False (*xiv*) True (*xv*) False

3. (*d*) **4.** (*d*) **5.** (*c*) **6.** (*d*) **7.** (*b*) **8.** (*c*)

9. (*c*) **10.** (*d*) **11.** (*a*) **12.** (*c*) **13.** (*d*) **14.** (*c*)

15. (*d*) **16.** (*d*) **17.** (*c*) **18.** (*d*) **19.** (*c*) **20.** (*c*)

21. (*d*)

HIGHER ORDER THINKING SKILLS

1. 45° **2.** $\sqrt{3} : 1$

CHAPTER TEST

2. Yes ; because BD = EF and CD = EF (opp. side of a || gm are equal)

3. 55° **4.** 75°

EXERCISE 10

1. 12.8 cm

2. (*i*) False ; because ar (ΔSQR) = 90 cm^2 and ar (ΔASR) < ar (ΔSQR).

(*ii*) True ; because if side of $\Delta ABC = a$, then side of $\Delta BDE = \frac{a}{2}$,

so $\frac{ar(\Delta BDE)}{ar(\Delta ABC)} = \frac{\frac{\sqrt{3}}{4}\left(\frac{a}{2}\right)^2}{\frac{\sqrt{3}}{4}a^2} = \frac{1}{4}$.

(*iii*) False ; because ar (ΔDPC) = $\frac{1}{2}$ ar (|| gm ABCD) = ar (|| gm EFGD).

10. (*i*) 54 cm^2 (*ii*) 27 cm^2 **11.** 8 cm^2

VERY SHORT ANSWER TYPE QUESTIONS

1. 15 cm **2.** 35 cm^2 **3.** 20 cm^2 **4.** 35 cm^2 **5.** 5 cm

OBJECTIVE TYPE QUESTIONS

1. (*i*) equal area (*ii*) equal (*iii*) equal (*iv*) the corresponding height
(*v*) half of (*vi*) equal (*vii*) the same parallels

2. (*i*) False (*ii*) True (*iii*) False (*iv*) False

3. (*b*) **4.** (*a*) **5.** (*d*) **6.** (*b*) **7.** (*b*) **8.** (*a*)

9. (*c*) **10.** (*a*) **11.** (*d*) **12.** (*d*) **13.** (*b*) **14.** (*c*)

VALUE BASED QUESTION

Three parts ; parts are in shapes of triangles. If the farmer sows wheat in the triangular region APQ, then he must sow pulses in the other two triangular regions ASP and AQR. Wheat and pulses provide us required calories and nutrients.

CHAPTER TEST

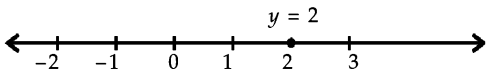
3. 16 cm^2 **4.** 30 cm^2 **6.** 12 cm^2

MODEL QUESTION PAPER 4

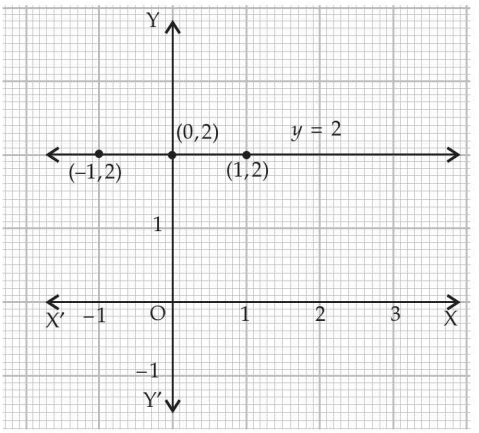
1. $\frac{43}{5}$ **2.** 18 cm **3.** 130°

5. Solution is $y = 2$

(*i*) It represents a point on the number line shown in the figure given below:



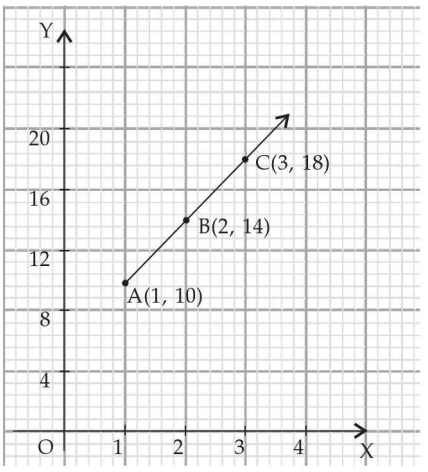
(*ii*) It represents a line parallel to x-axis shown in the adjoining figure.



9. $y = 4x + 6$.

Scale : Take 1 cm on x-axis = 1 km,
1 cm on y-axis = ₹ 4

As $x \geq 1$, the graph is the ray AB.



EXERCISE 11.1

1. (*i*) True ; because equal chords of congruent circles subtend equal angles at their respective centres.

(*ii*) True ; because AB will be a diameter.

(*iii*) True ; because the distances from the centres of two chords are equal.

(*iv*) False ; because the longer chord is at smaller distance from the centre.

(*v*) False ; because the angles will be equal only if AB = AC.

2. 1 : 1 **3.** 16 cm **4.** 12 cm **5.** 5 cm **6.** 26 cm

7. (*i*) 1 cm (*ii*) 7 cm **8.** $2\sqrt{3}$ cm **9.** 28 cm.

EXERCISE 11.2

1. (*i*) False ; because if two points lie in the same segment (major or minor) only, then the angles will be equal otherwise they are not equal).

(*ii*) True; because angle in a semicircle is a right angle, so $\angle C = 90°$ *i.e.* in ΔABC, $\angle C = 90°$ which implies $AC^2 + BC^2 = AB^2$.

(*iii*) False ; because there can be many points D such that $\angle BDC = 60°$ and each such point cannot be the centre of circle through A, B, C.

3. 50° **4.** 80° **5.** 50° **6.** 125°

7. (*i*) 110° (*ii*) 60° (*iii*) 20° (*iv*) 40° **8.** 25 **9.** 55° **10.** 70°.

EXERCISE 11.3

1. (*i*) False ; because $\angle A + \angle C = 90° + 95° = 185° \neq 180°$

(*ii*) True ; because the line segment BC makes equal angles (each = 45°) at two other points A and D on the same side of the line segment BC, therefore, the points A, B, C and D are concyclic.

2. (*i*) 75° (*ii*) 105° **3.** (*i*) 110° (*ii*) 70° **4.** 80°

5. (*i*) 32° (*ii*) 148° **6.** 30° **7.** (*i*) 150° (*ii*) 105° (*iii*) 75° (*iv*) 15°

8. (*i*) 55° (*ii*) 105°.

VERY SHORT ANSWER TYPE QUESTIONS

1. $2r$ units **2.** 120° **3.** 30° **4.** 70° **5.** 18°

6. 95° **7.** 60° **8.** 35° **9.** $x = 36°$, $y = 20$ **10.** 80°

OBJECTIVE TYPE QUESTIONS

1. (*i*) interior (*ii*) exterior (*iii*) minor (*iv*) supplementary (*v*) greater

2. (*i*) diameter (*ii*) inside (*iii*) three (*iv*) longest, equal (*v*) semicircle

(*vi*) two bounding radii (*vii*) the corresponding chord

(*viii*) same centre, different (*ix*) same radius (*x*) equal (*xi*) equal, equal

(*xii*) equal, equal (*xiii*) bisects, perpendicular (*xiv*) right angle

3. (*i*) True (*ii*) True

(*iii*) False; A circle has infinitely many number of equal chords.

(*iv*) False; There is one and one circle passing through three non-collinear points.

(*v*) True (*vi*) True (*vii*) True

(*viii*) False; The angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.

(*ix*) True (*x*) True

(*xi*) False; A diameter of a circle is a chord of the circle which passes through its centre.

4. (*i*) True; because a circle is the collection of all points in a plane which are at a fixed distance from a fixed point in the plane. Therefore, a circle is a plane figure.

(*ii*) False; because a circle is the collection of all points in a plane which are at a fixed distance from a fixed point in the plane. Therefore, a circle is a closed curve whereas the circumference is the length of the whole arc of the circle, so circumference is a real number.

(*iii*) False; because a diameter of a circle is the longest chord, therefore, the length of a chord of a circle cannot be longer than its diameter.

5. (*c*) **6.** (*d*) **7.** (*a*) **8.** (*d*) **9.** (*c*) **10.** (*a*)
11. (*b*) **12.** (*d*) **13.** (*c*) **14.** (*a*) **15.** (*b*) **16.** (*a*)
17. (*b*) **18.** (*c*) **19.** (*d*)

CHAPTER TEST

1. (*i*) 24 cm (*ii*) $12\sqrt{5}$ cm (*iii*) $6\sqrt{5}$ cm **2.** 24 cm **3.** $2\sqrt{29}$ cm **4.** 15 cm
7. 115° **8.** 70 **9.** 100°, 80° **10.** 10°
11. $\angle BDC = 80°$; $\angle BEC = 100°$ **12.** (*i*) 50° (*ii*) 40°
13. $\angle ABD = 20°$, $\angle BDC = 40°$ **14.** (*i*) 110° (*ii*) 60° (*iii*) 20° (*iv*) 40°
15. (*i*) 55° (*ii*) 55° (*iii*) 100° **16.** (*i*) 20° (*ii*) 65° **17.** 35°
18. (*i*) 40° (*ii*) 140° (*iii*) 50°

EXERCISE 12.1

1. 55° each **4.** 2.8 cm **5.** Yes **6.** Yes

9. (*i*) True ; because $67.5° = \frac{135°}{2} = \frac{1}{2}(90° + 45°)$, which can be constructed.

(*ii*) False ; because $42.5° = \frac{85°}{2}$ and angle of 85° cannot be constructed.

EXERCISE 12.2

1. (*i*) False ; because $\angle B + \angle C = 105° + 90° = 195° > 180°$.

(*ii*) True ; because $\angle B + \angle C = 60° + 45° = 105° < 180°$.

(*iii*) False ; because BC + AC must be greater than AB which is not so.

(*iv*) True ; because AC – AB < BC *i.e.* AC < AB + BC.

2. 20° each

OBJECTIVE TYPE QUESTIONS

1. (*i*) sides (*ii*) included (*iii*) greater **2.** (*c*) **3.** (*a*) **4.** (*b*)
5. (*c*) **6.** (*d*) **7.** (*a*)

EXERCISE 13.1

1. False ; because $\sqrt{3}a = 6\sqrt{3} \Rightarrow a = 6$, so edge of cube = 6 cm.
2. (*i*) 5.45 m^2 (*ii*) ₹ 109 **3.** (*i*) 4250 cm^2 (*ii*) 320 cm
4. 100 **5.** 180 cm^3 **6.** 405 cm^3 **7.** 4.75 m **8.** ₹ 4320 **9.** 135000 litres
10. 2 m **11.** 16000 **12.** 9.6 m ; 3686.4 m^3 **13.** 864 cm^2 ; 1728 cm^3
14. (*i*) $9\sqrt{3}$ cm (*ii*) 729 cm^3 **15.** 4 cm **16.** 2560 cm^3 **17.** ₹ 1350

EXERCISE 13.2

1. (*i*) True; because if r and h are radius and height respectively of given cylinder then its curved surface = $2\pi rh$ and curved surface of new cylinder = $2\pi \times 2r \times \frac{h}{2} = 2\pi rh$.

(*ii*) True; because if r and h are radius and height respectively of given cylinder then its volume = $\pi r^2 h$ and volume of new cylinder = $\pi(2r)^2 \times \frac{h}{2} = 2(\pi r^2 h)$ = 2 × volume of given cylinder.

2. 550 cm^2 **3.** 1584 m^2 **4.** (*i*) 13200 cm^2 (*ii*) 115.5 litres

5. (*i*) $62\frac{6}{7}$ m^3 (*ii*) ₹ 6285.70 **6.** 49.5 kg **7.** 739.2 litres **8.** 2 cm **9.** ₹ 68.75

10. 7.48 m^2 **11.** 17.6 m^3 **12.** 60 **13.** 616 cm^3 **14.** (*i*) 40 cm (*ii*) 38500 cm^3

15. Volume of wood = 5.28 cm^3 ; volume of graphite = 0.11 cm^3 **16.** 8800 cm^3

EXERCISE 13.3

1. (*i*) False; because if r and h are the radius and height respectively of the given cone then its volume = $\frac{1}{3}\pi r^2 h$ and the volume of the new cone = $\frac{1}{3}\pi \left(\frac{r}{2}\right)^2 (2h) = \frac{1}{6}\pi r^2 h$.

(*ii*) True; because if r and h are the radius and height respectively of the given cylinder and cone then

volume of cylinder = $\pi r^2 h = 3\ (\frac{1}{3}\ \pi r^2 h)$ = 3 × volume of cone.

(*iii*) True; because a right circular cone is formed only by revolving a right triangle about one of its sides (other than hypotenuse).

2. 220 cm^2 **3.** 165 cm^2 **4.** (*i*) 7 cm (*ii*) 462 cm^2 **5.** (*i*) 264 cm^3 (*ii*) 154 cm^3

6. (*i*) 1.232 litres (*ii*) $\frac{11}{35}$ litres **7.** 38.5 kilolitres **8.** 8 cm

9. 10 cm **10.** ₹ 1155 **11.** (*i*) 26 m (*ii*) ₹ 137280 **12.** 5500 cm^2

13. 1232 cm^3 **14.** (*i*) 50 cm (*ii*) 2816 cm^2 **15.** 301.44 cm^3 ; 188.4 cm^2

EXERCISE 13.4

1. (*i*) 2464 cm^2 (*ii*) 1386 cm^2 (*iii*) 394.24 cm^2

2. (*i*) 1.05 m^3 (approx.) (*ii*) 5887.32 cm^3 (approx.)

3. (*i*) 1386 cm^2 (*ii*) 38.5 cm^2 **4.** (*i*) $11498\frac{2}{3}$ cm^3 (*ii*) 0.004851 m^3

5. 3.85 kg (nearly) **6.** 345.39 g (approx.) **7.** 7 cm

8. (*i*) 2772 cm^2 (*ii*) 4158 cm^2 **9.** ₹ 27.72 **10.** 1 : 4 **11.** $\frac{1}{64}$

12. 30.48 cm^3 (approx.) **13.** $179\ \frac{2}{3}$ cm^3 **14.** 22.46 mm^3 (approx.)

15. 8 **16.** $89\frac{5}{6}$ cm^3 **17.** 668.66 kilolitres (approx.)

18. 942 cm^2 ; $2093\ \frac{1}{3}\ cm^3$

19. (*i*) True; because if diameter of sphere = $2r$ then diameter of cylinder is $2r$ and its height is $2r$. So volume of sphere

= $\frac{4}{3}\pi r^3 = \frac{2}{3}(2\ \pi r^3) = \frac{2}{3}(\pi r^2 \times 2r) = \frac{2}{3}$ of volume of cylinder.

(*ii*) True; because the largest cone that can be fitted in a cube of edge $2r$ has diameter $2r$ *i.e.* radius r and height $2r$, so its volume = $\frac{1}{3}\pi r^2 \times 2r = \frac{2}{3}\pi r^3$ = volume of a hemisphere of radius r.

(*iii*) True; because if radius of hemisphere is r then radius of cone is r and its height is r, radius of cylinder is r and its height is r. So

volume of cone : volume of hemisphere : volume of cylinder

$$= \frac{1}{3}\pi r^2 \times r : \frac{2}{3}\pi r^3 : \pi r^2 \times r = 1 : 2 : 3.$$

VERY SHORT ANSWER TYPE QUESTIONS

1. 150 cm^2 **2.** 125 cm^3 **3.** 64 **4.** 3.5 m **5.** 132 cm^2

6. 8 cm **7.** 12 cm **8.** 770 cm^3 **9.** 288 π cm^3 **10.** 462 cm^2

OBJECTIVE TYPE QUESTIONS

1. (*i*) total, square (*ii*) volume, cubic (*iii*) 96 cm^2, 64 cm^3 (*iv*) $75\sqrt{3}$ cm

(*v*) 27 cm^3 (*vi*) 6 units (*vii*) $\pi r^2 h$, $2\pi(r + h)r$ (*viii*) $\frac{1}{3}\pi r^2 h$, $\pi r(l + r)$ (*ix*) πd^2

(*x*) 200π cm^3 (*xi*) 144π cm^3 (*xii*) 220 cm^2 (*xiii*) 616 cm^2 (*xiv*) 462 cm^2

(*xv*) 66 cm^3 (*xvi*) 420 cm^2 (*xvii*) 7 cm (*xviii*) 13 cm

2. (*c*) **3.** (*a*) **4.** (*a*) **5.** (*b*) **6.** (*c*) **7.** (*b*)

8. (*c*) **9.** (*c*) **10.** (*c*) **11.** (*b*) **12.** (*d*) **13.** (*b*)

14. (*b*) **15.** (*b*) **16.** (*a*) **17.** (*c*) **18.** (*d*) **19.** (*b*)

20. (*a*) **21.** (*d*) **22.** (*a*)

VALUE BASED QUESTIONS

1. 47 m^2 ; values shown by Sheela are kindness and helpfulness.

2. Cylinder has greater capacity by 85 cm^3; values depicted are honesty and fairness towards customers.

3. 1232 m^3 ; Donating the tent to shelterless people shows her generosity.

HIGHER ORDER THINKING SKILLS (HOTS)

1. 152000 cm^3; 16800 cm^2

2. Length = 6 m, breadth = 4 m and height = 3.5 m

3. Outer radius = 2.5 cm and inner radius = 1.5 cm

4. Height = 12 cm, total surface area = 144.8 cm^2 **5.** ₹ 477.40

CHAPTER TEST

1. 94 m^2; 60 m^3 ; $5\sqrt{2}$ m **2.** 70 cm **3.** 2560 cm^3 ; $12\sqrt{5}$ cm

4. 6.4 m **5.** 648 cm^3 ; 504 cm^2 **6.** ₹ 775.50 **7.** 4620 cm^3 ; 1260 cm^3

8. (*i*) 40 cm (*ii*) 38500 cm^3 **9.** 22 : 15 **10.** 30 cm, 4620 cm^3

11. 1884 cm^3 ; 1507.2 cm^2 **12.** 416 cm^3 **13.** (*i*) 17.5 cm (*ii*) 18.5 cm

14. 726 m ; ₹ 58080 **15.** 3.5 cm ; 154 cm^2 **16.** 19404 cm^3

17. (*i*) 27 : 343 (*ii*) 9 : 49 **18.** 2.1 cm **19.** 270 **20.** 64

MODEL QUESTION PAPER 5

1. 42° **2.** 16 cm **4.** $9\sqrt{3}$ cm **5.** 20° **7.** 30 cm; 4620 cm^3

8. 1884 cm^3 ; 1507.2 cm^2 **10.** 64

EXERCISE 14.1

1.

No. of heads	*Tally marks*	*Frequency*
0	𝍸 I	6
1	𝍸 𝍸	10
2	𝍸 III	9
3	𝍸	5
Total		30

2.

Height (in cm)	*Tally marks*	*Frequency*
138	II	2
139	I	1
140	IIII	4
146	III	3
148	𝍸 I	6
150	𝍸	5
152	III	3
153	III	3
154	I	1
160	II	2
Total		30

3.

Score	*Tally marks*	*No. of students (Frequency)*
48	III	3
58	III	3
64	IIII	4
66	𝍸 II	7
69	𝍸 I	6
71	III	3
73	II	2
81	I	1
83	II	2
84	II	2
Total		33

4. (*i*)

No. of hours	*Tally marks*	*No. of children (Frequency)*
0 – 5	𝍸 𝍸	10
5 – 10	𝍸 𝍸 |||	13
10 – 15	𝍸	5
15 – 20	||	2
Total		30

(*ii*) 2

5. (*i*)

Relative humidity (%)	*Tally marks*	*Frequency*
84 – 86	|	1
86 – 88	|	1
88 – 90	||	2
90 – 92	||	2
92 – 94	𝍸 ||	7
94 – 96	𝍸 |	6
96 – 98	𝍸 ||	7
98 – 100	||||	4
Total		30

(*ii*) Rainy season (*iii*) 14.3

[Hint: (*iii*) Range = max. value – min. value = 99.2 – 84.9 = 14.3]

6. (*i*)

Height (in cm)	*Tally marks*	*No. of students (Frequency)*
150 – 155	𝍸 𝍸 ||	12
155 – 160	𝍸 ||||	9
160 – 165	𝍸 𝍸 ||||	14
165 – 170	𝍸 𝍸	10
170 – 175	𝍸	5
Total		50

(*ii*) 70% students have height less than 165 cm.

[Hint: No. of students whose height is less than 165 cm = 12 + 9 + 14 = 35

Percentage of these students = $\left(\frac{35}{50} \times 100\right)\%$ = 70%]

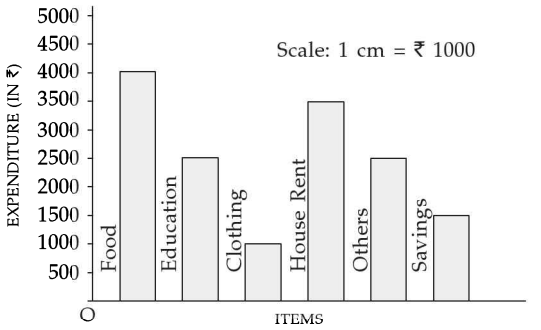
7.

Classes	23.5–27.5	27.5–31.5	31.5–35.5	35.5–39.5	39.5–43.5
Frequency	4	7	4	4	11

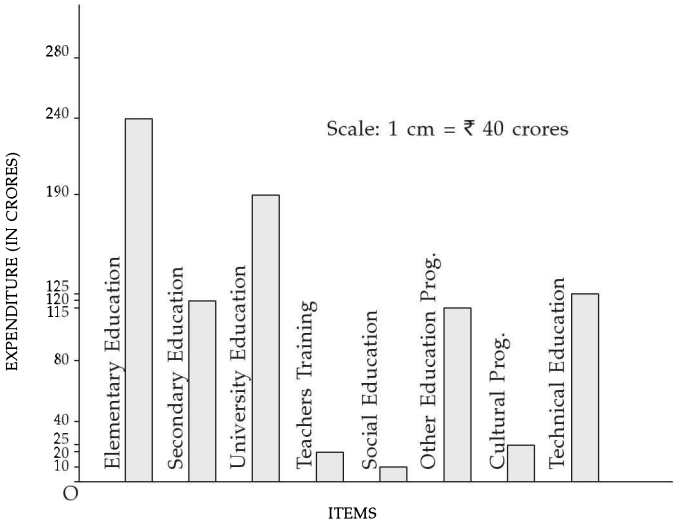
8. No ; because the number of children who watch TV for 10 or more hours a week = 4 + 2 = 6.

EXERCISE 14.2

1.



2.



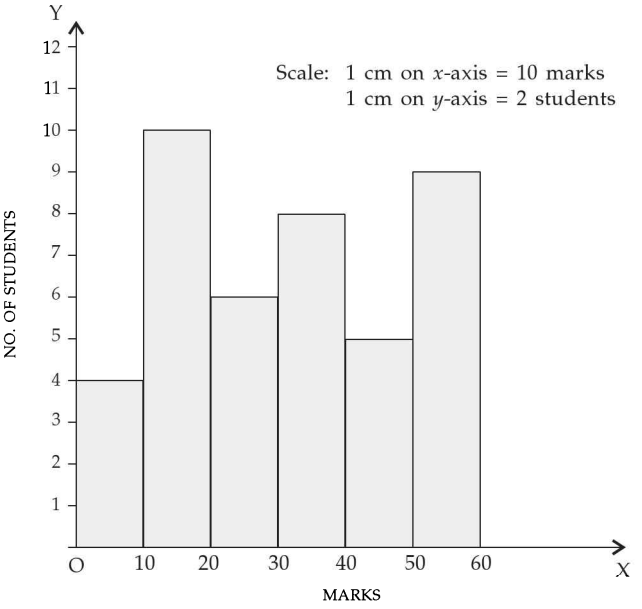
EXERCISE 14.3

1.

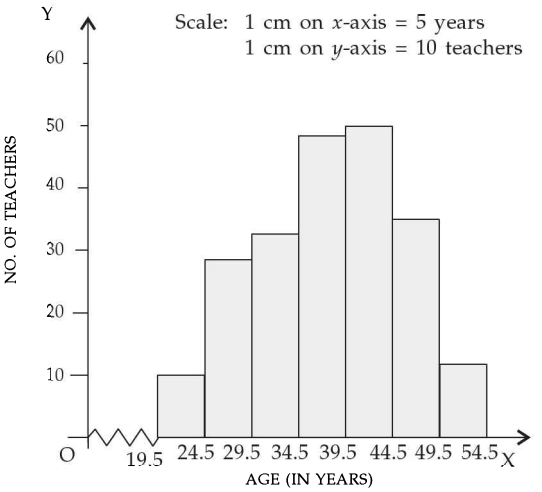
Class intervals	150–200	200–250	250–300	300–350	350–400
Frequency	50	30	35	20	10
Total					145

2. It is not correct ; because the classes are of unequal size, so it is a case of histogram with varying width but the graph is according to uniform width.

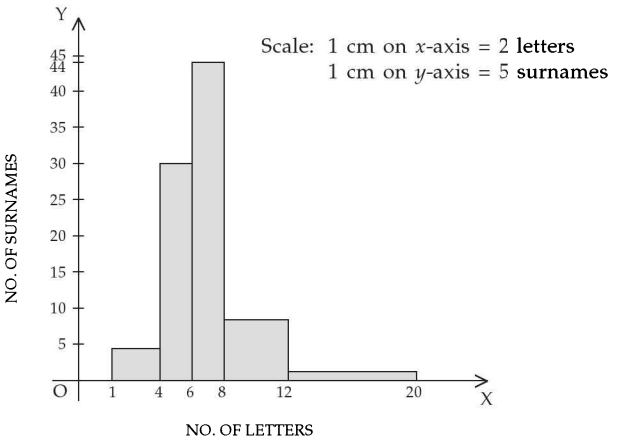
3.


Y
12
11
10
9
8
7
6
5
4
3
2
1
Scale: 1 cm on x-axis = 10 marks
1 cm on y-axis = 2 students
NO. OF STUDENTS
O
10
20
30
40
50
60
X
MARKS

4.


Y
60
50
40
30
20
10
Scale: 1 cm on x-axis = 5 years
1 cm on y-axis = 10 teachers
NO. OF TEACHERS
O
19.5
24.5
29.5
34.5
39.5
44.5
49.5
54.5
X
AGE (IN YEARS)

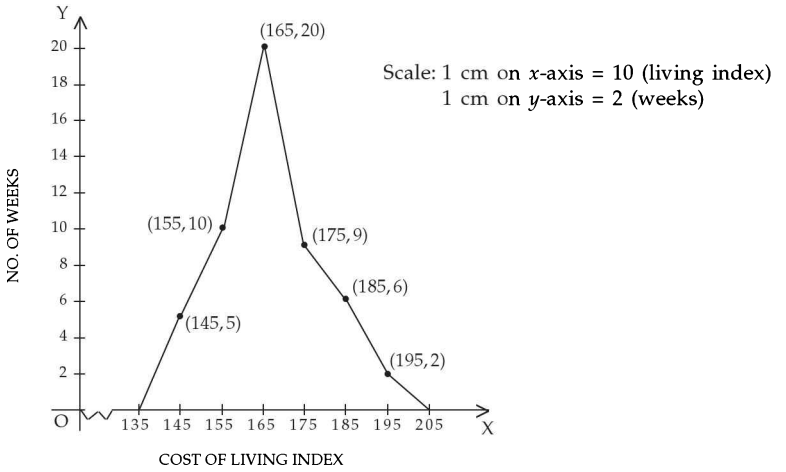
5. (*i*)


Y
45
44
40
35
30
25
20
15
10
5
Scale: 1 cm on x-axis = 2 letters
1 cm on y-axis = 5 surnames
NO. OF SURNAMES
O
1
4
6
8
12
20
X
NO. OF LETTERS

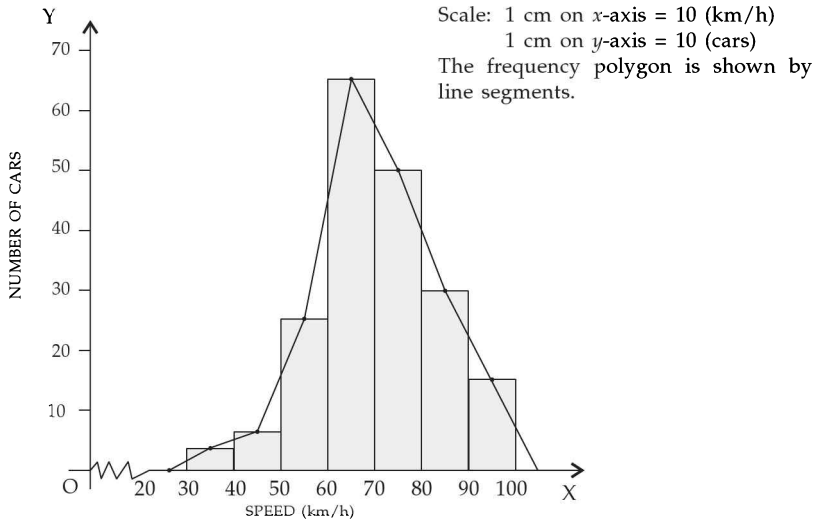
(*ii*) 6 – 8

EXERCISE 14.4

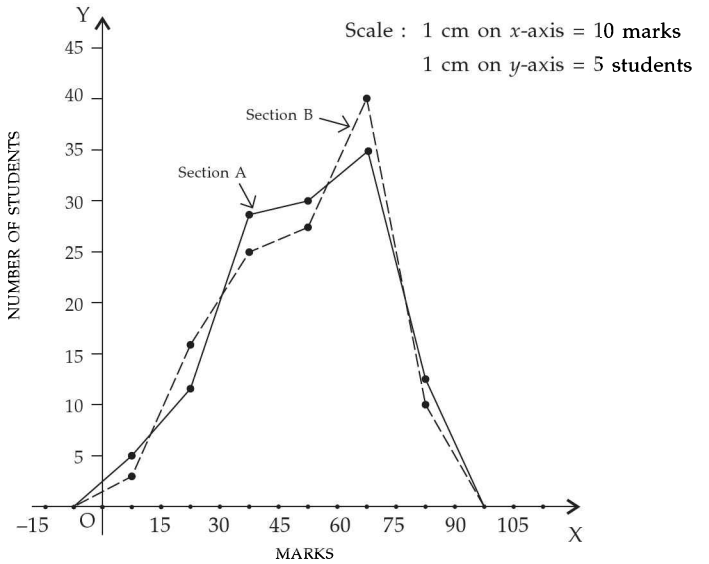
1.



2.



3.



Frequency polygon of section A is shown by thick line segments and frequency polygon of section B is shown by dotted line segments.

Overall performance of section A is better than performance of section B but more students of section B get higher marks than students of section A.

EXERCISE 14.5

1. 13 hours **2.** 2780 **3.** 6.5 **4.** $\frac{41}{6}$ **5.** 8 **6.** 27

7. 13 **8.** 45.9 **9.** 42 **10.** 8.05 **11.** 2.55 **12.** 20

13. 12 **14.** 11 **15.** 20 **16.** 14

17. Mean = 2.8, median = 3, mode = 3 **18.** Mean = 54.8, median = 52, mode = 52

19. Data has to be arranged in ascending (or descending) order before finding the median.

20. No ; because the data has not been arranged in ascending (or descending) order before finding the median.

VERY SHORT ANSWER TYPE QUESTIONS

1. 8 **2.** 15.5 **3.** 3.5 **4.** 16 **5.** 280

6. 2.5 **7.** $\frac{14}{3}$ **8.** 12 **9.** 26 **10.** 15

OBJECTIVE TYPE QUESTIONS

1. (*i*) primary (*ii*) secondary (*iii*) variable (*iv*) continuous
(*v*) difference (*vi*) frequency (*vii*) class size (or class width)
(*viii*) 20 (*ix*) total number (*x*) $\left(\frac{n+1}{2}\right)$ th observation
(*xi*) $\frac{\frac{n}{2}\text{th observation} + \left(\frac{n}{2}+1\right)\text{th observation}}{2}$ (*xii*) successive

2. (*b*) **3.** (*b*) **4.** (*b*) **5.** (*c*) **6.** (*b*)

7. (*b*) **8.** (*c*) **9.** (*b*) **Hint:** class size = 10, range = 98.

10. (*b*) **11.** (*d*) **12.** (*a*) **13.** (*c*) **14.** (*d*) **15.** (*c*)

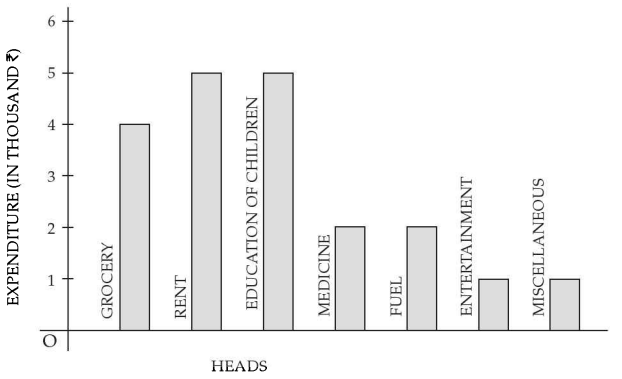
16. (*b*) **17.** (*b*) **18.** (*b*) **19.** (*c*) **20.** (*c*) **21.** (*c*)

22. (*b*) **23.** (*b*)

24. (*a*) **Hint:** As the median depends only on 5th and 6th entries, and there is no change in these entries, therefore, there is no change in the value of the median.

VALUE BASED QUESTIONS

1.

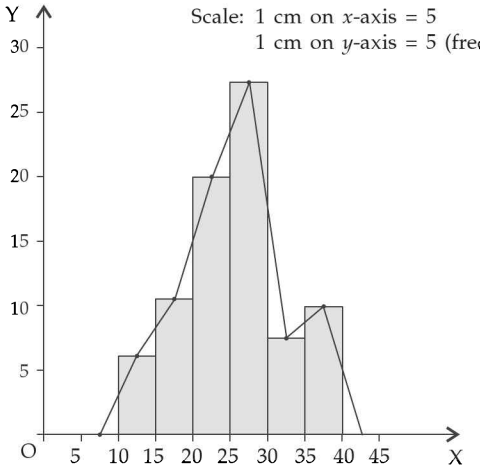


With proper education, children will grow up as good citizens which leads to the progress of the family and the country.

2. ₹ 3000; Giving scholarship to poor intelligent students is a noble work to uplift the weaker sections of society.

HIGHER ORDER THINKING SKILLS (HOTS)

1.



2. $\dfrac{\sum_{i=1}^{n} n_i \bar{x}_i}{\sum_{i=1}^{n} n_i}$

CHAPTER TEST

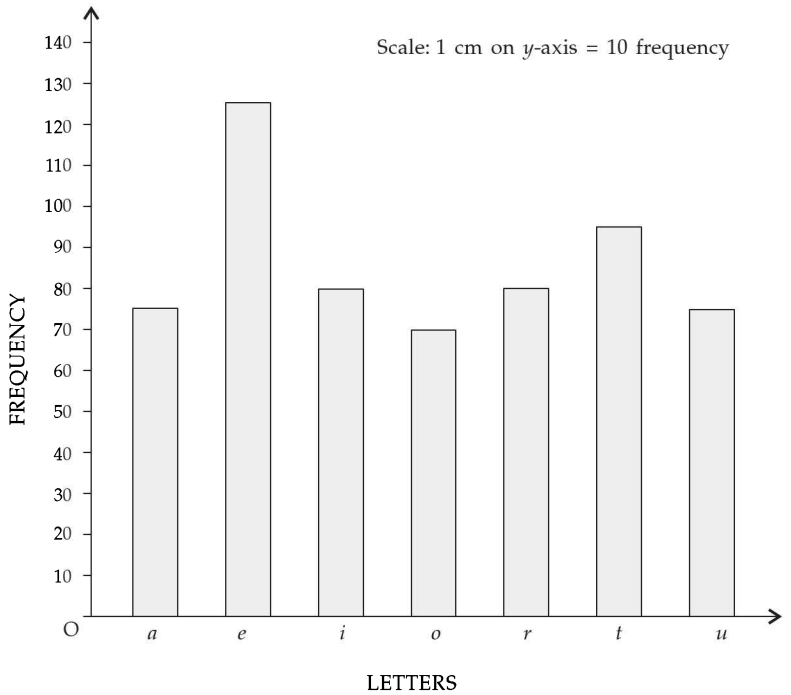
1.

Classes	0–9	10–19	20–29	30–39	40–49	50–59	60–69	70–79	80–89	90–99
Frequency	4	7	5	10	5	8	5	8	5	3

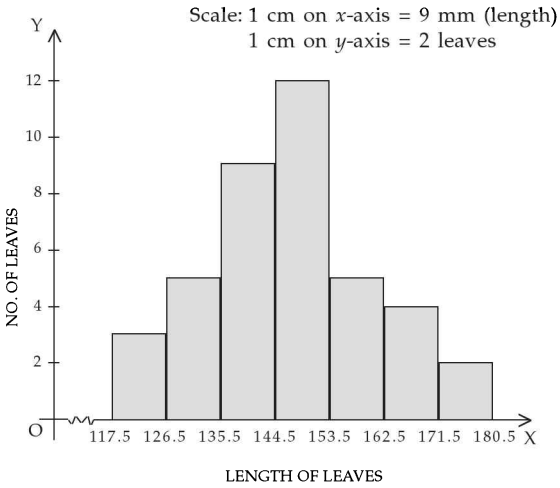
2.

Classes	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80	80–90	90–100
Frequency	4	7	5	10	5	8	5	8	5	3

3.



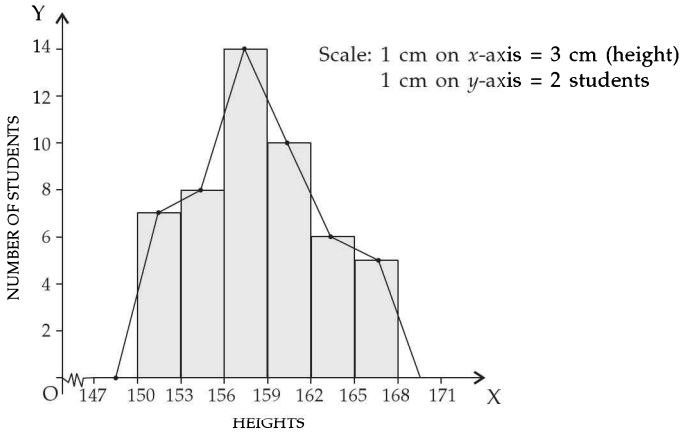
4. (*i*)



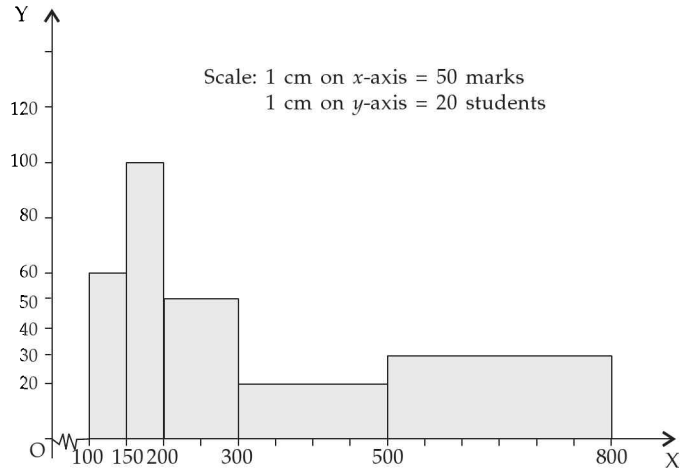
(*ii*) Frequency polygon

(*iii*) No; in fact, maximum number of leaves are of length between 145 mm and 153 mm (both inclusive).

5.



6.



7. 35 years **8.** $8\frac{2}{7}$ **9.** 152.5 cm **10.** Mean = 54.8, median = 52

11. 64 **12.** Mean = 75.64, median = 77, mode = 85

EXERCISE 15

1. $\frac{4}{5}$ **2.** (*i*) $\frac{27}{40}$ (*ii*) $\frac{13}{40}$ **3.** (*i*) 0.455 (*ii*) 0.545

4. (*i*) 0.21 (*ii*) 0.55 (*iii*) 0.24

5. P(1) = 0.179, P(2) = 0.15, P(3) = 0.157, P(4) = 0.149, P(5) = 0.175, P(6) = 0.19

6. (*i*) $\frac{19}{60}$ (*ii*) $\frac{407}{750}$ (*iii*) $\frac{211}{1500}$ **7.** $\frac{7}{11}$ **8.** 0.1

9. (*i*) $\frac{9}{40}$ (*ii*) $\frac{31}{40}$ (*iii*) 0

10. (*i*) 0.06 (*ii*) 0.19 (*iii*) 0.0075

11. (*i*) 0.675 (*ii*) 0.325 (*iii*) 0.135 (*iv*) 0.66

VERY SHORT ANSWER TYPE QUESTIONS

1. $\frac{7}{10}$ **2.** $\frac{3}{5}$ **3.** $\frac{7}{15}$ **4.** $\frac{4}{7}$ **5.** $\frac{11}{50}$

OBJECTIVE TYPE QUESTIONS

1. (*i*) uncertainty (*ii*) outcomes (*iii*) occurred (or happened)

2. (*i*) experiment (*ii*) events

3. (*a*) **4.** (*c*) **5.** (*c*) **6.** (*b*) **7.** (*c*) **8.** (*d*)

9. (*c*) **10.** (*d*)

VALUE BASED QUESTIONS

1. (*i*) $\frac{3}{38}$

(*ii*) The event that a student weighs less than 30 kg; the event that a student weighs 30 kg or more than 30 kg.

(*iii*) Yes; obesity may create many physical problems. It is advisable for the students to keep weight under control.

2. (*i*) $\frac{9}{50}$ (*ii*) $\frac{41}{200}$

(*iii*) $\frac{123}{200}$; Cycling should be promoted as it is good for health, no pollution and saves petrol/diesel.

CHAPTER TEST

1. No; because the number of trials in which the event can happen cannot be negative and the total number of trials is always positive.

2. No; because the number of trials in which the event can happen cannot be greater than the total number of trials.

3. $\frac{9}{25}$ **4.** (*i*) $\frac{4}{7}$ (*ii*) $\frac{59}{350}$ (*iii*) $\frac{669}{700}$

5. (*i*) 0.25 (*ii*) 0.75 (*iii*) 0.73 (*iv*) 0 **6.** (*i*) $\frac{1}{4}$ (*ii*) $\frac{11}{20}$

7. (*i*) $\frac{29}{2400}$ (*ii*) $\frac{579}{2400}$ (*iii*) $\frac{1}{240}$ (*iv*) $\frac{1}{96}$ (*v*) $\frac{1031}{1200}$

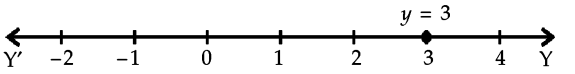
MODEL QUESTION PAPER 6

1. 50° **2.** 600 cm^3 **3.** 3 units **4.** $\frac{9}{40}$ **5.** (2, 3)

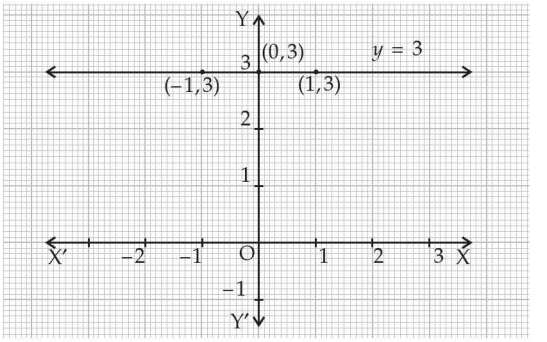
6. (*i*) 54 cm^2 (*ii*) 27 cm^2 **7.** 115 **8.** 20 cm **9.** (*i*) 7 cm (*ii*) 462 cm^2

10. 30 **11.** (0, 4)

12. (*i*) It represents a point on the number line shown in the adjoining figure.



(*ii*) It represents a line parallel to *x*-axis shown in the adjoining figure.



15. 60° **18.** 4000 m^3 **19.** (*i*) 26 m (*ii*) ₹ 137280

20. Mean = 15.5, median = 11.5, mode = 10

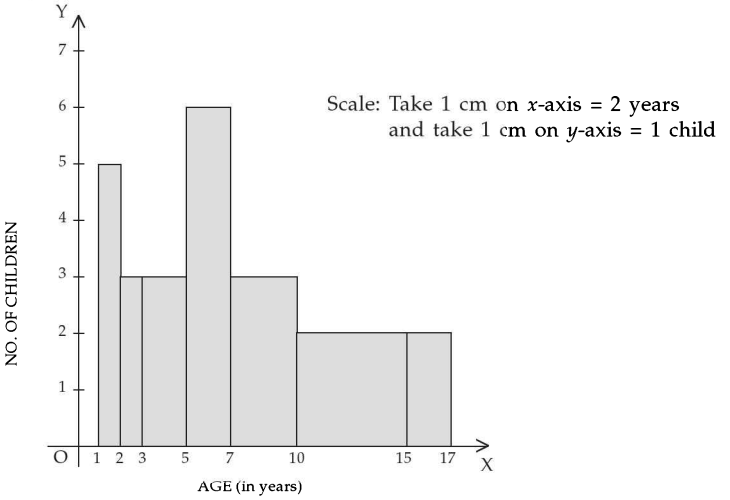
21. $5x - y + 3 = 0, x \geq 1$ **22.** $a = \frac{1}{4}, b = \frac{1}{6}$

27. (*i*) 59.4 m^2 (*ii*) 95.04 m^2

28. 1232 m^3; Donating the tent to a shelterless family shows her generosity

29. (*ii*) A

30.



31. (*i*) $\frac{43}{500}$ (*ii*) $\frac{127}{500}$ (*iii*) $\frac{47}{125}$